Exchange Rates and Multicommodity International Trade: Insights from Spatial Price Equilibrium Modeling with Policy Instruments via Variational Inequalities

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WORKSHOP: PANOPTIC VIEW ON GLOBAL OPTIMIZATION

University of Florida, Gainesville, FL, March 9 and 10, 2023

Acknowledgment and Dedication

The authors acknowledge the partnership established between the University of Massachusetts Amherst and the Kyiv School of Economics, which provided support for this collaboration.

This presentation is dedicated to all Ukrainian farmers who have sacrificed so much as the war continues in Ukraine.

Thanks to Professor Panos Pardalos!

Outline of Presentation

- Background and Motivation
- Literature Review
- The Multicommodity International Trade Spatial Price Equilibrium Model
- Illustrative Examples
- The Algorithm
- Larger Numerical Examples
- Insights and Summary

Background and Motivation

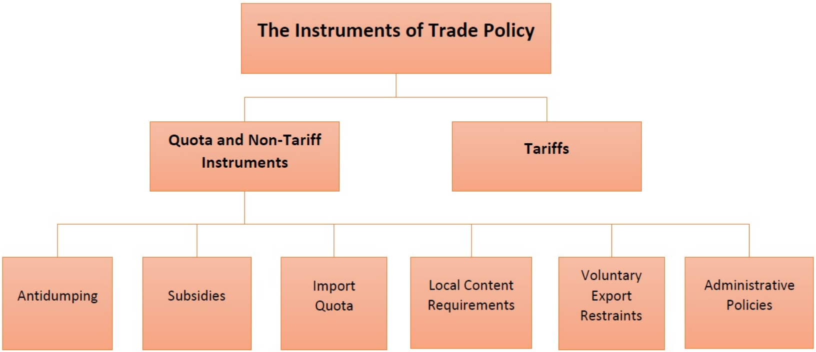
International Trade

International trade provides us with commodities throughout the year and has benefits for producers and consumers alike.

International Trade and Policies

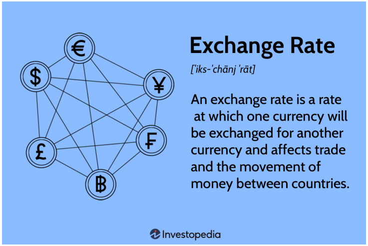
- **Nations engage in trade to increase their productivity levels, employment rates, and general economic welfare.**
- **The increased level of world trade and competition has garnered the attention of government policy makers.**
- Trade policy instruments such as tariffs, subsidies, and quotas have become highly relevant as the world continues to battle the impacts of the COVID-19 pandemic and millions on the planet suffer from hunger and growing food insecurity.

Identifying quantitatively the **impacts of trade policies and exchange rates** on international trade can provide trade and regulatory bodies with valuable information on product trade volumes and producer and consumer prices.



Exchange Rates

Exchange rates represent the value (price) of one currency relative to another currency.



They are important economic parameters in international trade, with changes in the exchange rate affecting the decision-making of individuals, businesses, and governments.

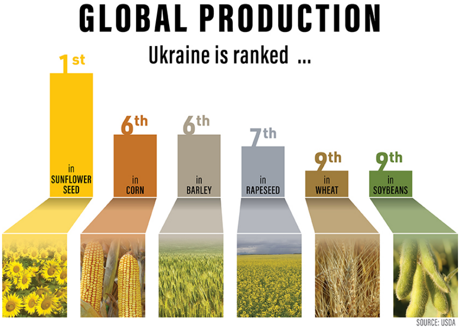
Exchange Rates

• The US dollar has gotten stronger over the past year, with the greatest rate of increase occurring since the Russian invasion of Ukraine on February 24, 2022, with the war still ongoing.

• In contrast, during the first year of the COVID-19 pandemic, the dollar weakened with respect to the euro, the British pound, and the yen.

Additional Motivation

This work is inspired by Russia's war on Ukraine and the need to assess its impacts on agricultural trade as well as on food insecurity and the need to provide more general computable models for assessing the impacts of trade policies and exchange rates on international trade.



Our Contributions in This Paper

In this paper, we harness the powerful theory of variational inequalities to construct a model with the following features:

- Multiple commodities;
- Multiple routes from origin nodes to destination nodes in the same or different countries;
- Exchange rates and the formula for their computation along trade routes;
- Inclusion of policies in the form of tariffs, subsidies, and quotas;
- The underlying economic functions can be nonlinear and asymmetric. Hence, our transportation cost functions capture congestion.

Literature Review

Literature Review

The intellectual foundations of our work lie in the contributions of Samuelson (1952) and Takayama and Judge (1964, 1971) to spatial price equilibrium(SPE) modeling with notice of the following works that utilize variational inequality theory: Florian and Los (1982), Dafermos and Nagurney (1984), Harker and Friesz (1986), Nagurney, Thore, and Pan (1996), Daniele (2004).

None of these models incorporated exchange rates!

• Variational inequality formulations of SPE problems with trade instruments have been constructed by Nagurney, Nicholson, and Bishop (1996), Nagurney, Besik, and Dong (2019), and Nagurney, Salarpour, and Dong (2022).

• Other variational inequality models of spatial price equilibrium problems have included product quality (Li, Nagurney, and Yu (2018)) and product perishability (Nagurney and Aronson (1989)), the latter even with policy instruments in Nagurney (2022).

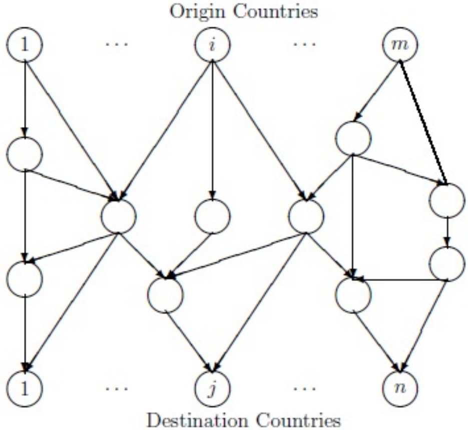
Literature Review

• There have been several imperfectly competitive variational inequality models developed with exchange rates (e.g., Liu and Nagurney (2011), Cruz (2013)). In these, unlike this paper, imperfectly competitive models, the paths consisted of single links and would not allow for transportation through different countries.

• Devadoss and Sabala (2020) emphasize that, to the best of their knowledge, no study until theirs had previously used the spatial price equilibrium model to analyze the effects of exchange rate changes. Their study focused on the yuan-dollar exchange rate and cotton markets and proposed a single commodity spatial price equilibrium model, with a single route, consisting of a single link, between countries.

The Multicommodity International Trade Model

The Multicommodity International Trade Model



The network topology is denoted by the graph $G = [N, L]$, where N is the set of nodes, L is the set of links, and P is the set of paths. There are H commodities, with a typical one denoted by h.

The Multicommodity International Trade Model

Each path p represents a trade route. Intermediate nodes in the network, which are transit points, also correspond to countries.

Let P_{ij} denote the set of paths connecting the pair of origin/destination country nodes (i, j). The paths are acyclic.

A typical link is denoted by a and represents transport from a country node at which the link originates to the node denoting the country at which the link terminates.

A trade route can entail transportation through multiple countries, depending on the application, and via different modes, such as rail, truck, air, or water (sea, river, etc.).

The Exchange Rates

Associated with each link $a \in L$ is an exchange rate e_a, reflecting the exchange rate from the country (node) that the link emanates from to the country (node) that it terminates in.

Also, associated with each pair of origin/destination countries (i, j) is the exchange rate e_{ij} for $i = 1, \ldots, m$; $j = 1, \ldots, n$.

The Trade Policies

There is a nonnegative subsidy associated with commodity h and imposed by the government in country i, which is denoted by sub_i^h for $h = 1, \ldots, H$; $i = 1, \ldots, m$.

The unit tariff levied by country j on commodity h from country i is denoted by τ_{ij}^h for $h = 1, \ldots, H$; $i = 1, \ldots, m$; $j = 1, \ldots, n$. Tariffs within a country are not imposed; hence, $\tau_{ii}^h = 0$, $\forall i$, $\forall h$.

In addition, there are capacities, which can represent quotas, where $\bar{Q}_p^h$; $h = 1, \ldots, H$; $p \in P$, denotes the bound on the commodity shipment of commodity h on path p.

Variables and Constraints

All commodity path flows, for all commodities h, and all paths p, must be nonnegative:

$$Q_p^h \geq 0, \quad \forall h, \forall p \in P. \tag{1}$$

The flow on a link a of commodity h, in turn, is equal to the sum of the path flows of the commodity h that use the link:

$$f_a^h = \sum_{p \in P} Q_p^h \delta_{ap}, \quad \forall h, \forall a \in L. \tag{2}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and is 0, otherwise.

Variables and Constraints

The supply of commodity h produced in country i, s_i^h, is equal to the shipments of the commodity from the country to all destination countries:

$$s_i^h = \sum_{p \in P^i} Q_p^h, \quad h = 1, \dots, H; i = 1, \dots, m, \tag{3}$$

whereas the demand for commodity h in country j, d_j^h, is equal to the shipments of the commodity from all origin countries to that country:

$$d_j^h = \sum_{p \in P_j} Q_p^h, \quad h = 1, \dots, H; j = 1, \dots, n. \tag{4}$$

$Q \in R_+^{n_P}$ is the vector of commodity shipments with $s \in R_+^{Hm}$ being the vector of commodity supplies and $d \in R_+^{Hn}$ being the vector of commodity demands.

Supply Price Functions

The supply price function for commodity h of country i is denoted by π_i^h and we have that:

$$\pi_i^h = \pi_i^h(s), \quad h = 1, \ldots, H; i = 1, \ldots, m. \qquad (5a)$$

With notice of the conservation of flow equations (3), we may define new supply price functions $\tilde{\pi}_i^h$; $h = 1, \ldots, H$; $i = 1, \ldots, m$, such that

$$\tilde{\pi}_i^h(Q) \equiv \pi_i^h(s). \qquad (5b)$$

Demand Price Functions

The demand price functions, in turn, are:

$$\rho_j^h = \rho_j^h(d), \quad h = 1, \ldots, H; j = 1, \ldots, n, \tag{6a}$$

where ρ_j^h denotes the demand price for commodity h in country j. Making use now of conservation of flow equations (4), we construct equivalent demand price functions $\tilde{\rho}_j^h$; $h = 1, \ldots, H$; $j = 1, \ldots, n$, as follows:

$$\tilde{\rho}_j^h(Q) \equiv \rho_j^h(d). \tag{6b}$$

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Transportation Cost Functions

With each link $a \in L$, and commodity h, we associate a unit transportation cost c_a^h such that

$$c_a^h = c_a^h(f), \quad \forall h, \forall a \in L. \tag{7a}$$

Because of the conservation of flow equations (2), we can define link unit transportation cost functions $\tilde{c}_a^h(Q)$, $\forall a \in L$, $\forall h$, as:

$$\tilde{c}_a^h(Q) \equiv c_a^h(f). \tag{7b}$$

The Effective Exchange Rate

Observe that, in order to appropriately quantify the effective transportation cost on a link a for a commodity h, if a commodity makes use of the link on a path from an origin country node to a destination country node, one needs to calculate the effective exchange rate associated with the commodity on link a being transported onward on path p, which is denoted by e_a^p. Note that e_a^p is the product of the exchange rates on the links on path p that include and follow link a on that path, and is given by:

$$e_a^p \equiv \begin{cases} \prod_{b \in \{a' \geq a\}_p} e_b, & \text{if } \{a' \geq a\}_p \neq \emptyset, \\ 0, & \text{if } \{a' \geq a\}_p = \emptyset, \end{cases} \tag{8}$$

where $\{a' \geq a\}_p$ denotes the set of the links including and following link a in path p, and $\emptyset$ denotes the null set.

The Effective Transportation Cost

The true transportation cost then on link a, $a \in L$, for commodity h; $h = 1, \ldots, H$, when it is used in a path p, is given by the expression:

$$\tilde{c}_a^{hp} = \tilde{c}_a^h(Q)e_a^p. \tag{9}$$

The effective transportation cost on a path, $\tilde{C}_p^h$, $\forall p \in P$, for commodity h; $h = 1, \ldots, H$, is then calculated as:

$$\tilde{C}_p^h = \sum_{a \in L} \tilde{c}_a^{hp} \delta_{ap}; \tag{10}$$

that is, the effective transportation cost on a path, which represents a trade route, is equal to the sum of the effective transportation costs for the commodity on links that make up the path.

Equilibrium Conditions

Definition 1: The Multicommodity International Trade Equilibrium Conditions

A multicommodity path trade flow pattern $Q^* \in R_+^{n_P}$ *is an international trade spatial price network equilibrium pattern under subsidies and tariffs with explicit exchange rates and capacities if the following conditions hold: For all pairs of country origin and destination nodes:* (i,j); $i = 1, \dots, m$; $j = 1, \dots, n$, *and all paths* $p \in P_{ij}$ *as well as all commodities* h; $h = 1, \dots, H$:

$$(\tilde{\pi}_i^h(Q^*) - sub_i^h + \tau_{ij}^h)e_{ij} + \tilde{C}_p^h(Q^*) \begin{cases} \leq \tilde{\rho}_j^h(Q^*), & \text{if} \quad Q_p^{h*} = \bar{Q}_p^h, \\ = \tilde{\rho}_j^h(Q^*), & \text{if} \quad 0 < Q_p^{h*} < \bar{Q}_p^h, \\ \geq \tilde{\rho}_j^h(Q^*), & \text{if} \quad Q_p^{h*} = 0. \end{cases} \tag{11}$$

Variational Inequality Formulation and Existence

Theorem 1: Variational Inequality Formulation of the Multicommodity International Trade Equilibrium Conditions

A multicommodity path trade flow pattern $Q^* \in K$, where $K \equiv \{Q | 0 \leq Q \leq \bar{Q}\}$ is a multicommodity international trade spatial price network equilibrium pattern with exchange rates and under subsidies and tariffs and capacities, according to Definition 1, if and only if it satisfies the variational inequality:

$$\sum_{h=1}^{H}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p\in P_{ij}}\left[(\tilde{\pi}_i^h(Q^*) - sub_i^h + \tau_{ij}^h)e_{ij} + \tilde{C}_p^h(Q^*) - \tilde{\rho}_j^h(Q^*)\right]$$

$$\times\left[Q_p^h - Q_p^{h*}\right] \geq 0, \quad \forall Q \in K. \qquad (12)$$

Existence of an equilibrium solution Q^* is guaranteed since the feasible set K is compact.

Variational Inequality Formulation

Standard Form

Variational inequality (12) is now put into standard form (cf. Nagurney (1999)), VI$(F, \mathcal{K})$, where one seeks to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{13}$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

Specifically, we define $X \equiv Q$, $\mathcal{K} \equiv K$, and $\mathcal{N} = Hn_P$. Plus, $F(X)$ consists of the elements $F^h_p(X) \equiv \left[(\tilde{\pi}^h_i(Q) - sub^h_i + \tau^h_{ij})e_{ij} + \tilde{C}^h_p(Q) - \tilde{\rho}^h_j(Q)\right]$, $\forall h$, $\forall i, j$, $\forall p \in P_{ij}$. Clearly, VI (12) can be put into standard form (13).

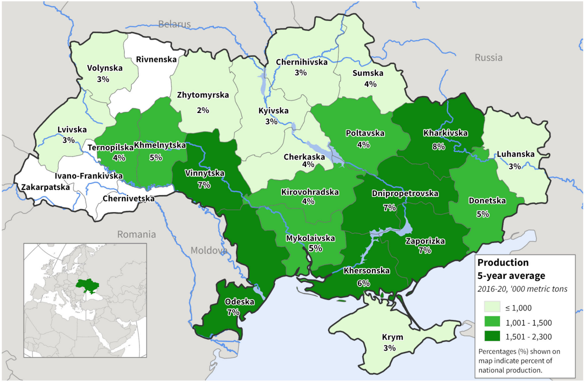
Illustrative Examples

Illustrative Examples

The examples focus on wheat commodity flows from Ukraine to Lebanon before and after the invasion of Ukraine by Russia on February 24, 2022.

The local currency codes are: UAH for Ukrainian hryvnia, MDL for the Moldovan leu, RON for the Romanian leu, LBP for the Lebanese pound, and USD for the United States dollar.

Ukraine: Wheat Production



Illustrative Examples

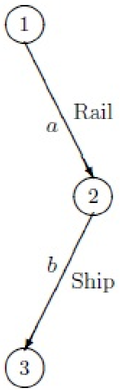
The unit of flow is a ton of wheat.

In these examples, there are no commodity path flow capacities.

These illustrative examples are stylized but, nevertheless, are grounded in real data.




Example 1 - Pre-Invasion Scenario



The network topology for Illustrative Example 1 with nodes 1 and 2 representing Ukraine and node 3 corresponding to Lebanon. There is a single path $p_1 = (a, b)$, where link a corresponds to transport to the Black Sea ports via rail inside Ukraine, and link b represents maritime transport from the Black Sea ports to Lebanon.

Example 1 - Pre-Invasion Scenario

The exchange rate data for Example 1 is drawn from early January 2022:

$$e_{13} = 55.0581, \quad e_a = 1.0000, \quad e_b = 55.0581.$$

The supply price function in Ukraine in hryvnia is:

$$\pi_1 = \pi_1(s_1) = .000136 s_1 + 7,001.60.$$

The transportation cost functions in local currencies are:

$$c_a = c_a(f_a) = .000278 f_a + 954.80, \quad c_b = c_b(f_b) = .000278 f_b + 1,091.20.$$

The demand price function in Lebanon in Lebanese pounds is:

$$\rho_3 = \rho_3(d_3) = -.15 d_3 + 602,344.00.$$

The effective exchange rates are:

$$e_a^{p_1} = e_a e_b = 55.0581, \quad e_b^{p_1} = 55.0581,$$

so that the effective link costs are:

$$\tilde{c}_a^{p_1} = e_a^{p_1} \tilde{c}_a = 55.0581 \tilde{c}_a, \quad \tilde{c}_b^{p_1} = e_b^{p_1} \tilde{c}_b = 55.0581 \tilde{c}_b.$$

Example 1 - Pre-Invasion Scenario

The effective path cost on path p_1 is:

$$\tilde{C}_{p_1} = \tilde{c}_a^{p_1} + \tilde{c}_b^{p_1}.$$

Applying the international trade spatial price equilibrium conditions (11), under the assumption of no tariff and no subsidy, and no quota, and assuming that $Q^*_{p_1} > 0$, we have that:

$$\tilde{\pi}_1(Q^*_{p_1})e_{13} + \tilde{C}_{p_1}(Q^*_{p_1}) = \tilde{\rho}_3(Q^*_{p_1}),$$

which, in turn, reduces to:

$$.1881 Q^*_{p_1} = 104,200.3344,$$

with solution:

$$Q^*_{p_1} = 553,962.4370.$$

The $553,962.4370$ tons of wheat flow is quite reasonable, since, in 2021, Lebanon imported $520,000$ tons of wheat from Ukraine, and an even greater harvest was expected in 2022.

Example 1 - Pre-Invasion Scenario

The above wheat commodity flow pattern results in a supply and a demand of:

$$s_1^* = d_3^* = Q_{p_1}^* = 553,962.4370.$$

The supply and demand prices are:

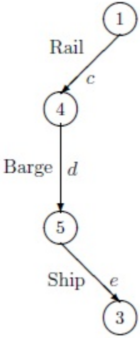
$$\pi_1(s_1^*) = \tilde{\pi}_1(Q_{p_1}^*) = 7,076.9388 \text{ UAH} = \$257.7002,$$
$$\rho_3(d_3^*) = \tilde{\rho}_3(Q_{p_1}^*) = 519,249.6344 \text{ LBP} = \$343.4190.$$

The incurred transportation costs are:

$$\tilde{c}_a = 1,108.8015 \text{ UAH} = \$40.3759, \quad \tilde{c}_b = 1,245.2015 \text{ UAH} = \$45.3428.$$

The supply price of \$257.7002 **per ton of wheat in Ukraine (at the farmer level) and the demand price of** \$343.4190 **in Lebanon are close to the reported prices in 2021. Farmers in Ukraine could get about** \$270 **per ton of wheat before the invasion. The transportation cost pre-invasion for a ton of wheat in Ukraine to a port was about** \$40**, as is the result in this example.**

Example 2 - Invasion Scenario



In Example 2, we consider the invasion scenario after February 24, 2022, but before the Black Sea Grain Initiative, which took effect in late July. During this period, essentially no grain was shipped from Ukraine using a Black Sea route as in Example 1. There is a single path $p_2 = (c, d, e)$. Nodes 1, 3, 4, and 5 denote Ukraine, Lebanon, Moldova, and Romania, respectively.

Example 2 - Invasion Scenario

The exchange rates for Example 2 are obtained from early July; that is, after the invasion but before the Black Sea Grain Initiative:

$$e_{13} = 51.6836, \quad e_c = .6528, \quad e_d = .2523, \quad e_e = 313.6980,$$

The exchange rates were essentially the same on July 20, 2022.

The supply price function in Ukraine in hryvnia is:

$$\pi_1(s) = \pi_1(s_1) = .002673s_1 + 2,806.30.$$

The difference in supply price function compared to the function in Example 1 is due to the damages because of the war.

Example 2 - Invasion Scenario

The transportation cost functions in local currencies are:

$$c_c = .002768 f_c + 6,546.50, \quad c_d = .002172 f_d + 2,324.60, \quad c_e = .000257 f_e + 345.40.$$

The difference in the cost function on link c in this example and the cost function on link a in Example 1, with both entailing rail transportation in Ukraine, is due to the different rail gauges used in Ukraine and Moldova, which necessitates including loading and unloading costs. Loading and unloading costs are also accounted for in the cost function on link d.

The demand price function in Lebanon in Lebanese pounds is:

$$\rho_3(d) = \rho_3(d_3) = -.17 d_3 + 793,747.50.$$

Note that due to the food security issues in Lebanon and concerns over the availability of Ukrainian wheat because of the war, the demand price function is different from the one in Example 1.

Example 2 - Invasion Scenario

According to the international trade spatial price equilibrium conditions (11), and assuming that $Q^*_{p_2} > 0$, we have that:

$$\tilde{\pi}_1(Q^*_{p_2})e_{13} + \tilde{C}_{p_2}(Q^*_{p_2}) = \tilde{\rho}_3(Q^*_{p_2}),$$

the solution of which yields:

$$Q^*_{p_2} = 25,780.2589.$$

The wheat flow of $25,780.2589$ tons is reasonable since, without access to deep-sea ports on the Black Sea, Ukraine can, at most, export around 10% of what it used to.

This commodity flow results in a supply and a demand of:

$$s^*_1 = d^*_3 = Q^*_{p_2} = 25,780.2589.$$

Example 2 - Invasion Scenario

The supply and demand prices are:

$$\pi_1(s_1^*) = \tilde{\pi}_1(Q^*) = 2,875.2106 \text{ UAH} = \$98.2813,$$

$$\rho_3(d_3^*) = \tilde{\rho}_3(Q^*) = 789,364.8559 \text{ LBP} = \$522.0667.$$

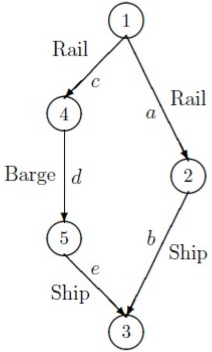
The incurred transportation link costs are:

$$\tilde{c}_c = 6,617.8597 \text{ UAH} = \$226.2137, \quad \tilde{c}_d = 2,380.5947 \text{ MDL} = \$124.6358,$$

$$\tilde{c}_e = 352.0255 \text{ RON} = \$73.0358.$$

The transportation cost of grain inside Ukraine has jumped to about \$200. Furthermore, because of the ongoing war, Ukrainian farmers are earning approximately \$100 per ton of wheat, which is similar to the supply price of \$98.2813 in this example. Moreover, with the continuing food crisis in Lebanon, and, as a result of the war, the price of wheat in Lebanon has gone up to more than \$500 per ton.

Example 3 - Black Sea Grain Initiative in Place



The network topology for Example 3, where we consider the post-July 22 Black Sea Grain Initiative scenario with maritime transportation from several of the Ukrainian Black Sea ports being, again, possible. In the network above, the nodes and the links correspond to the same countries and modes of transportation as in Examples 1 and 2.

Example 3 - Black Sea Grain Initiative in Place

The exchange rates on links are from late August; that is, after the Black Sea Grain Initiative:

$$e_a = 1.0000, \quad e_b = 41.3469, \quad e_c = .5291,$$

$$e_d = .2521, \quad e_e = 309.8670, \quad e_{13} = 41.3469.$$

The exchange rates were essentially the same on July 22, 2022.

The supply price function in Ukrainian hryvnia is now:

$$\pi_1(s) = \pi_1(s_1) = .000167 s_1 + 3,364.60.$$

Observe that due to the damages by the ongoing war, the supply price function, again, changes from the ones in Examples 1 and 2.

Example 3 - Black Sea Grain Initiative in Place

The transportation cost functions in local currencies are:

$$c_a = .000217f_a + 7,144.80, \quad c_b = .000246f_b + 7,423.10,$$

$$c_c = .003284f_c + 8,304.80, \quad c_d = .003097f_d + 2,397.50,$$

$$c_e = .000428f_e + 361.20.$$

The damages to the transportation infrastructure, and the congestion associated with products to be exported after the placement of the Black Sea Grain Initiative, result in different transportation cost functions from the previous examples.

The demand price function in Lebanese pounds is:

$$\rho_3(d) = \rho_3(d_3) = -.082d_3 + 796,162.50.$$

The demand price function is different from Example 2, which is in correspondence to the food security issues in Lebanon.

Example 3 - Black Sea Grain Initiative in Place

The equilibrium conditions (11) are, for this example, assuming positive commodity shipments:

$$\tilde{\pi}_1(Q^*)e_{13} + \tilde{C}_{p_1}(Q^*_{p_1}) = \tilde{\rho}_3(Q^*), \quad \tilde{\pi}_1(Q^*)e_{13} + \tilde{C}_{p_2}(Q^*_{p_2}) = \tilde{\rho}_3(Q^*).$$

The solution of the above system of equations yields a negative path flow on path p_2, which is infeasible. Therefore, path p_2 is not used. Then, one has that: $Q^*_{p_1} = 506,566.8120$ and $Q^*_{p_2} = 0.0000$, with the commodity flows, again, in tons.

The supply is similar to what Ukraine used to export to Lebanon pre-war. With the availability of maritime transportation from Ukraine on the Black Sea, the wheat flow on path p_2 is at 0.0000, which is due to the inefficiency of transporting the grain to a Middle Eastern country by such a route and composition of modes.

Example 3 - Black Sea Grain Initiative in Place

This wheat commodity flow pattern results in the following supply and demand:

$$s_1^* = d_3^* = Q_{p_1}^* + Q_{p_2}^* = 506,566.8120,$$

with the supply and demand prices per ton now being:

$$\pi_1(s_1^*) = \tilde{\pi}_1(Q^*) = 3,449.1966 \text{ UAH} = \$94.3212,$$

$$\rho_3(d_3^*) = \tilde{\rho}_3(Q^*) = 754,624.0214 \text{ LBP} = \$499.0899.$$

The incurred transportation costs are:

$$\tilde{c}_a = 7,254.7249 \text{ UAH} = \$198.3867, \quad \tilde{c}_b = 7,547.7154 \text{ UAH} = \$206.3988,$$

$$\tilde{c}_c = 8,304.8000 \text{ UAH} = \$227.1019, \quad \tilde{c}_d = 2,397.5000 \text{ MDL} = \$123.9018,$$

$$\tilde{c}_e = 361.2000 \text{ RON} = \$74.0245.$$

Although the initiative has facilitated the transportation of wheat, the war, and nearly full storage have kept the prices high. The supply price at $\$94.3212$ **and the demand price at** $\$499.0899$**, along with the** $\$206.3988$ **transportation cost on link** b**, reflect these issues and are preventing the demand market prices from falling.**

Example 4 - Example 3 Data with Subsidy

In Example 4, we, again, consider the post-July 22 Black Sea Grain Initiative scenario with maritime transportation via the Black Sea from Ukraine possible; however, a subsidy is introduced in this example, and the impact is quantified. We consider the effect of the subsidy $sub_1 = 1,000.00$ in hryvnia on Ukrainian wheat shipped to Lebanon:

In the solution of this example, again, only path p_1 is used, and one has that: $Q^*_{p_1} = 889,408.4787$ and $Q^*_{p_2} = 0.0000$.

This wheat commodity flow pattern results in the following supply and demand:

$$s^*_1 = d^*_3 = Q^*_{p_1} + Q^*_{p_2} = 889,408.4787,$$

with the following supply and demand prices:

$$\pi_1(s^*_1) = \tilde{\pi}_1(Q^*) = 3,513.1312 \text{ UAH} = \$96.0696,$$

$$\rho_3(d^*_3) = \tilde{\rho}_3(Q^*) = 723,231.0047 \text{ LBP} = \$478.3273.$$

Example 4 - Example 3 Data with Subsidy

The incurred transportation costs are:

$$\tilde{c}_a = 7,337.8016 \text{ UAH} = \$200.6585, \quad \tilde{c}_b = 7,641.8944 \text{ UAH} = \$208.9742,$$

$$\tilde{c}_c = 8,304.8000 \text{ UAH} = \$227.1019, \quad \tilde{c}_d = 2,397.5000 \text{ MDL} = \$123.9018,$$

$$\tilde{c}_e = 361.2000 \text{ RON} = \$74.0245.$$

The subsidy increases the quantity of wheat shipment, and increases the price that farmers can expect to get for a ton of wheat to $96.0696**, which is of value as the current low supply prices threaten the farmers' ability to buy seed and equipment for the next harvest season.**

The subsidy also helps to reduce the demand price to $478.3273**, which can be of significant importance in countering the food crisis and associated food insecurity in Lebanon.**

The Algorithm

The Algorithm

Modified Projection Method (Korpelevich (1977))

Step 0: Initialization

Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $t := 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation

Compute $\bar{X}^t$ by solving the variational inequality subproblem:

$$\langle \hat{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \hat{X}^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{14}$$

Step 2: Adaptation

Compute X^t by solving the variational inequality subproblem:

$$\langle X^t + (\beta F(\hat{X}^t) - X^{t-1}), X - X^t \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{15}$$

Step 3: Convergence Verification

If $|X^t - X^{t-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $t := t + 1$ and go to Step 1.

The Algorithm

Explicit Formulae at Iteration t for the Multicommodity Product Path Flows in Step 1

The algorithm results in the following closed form expressions for (14) for the multicommodity product path flows in Step 1 for the solution of variational inequality (12):

$$\bar{Q}_p^{ht} = \max\{0, \min\{\bar{Q}_p^h, Q_p^{ht-1} + \beta(\tilde{\rho}_j^h(Q^{t-1}) - (\tilde{\pi}_i^h(Q^{t-1}) + sub_i^h - \tau_{ij}^h)e_{ij} - \tilde{C}_p^h(Q^{t-1}))\}\},$$

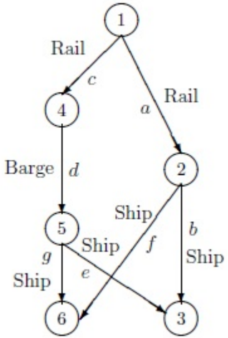
$$\forall h, \forall p. \qquad (16)$$

The explicit formulae for (15) in Step 2 readily follow.

Larger Numerical Examples

Larger Numerical Examples

The examples focus on commodity flows from Ukraine to Lebanon and Egypt, after the Black Sea Grain Initiative of July 22, 2022.



The figure above represents the network topology for the numerical Examples. Nodes 1 and 2 correspond to Ukraine, node 3 represents Lebanon, node 4 represents Moldova, node 5 is Romania, and node 6 denotes Egypt.

Example 5 - Brokered Agreement - Two Demand Markets

In this example, the commodity is, again, that of wheat. The local currency code for the Egyptian pound is EGP.

There are four paths:

$$p_1 = (a, b), \quad p_2 = (c, d, e), \quad p_3 = (a, f), \quad p_4 = (c, d, g).$$

The exchange rates are derived from late August, and are as follows:

$$e_a = 1.0000, \quad e_b = 41.3469, \quad e_c = .5291, \quad e_d = .2521, \quad e_e = 309.8670,$$

$$e_f = .5236, \quad e_g = 3.9415, \quad e_{13} = 41.3469, \quad e_{16} = .5236.$$

The functions are as in Examples 3 and 4, with the demand price function for wheat in Egypt in Egyptian pounds given by:

$$\rho_6(d) = \rho_6(d_6) = -.000216d_6 + 10,000.60.$$

Example 5 - Brokered Agreement - Two Demand Markets

The modified projection method computes the following commodity path flow pattern in tons of wheat:

$$Q^*_{p_1} = 302,029.3750, \quad Q^*_{p_2} = 0.0000, \quad Q^*_{p_3} = 1,390,388.5000, \quad Q^*_{p_4} = 0.0000.$$

One can see, from this solution, how important having an unblocked maritime route on the Black Sea is.

The demand market prices at equilibrium in the local currencies are:

$$\rho_3(d^*) = 771,396.0912 \text{ LBP} = \$510.1826, \quad \rho_6(d^*) = 9,700.2754 \text{ EGP} = \$506.5417.$$

The lower wheat commodity flow as compared to that in Example 3 shows that Egypt essentially "competes" with Lebanon for wheat. This result also supports the negative effects of war-induced higher prices compared to the pre-war prices (e.g., around \$343 **for Lebanon, as shown in Example 1). The demand price of wheat in Egypt is in accord with the post-war reported prices of more than** \$470 **as compared to less than** \$300 **pre-war.**

Example 6 - Example 5 with a Subsidy

In order to support farmers, we assume that the Ukrainian government is now subsidizing farmers so that $sub_1 = 1000$ in hryvnia.

The modified projection method now yields the following equilibrium commodity path flow pattern of the wheat in tons:

$$Q^*_{p_1} = 557,759.6250, \quad Q^*_{p_2} = 0.0000,$$

$$Q^*_{p_3} = 2,254,257.0000, \quad Q^*_{p_4} = 0.0000.$$

The most efficient paths only are, again, used, and we see a big increase in the wheat flow on paths p_1 and p_3, almost similar to the pre-war import levels.

Example 6 - Example 5 with a Subsidy

The demand market prices at equilibrium in the local currencies are:

$$\rho_3(d^*) = 750,426.1875 \text{ LBP} = \$496.3136,$$

$$\rho_6(d^*) = 9,513.6797 \text{ EGP} = \$496.7978.$$

The demand market price of wheat decreases in both countries under the subsidy, which benefits consumers. The farmers in Ukraine now sell the wheat at a lower price (i.e., under $105) relative to the pre-war supply price of $257.7002 in Example 1, but now export a greater volume as compared to Example 5. The greater volume of wheat can assist in reducing food insecurity in Lebanon and Egypt, which rely heavily on Ukrainian wheat.

Example 7 - Two Demand Markets and Two Commodities

The network topology remains as in Example 6, but now we have two commodities: wheat and corn. Here, the subscript $h = 1$ refers to wheat and the subscript $h = 2$ corresponds to corn. For wheat, the supply and demand price functions are as in Example 6.

The supply price function for corn in Ukrainian hryvnia is:

$$\pi_1^2(s) = \pi_1^2(s_1^1, s_1^2) = .000054 s_1^1 + .000109 s_1^2 + 4022.50.$$

The demand price functions for corn are:

$$\rho_3^2(d) = \rho_3^2(d_3^2) = -.43 d_3^2 + 718,256.40,$$

$$\rho_6^2(d) = \rho_6^2(d_6^2) = -.000308 d_6^2 + 9,900.50.$$

Example 7 - Two Demand Markets and Two Commodities

The unit transportation costs on links in local currencies are:

$$c_a^1(f) = .000217 f_a^1 + 0.000043 f_a^2 + 7,144.80, \quad c_a^2(f) = .000047 f_a^1 + 0.000236 f_a^2 + 7,013.60,$$

$$c_b^1(f) = .000246 f_b^1 + .000049 f_b^2 + 7,423.10, \quad c_b^2(f) = .000052 f_b^1 + 0.000251 f_b^2 + 7,248.30,$$

$$c_c^1(f) = .003284 f_c^1 + 0.000655 f_c^2 + 8,304.80, \quad c_c^2(f) = .000639 f_c^1 + .003196 f_c^2 + 8,0096.60,$$

$$c_d^1(f) = .003097 f_d^1 + .000622 f_d^2 + 2,397.50, \quad c_d^2(f) = .000575 f_d^1 + .002878 f_d^2 + 2,251.40,$$

$$c_e^1(f) = .000428 f_e^1 + .000086 f_e^2 + 361.20, \quad c_e^2(f) = .000093 f_e^1 + .000461 f_e^2 + 352.50,$$

$$c_f^1(f) = .000246 f_f^1 + .000049 f_f^2 + 7,023.60, \quad c_f^2(f) = .000051 f_f^1 + .000254 f_f^2 + 6,892.50,$$

$$c_g^1(f) = .000428 f_g^1 + .000086 f_g^2 + 335.20, \quad c_g^2(f) = .000088 f_g^1 + .000441 f_g^2 + 326.80.$$

Example 7 - Two Demand Markets and Two Commodities

The modified projection method yields the following multicommodity equilibrium flow pattern in tons of wheat and corn, respectively:

$$Q_{p_1}^{1*} = 285,284.5625, \quad Q_{p_2}^{1*} = 0.0000, \quad Q_{p_3}^{1*} = 1,288,246.2500, \quad Q_{p_4}^{1*} = 0.0000.$$

$$Q_{p_1}^{2*} = 19,948.1738, \quad Q_{p_2}^{2*} = 0.0000, \quad Q_{p_3}^{2*} = 630,883.1250, \quad Q_{p_4}^{2*} = 0.0000.$$

The results for Example 7 further substantiate the importance of maritime routes over the Black Sea for exporting agricultural products from Ukraine, which would even be more the case if maritime freight rates are to decrease. The more efficient paths p_1 and p_3 are, again, in use for wheat shipments; however, one can see that the flows have decreased compared to Example 5, which is the impact of having another type of grain in the trade network.

Example 7 - Two Demand Markets and Two Commodities

The prices at equilibrium in the local currencies for a ton of wheat in Lebanon and Egypt are:

$$\rho_3^1(d^*) = 772,769.1875 \text{ LBP} = \$511.0907, \quad \rho_6^1(d^*) = 9,722.3388 \text{ EGP} = \$507.6939,$$

and for corn:

$$\rho_3^2(d^*) = 772,678.6875 \text{ LBP} = \$511.0308, \quad \rho_6^2(d^*) = 9,706.1885 \text{ EGP} = \$506.8505.$$

The demand prices for wheat are nearly similar to those in Example 5, as both countries highly depend on Ukrainian wheat imports. Furthermore, one can observe the similarly war-induced higher demand prices for corn, although less than wheat. It is also worth noting that even at an increased supply price of around \$114 at the farm's gate, Ukraine remains the supplier providing the cheapest corn in the world.

Example 8 - Example 7 with Quotas on Commodity Flows

This scenario is inspired by slowdowns in the processing of shipments of agricultural products even after the passage of the Black Sea Grain Initiative. Pre-war, there used to be around 40 inspections a day, but now, the number has decreased to, on the average, 5 inspections per day.

Example 8 has data identical to that in Example 7 except that now commodity path flow quotas are imposed:

$$\bar{Q}^1_{p_1} = \bar{Q}^1_{p_2} = 200,000.0000, \quad \bar{Q}^1_{p_3} = \bar{Q}^1_{p_4} = 100,000.0000,$$

$$\bar{Q}^2_{p_1} = \bar{Q}^2_{p_2} = 15,000.0000, \quad \bar{Q}^2_{p_3} = \bar{Q}^2_{p_4} = 600,000.0000.$$

Example 8 - Example 7 with Quotas on Commodity Flows

The modified projection method now converges to the following equilibrium multicommodity flow pattern:

$$Q^{1*}_{p_1} = 200,000.0000, \quad Q^{1*}_{p_2} = 0.0000, \quad Q^{1*}_{p_3} = 100,000.0000, \quad Q^{1*}_{p_4} = 14,006.8184,$$

$$Q^{2*}_{p_1} = 15,000.0000, \quad Q^{2*}_{p_2} = 0.0000, \quad Q^{2*}_{p_3} = 600,000.0000, \quad Q^{2*}_{p_4} = 0.0000.$$

From the numerical results, we see that the wheat flows are at their capacities on paths: p_1 and p_3, and the same paths are at their capacities for corn: p_1, p_3. In the case of wheat, we now have positive flow on path p_4, which was not the case in Example 7, but can be compared to Example 2. Clearly, the demand is sufficiently high that an alternative route is now being used.

It can also be observed that the alternative route is not used for corn, which, again, is in accord with the high dependence of Lebanon and Egypt on Ukrainian wheat compared to corn.

Example 8 - Example 7 with Quotas on Commodity Flows

The demand market prices at equilibrium in the local currencies for a ton of wheat and corn in Lebanon and Egypt are:

$$\rho_3^1(d^*) = 779,762.5000 \text{ LBP} = \$515.7159, \quad \rho_6^1(d^*) = 9,975.9746 \text{ EGP} = \$520.9386$$

$$\rho_3^2(d^*) = 774,806.3750 \text{ LBP} = \$512.4380, \quad \rho_6^2(d^*) = 9,715.7002 \text{ EGP} = \$507.3472.$$

We see that the demand market prices of both wheat and corn rise in both Lebanon and Egypt under the quota regime as compared to the respective prices in Example 7. However, the increase in the price of wheat is greater than that of corn, which can be traced back to the criticality of wheat as the main source of calories in the two countries.

Example 8 - Example 7 with Quotas on Commodity Flows

Additionally, this example indicates that countries whose grain imports mostly consist of grain for food (e.g., wheat) and not much grain for feed (e.g., corn), are especially vulnerable to higher wheat prices.

The supply price of wheat is at around $95 relative to the respective supply price of about $100 in Example 7. Similarly, the supply price of corn decreases from around $114 in Example 7 to about $112 in this example. Having quotas, or equivalently, a reduction in capacity on routes has a negative impact on farmers as well as on consumers in terms of prices and commodity availability.

Insights and Summary

Insights

• The importance of having alternative routes in countering disruptions and congestion is evident in the examples.

• The results strongly confirm the need for efficient transportation routes for trade, as, for example, for the export of grain via maritime transport from the Black Sea ports in the case of Ukraine.

• The examples show the benefits of subsidies for agricultural trade for both farmers and consumers.

• The impact of the exchange rates on the grain commodity flows and on producer and consumer prices are revealed in the examples for different periods: pre-invasion, following the invasion, and after the Black Sea Grain Initiative.

• The examples demonstrate the importance of the Ukrainian grain, and its relevance to global food security.

Summary

• International trade of commodities is essential to both producers and consumers. Various recent events of historical significance, from the COVID-19 pandemic to Russia's war on Ukraine, have demonstrated the criticality of trade for the availability of various commodities.

• Our model allows for multiple routes for commodities from origin countries to destination countries, and these routes, in turn, can consist of multiple transportation links through different countries. We show how exchange rates affect costs on links and paths as well as supply prices, and formalize the definition of the multicommodity spatial price equilibrium with exchange rates and under policies such as tariffs, subsidies, and quotas on commodity path flows.

• The governing equilibrium conditions are formulated as a variational inequality problem in commodity path flows, and the existence of an equilibrium is established.

Summary

• The numerical examples, for which complete input and output data are reported, are drawn from Russia's war on Ukraine, both illustrative examples as well as larger-scale ones, which are solved using an implemented computational scheme.

• The flexibility of the modeling and algorithmic framework allows for quantitatively investigating the impacts of different scenarios with features of exchange rates plus various policies and the addition/deletion of markets and trade routes.

More Wonderful Memories Thanks to Professor Panos Pardalos!

Thank You!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