Integrated Crop and Cargo War Risk Insurance: Application to Ukraine

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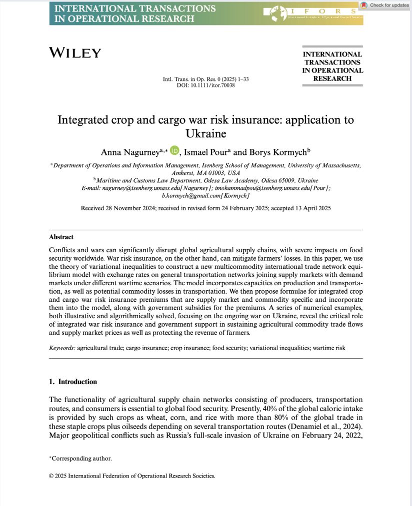

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Abstract

Conflicts and wars can significantly disrupt global agricultural supply chains, with severe impacts on food security worldwide. War risk insurance, on the other hand, can mitigate farmers' losses. In this paper, we use the theory of variational inequalities to construct a new multicommodity international trade network equilibrium model with exchange rates on general transportation networks joining supply markets with demand markets under different wartime scenarios. The model incorporates capacities on production and transportation, as well as potential commodity losses in transportation. We then propose formulae for integrated crop and cargo war risk insurance premiums that are supply market and commodity specific and incorporate them into the model, along with government subsidies for the premiums. A series of numerical examples, both illustrative and algorithmically solved, focusing on the ongoing war on Ukraine, reveal the critical role of integrated war risk insurance and government support in sustaining agricultural commodity trade flows and supply market prices as well as protecting the revenue of farmers.

Keywords: agricultural trade; cargo insurance; crop insurance; food security; variational inequalities; wartime risk

1. Introduction

The functionality of agricultural supply chain networks consisting of producers, transportation routes, and consumers is essential to global food security. Presently, 40% of the global caloric intake is provided by such crops as wheat, corn, and rice with more than 80% of the global trade in these staple crops plus oilseeds depending on several transportation routes (Denamiel et al., 2024). Major geopolitical conflicts such as Russia's full-scale invasion of Ukraine on February 24, 2022,

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Outline of Presentation

- **Background, Motivation, Literature Review, and Our Contributions**
- **Integrated Crop and Cargo War Risk Insurance**
- **Numerical Examples, Sensitivity Analysis, and Policy Implications**
- **Key Insights and Conclusions**

Background, Motivation, Literature Review, and Our Contributions

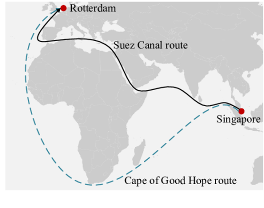
Global Agricultural Trade: Importance

- **Agricultural supply chains** are essential for global food security.
- **Staples (wheat, corn, rice)** deliver approximately **40%** of global calories.
- Over **80%** of the trade of these commodities relies on **established transportation routes**.



Motivation: Impacts of War on Supply Chains

- **Pre-invasion: 10%** of the global wheat exports, **15%** of the global corn and barley exports, and **50%** of global sunflower oil exports, with **90%** of these agricultural commodities exported through Ukraine's deepwater Black Sea ports.
- **Post-invasion:** Significant challenges to the trade of such essential agricultural commodities; increases in prices due to **heightened risk**, **transportation delays**, and increased **production** and **transportation costs**.




Integrating Crop and Cargo Insurance in Conflict Zones

- **Insurance policies** are an integral part of risk management in agricultural supply chains.
- Insurance professionals consider **marine insurance** – hull and cargo – to be among the **oldest forms of insurance**, dating back to the Phoenicians trading in the Mediterranean (around **1200 BC**), with the first formal policy established in **1350**.
- **Traditional crop insurance** shields farmers; **cargo insurance** protects transportation of commodities.
- In war, both **production** and **transportation** face significant risks.
- **War risk insurance**, according to Kagan (2021), covers losses due to **war, invasions, strikes, and terrorism**.
- An **integrated approach** can help maintain **trade flows** and support **food security**.

Literature Review: Insurance Methodologies and Catastrophic Risk

- **Mathematical Programming in Insurance:**
  - **Samson and Thomas (1985), von Lanzenauer and Wright (1991), Brockett and Xia (1995):** Overview of linear programming, network optimization, and game theory applied to insurance challenges.
- **Risk Reduction and Catastrophe:**
  - **Ermoliev et al. (2000):** Emphasized the synergy between risk reduction measures and insurance mechanisms in managing rare, catastrophic events.
  - **Lodree Jr. and Taskin (2008):** Introduced an insurance risk management framework for disaster relief and supply chain disruption.
  - **Kalfin et al. (2022):** Provided a systematic review on insurance as a tool for sustainable economic recovery after disasters.
  - **Fan et al. (2024b) and Zbib et al. (2024):** Developed stochastic programming and mutual catastrophe insurance frameworks.

Literature Review: Agricultural and Maritime Insurance

- **Crop Insurance Theory:**
  - **Ahsan et al. (1982):** Developed a theory of crop insurance, and its role in risk spreading and the challenges due to imperfect information.
  - **Myers (1988):** Evaluated the benefits of ideal contingency markets, while noting potential trade-offs for farmers and consumers.

- **Design and Calibration of Insurance Products:**
  - **Mahul and Wright (2003):** Analyzed the design of optimal crop revenue insurance, considering basis risk and indemnity schedules.
  - **Fan et al. (2024a):** Examined different agricultural subsidy schemes and their impact on output and wealth distribution.

- **Maritime and Cargo Insurance Reviews:**
  - **Ksciuk et al. (2023):** Provided a literature review on uncertainty in maritime ship routing and scheduling, emphasizing OR's role in risk mitigation.
  - **Ellili et al. (2023):** Conducted a bibliometric analysis of marine insurance literature, identifying key trends and areas for future research.

Literature Review: Subsidies & Spatial Price Equilibrium Models

- **Spatial Price Equilibrium Models and Variational Inequalities:**
  - **Nagurney (1999):** Pioneered the use of variational inequalities in network economics, providing the theoretical foundation for our model.
  - **Nagurney et al. (2023)** and **Nagurney, Pour, and Samadi (2024):** Extended spatial price equilibrium models to include various factors such as exchange rates and network capacities.

- **Incorporating Government Subsidies:**
  - **Nagurney (2023); Nagurney and Besedina (2023):** Developed models incorporating consumer subsidies and non-tariff measures.
  - **Nagurney et al. (2023), Nagurney, Salarpour, and Dong (2022):** Addressed policy impacts (e.g., subsidies) on spatial price equilibrium in the context of essential goods and health products.
  - **Nagurney, Daniele, and Cappello (2021):** Demonstrated subsidy effects in the context of human migration, highlighting the broader applicability of subsidy-based interventions.

Main Contributions

- **Integrated Framework:**
  - First integrated model that combines crop insurance and cargo insurance under war risk.
  - Accounts for both production disruptions and transportation losses.
- **Network Equilibrium Model:**
  - Develops a multicommodity international trade network equilibrium model using variational inequality (VI) formulation.
  - Incorporates production capacities, transportation constraints, commodity loss multipliers, and exchange rate effects.
- **Insurance Premium Formulation:**
  - Derives explicit formulas for integrated war risk insurance premiums as the expected drop in supply price under war scenarios.
  - Includes a framework for incorporating government subsidies that reduce the effective premium for farmers.
- **Numerical Validation:**
  - Provides comprehensive numerical examples and algorithmic solutions for both single-commodity and multi-commodity cases.
  - Provides sensitivity analysis of the impact of varying subsidy levels.

Integrated Crop and Cargo War Risk Insurance

An International Trade Network Topology

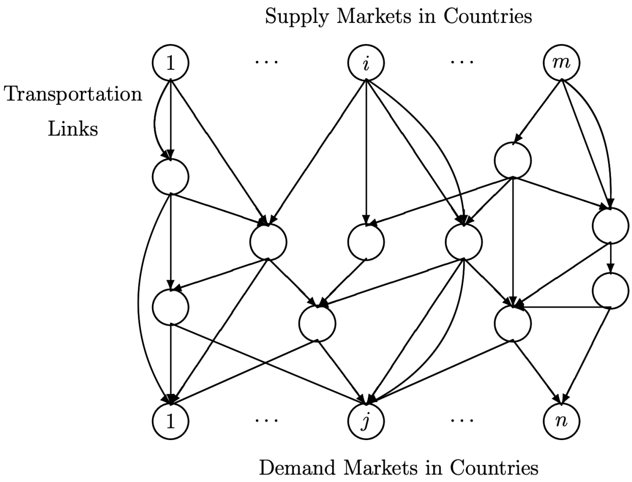


Figure 1: An International Trade Network Topology

Model Notation: Parameters

Parameters

$u_i^{s\xi_l}$	upper bound on the supply of the commodities at supply market i; $i = 1, \ldots, m$ under war scenario ξ_l; $l = 1, \ldots, \omega$.
$u_{ijr}^{Q\xi_l}$	upper bound on the transportation of all the commodities from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ on route r; $r = 1, \ldots, n_{ij}$ under war scenario ξ_l; $l = 1, \ldots, \omega$.
$\alpha_{ijr}^{\xi_l}$	the route r flow multiplier which quantifies how much of all the commodities remain after being transported on route r; $r = 1, \ldots, n_{ij}$ under war scenario ξ_l; $l = 1, \ldots, \omega$.
$e_{ij}^{\xi_l}$	the exchange rate from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ under scenario ξ_l; $l = 1, \ldots, \omega$.
σ_i^k	fraction of the premium for supply market i; $i = 1, \ldots, m$, and commodity k; $k = 1, \ldots, K$ covered by an authority with the values lying between 0 and 1.

Model Notation: Variables

Variables

$s_i^{k\xi_l}$	the supply of the commodity k; $k = 1, \ldots, K$ at supply market i; $i = 1, \ldots, m$ under war scenario ξ_l; $l = 1, \ldots, \omega$. Group all the supplies at war scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $s^{\xi_l} \in R_+^{Km}$.
$d_j^{k\xi_l}$	the demand for the commodity k; $k = 1, \ldots, K$ at demand market j; $j = 1, \ldots, n$ under war scenario ξ_l; $l = 1, \ldots, \omega$. Group all the demands at scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $d^{\xi_l} \in R_+^{Kn}$.
$Q_{ijr}^{k\xi_l}$	the shipment of the commodity k; $k = 1, \ldots, K$ from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ on route r; $r = 1, \ldots, n_{ij}$ under war scenario ξ_l; $l = 1, \ldots, \omega$. Group all the commodity shipments at scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $Q^{\xi_l} \in R_+^{KP}$.
$\lambda_i^{s\xi_l}$	the Lagrange multiplier associated with the production capacity constraint at supply market i; $i = 1, \ldots, m$ under war scenario ξ_l; $l = 1, \ldots, \omega$. Group all these Lagrange multipliers at scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $\lambda^{s\xi_l} \in R_+^m$.
$\lambda_{ijr}^{Q\xi_l}$	the Lagrange multiplier associated with the transportation capacity constraint on route r; $r = 1, \ldots, n_{ij}$ joining supply market i; $i = 1, \ldots, m$ and demand market j; $j = 1, \ldots, n$ under war scenario ξ_l; $l = 1, \ldots, \omega$. Group all these Lagrange multipliers at scenario ξ_l; $l = 1, \ldots, \omega$ into the vector $\lambda^{Q\xi_l} \in R_+^P$.

Model Notation: Functions

Functions

$\pi_i^{k\xi_l}(s^{\xi_l})$	the supply price function for commodity k; $k = 1, \ldots, K$ at supply market i; $i = 1, \ldots, m$ under war scenario ξ_l; $l = 1, \ldots, \omega$.
$\rho_j^{k\xi_l}(d^{\xi_l})$	the demand price function for commodity k; $k = 1, \ldots, K$ at demand market j; $j = 1, \ldots, n$ under war scenario ξ_l; $l = 1, \ldots, \omega$.
$c_{ijr}^{k\xi_l}(Q^{\xi_l})$	the unit transportation cost associated with transporting the commodity k; $k = 1, \ldots, K$ from supply market i; $i = 1, \ldots, m$ to demand market j; $j = 1, \ldots, n$ via route r; $r = 1, \ldots, n_{ij}$ under war scenario ξ_l; $l = 1, \ldots, \omega$.

Flow Conservation, Capacity Constraints, and Redefining Price Functions

- Flow Conservation

$$s_i^{k\xi_l} = \sum_{j=1}^{n}\sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l}, \quad \forall\, k,\, i,\, l, \qquad d_j^{k\xi_l} = \sum_{i=1}^{m}\sum_{r=1}^{n_{ij}} \alpha_{ijr}^{\xi_l} Q_{ijr}^{k\xi_l}, \quad \forall\, k,\, j,\, l.$$

- Capacity Constraints

$$\sum_{k=1}^{K}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l} \leq u_i^{s\xi_l}, \quad \forall\, i,\, l, \qquad \sum_{k=1}^{K} Q_{ijr}^{k\xi_l} \leq u_{ijr}^{Q\xi_l}, \quad \forall\, i,\, j,\, r,\, l.$$

- Redefining Price Functions in Terms of Shipments Instead of expressing the supply and demand price functions as functions of the production s^{ξ_l} and demand d^{ξ_l} variables, we redefine them in terms of the shipment vector Q^{ξ_l}. Specifically, we define:

$$\tilde{\pi}_i^{k\xi_l}(Q^{\xi_l}) \equiv \pi_i^{k\xi_l}(s^{\xi_l}), \quad \text{for } k = 1, \ldots, K \text{ and } i = 1, \ldots, m,$$

$$\tilde{\rho}_j^{k\xi_l}(Q^{\xi_l}) \equiv \rho_j^{k\xi_l}(d^{\xi_l}), \quad \text{for } k = 1, \ldots, K \text{ and } j = 1, \ldots, n.$$

Equilibrium Conditions

Definition 1: Equilibrium Conditions Under Capacity Reductions and Commodity Losses

A multicommodity shipment and Lagrange multiplier pattern $(Q^{\xi_l *}, \lambda^{s\xi_l *}, \lambda^{Q\xi_l *}) \in \mathcal{K}^{\xi_l}$, where

$$\mathcal{K}^{\xi_l} \equiv \{(Q^{\xi_l}, \lambda^{s\xi_l}, \lambda^{Q\xi_l}) | (Q^{\xi_l}, \lambda^{s\xi_l}, \lambda^{Q\xi_l}) \in R_+^{KP+m+P}\}$$

is a multicommodity international trade network equilibrium under capacity reductions and commodity losses in war scenario ξ_l; $l = 1, \dots, \omega$, if the following conditions hold: for all commodities k; $k = 1, \dots, K$; for all supply and demand market pairs: (i, j); $i = 1, \dots, m$; $j = 1, \dots, n$, and for all routes r; $r = 1, \dots, n_{ij}$:

$$(\tilde{\pi}_i^{k\xi_l}(Q^{\xi_l *}) + c_{ijr}^{k\xi_l}(Q^{\xi_l *}))e_{ij}^{\xi_l} + \lambda_i^{s\xi_l *} + \lambda_{ijr}^{Q\xi_l *} \begin{cases} = \alpha_{ijr}^{\xi_l} \tilde{\rho}_j^{k\xi_l}(Q^{\xi_l *}), & \text{if } Q_{ijr}^{k\xi_l *} > 0, \\ \geq \alpha_{ijr}^{\xi_l} \tilde{\rho}_j^{k\xi_l}(Q^{\xi_l *}), & \text{if } Q_{ijr}^{k\xi_l *} = 0; \end{cases} \quad (1)$$

Equilibrium Conditions

for all commodities k; $k = 1, \ldots, K$, and for all supply markets i; $i = 1, \ldots, m$:

$$u_i^{s\xi_l} \begin{cases} = \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l *}, & \text{if } \lambda_i^{s\xi_l *} > 0, \\ \geq \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l *}, & \text{if } \lambda_i^{s\xi_l *} = 0; \end{cases} \quad (2)$$

for all commodities k; $k = 1, \ldots, K$, and for all supply and demand markets (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, and for all routes r; $r = 1, \ldots, n_{ij}$:

$$u_{ijr}^{Q\xi_l} \begin{cases} = \sum_{k=1}^{K} Q_{ijr}^{k\xi_l *}, & \text{if } \lambda_{ijr}^{Q\xi_l *} > 0, \\ \geq \sum_{k=1}^{K} Q_{ijr}^{k\xi_l *}, & \text{if } \lambda_{ijr}^{Q\xi_l *} = 0. \end{cases} \quad (3)$$

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation of the International Trade Network Equilibrium Conditions Under Capacity Reductions and Commodity Losses

A multicommodity shipment and Lagrange multiplier pattern $(Q^{\xi_l *}, \lambda^{s\xi_l *}, \lambda^{Q\xi_l *}) \in \mathcal{K}^{\xi_l}$ *for each* ξ_l*;* $l = 1, \ldots, \omega$*, is an international trade network equilibrium under capacity disruptions and commodity losses, according to Definition 1, if and only if it satisfies the variational inequality:*

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\left[(\tilde{\pi}_i^{k\xi_l}(Q^{\xi_l *}) + c_{ijr}^{k\xi_l}(Q^{\xi_l *}))e_{ij}^{\xi_l} + \lambda_i^{s\xi_l *} + \lambda_{ijr}^{Q\xi_l *} - \alpha_{ijr}^{\xi_l}\tilde{\rho}_j^{k\xi_l}(Q^{\xi_l *})\right] \times (Q_{ijr}^{k\xi_l} - Q_{ijr}^{k\xi_l *})$$

$$+\sum_{i=1}^{m}\left[u_i^{s\xi_l} - \sum_{k=1}^{K}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_l *}\right]\times(\lambda_i^{s\xi_l} - \lambda_i^{s\xi_l *}) + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\left[u_{ijr}^{Q\xi_l} - \sum_{k=1}^{K} Q_{ijr}^{k\xi_l *}\right]\times(\lambda_{ijr}^{Q\xi_l} - \lambda_{ijr}^{Q\xi_l *}) \geq 0,$$

$$\forall (Q^{\xi_l}, \lambda^{s\xi_l}, \lambda^{Q\xi_l}) \in \mathcal{K}^{\xi_l}. \quad (4)$$

War Risk Insurance Premiums

Definition 2

The integrated **crop and cargo war risk insurance premium** for commodity k at supply market i is defined as

$$IP_i^k = \sum_{l=1}^{\omega} \left[\tilde{\pi}_i^{k\xi_0}(Q^{\xi_0 *}) - \tilde{\pi}_i^{k\xi_l}(Q^{\xi_l *}) \right] p_{\xi_l}, \tag{5}$$

where:

- $\tilde{\pi}_i^{k\xi_0}(Q^{\xi_0 *})$ is the equilibrium supply price under the baseline scenario ξ_0,
- $\tilde{\pi}_i^{k\xi_l}(Q^{\xi_l *})$ is the equilibrium price under war scenario ξ_l,
- p_{ξ_l} is the discrete probability of scenario ξ_l.

Equilibrium with War Insurance Premiums

Definition 3: The International Trade Network Equilibrium Conditions Under the War Insurance Premiums

A multicommodity shipment and Lagrange multiplier pattern $(Q^{\xi_0 **}, \lambda^{s\xi_0 **}, \lambda^{Q\xi_0 **}) \in \mathcal{K}^{\xi_0}$, where

$$\mathcal{K}^{\xi_0} \equiv \{(Q^{\xi_0}, \lambda^{s\xi_0}, \lambda^{Q\xi_0}) | (Q^{\xi_0}, \lambda^{s\xi_0}, \lambda^{Q\xi_0}) \in R_+^{KP+m+P}\}$$

is a multicommodity international trade network equilibrium under the war insurance premiums, if the following conditions hold: for all commodities k; $k = 1, \ldots, K$; for all supply and demand market pairs: (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$, and for all routes r; $r = 1, \ldots, n_{ij}$:

$$(\tilde{\pi}_i^{k\xi_0}(Q^{\xi_0 **}) + c_{ijr}^{k\xi_0}(Q^{\xi_0 **}))e_{ij}^{\xi_0} + IP_i^k(1 - \sigma_i^k) + \lambda_i^{s\xi_0 **} + \lambda_{ijr}^{Q\xi_0 **} \begin{cases} = \alpha_{ijr}^{\xi_0} \tilde{\rho}_j^{k\xi_0}(Q^{\xi_0 **}), & \text{if } Q_{ijr}^{k\xi_0 **} > 0, \\ \geq \alpha_{ijr}^{\xi_0} \tilde{\rho}_j^{k\xi_0}(Q^{\xi_0 **}), & \text{if } Q_{ijr}^{k\xi_0 **} = 0; \end{cases} \quad (6)$$

Equilibrium with War Insurance Premiums

for all commodities k; $k = 1, \dots, K$, and for all supply markets i; $i = 1, \dots, m$:

$$u_i^{s\xi_0} \begin{cases} = \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_0 **}, & \text{if } \lambda_i^{s\xi_0 **} > 0, \\ \geq \sum_{k=1}^{K} \sum_{j=1}^{n} \sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_0 **}, & \text{if } \lambda_i^{s\xi_0 **} = 0; \end{cases} \tag{7}$$

for all commodities k; $k = 1, \dots, K$, and for all supply and demand markets (i, j); $i = 1, \dots, m$; $j = 1, \dots, n$, and for all routes r; $r = 1, \dots, n_{ij}$:

$$u_{ijr}^{Q\xi_0} \begin{cases} = \sum_{k=1}^{K} Q_{ijr}^{k\xi_0 **}, & \text{if } \lambda_{ijr}^{Q\xi_0 **} > 0, \\ \geq \sum_{k=1}^{K} Q_{ijr}^{k\xi_0 **}, & \text{if } \lambda_{ijr}^{Q\xi_0 **} = 0. \end{cases} \tag{8}$$

VI Formulation with War Insurance Premiums

Theorem 2: Variational Inequality Formulation of the International Trade Network Equilibrium Conditions Under the War Insurance Premiums

A multicommodity shipment and Lagrange multiplier pattern $(Q^{\xi_0 **}, \lambda^{s\xi_0 **}, \lambda^{Q\xi_0 **}) \in \mathcal{K}^{\xi_0}$ *is an international trade network equilibrium under war insurance premiums, according to Definition 2, if and only if it satisfies the variational inequality:*

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\Big[(\tilde{\pi}_i^{k\xi_0}(Q^{\xi_0 **}) + c_{ijr}^{k\xi_0}(Q^{\xi_0 **}) + IP_i^k(1-\sigma_i^k) + \lambda_i^{s\xi_0 **} + \lambda_{ijr}^{Q\xi_0 **} - \alpha_{ijr}^{\xi_l}\tilde{\rho}_j^{k\xi_0}(Q^{\xi_0 **})\Big]$$

$$\times (Q_{ijr}^{k\xi_0} - Q_{ijr}^{k\xi_0 **})$$

$$+\sum_{i=1}^{m}\Big[u_i^{s\xi_0} - \sum_{k=1}^{K}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}} Q_{ijr}^{k\xi_0 **}\Big] \times (\lambda_i^{s\xi_0} - \lambda_i^{s\xi_0 **}) + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{n_{ij}}\Big[u_{ijr}^{Q\xi_0} - \sum_{k=1}^{K} Q_{ijr}^{k\xi_0 **}\Big]$$

$$\times (\lambda_{ijr}^{Q\xi_0} - \lambda_{ijr}^{Q\xi_0 **}) \geq 0, \quad \forall (Q^{\xi_0}, \lambda^{s\xi_0}, \lambda^{Q\xi_0}) \in \mathcal{K}^{\xi_0}. \tag{9}$$

Numerical Examples, Sensitivity Analysis, and Policy Implications

Illustrative Example: Baseline Scenario (ξ_0)

Illustrative Exampl consists of a single commodity, that is wheat, a single supply market, say, Ukraine, and a single demand market - that of Lebanon. There is one route connecting the supply market with the demand market which includes a maritime link on the Black Sea. For simplicity, the functions are in US dollars. The baseline ξ_0 nondisrupted scenario is prior to the full-scale invasion of February 24, 2022.

Baseline Model Functions and Parameters

Supply Price, Transportation Cost, and Demand Price Functions:

$$\pi_1^{1\xi_0}(s^{\xi_0}) = 0.0002\, s_1^{1\xi_0} + 170, \quad c_{111}^{1\xi_0}(Q^{\xi_0}) = 0.0001\, Q_{111}^{1\xi_0} + 30, \quad \rho_1^{1\xi_0}(d^{\xi_0}) = -0.0001\, d_1^{1\xi_0} + 400.$$

Upper Bounds, Route Flow Multiplier, and Exchange Rate:

$$u_1^{s\xi_0} = u_{111}^{Q\xi_0} = 1,000,000, \quad \alpha_{111}^{\xi_0} = 1, \quad e_{11}^{\xi_0} = 1.$$

Solving the variational inequality:

$$s_1^{1\xi_0} = Q_{111}^{1\xi_0 *} = d_1^{1\xi_0} = 500,000,$$

with both Lagrange multipliers equal to 0, and the following equilibrium values:

$$\pi_1^1 = 270, \quad c_{111}^1 = 80, \quad \rho_1^1 = 350 \quad (\text{since } \alpha_{111}^{\xi_0}\rho_1^1 = 350).$$

Illustrative Example: War Scenario ξ_1 (Low Damage)

War Scenario ξ_1 Parameters

Capacities:

$$u_1^{s\xi_1} = 500,000, \quad u_{111}^{Q\xi_1} = 500,000.$$

Loss Multiplier:

$$\alpha_{111}^{\xi_1} = 0.9.$$

The variational inequality solution for scenario ξ_1 gives:

$$s_1^{1\xi_1*} = Q_{111}^{1\xi_1*} = 419,947.51,$$

and the effective demand is:

$$d_1^{1\xi_1*} = 0.9 \times 419,947.51 = 377,952.76.$$

The resulting equilibrium values are:

$$\pi_1^1 = 253.99, \quad c_{111}^1 = 71.99, \quad \rho_1^1 = 362.20,$$

with $\alpha_{111}^{\xi_1}\rho_1^1 = 325.98$.

Illustrative Example: War Scenario ξ_2 (High Damage)

War Scenario ξ_2 Parameters

Capacities:

$$u_1^{s\xi_2} = 300,000, \quad u_{111}^{Q\xi_2} = 300,000.$$

Loss Multiplier:

$$\alpha_{111}^{\xi_2} = 0.8.$$

The equilibrium solution for scenario ξ_2 is:

$$s_1^{1\xi_2*} = Q_{111}^{1\xi_2*} = 300,000,$$

with the effective demand:

$$d_1^{1\xi_2*} = 0.8 \times 300,000 = 240,000.$$

Further, the equilibrium values are:

$$\pi_1^1 = 230, \quad c_{111}^1 = 60, \quad \rho_1^1 = 376,$$

and $\alpha_{111}^{\xi_2}\rho_1^1 = 300.8$, with supply Lagrange multiplier being equal to 10.8, the other Lagrange multiplier being equal to 0.

Sensitivity Analysis: Impact of Government Subsidies

We now incorporate government subsidization on the insurance premium. Let σ_1^1 denote the subsidy fraction. The effective premium becomes:

$$\text{Effective Premium} = IP_1^1(1 - \sigma_1^1).$$

For various subsidy levels:

- $\sigma_1^1 = 0$ (0% subsidy):

$$IP_1^1(1 - 0) = 28.005 \text{ USD/ton}, \quad s_1^{1\xi_0**} = 429,986.88 \, \text{tons}.$$

- $\sigma_1^1 = 0.25$ (25% subsidy):

$$IP_1^1(1 - 0.25) = 28.005 \times 0.75 = 21.004 \text{ USD/ton}, \quad s_1^{1\xi_0**} = 447,490.16 \, \text{tons}.$$

- $\sigma_1^1 = 0.5$ (50% subsidy):

$$IP_1^1(1 - 0.5) = 28.005 \times 0.5 = 14.003 \text{ USD/ton}, \quad s_1^{1\xi_0**} = 464,993.44 \, \text{tons}.$$

- $\sigma_1^1 = 0.75$ (75% subsidy):

$$IP_1^1(1 - 0.75) = 28.005 \times 0.25 = 7.001 \text{ USD/ton}, \quad s_1^{1\xi_0**} = 482,496.72 \, \text{tons}.$$

These results illustrate that greater government subsidization significantly lowers the effective insurance premium, thereby supporting higher production levels and mitigating the negative impact of war scenarios on the supply chain and food security.

Algorithmically Solved Numerical Examples

- Now we present numerical examples for our model solved via the modified projection method (Korpelevich (1977)).
- The algorithm was implemented in FORTRAN on a Linux system. Convergence is achieved if the absolute change between successive iterations is less than or equal to 0.01.
- Our examples examine the commodities of **wheat** (commodity 1) and **corn** (commodity 2) with the supply market in Ukraine and the demand markets in Lebanon and Egypt.
- Two transportation routes are considered from Ukraine to each demand market.

Example 1 — Scenario ξ_0 (Baseline Pre-War)

- **Exchange Rates:**

$$e_{11}^{\xi_0} = 55.0581, \quad e_{12}^{\xi_0} = 0.5714,$$

$$\text{USD/UAH} = 27.4619, \quad \text{USD/LBP} = 1,512.0000, \quad \text{USD/EGP} = 15.7300.$$

- **Supply Price Functions (in UAH/ton), Transportation Cost Functions (in UAH/ton), and Demand Price Functions (in local currencies per metric ton):**

$$\pi_1^{1\xi_0}(s^{\xi_0}) = 0.000136\, s_1^{1\xi_0} + 0.000068\, s_1^{2\xi_0} + 7001.60,$$

$$\pi_1^{2\xi_0}(s^{\xi_0}) = 0.000073\, s_1^{1\xi_0} + 0.000142\, s_1^{2\xi_0} + 6728.20.$$

$$c_{111}^{1\xi_0}(Q^{\xi_0}) = 0.000556\, Q_{111}^{1\xi_0} + 2046.80, \quad c_{112}^{1\xi_0}(Q^{\xi_0}) = 0.007512\, Q_{112}^{1\xi_0} + 10984.60,$$

$$c_{121}^{1\xi_0}(Q^{\xi_0}) = 0.000185\, Q_{121}^{1\xi_0} + 2046.80, \quad c_{122}^{1\xi_0}(Q^{\xi_0}) = 0.007312\, Q_{122}^{1\xi_0} + 10984.60,$$

$$c_{111}^{2\xi_0}(Q^{\xi_0}) = 0.005566\, Q_{111}^{2\xi_0} + 2046.80, \quad c_{112}^{2\xi_0}(Q^{\xi_0}) = 0.006812\, Q_{112}^{2\xi_0} + 10984.60,$$

$$c_{121}^{2\xi_0}(Q^{\xi_0}) = 0.001259\, Q_{121}^{2\xi_0} + 2046.80, \quad c_{122}^{2\xi_0}(Q^{\xi_0}) = 0.007012\, Q_{122}^{2\xi_0} + 10984.60.$$

$$\rho_1^{1\xi_0}(d^{\xi_0}) = -0.15\, d_1^{1\xi_0} + 602344.00, \quad \rho_1^{2\xi_0}(d^{\xi_0}) = -0.68\, d_1^{2\xi_0} + 574560.00,$$

$$\rho_2^{1\xi_0}(d^{\xi_0}) = -0.000475\, d_2^{1\xi_0} + 6290.00, \quad \rho_2^{2\xi_0}(d^{\xi_0}) = -0.000758\, d_2^{2\xi_0} + 5980.00.$$

- **Capacities:**

$$u_1^{s\xi_0} = 5{,}000{,}000, \quad u_{111}^{Q\xi_0} = 5{,}000{,}000, \quad u_{112}^{Q\xi_0} = 500{,}000,$$

$$u_{121}^{Q\xi_0} = 5{,}000{,}000, \quad u_{122}^{Q\xi_0} = 500{,}000.$$

Example 2 (Scenario ξ_1) and Example 3 (Scenario ξ_2)

- **Example 2 — Scenario ξ_1 (Maritime Blockade):** Full-scale invasion leads to blockade/mining of maritime routes.
  - Maritime route capacities reduced to 0.00.
  - All other data (exchange rates, price functions, capacities) remain as in Scenario ξ_0.

- **Example 3 — Scenario ξ_2 (Wartime with Reduced Production and Economic Deterioration):** Worsening war scenario with additional reductions.
  - Supply capacity curtailed to 1,000,000 metric tons.
  - Modified supply price, transportation cost, and demand price functions.
  - Changes in exchange rates reflecting economic deterioration.

Numerical Example Set 1

Table 2: Equilibrium Commodity Shipments for Numerical Examples in Set 1

	Scenario		
Equilibrium Commodity Flows	ξ_0	ξ_1	ξ_2
$Q_{111}^{1\xi_l*}$	477,085.5938	–	477,651.1563
$Q_{112}^{1\xi_l*}$	0.0000	216,433.1406	0.0000
$Q_{121}^{1\xi_l*}$	1,605,672.50000	–	552,348.4375
$Q_{122}^{1\xi_l*}$	0.0000	500,000.00	0.0000
$Q_{111}^{2\xi_l*}$	79,128.0781	–	0.0000
$Q_{112}^{2\xi_l*}$	0.0000	0.0000	0.0000
$Q_{121}^{2\xi_l*}$	560,130.3750	–	0.0000
$Q_{122}^{2\xi_l*}$	0.0000	0.0000	0.0000
Equilibrium Supply Prices in USD	ξ_0	ξ_1	ξ_2
$\pi_1^{1\xi_l}(s^{\xi_l*})$	266.8542	258.5048	95.7269
$\pi_1^{2\xi_l}(s^{\xi_l*})$	253.8432	246.9056	111.9949
Equilibrium Demand Prices in USD	ξ_0	ξ_1	ξ_2
$\rho_1^{1\xi_l}(d^{\xi_l*})$	351.0457	376.9041	482.1526
$\rho_2^{1\xi_l}(d^{\xi_l*})$	351.3862	380.0000	508.5239
$\rho_1^{2\xi_l}(d^{\xi_l*})$	344.4132	384.7743	475.0372
$\rho_2^{2\xi_l}(d^{\xi_l*})$	353.1436	380.1653	516.9973

Insurance Premium Calculation: Numerical Example Set 1

Assumptions:

- Two wartime scenarios with associated probabilities:

$$p_{\xi_1} = 0.5, \quad p_{\xi_2} = 0.5.$$

- Calculations based on supply market prices (in USD).

Premiums for Wheat and Corn:

$$IP_1^1 = \Big[(266.8542 - 258.5048) \times 0.5 + (266.8542 - 95.7269) \times 0.5\Big]$$
$$= 89.7384\,(\$),$$

$$IP_1^2 = \Big[(253.8432 - 246.9056) \times 0.5 + (253.8432 - 111.9949) \times 0.5\Big]$$
$$= 74.3930\,(\$).$$

In Ukrainian hryvnia,

$$IP_1^1 = 2,464.43, \quad IP_1^2 = 2,042.89.$$

Observation: Without government subsidies ($\sigma_1^1 = 0$ and $\sigma_1^2 = 0$), these premiums yield zero commodity shipments.

Economic Outcomes: 50% vs. 75% Subsidization (Set 1)

Commodity Shipments and Revenues:

- **Wheat:**
  - 50% Subsidy: 760,694 metric tons (164,251 to Lebanon; 596,443 to Egypt)
  - 75% Subsidy: 1,421,501 metric tons (320,679 to Lebanon; 1,100,822 to Egypt)
  - Revenue: $197.29 million at 50% vs. $374.52 million at 75%
- **Corn:**
  - 50% Subsidy: 253,958 metric tons (30,557 to Lebanon; 223,401 to Egypt)
  - 75% Subsidy: 446,591 metric tons (54,844 to Lebanon; 391,747 to Egypt)
  - Revenue: $63.07 million at 50% vs. $112.13 million at 75%

Additional 25% subsidy increases:

- Wheat by approximately 660,807 metric tons ($\sim$ $177.23 million).
- Corn by approximately 192,633 metric tons ($\sim$ $49.06 million).

Conclusion: Higher subsidization significantly boosts commodity shipments and revenue, thereby strengthening food security and supporting farmer incomes during wartime.

Numerical Example Set 2: With Commodity Losses

- Retain baseline Scenario ξ_0 for pre-war.
- Modify wartime scenarios by setting all route multiplier $\alpha_{ijr} = 0.9$ (scenarios ξ_3 and ξ_4)

Table 3: Equilibrium Commodity Shipments for Numerical Examples in Set 2

	Scenario	
Equilibrium Commodity Flows	ξ_3	ξ_4
$Q_{111}^{1\xi_l*}$	–	26,877.5488
$Q_{112}^{1\xi_l*}$	93,835.6094	0.0000
$Q_{121}^{1\xi_l*}$	–	0.0000
$Q_{122}^{1\xi_l*}$	408,930.5938	0.0000
$Q_{111}^{2\xi_l*}$	–	0.0000
$Q_{112}^{2\xi_l*}$	0.0000	0.0000
$Q_{121}^{2\xi_l*}$	–	0.0000
$Q_{122}^{2\xi_l*}$	0.0000	0.0000
Equilibrium Supply Prices in USD	ξ_3	ξ_4
$\pi_1^{1\xi_l}(s^{\xi_l*})$	257.4467	92.1079
$\pi_1^{2\xi_l}(s^{\xi_l*})$	246.3377	110.0523
Equilibrium Demand Prices in USD	ξ_3	ξ_4
$\rho_1^{1\xi_l}(d^{\xi_l*})$	389.9975	524.1627
$\rho_2^{1\xi_l}(d^{\xi_l*})$	388.7592	522.2245
$\rho_1^{2\xi_l}(d^{\xi_l*})$	380.0000	475.0373
$\rho_2^{2\xi_l}(d^{\xi_l*})$	380.1653	516.9974

Insurance Premium Calculation: Numerical Example Set 2

Assumptions:

- Scenarios ξ_3 and ξ_4 each have probability 0.5.
- The baseline scenario remains ξ_0.

Premium Calculation using Supply Market Prices (in USD):

$$IP_1^1 = \Big[(266.8542 - 257.4467) \times 0.5 + (266.8524 - 92.1079) \times 0.5\Big] = 92.1210,$$

$$IP_1^2 = \Big[(253.8432 - 246.3377) \times 0.5 + (253.8432 - 110.0523) \times 0.5\Big] = 75.6482,$$

and premiums in UAH are:

$$IP_1^1 = 2,529.8170, \quad IP_1^2 = 2,077.4433.$$

Observation: These premiums are higher than in Set 1 due to commodity losses during transportation.

Economic Outcomes: 50% vs. 75% Subsidization (Set 2)

Commodity Shipments and Revenues:

- **Wheat:**
  - 50% Subsidy: 724,999 metric tons (155,809 to Lebanon; 569,190 to Egypt)
  - 75% Subsidy: 1,403,805 metric tons (316,493 to Lebanon; 1,087,312 to Egypt)
  - Revenue: $187.90 million at 50% vs. $369.21 million at 75%
- **Corn:**
  - 50% Subsidy: 252,015 metric tons (30,312 to Lebanon; 221,703 to Egypt)
  - 75% Subsidy: 443,533 metric tons (54,458 to Lebanon; 389,074 to Egypt)
  - Revenue: $62.47 million at 50% vs. $111.34 million at 75%

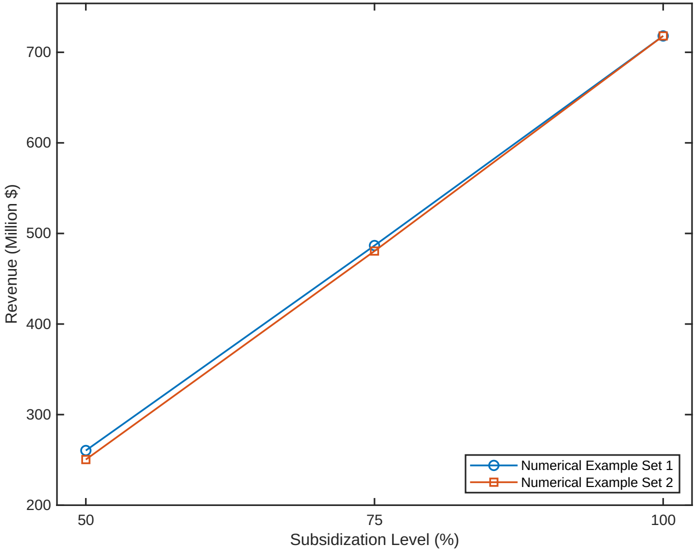
Additional 25% Subsidy Increases:

- Wheat by approximately 678,806 metric tons ($\sim$ $181.31 million).
- Corn by approximately 191,518 metric tons ($\sim$ $48.87 million).

Conclusion: Higher subsidization significantly boosts commodity shipments and revenue, thereby strengthening food security and supporting farmer incomes during wartime.

Combined Revenue for Corn and Wheat

- Combined Revenue for Corn and Wheat Under Subsidization Levels of 50%, 75%, and 100%

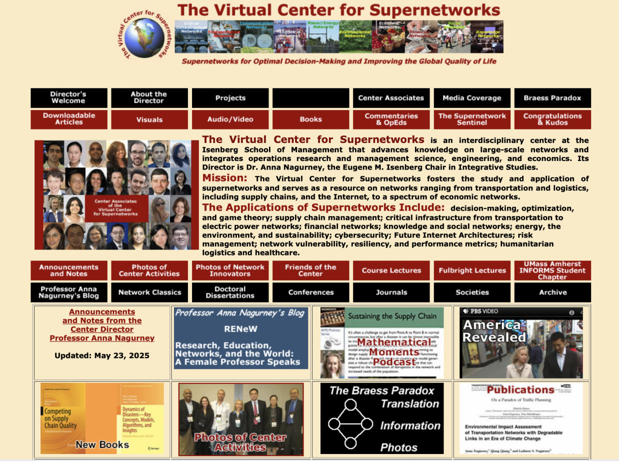


Key Insights and Conclusions

Key Insights and Conclusions

- **Integrated Framework:** Our model combines production capacities, transportation capacities, commodity loss multipliers, and exchange rate effects into a unified framework.
- **VI formulation:** The variational inequality formulation rigorously characterizes equilibrium conditions for both baseline and war scenarios.
- **Quantitative Insurance Premiums:** Integrated crop and cargo insurance premium formulas provide a quantitative measure of the expected loss in supply prices due to disruptions.
- **Subsidy Impact:** Sensitivity analysis shows that increased government subsidy lowers the effective premium burden, thereby supporting higher production and trade flows.

Thank You Very Much!



More information on our work can be found on the Supernetwork Center site: https://supernet.isenberg.umass.edu/