Multiclass, Multicriteria TNE Model to improve transit activity in San Juan Metropolitan Area

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Project Goals

- Highlight the importance of Multiclass, Multicriteria Network Equilibrium Models

- Describe the Multiclass, Multicriteria TNE Model developed by Nagurney & Dong (2000)

- Describe current and proposed traffic network in the San Juan Metropolitan Area

- Discuss how to apply the model to study this particular network
Multiclass, Multicriteria TNE Models

- Allow weighting decision-making criteria

- Travelers consider several criteria to choose their optimal travel path → Quandt (1967)

- Uncongested model → Dial (1979)

- Congested model, infinite-dimensionlal VI formulation of multiclass, multicriteria TNE problems, qualitative properties → Dafermos (1981)
Multiclass, Multicriteria TNE Model
(Nagurney & Dong 2000)

- Allow weighting decision-making criteria which are class and link dependent
  - travel time and travel cost

- Deals with demand functions that are not separable

- Qualitative analysis

- Computational procedures (VI)
Multiclass, Multicriteria TNE Model
(Nagurney & Dong 2000)

- $k = \text{classes of travelers in the network with a class denoted by } i$
- $f_{ai} = \text{flow of class } i \text{ on link } a$
- $x_{ip} = \text{nonnegative flow of class } i \text{ on path } p$

\[
f_a^i = \sum_{p \in P} x_p^i \delta_{ap}, \quad \forall i, \forall a
\]

\[
f_a = \sum_{i=1}^{k} f_a^i, \quad \forall a \in L
\]
Assume as given, a travel time function and a travel cost function associated with each link $a$.

\[ t_a = t_a(f) \quad \forall a \in L \]

\[ c_a = c_a(f) \quad \forall a \in L \]
Generalized Cost Functions

- Generalized cost of class $i$ associated with each link $a$, 

\[ u^i_a = w^i_{1a} t_a + w^i_{2a} c_a \quad \forall i, \forall a \]

- Where:

\[ t_a = t_a (f) \quad \forall a \in L \]
\[ c_a = c_a (f) \quad \forall a \in L \]

$w^i_{1a} =$ weight associated with class $i$’s travel time on link $a$
$w^i_{2a} =$ weight associated with class $i$’s travel cost on link $a$
Additional Comments

- Generalized cost of class $i$ associated with traveling on path $p$,

$$v^i_p = \sum_{a \in L} u^i_a(\widetilde{f}) \delta_{ap}, \quad \forall i, \forall p$$

- Travel demand of class $i$ traveler between O/D pair $w$,

$$d^i_w = \sum_{p \in P_w} x^i_p, \quad \forall i, \forall w$$

- Travel disutility associated with class $i$ traveler between O/D pair $w$,

$$d^i_w = d^i_w(\lambda), \quad \forall i, \forall w$$
Traffic Network Equilibrium Conditions

\[ v_p^i (\tilde{f}^*) \begin{cases} = \lambda_w^* & \text{if } x_p^i > 0 \\ \geq \lambda_w^* & \text{if } x_p^i = 0 \end{cases} \]

\[ d_w^i (\lambda^*) \begin{cases} = \sum_{p \in P_w} x_p^i & \text{if } \lambda_w^* > 0 \\ \leq \sum_{p \in P_w} x_p^i & \text{if } \lambda_w^* = 0 \end{cases} \]
VI Formulation

\[ K \equiv \{(\tilde{f}, d, \lambda) | \lambda \geq 0 \quad \text{and} \quad \exists \tilde{x} \geq 0\} \]

\[
\sum_{i=1}^{k} \sum_{a \in L} u^i_a (\tilde{f}^i - f^i_a) - \sum_{i=1}^{k} \sum_{w \in W} \lambda^*_w (d^i_w - d^i_w^*) + \sum_{i=1}^{k} \sum_{w \in W} (d^i_w^* - d^i_w (\lambda^*) \times (\lambda^*_w - \lambda^*_w) \geq 0, \quad \forall (\tilde{f}, d, \lambda) \in K
\]
Applying the Model
San Juan Metropolitan Area
Case Study
San Juan Metropolitan Area

- Area = 400 mi$^2$
- 1.4 million residents (35% of the total population), generate 3.2 million trips a day
- 63% of the jobs are in the metro area
- It is expected an increase of 45% in the number of trips by 2010

Source: Department of Transportation & Public Works
PR-52 Northbound
A.M. Peak Hour
Traffic Network Analyzed
Simplified Network

Train Station

Park & Ride

A
B
C

52
a
b
c
Generalized cost functions should reflect the two alternatives available to travelers.

- Alternative 1: using link \(c\) for private vehicle
- Alternative 2: using links \(a\) and \(b\) for transit or HOV
The travel time for Alternative 1 (private vehicle) should include a congested factor and should reflect driver’s comfort when compared with Alternative 2.

Travel time functions for Alternative 2 (park & ride) should reflect the time required to park, walk and wait and the higher speed that transit and HOV vehicles can reach.
Link Travel Cost Functions

\[ c_a = c_a(f) \quad \forall a \in L \]

- The cost functions for Alternative 1 (private vehicle) should include costs such as fuel and tolls.
- Travel time functions for Alternative 2 should reflect the cost to park and transit fare.
Travelers

- For this example, there are 2 classes

- Travelers in class 1 are highly interested in minimize travel time and relatively low interest in travel cost

- Travelers in class 2 are interested in minimize both travel time and travel cost
Members of a class of travelers perceive their generalized cost on a route as a weighting of travel time and travel cost. This can be represented as weight factors as follows:

\[ w_{1a} = \text{weight associated with class 1 travel time on link } a \]
\[ w_{2a} = \text{weight associated with class 2 travel cost on link } a \]
- We have three links and two classes. We can specify weight for each link:

<table>
<thead>
<tr>
<th>Class</th>
<th>Travel Time</th>
<th>Travel Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Weight on Link a</td>
<td>Weight on Link b</td>
</tr>
<tr>
<td>1</td>
<td>( w^{1}_{1a} = .75 )</td>
<td>( w^{1}_{1b} = .75 )</td>
</tr>
<tr>
<td>2</td>
<td>( w^{2}_{1a} = .5 )</td>
<td>( w^{2}_{1b} = .5 )</td>
</tr>
</tbody>
</table>
## Generalized Cost Functions

$$u^i_a = w^i_{1a} t_a + w^i_{2a} c_a \quad \forall i, \forall a$$

<table>
<thead>
<tr>
<th>Link</th>
<th>Class 1</th>
<th>Class 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$u^1_a = 0.75 , t_a(f) + 0.25 , c_a(f)$</td>
<td>$u^2_a = 0.5 , t_a(f) + 0.5 , c_a(f)$</td>
</tr>
<tr>
<td>b</td>
<td>$u^1_b = 0.75 , t_b(f) + 0.25 , c_b(f)$</td>
<td>$u^2_b = 0.5 , t_b(f) + 0.5 , c_b(f)$</td>
</tr>
<tr>
<td>c</td>
<td>$u^1_c = 0.75 , t_c(f) + 0.25 , c_c(f)$</td>
<td>$u^2_c = 0.5 , t_c(f) + 0.5 , c_c(f)$</td>
</tr>
</tbody>
</table>
Conclusions & Recommendations

- The model allow to evaluate a traffic network considering that travelers use several criteria to choose their optimal travel path.

- This can be represented by weights that are class and link-dependent.

- The model deal with general and not separable demand functions.

- Data needed to construct meaningful travel cost and travel time functions.

- Information obtained with this model could provide useful insight into the planning process.
References


- Puerto Rico Department of Transportation & Public Works Website. [http://www.dtop.gov.pr](http://www.dtop.gov.pr)

- Puerto Rico Integrated Transportation Alternative System Website. [http://www.ati.gobierno.pr](http://www.ati.gobierno.pr)