A Network Economic Game Theory Model of a Service-Oriented Internet with Price and Quality Competition in Both Content and Network Provision

Sara Saberi[1], Anna Nagurney[1], and Tilman Wolf[2]
[1]Department of Operations and Information Management, [2]Department of Electrical and Computer Engineering
University of Massachusetts, Amherst, Massachusetts

The Virtual Center for Supernetworks
Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

Introduction

- Advances in the Internet and other telecommunication networks bring about new applications and services.
- Key challenge is how to price and bill.
- Price is not the only factor and Quality of Service (QoS) comes into play.
- Networking research community is designing new architectures for the next generation Internet.
- Economic relationships are far more mysterious than the underlying technology.
- FIA is expected to be service-oriented with services at different quality levels and different costs.
- **ChoiceNet** project is a new network architecture for future Internet.

The Network of Oligopoly Model

- Demand function: $d_{ijk} = d_{ijk}(p_s, q_s, p_c, q_c), \quad \forall i,j,k$

Content Providers (CP)

- The production cost for CP_i:

$$CC_i = CC_i(SCP_i, q_{c_i}), \quad i = 1,\ldots,M$$

- The utility of each CP:

$$U_{CP_i} = \sum_{j=1}^{N}(p_{c_i} - p_{t_j})\sum_{k=1}^{O} d_{ijk} - CC_i(SCP_i, q_{c_i})$$

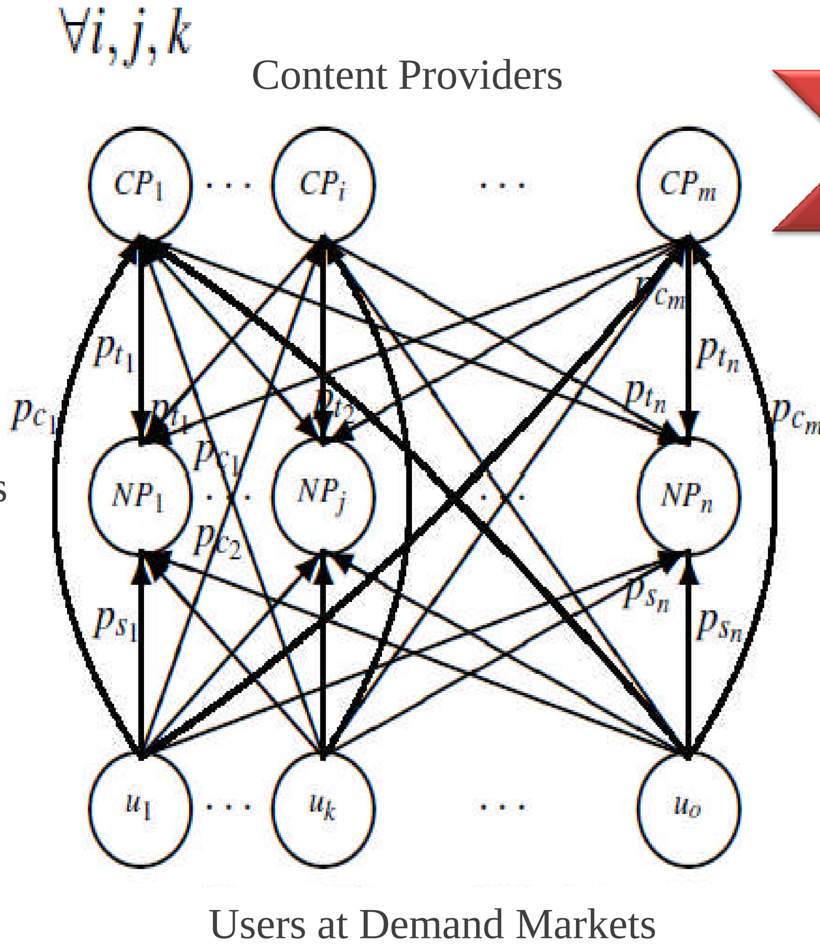
Network Providers (NP)

- The transmission cost of NP_j:

$$CS_j = CS_j(TNP_j, q_{s_j}), \quad j = 1,\ldots,N$$

- The utility function:

$$U_{NP_j} = (p_{s_j} + p_{t_j})\left(\sum_{i=1}^{M}\sum_{k=1}^{O} d_{ijk}\right) - CS_j(TNP_j, q_{s_j})$$



Nash Equilibrium

Definition: Nash Equilibrium in Price and Quality

$$U_{CP_i}(p^*_{c_i}, \hat{p^*_{c_i}}, q^*_{c_i}, \hat{q^*_{c_i}}, p^*_s, q^*_s) \geq U_{CP_i}(p_{c_i}, \hat{p^*_{c_i}}, q_{c_i}, \hat{q^*_{c_i}}, p^*_s, q^*_s), \quad \forall (p_{c_i}, q_{c_i}) \in \mathcal{K}^1_i$$

$$\hat{p^*_{c_i}} \equiv (p^*_{c_1}, \ldots, p^*_{c_{i-1}}, p^*_{c_{i+1}}, \ldots, p^*_{c_m}) \text{ and } \hat{q^*_{c_i}} \equiv (q^*_{c_1}, \ldots, q^*_{c_{i-1}}, q^*_{c_{i+1}}, \ldots, q^*_{c_m})$$

$$U_{NP_j}(p^*_c, q^*_c, p^*_{s_j}, \hat{p^*_{s_j}}, q^*_{s_j}, \hat{q^*_{s_j}}) \geq U_{NP_j}(p_{s_j}, p^*_c, q^*_c, \hat{p^*_{s_j}}, q_{s_j}, \hat{q^*_{s_j}}), \quad \forall (p_{s_j}, q_{s_j}) \in \mathcal{K}^2_j$$

$$\hat{p^*_{s_j}} \equiv (p^*_{s_1}, \ldots, p^*_{s_{j-1}}, p^*_{s_{j+1}}, \ldots, p^*_{s_n}) \text{ and } \hat{q^*_{s_j}} \equiv (q^*_{s_1}, \ldots, q^*_{s_{j-1}}, q^*_{s_{j+1}}, \ldots, q^*_{s_n})$$

Variational Inequality Formulation of Nash Equilibrium

$$\sum_{i=1}^{M}\left[-\sum_{j=1}^{N}\sum_{k=1}^{O} d_{ijk} - \sum_{j=1}^{N}\sum_{k=1}^{O}\frac{\partial d_{ijk}}{\partial p_{c_i}} \times (p^*_{c_i} - p_{t_j}) + \frac{\partial f_{c_i}(SCP_i, q^*_{c_i})}{\partial SCP_i} \cdot \frac{\partial SCP_i}{\partial p_{c_i}}\right] \times (p_{c_i} - p^*_{c_i})$$

$$+\sum_{i=1}^{M}\left[-\sum_{j=1}^{N}\sum_{k=1}^{O}\frac{\partial d_{ijk}}{\partial q_{c_i}} \times (p^*_{c_i} - p_{t_j}) + \frac{\partial f_{c_i}(SCP_i, q^*_{c_i})}{\partial q_{c_i}}\right] \times (q_{c_i} - q^*_{c_i})$$

$$+\sum_{j=1}^{N}\left[-\sum_{i=1}^{M}\sum_{k=1}^{O} d_{ijk} - \sum_{i=1}^{M}\sum_{k=1}^{O}\frac{\partial d_{ijk}}{\partial p_{s_j}} \times (p^*_{s_j} + p_{t_j}) + \frac{\partial f_{s_j}(TNP_j, q^*_{s_j})}{\partial TNP_j} \cdot \frac{\partial TNP_j}{\partial p_{s_j}}\right] \times (p_{s_j} - p^*_{s_j})$$

$$+\sum_{j=1}^{N}\left[-\sum_{j=1}^{N}\sum_{k=1}^{O}\frac{\partial d_{ijk}}{\partial q_{s_j}} \times (p^*_{s_j} + p_{t_j}) + \frac{\partial f_{s_j}(TNP_j, q^*_{s_j})}{\partial q_{s_j}}\right] \times (q_{s_j} - q^*_{s_j}) \geq 0,$$

$$\forall (p_c, q_c, p_s, q_s) \in \mathcal{K}^3.$$

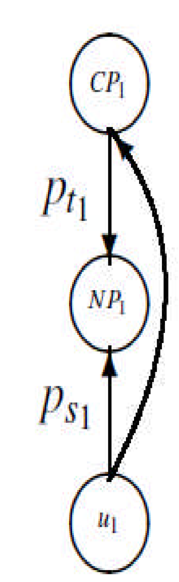
Basic Model

- The demand function:

$$d_{111} = d_0 - \alpha p_{s_1} - \beta p_{c_1} + \gamma q_{s_1} + \delta q_{c_1}$$

- Quality defined as the "expected delay," (Kleinrock function):

$$q_{s_1} = \frac{1}{\sqrt{Delay}} = \sqrt{b(d, q_{s_1}) - d_{111}}.$$



- The utility of the network and content providers:

$$U_{NP_1} = (p_{s_1} + p_{t_1} - R)d_{111} - Rq^2_{s_1}$$

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} - Kq^2_{c_1}$$

Theorem

- The network provider will benefit from charging the content provider if: $4\alpha R - \gamma^2 > 0, \text{ and } \alpha > \beta.$

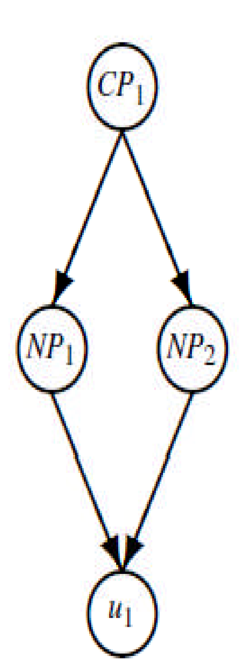
Numerical Examples

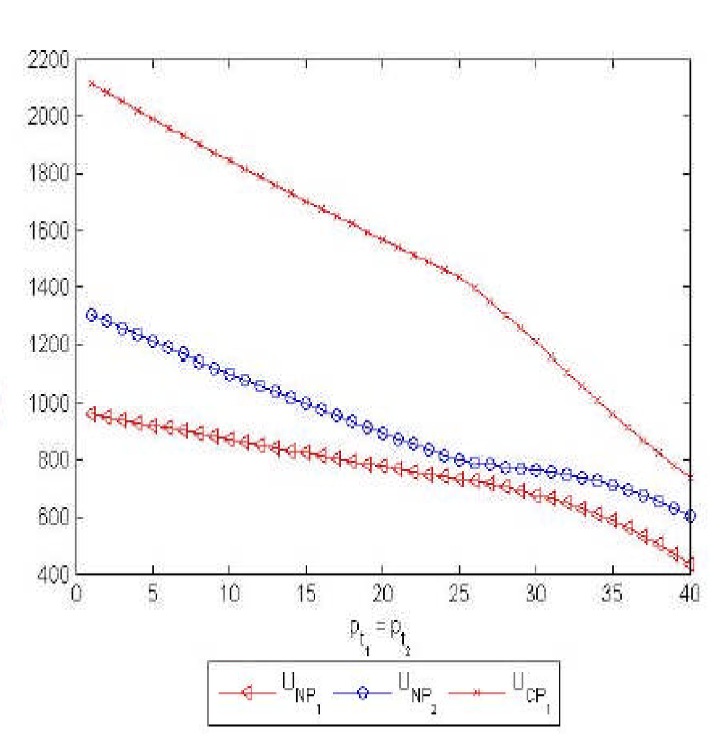
- We recall the Euler method for the solution of the Variational Inequality Problem

In the service-oriented Internet model, let $F(X) = -\nabla U(p_c, q_c, p_s, q_s)$ be strictly monotone at any equilibrium pattern. Also, assume that F is uniformly Lipschitz continuous. Then, there exists a unique equilibrium price and quality pattern $(p^*_c, q^*_c, p^*_s, q^*_s) \in \mathcal{K}$ and any sequence generated by the Euler method, where $\{a_\tau\}$ satisfies $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$ converges to $(p^*_c, q^*_c, p^*_s, q^*_s)$.

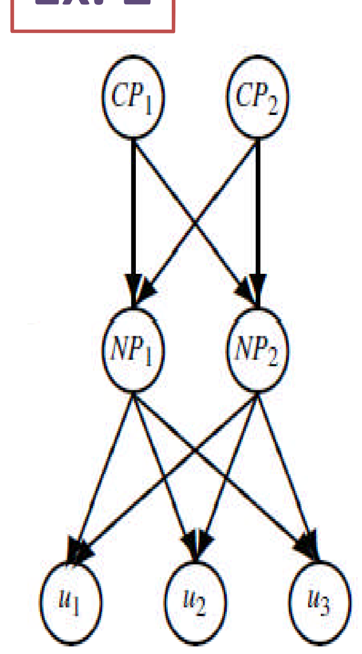
Effect of different Transfer Price Values on Utility

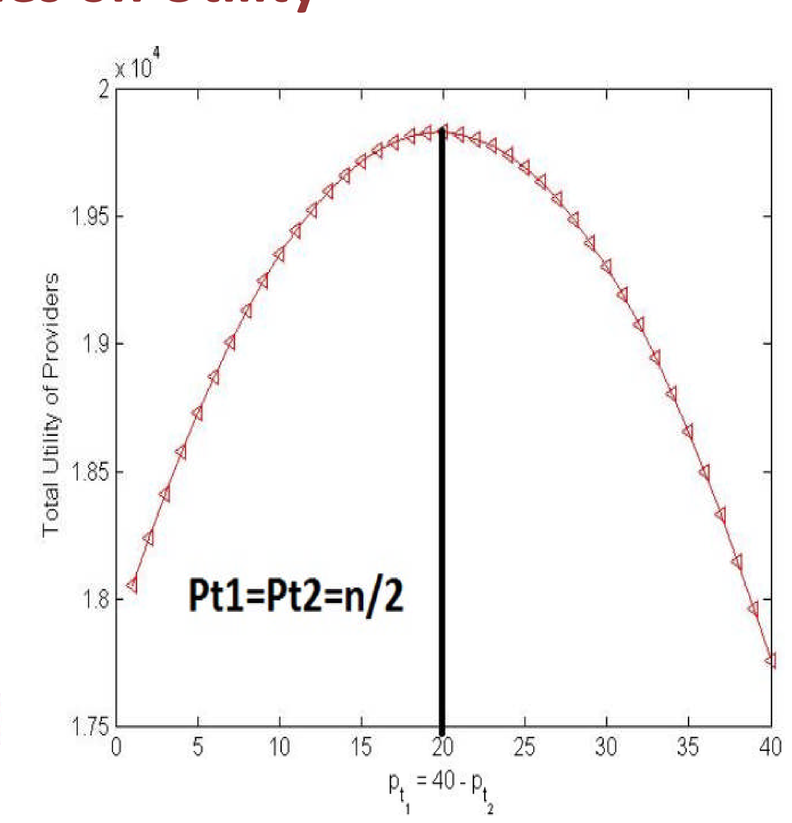
Ex. 1





Ex. 2





Conclusion

- Analyses showed that the network provider will benefit from charging the content provider if the user is more sensitive toward the network provider's fee.
- Sensitivity analysis shows that the overall effect of implementing network neutrality regulations (e.g., having $p_{tj} = 0$) may still be both positive and negative depending.

Acknowledgments

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Paper

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