Fashion Supply Chain Management through Cost and Time Minimization from a Network Perspective

Anna Nagurney
John F. Smith Memorial Professor
and
Min Yu
Doctoral Student

Department of Finance and Operations Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

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Outline

- Background and Motivation
- An Overview of the Relevant Literature
- The Fashion Supply Chain Management Model
- Single Product Fashion Supply Chain Examples
- Multiproduct Fashion Supply Chain Examples
- Summary and Conclusions
This talk is based on the paper:

Background and Motivation

The volatile demand forces fashion companies to react to new trends and consumer requirements quickly (see Christopher, Lowson, and Peck (2004), and Gustafson, von Schmiesing-Korff, and Ng (2004)).

The fashion and apparel industry is notable for its short product life cycle, low predictability, high volatility, and tremendous product variety (see Bruce, Daly, and Towers (2004), Christopher, Lowson, and Peck (2004), Şen (2008), and Sull and Turconi (2008)).
Time is one crucial factor in the fashion industry.

Below is a list of fashion supply chain management strategies.

- Fast Fashion,
- Just-in-Time Logistics,
- Supply Chain Coordination,
- Quick Response Strategies,
- Vendor-Managed Inventory, and
- Lean and Agile Supply Chains.
Fast Fashion

Fashion retailers, such as Benetton, H&M, Topshop, and Zara have revolutionized the fashion industry by following the fast fashion strategy, in which retailers respond to shifts in the market within just a few weeks (Sull and Turconi (2008)).

Since 2003, Zara, a leader in ‘fast fashion’, has been able to limit its delivery time to stores to within 15 days after the design (Gustafson, von Schmiesing-Korff, and Ng (2004)).
Fast fashion is aiming at satisfying the desire for trendy, short-cycle, and relatively inexpensive clothing (Doeringer and Crean (2006)), resulting in, on the average, 9% higher profit margins than those of traditional retailers (Sull and Turconi (2008)).

Supply chains are central to the fast fashion strategy, which make it possible to obtain fabrics, to manufacture samples, and to start shipping products with far shorter lead times than those of the conservative rivals (Doeringer and Crean (2006)).
Obviously, superior time performance cannot be achieved without some sacrifice of the operational cost.

Clearly, in the case of fashion products, an appropriate supply chain management framework must capture both the operational (and other) cost dimension as well as the time dimension.
An Overview of the Relevant Literature

- Time Issues in Supply Chain Management
An Overview of the Relevant Literature

Fashion Supply Chains


- Ferdows, Lewis, and Machuca (2004) recognized the **nonlinear relationship** between capacity and time in the context of the fashion industry and fast response, hence, an appropriate model for fashion supply chain management must be able to handle such nonlinearities.
The fashion firm is involved in the production, storage, and distribution of multiple fashion products and is seeking to determine its optimal multiproduct flows to its demand points (markets) under total cost minimization and total time minimization, with the latter objective function weighted by the fashion firm.
The Fashion Supply Chain Network Topology

Firm

\[\text{Demand Points} \]
Demands, Path Flows, and Link Flows

Let $d^j_k$ denote the demand for product $j$; $j = 1, \ldots, J$, at demand point $R_k$. Let $x^j_p$ denote the nonnegative flow of product $j$ on path $p$. Let $f^j_a$ denote the flow of product $j$ on link $a$.

The following conservation of flow equations

\[ \sum_{p \in P_{R_k}} x^j_p = d^j_k, \quad j = 1, \ldots, J; \quad k = 1, \ldots, n_R. \]  

(1)

\[ f^j_a = \sum_{p \in P} x^j_p \delta_{ap}, \quad j = 1, \ldots, J; \quad \forall a \in L. \]  

(2)
Operational Cost

The unit cost on a link $a$ associated with product $j$ is denoted by $c^j_a$.

$$c^j_a = c^j_a(f^1_a, \ldots, f^J_a), \quad j = 1, \ldots, J; \quad \forall a \in L. \quad (3)$$

Let $C^j_p$ denote the unit operational cost associated with product $j$; $j = 1, \ldots, J$, on a path $p$, where

$$C^j_p = \sum_{a \in L} c^j_a \delta_{ap}, \quad j = 1, \ldots, J; \quad \forall p \in P. \quad (4)$$

Then, the total operational cost for product $j$; $j = 1, \ldots, J$, on path $p$; $p \in P$ can be expressed as:

$$\hat{C}^j_p(x) = C^j_p(x) \times x^j_p, \quad j = 1, \ldots, J; \quad \forall p \in P. \quad (5)$$
Let \( t^j_a \) denote the average unit time consumption for product \( j \); \( j = 1, \ldots, J \), on link \( a, a \in L \).

\[
\left. \begin{array}{c}
t^j_a = t^j_a(f^1_a, \ldots, f^J_a), \quad j = 1, \ldots, J, \quad \forall a \in L.
\end{array} \right\} \tag{6}
\]

Therefore, the average unit time consumption for product \( j \) on path \( p \) is:

\[
T^j_p = \sum_{a \in L} t^j_a \delta_{ap}, \quad j = 1, \ldots, J, \quad \forall p \in P,
\tag{7}
\]

with the total time consumption for product \( j \) on path \( p \) given by:

\[
\hat{T}^j_p(x) = T^j_p(x) \times x^j_p, \quad j = 1, \ldots, J; \quad \forall p \in P.
\tag{8} \]
The fashion firm is seeking to determine its optimal multiproduct flows to its demand points (markets) under total cost minimization and total time minimization.

Minimize \[ J \sum_{j=1}^{J} \sum_{p \in P} \hat{C}_p(x) + \omega \sum_{j=1}^{J} \sum_{p \in P} \hat{T}_p(x) \] (9)

Minimize \[ J \sum_{j=1}^{J} \sum_{a \in L} \hat{c}_a(x) + \omega \sum_{j=1}^{J} \sum_{a \in L} \hat{t}_a(x), \] (10)

where \( \hat{c}_a \equiv c_a(f_1^a, \ldots, f_J^a) \times f_j^a \) and the \( \hat{t}_a \equiv t_a(f_1^a, \ldots, f_J^a) \times f_j^a \). The total link cost functions \( \hat{c}_a \) and total time functions \( \hat{t}_a \) are assumed to be convex and continuously differentiable, for all products \( j \) and all links \( a \in L \).
Determine the vector of optimal path flows, $x^* \in K$, such that:

$$
\sum_{j=1}^{J} \sum_{p \in P} \left[ \frac{\partial \hat{C}_p^j(x^*)}{\partial x_p^j} + w \frac{\partial \hat{T}_p^j(x^*)}{\partial x_p^j} \right] \times (x_p^j - x_p^{j*}) \geq 0, \quad \forall x \in K, \quad (11)
$$

where

$$
K \equiv \{x| (1) \text{ is satisfied, and } x \geq 0\},
$$

$$
\frac{\partial \hat{C}_p^j(x)}{\partial x_p^j} \equiv \sum_{l=1}^{J} \sum_{a \in L} \frac{\partial \hat{c}_a^l(f_1^a, \ldots, f_J^a)}{\partial f_a^j} \delta_{ap},
$$

$$
\frac{\partial \hat{T}_p^j(x)}{\partial x_p^j} \equiv \sum_{l=1}^{J} \sum_{a \in L} \frac{\partial \hat{t}_a^l(f_1^a, \ldots, f_J^a)}{\partial f_a^j} \delta_{ap}.
$$
Determine the vector of optimal link flows, \( f^* \in K^1 \), such that:

\[
\sum_{j=1}^{J} \sum_{l=1}^{J} \sum_{a \in L} \left[ \frac{\partial \hat{c}_a^l(f_a^{1*}, \ldots, f_a^{J*})}{\partial f_a^j} + \omega \frac{\partial \hat{t}_a^l(f_a^{1*}, \ldots, f_a^{J*})}{\partial f_a^j} \right] \times (f_a^j - f_a^{j*}) \geq 0,
\]

\( \forall f \in K^1, \)  

(12)

where \( K^1 \equiv \{ f | (1), \text{ and (2) are satisfied, and } x \geq 0 \} \).
The existence of solutions to (11) and (12) is guaranteed since the underlying feasible sets, $K$ and $K^1$, are compact and the corresponding functions of marginal total costs and marginal total time are continuous, under the above assumptions.

If the total link cost functions and the total time functions are strictly convex, then the solution to (12) is guaranteed to be unique.
Single Product Fashion Supply Chain Examples

Firm

1

M_1

1 2

3 4 5 6

D_{1,1}

7

D_{1,2}

9 10 11 12

R_1

M_2

D_{2,1}

D_{2,2}

R_2

University of Massachusetts Amherst Fashion Supply Chain Management
The manufacturing plant $M_1$ is located in the U.S., while the manufacturing plant $M_2$ is located off-shore and has lower operating cost. The average manufacturing time consumption of one unit of product is identical at these two plants, while the related costs vary mainly because of the different labor costs.

The demands for this fashion product at the demand points are:

$$d_1 = 100, \quad d_2 = 200,$$
### Table: Total Link Operational Cost and Total Time Functions

<table>
<thead>
<tr>
<th>Link $a$</th>
<th>$\hat{c}_a(f_a)$</th>
<th>$\hat{t}_a(f_a)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10f_1^2 + 10f_1$</td>
<td>$f_1^2 + 10f_1$</td>
</tr>
<tr>
<td>2</td>
<td>$f_2^2 + 5f_2$</td>
<td>$f_2^2 + 10f_2$</td>
</tr>
<tr>
<td>3</td>
<td>$f_3^2 + 3f_3$</td>
<td>$.5f_3^2 + 5f_3$</td>
</tr>
<tr>
<td>4</td>
<td>$f_4^2 + 4f_4$</td>
<td>$.5f_4^2 + 7f_4$</td>
</tr>
<tr>
<td>5</td>
<td>$2f_5^2 + 30f_5$</td>
<td>$.5f_5^2 + 25f_5$</td>
</tr>
<tr>
<td>6</td>
<td>$2f_6^2 + 20f_6$</td>
<td>$.5f_6^2 + 15f_6$</td>
</tr>
<tr>
<td>7</td>
<td>$.5f_7^2 + 3f_7$</td>
<td>$f_7^2 + 5f_7$</td>
</tr>
<tr>
<td>8</td>
<td>$f_8^2 + 3f_8$</td>
<td>$f_8^2 + 2f_8$</td>
</tr>
<tr>
<td>9</td>
<td>$f_9^2 + 2f_9$</td>
<td>$f_9^2 + 5f_9$</td>
</tr>
<tr>
<td>10</td>
<td>$2f_{10}^2 + f_{10}$</td>
<td>$f_{10}^2 + 3f_{10}$</td>
</tr>
<tr>
<td>11</td>
<td>$f_{11}^2 + 5f_{11}$</td>
<td>$f_{11}^2 + 2f_{11}$</td>
</tr>
<tr>
<td>12</td>
<td>$f_{12}^2 + 4f_{12}$</td>
<td>$f_{12}^2 + 4f_{12}$</td>
</tr>
</tbody>
</table>
Sensitivity Analysis

Figure: Optimal Link Flows on Manufacturing Links 1 and 2 as $\omega$ Increases
Sensitivity Analysis

Figure: Optimal Link Flows on Transportation Links 3, 4, 5, and 6 as $\omega$ Increases
Figure: Optimal Link Flows on Storage Links 7 and 8 as \( \omega \) Increases
Figure: Optimal Link Flows on Transportation Links 9, 10, 11, and 12 as $\omega$ Increases
Sensitivity Analysis

Figure: Minimal Total Costs and Minimal Total Times as $\omega$ Increases
The fashion firm was assumed to provide two different fashion products with the same supply chain network topology.

The demands for the two fashion products at the demand points are:

\[ d_1^1 = 100, \quad d_1^2 = 200, \quad d_2^1 = 300, \quad d_2^2 = 400. \]
### Total Link Operational Cost and Total Time Functions for Product 1

<table>
<thead>
<tr>
<th>Link a</th>
<th>(\hat{c}^1_a(f^1_a, f^2_a))</th>
<th>(\hat{t}^1_a(f^1_a, f^2_a))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(10(f^1_1)^2 + 1f^1_1f^2_1 + 10f^1_1)</td>
<td>(1(f^1_1)^2 + .3f^1_1f^2_1 + 10f^1_1)</td>
</tr>
<tr>
<td>2</td>
<td>((f^1_2)^2 + .4f^1_2f^2_2 + 5f^1_2)</td>
<td>((f^1_2)^2 + .3f^1_2f^2_2 + 10f^1_2)</td>
</tr>
<tr>
<td>3</td>
<td>((f^1_3)^2 + .3f^1_3f^2_3 + 3f^1_3)</td>
<td>(.5(f^1_3)^2 + .2f^1_3f^2_3 + 5f^1_3)</td>
</tr>
<tr>
<td>4</td>
<td>((f^1_4)^2 + .2f^1_4f^2_4 + 4f^1_4)</td>
<td>(.5(f^1_4)^2 + .2f^1_4f^2_4 + 7f^1_4)</td>
</tr>
<tr>
<td>5</td>
<td>(2(f^1_5)^2 + .25f^1_5f^2_5 + 30f^1_5)</td>
<td>(.5(f^1_5)^2 + .1f^1_5f^2_5 + 25f^1_5)</td>
</tr>
<tr>
<td>6</td>
<td>(2(f^1_6)^2 + .3f^1_6f^2_6 + 20f^1_6)</td>
<td>(.5(f^1_6)^2 + .1f^1_6f^2_6 + 15f^1_6)</td>
</tr>
<tr>
<td>7</td>
<td>(.5(f^1_7)^2 + .1f^1_7f^2_7 + 3f^1_7)</td>
<td>((f^1_7)^2 + .5f^1_7f^2_7 + 5f^1_7)</td>
</tr>
<tr>
<td>8</td>
<td>((f^1_8)^2 + .1f^1_8f^2_8 + 3f^1_8)</td>
<td>((f^1_8)^2 + .5f^1_8f^2_8 + 2f^1_8)</td>
</tr>
<tr>
<td>9</td>
<td>((f^1_9)^2 + .5f^1_9f^2_9 + 2f^1_9)</td>
<td>((f^1_9)^2 + .2f^1_9f^2_9 + 5f^1_9)</td>
</tr>
<tr>
<td>10</td>
<td>(2(f^1_{10})^2 + .3f^1_{10}f^2_{10} + 1f^1_{10})</td>
<td>((f^1_{10})^2 + .4f^1_{10}f^2_{10} + 3f^1_{10})</td>
</tr>
<tr>
<td>11</td>
<td>((f^1_{11})^2 + .6f^1_{11}f^2_{11} + 5f^1_{11})</td>
<td>((f^1_{11})^2 + .25f^1_{11}f^2_{11} + 2f^1_{11})</td>
</tr>
<tr>
<td>12</td>
<td>((f^1_{12})^2 + .7f^1_{12}f^2_{12} + 4f^1_{12})</td>
<td>((f^1_{12})^2 + .25f^1_{12}f^2_{12} + 4f^1_{12})</td>
</tr>
</tbody>
</table>
Table: Total Link Operational Cost and Total Time Functions for Product 2

<table>
<thead>
<tr>
<th>Link</th>
<th>( \hat{c}_a(f_1, f_2) )</th>
<th>( \hat{t}_a(f_1, f_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 8(f_1^2)^2 + 1f_1^1f_2^2 + 10f_1^2 )</td>
<td>( 1(f_2^2)^2 + .5f_1^1f_2^2 + 8f_1^2 )</td>
</tr>
<tr>
<td>2</td>
<td>( 1(f_2^2)^2 + .5f_2^1f_2^2 + 4f_2^2 )</td>
<td>( 1(f_2^2)^2 + .5f_2^1f_2^2 + 8f_2^2 )</td>
</tr>
<tr>
<td>3</td>
<td>( 1.5(f_3^2)^2 + .2f_3^1f_3^2 + 3f_3^2 )</td>
<td>( 1(f_3^2)^2 + .1f_3^1f_3^2 + 3f_3^2 )</td>
</tr>
<tr>
<td>4</td>
<td>( 1(f_4^2)^2 + .3f_4^1f_4^2 + 4f_4^2 )</td>
<td>( 1(f_4^2)^2 + .2f_4^1f_4^2 + 3f_4^2 )</td>
</tr>
<tr>
<td>5</td>
<td>( 2(f_5^2)^2 + .3f_5^1f_5^2 + 25f_5^2 )</td>
<td>( .8(f_5^2)^2 + .1f_5^1f_5^2 + 20f_5^2 )</td>
</tr>
<tr>
<td>6</td>
<td>( 3(f_6^2)^2 + .4f_6^1f_6^2 + 20f_6^2 )</td>
<td>( .8(f_6^2)^2 + .2f_6^1f_6^2 + 12f_6^2 )</td>
</tr>
<tr>
<td>7</td>
<td>( 1(f_7^2)^2 + .1f_7^1f_7^2 + 3f_7^2 )</td>
<td>( 1(f_7^2)^2 + .4f_7^1f_7^2 + 4f_7^2 )</td>
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<tr>
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<td>( 1(f_{10}^2)^2 + .3f_{10}^1f_{10}^2 + 6f_{10}^2 )</td>
</tr>
<tr>
<td>11</td>
<td>( 2(f_{11}^2)^2 + .5f_{11}^1f_{11}^2 + 8f_{11}^2 )</td>
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Summary and Conclusions

- We developed a fashion supply chain management model, using a network economics perspective, that allows for multiple fashion products.

- The model consists of two objective functions: total cost minimization, associated with supply chain network activities.

- A weighted objective function was constructed with the weighting factor, representing the monetary value of a unit of time, decided by the firm.

- We also provided the optimization model’s equivalent variational inequality formulation, with nice features for computational purposes.
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  - the effects of changes in the demand for its products on the total operations costs and time,
  - changes in the cost functions and the time functions on total supply chain network costs and time, and
  - the addition of various links (or their removal) on the values of the objective function(s).

Suggestions for Future Research:

- a fashion supply chain management model with price-sensitive and time-sensitive demands under oligopolistic competition,
- a fashion supply chain management model incorporating environmental concerns and associated trade-offs, and
- empirical research on large-scale fashion supply chain networks.
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Thank You!