DYNAMICS OF ELECTRIC POWER SUPPLY CHAIN NETWORKS UNDER RISK AND UNCERTAINTY

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Motivation

- The transformation of the electric power industry from a regulated to a competitive industry
  - In the US,
  - In the EU,
  - And in many other countries
- The dramatic increase in the number of market participants
  - By the end of 2000, approximately 20% of all electric utility generating capacity had been sold or transferred to unregulated companies
  - In the US alone as of February 1, 2005, more than 1200 companies were eligible to sell wholesale power at market-based rates (source: http://www.eia.doe.gov)
- Changes in electricity trading patterns

"[In recent years] the adequacy of the bulk power transmission system has been challenged to support the movement of power in unprecedented amounts and in unexpected directions" (North American Electric Reliability Council (1998))

"There is a critical need to be sure that reliability is not taken for granted as the industry restructures, and thus does not fall through the cracks" (Secretary of Energy Advisory Board's (SEAB) Task Force on Electric System Reliability (1998))

These Concerns have Stimulated Much Research Activity

- Schweppe et al., 1988
- Hogan, 1992
- Chao and Peck, 1996
- Wu et al., 1996
- Kahn, 1998
- Singh, 1999
- Jing-Yuan and Smeers, 1999
- Hobbs et al., 2000
- Day et al., 2002
- . . .

Despite all the Analytical Efforts...

- August 14, 2003 - The biggest power outage in US history occurred
  - Approximately 50 million people were left without electricity
  - 61,800 megawatts of electric load were affected
  - Estimates of total associated cost in the US range between $4 and $10 billion
  - In Canada, the gross domestic product was down 0.7%, there was a net loss of 18.9 million work hours
- September 2003
  - A major outage occurred in England
  - Significant outage initiated in Switzerland and cascaded over much of Italy

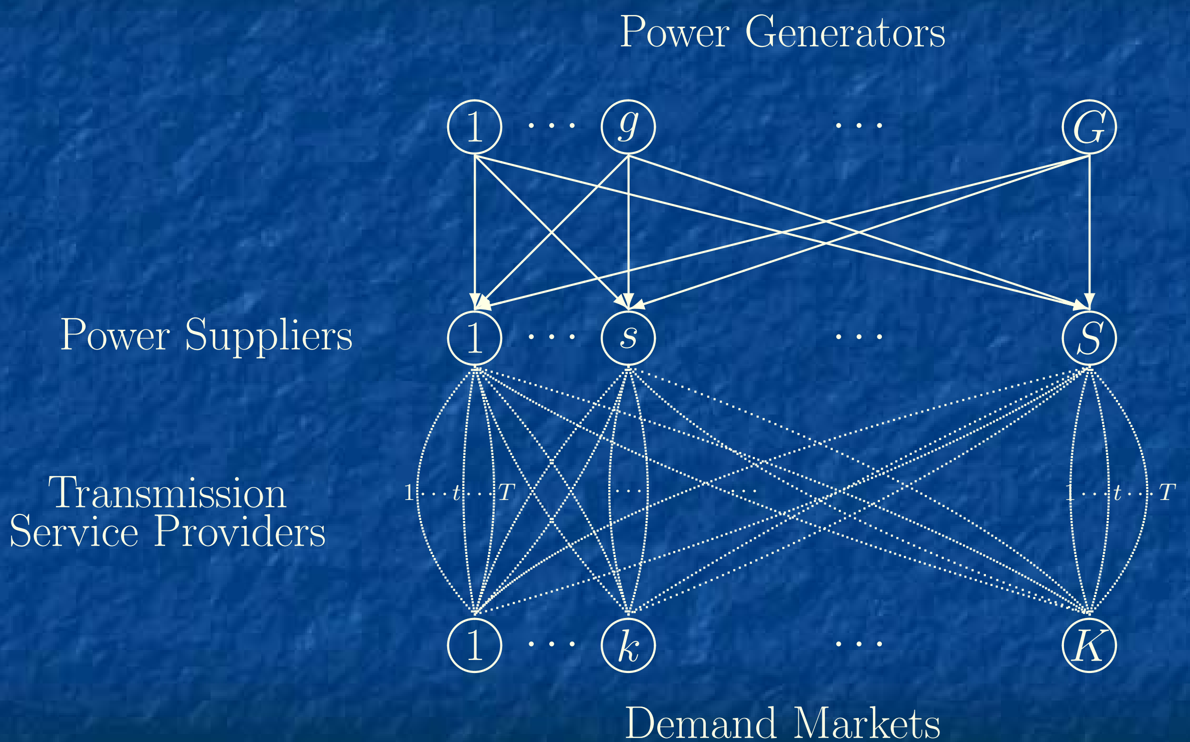
A Novel Approach

- Dynamic model
- Based on a supply chain network framework
- Several different types of decision-makers:
  - Power Generators
  - Power Suppliers
  - Consumers
  - with Multiple Transmission Service Providers
- Multicriteria decision-making
- Explicit modeling of the behavior of decision-makers
- Computation of dynamic trajectories of the evolution of the electric power flows, as well as the prices associated with various transactions

References

- Dupuis, P. and A. Nagurney (1993), "Dynamical Systems and Variational Inequalities," *Annals of Operations Research* **44**, 9-42
- Nagurney, A. and D. Zhang (1996), **Projected Dynamical Systems and Variational Inequalities with Applications**, Kluwer Academic Publishers, Boston, Massachusetts
- Nagurney, A. and D. Matsypura (2005), "A Supply Chain Network Perspective for Electric Power Generation, Supply, Transmission, and Consumption," to appear in **Advances in Computational Economics, Finance and Management Science**, E. J. Kontoghiorghes and C. Gatu, editors, Kluwer Academic Publishers, Dordrecht, The Netherlands.

The Electric Power Supply Chain Network



Notation

Flow between generator g and supplier s	q_{gs}
Flow between supplier s and demand market k through TSP t	q^t_{sk}
Generating cost function of generator g	f_g
Transaction cost function of generator g	c_{gs}
Risk function of generator g	r_g
Operating cost function of supplier s	c_s
Transaction cost function of supplier s	$\hat{c}_{gs}$
Transaction cost function of supplier s	c^t_{sk}
Risk function of supplier s	r_s
Demand function	$\hat{d}_k$
Expected demand function	d_k
Price at power generator g for supplier s	ρ_{1gs}
Price at power supplier s for demand market k	ρ^t_{2sk}
Price at demand market k	ρ_{3k}
Clearing price at supplier s	γ_s
Speed of adjustment	ϕ

Demand Market Price Dynamics

$$\dot{\rho}_{3k} = \begin{cases} \phi_k(d_k(\rho_3) - \sum_{s=1}^{S} \sum_{t=1}^{T} q_{sk}^t), & \text{if} \quad \rho_{3k} > 0 \\ \max\{0, \phi_k(d_k(\rho_3) - \sum_{s=1}^{S} \sum_{t=1}^{T} q_{sk}^t)\} & \text{if} \quad \rho_{3k} = 0 \end{cases}$$

The Dynamics of the Prices at the Power Suppliers

$$\dot{\gamma}_s = \begin{cases} \phi_s(\sum_{k=1}^K \sum_{t=1}^T q_{sk}^t - \sum_{g=1}^G q_{gs}), & \text{if} \quad \gamma_s > 0 \\ \max\{0, \phi_s(\sum_{k=1}^K \sum_{t=1}^T q_{sk}^t - \sum_{g=1}^G q_{gs})\}, & \text{if} \quad \gamma_s = 0 \end{cases}$$

Multicriteria Decision-Making Behavior of the Power Generators

$$\text{Maximize} \quad U_g = \sum_{s=1}^{S} \rho_{1gs} q_{gs} - f_g(Q^1) - \sum_{s=1}^{S} c_{gs}(Q^1) - \alpha_g r_g(Q^1)$$

subject to:

$$q_{gs} \geq 0, \qquad \forall s$$

Multicriteria Decision-Making Behavior of the Power Suppliers

$$\text{Maximize} \quad U_s = \sum_{k=1}^{K} \sum_{t=1}^{T} \rho_{2sk}^t q_{sk}^t - c_s(Q^1, Q^2) - \sum_{g=1}^{G} \rho_{1gs} q_{gs} - \sum_{g=1}^{G} \hat{c}_{gs}(Q^1)$$

$$- \sum_{k=1}^{K} \sum_{t=1}^{T} c_{sk}^t(Q^2) - \beta_s r_s(Q^1, Q^2)$$

subject to:

$$\sum_{k=1}^{K} \sum_{t=1}^{T} q_{sk}^t \leq \sum_{g=1}^{G} q_{gs}$$

$$q_{gs} \geq 0, \quad \forall g$$

$$q_{sk}^t \geq 0, \quad \forall k, \forall t$$

Dynamics of the Electricity Transactions between Power Generators and Power Suppliers

$$\dot{q}_{gs} = \begin{cases} \phi_{gs}(\gamma_s - \frac{\partial f_g(Q^1)}{\partial q_{gs}} - \frac{\partial c_{gs}(Q^1)}{\partial q_{gs}} - \alpha_g \frac{\partial r_g(Q^1)}{\partial q_{gs}} & \\ -\frac{\partial c_s(Q^1,Q^2)}{\partial q_{gs}} - \frac{\partial \hat{c}_{gs}(Q^1)}{\partial q_{gs}} - \beta_s \frac{\partial r_s(Q^1,Q^2)}{\partial q_{gs}}), & \text{if} \quad q_{gs} > 0 \\ \max\{0, \phi_{gs}(\gamma_s - \frac{\partial f_g(Q^1)}{\partial q_{gs}} - \frac{\partial c_{gs}(Q^1)}{\partial q_{gs}} - \alpha_g \frac{\partial r_g(Q^1)}{\partial q_{gs}} & \\ -\frac{\partial c_s(Q^1,Q^2)}{\partial q_{gs}} - \frac{\partial \hat{c}_{gs}(Q^1)}{\partial q_{gs}} - \beta_s \frac{\partial r_s(Q^1,Q^2)}{\partial q_{gs}})\}, & \text{if} \quad q_{gs} = 0 \end{cases}$$

Dynamics of the Electricity Transactions between Power Suppliers and Demand Markets

$$\dot{q}^t_{sk} = \begin{cases} \phi^t_{sk}(\rho_{3k} - \hat{c}^t_{sk}(Q^2) - \frac{\partial c_s(Q^1,Q^2)}{\partial q^t_{sk}} - \frac{\partial c^t_{sk}(Q^2)}{\partial q^t_{sk}} & \\ -\beta_s \frac{\partial r_s(Q^1,Q^2)}{\partial q^t_{sk}} - \gamma_s), & \text{if} \quad q^t_{sk} > 0 \\ \max\{0, \phi^t_{sk}(\rho_{3k} - \hat{c}^t_{sk}(Q^2) - \frac{\partial c_s(Q^1,Q^2)}{\partial q^t_{sk}} - \frac{\partial c^t_{sk}(Q^2)}{\partial q^t_{sk}} & \\ -\beta_s \frac{\partial r_s(Q^1,Q^2)}{\partial q^t_{sk}} - \gamma_s)\}, & \text{if} \quad q^t_{sk} = 0 \end{cases}$$

Projected Dynamical System

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0,$$

where

$F(X) \equiv (F_{gs}, F^t_{sk}, F_s, F_k)_{g=1,\ldots,G; s=1,\ldots,S; t=1,\ldots,T; k=1,\ldots,K}$,

$\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto $\mathcal{K}$ at X,

$\mathcal{K} \equiv R_+^{GS+SKT+S+K}$,

and $X_0 = (Q^{10}, Q^{20}, \gamma^0, \rho_3^0)$ is the initial point

VI Formulation

Since the feasible set $\mathcal{K}$ is a convex polyhedron, the set of stationary points of the projected dynamical system coincides with the set of solutions to the variational inequality problem given by: determine $X^* \in \mathcal{K}$, such that

$$\sum_{g=1}^{G}\sum_{s=1}^{S}\phi_{gs}\Big[\frac{\partial f_g(Q^{1*})}{\partial q_{gs}}+\frac{\partial c_{gs}(Q^{1*})}{\partial q_{gs}}+\alpha_g\frac{\partial r_g(Q^1)}{\partial q_{gs}}+\frac{\partial c_s(Q^{1*},Q^{2*})}{\partial q_{gs}}+\frac{\partial \hat{c}_{gs}(Q^{1*})}{\partial q_{gs}}$$

$$+\beta_s\frac{\partial r_s(Q^1,Q^2)}{\partial q_{gs}}-\gamma_s^*\Big]\times[q_{gs}-q_{gs}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\phi_{sk}^t\left[\frac{\partial c_s(Q^{1*},Q^{2*})}{\partial q_{sk}^t}+\frac{\partial c_{sk}^t(Q^{2*})}{\partial q_{sk}^t}+\hat{c}_{sk}^t(Q^{2*})+\beta_s\frac{\partial r_s(Q^1,Q^2)}{\partial q_{sk}^t}+\gamma_s^*-\rho_{3k}^*\right]$$

$$\times[q_{sk}^t-q_{sk}^{t*}]$$

$$+\sum_{s=1}^{S}\phi_s\left[\sum_{g=1}^{G}q_{gs}^*-\sum_{k=1}^{K}\sum_{t=1}^{T}q_{sk}^{t*}\right]\times[\gamma_s-\gamma_s^*]+\sum_{k=1}^{K}\phi_k\left[\sum_{s=1}^{S}\sum_{t=1}^{T}q_{sk}^{t*}-d_k(\rho_3^*)\right]\times[\rho_{3k}-\rho_{3k}^*]\geq 0,$$

$$\forall(Q^1,Q^2,\gamma,\rho_3)\in\mathcal{K}$$

The Algorithm

The Euler Method

Step 0: Initialization Set $X^0 = (Q^{10}, Q^{20}, \gamma^0, \rho_3^0) \in \mathcal{K}$. Let $\mathcal{T}$ denote an iteration counter and set $\mathcal{T} = 1$. Set the sequence $\{\alpha_{\mathcal{T}}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} \alpha_{\mathcal{T}} = \infty$, $\alpha_{\mathcal{T}} > 0$, and $\alpha_{\mathcal{T}} \to 0$, as $\mathcal{T} \to \infty$

Step 1: Computation Compute $X^{\mathcal{T}} = (Q^{1\mathcal{T}}, Q^{2\mathcal{T}}, \gamma^{\mathcal{T}}, \rho_3^{\mathcal{T}}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + \alpha_{\mathcal{T}} F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - X^{\mathcal{T}} \rangle \geq 0, \qquad \forall X \in \mathcal{K}$$

Step 2: Convergence Verification If $|X^{\mathcal{T}} - X^{\mathcal{T}-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1

Theoretical Result

The set of equilibria of the electric power supply chain network are *independent* of the speeds of adjustment

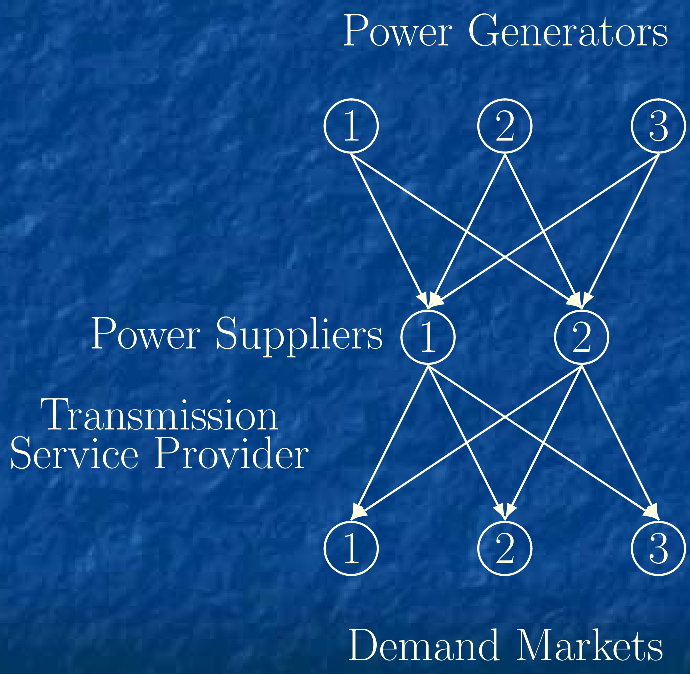
Theorem: *Assume that* $\mathcal{K}$ *is the convex polyhedron given by* R^N_+ *and that* $\phi \equiv (\phi_1, \ldots, \phi_N)$ *is a vector of positive terms. Then, the set of stationary points of the projected dynamical system given above is equivalent to the set of solutions to the variational inequality* $\mathrm{VI}(F', \mathcal{K})$ *where:*

$$F' \equiv (F'_1, \ldots, F'_N)$$

and

$$F \equiv (\phi_1 F'_1, \ldots, \phi_n F'_N)$$

Numerical Examples: Network Topology



Values of ϕ

Associated Variable	Parameter	Experiment 1	Experiment 2	Experiment 3	Experiment 4
q_{11}	ϕ_{11}	1	1	0.5	0.5
q_{12}	ϕ_{12}	1	1	1	0.7
q_{21}	ϕ_{21}	1	1	1	1
q_{22}	ϕ_{22}	1	1	1	1
q_{31}	ϕ_{31}	1	1	1	1
q_{32}	ϕ_{32}	1	1	1	1
q_{11}^1	ϕ_{11}^1	1	1	1	1
q_{12}^1	ϕ_{12}^1	1	1	1	1
q_{13}^1	ϕ_{13}^1	1	1	1	1
q_{21}^1	ϕ_{21}^1	1	1	1	1
q_{22}^1	ϕ_{22}^1	1	1	1	1
q_{23}^1	ϕ_{23}^1	1	1	1	1
γ_1	ϕ_1	1	1	1	0.5
γ_2	ϕ_2	1	1	1	0.3
ρ_{31}	ϕ_1	1	0.5	0.5	0.5
ρ_{32}	ϕ_2	1	0.5	0.5	0.5
ρ_{33}	ϕ_3	1	0.5	0.5	0.5

Example 1

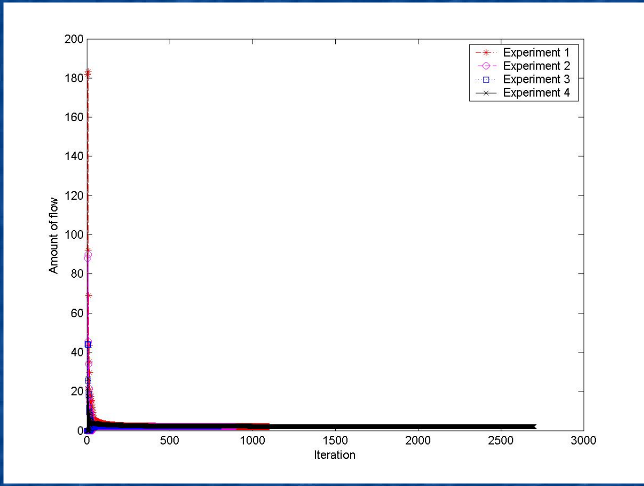
Power generating cost functions	$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1$ $f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2$ $f_3(q) = .5q_3^2 + .5q_1q_3 + 2q_3$
Transaction cost functions	$c_{11}(Q^1) = .5q_{11}^2 + 3.5q_{11}$ $c_{12}(Q^1) = .5q_{12}^2 + 3.5q_{12}$ $c_{21}(Q^1) = .5q_{21}^2 + 3.5q_{21}$ $c_{22}(Q^1) = .5q_{22}^2 + 3.5q_{22}$ $c_{31}(Q^1) = .5q_{31}^2 + 2q_{31}$ $c_{32}(Q^1) = .5q_{32}^2 + 2q_{32}$
Operating costs of power suppliers	$c_1(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i1})^2$ $c_2(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i2})^2$
Expected demand functions	$d_k(\rho_{3k}) = E(\hat{d}_k) = \frac{1}{2}\frac{1000}{(\rho_{3k}+1)}$

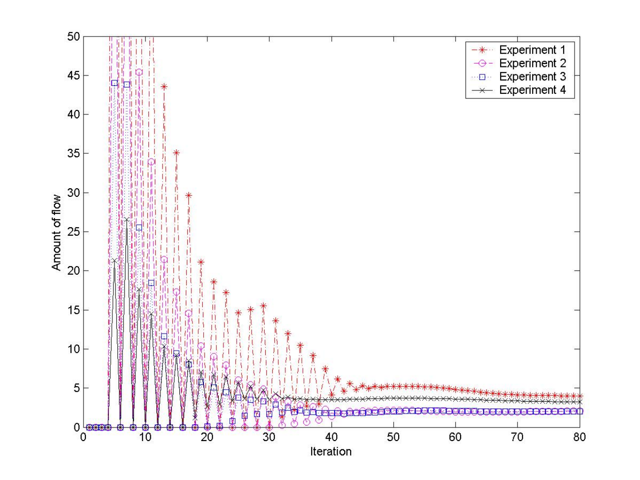
Solutions to Example 1

Computed Equilibrium Values

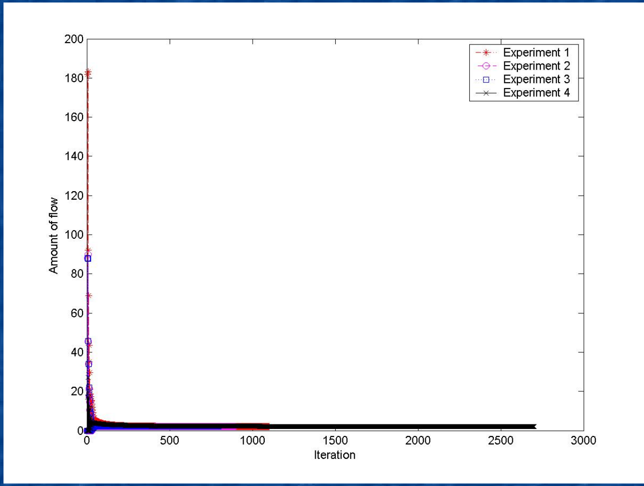
	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Number of Iterations	1101	892	794	2702
q_{11}^*	2.12	2.12	2.12	2.12
q_{12}^*	2.12	2.12	2.12	2.13
q_{21}^*	2.12	2.12	2.12	2.12
q_{22}^*	2.12	2.12	2.12	2.13
q_{31}^*	8.99	8.98	8.98	8.99
q_{32}^*	8.99	8.98	8.98	9.00
q_{11}^{*1}	4.41	4.41	4.41	4.41
q_{12}^{*1}	4.41	4.41	4.41	4.41
q_{13}^{*1}	4.41	4.41	4.41	4.41
q_{21}^{*1}	4.41	4.41	4.41	4.41
q_{22}^{*1}	4.41	4.41	4.41	4.41
q_{23}^{*1}	4.41	4.41	4.41	4.41
γ_1^*	46.32	46.27	46.27	46.37
γ_2^*	46.32	46.27	46.27	46.37
ρ_{31}^*	55.73	55.68	55.68	55.78
ρ_{32}^*	55.73	55.68	55.68	55.78
ρ_{33}^*	55.73	55.68	55.68	55.78

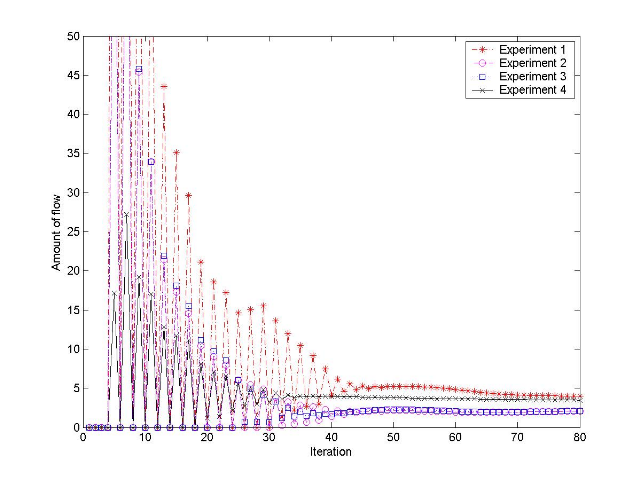
Dynamics of Electric Power Transaction q_{11}



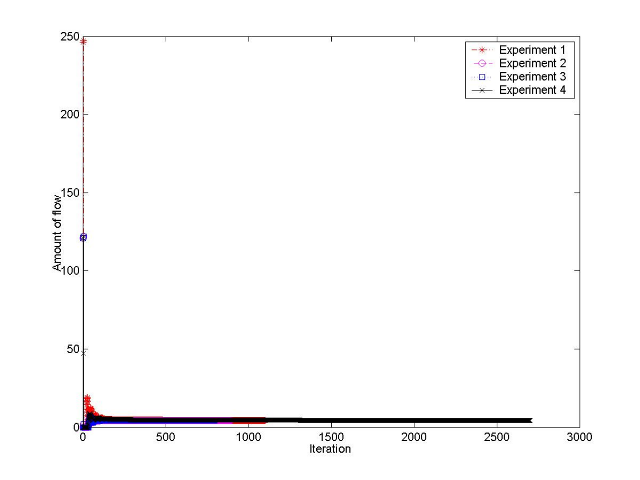


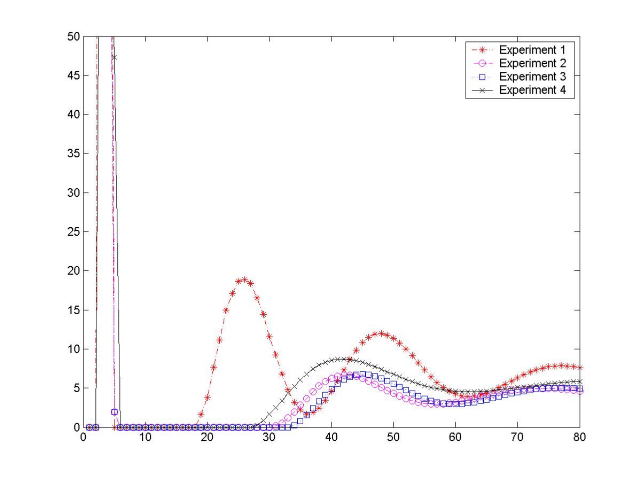
Dynamics of Electric Power Transaction q_{12}



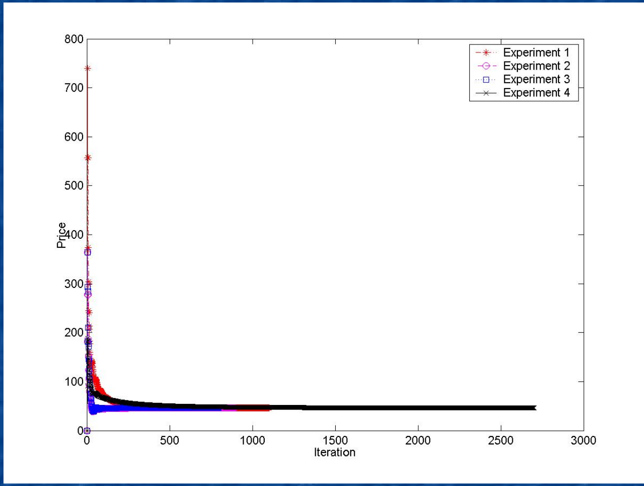


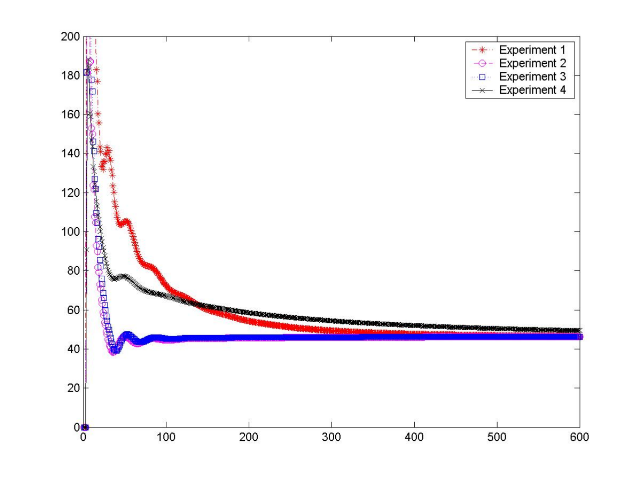
Dynamics of Electric Power Transaction q_{11}^1



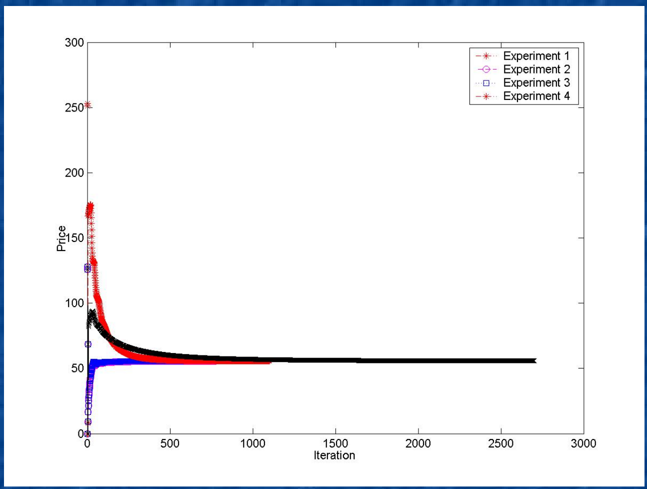


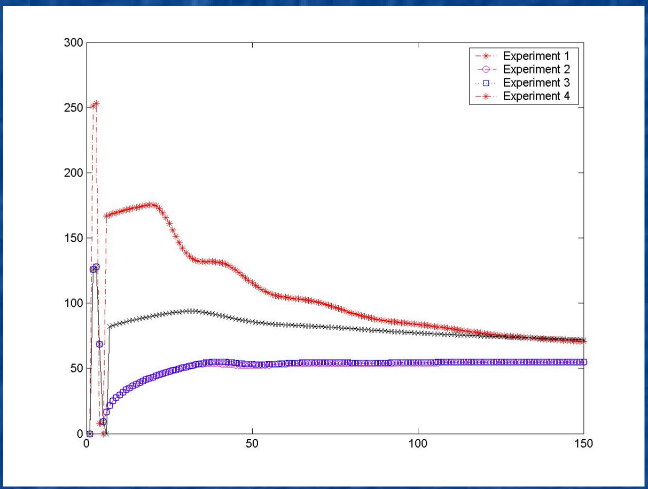
Dynamics of Price γ_1





Dynamics of Price ρ_{31}





Example 2

Power generating cost functions	$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1$ $f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2$ $f_3(q) = .5q_3^2 + .5q_1q_3 + 2q_3$
Transaction cost functions	$c_{11}(Q^1) = .5q_{11}^2 + 3.5q_{11}$ $c_{12}(Q^1) = .5q_{12}^2 + 3.5q_{12}$ $c_{21}(Q^1) = .5q_{21}^2 + 3.5q_{21}$ $c_{22}(Q^1) = .5q_{22}^2 + 3.5q_{22}$ $c_{31}(Q^1) = .5q_{31}^2 + 2q_{31}$ $c_{32}(Q^1) = .5q_{32}^2 + 2q_{32}$
Operating costs of power suppliers	$c_1(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i1})^2$ $c_2(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i2})^2$
Expected demand functions	$d_k(\rho_{3k}) = E(\hat{d}_k) = \frac{1}{2}\frac{1000}{(\rho_{3k}+1)}$
Risk function for generator 1 Associated weight	$r_1 = (\sum_{s=1}^2 q_{1s} - 2)^2$ $\alpha_1 = 1$

Solutions to Example 2

Computed Equilibrium Values

	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Number of Iterations	1117	926	829	2732
q_{11}^*	1.83	1.83	1.83	1.83
q_{12}^*	1.83	1.83	1.83	1.83
q_{21}^*	2.20	2.19	2.19	2.20
q_{22}^*	2.20	2.19	2.19	2.20
q_{31}^*	9.16	9.15	9.15	9.18
q_{32}^*	9.16	9.15	9.15	9.18
q_{11}^{*1}	4.40	4.39	4.39	4.40
q_{12}^{*1}	4.40	4.39	4.39	4.40
q_{13}^{*1}	4.40	4.39	4.39	4.40
q_{21}^{*1}	4.40	4.39	4.39	4.39
q_{22}^{*1}	4.40	4.39	4.39	4.39
q_{23}^{*1}	4.40	4.39	4.39	4.39
γ_1^*	46.51	46.46	46.46	46.57
γ_2^*	46.51	46.46	46.46	46.57
ρ_{31}^*	55.91	55.86	55.86	55.96
ρ_{32}^*	55.91	55.86	55.86	55.96
ρ_{33}^*	55.91	55.86	55.86	55.96

Example 3

Power generating cost functions	$f_1(q) = 2.5q_1^2 + q_1 q_2 + 2q_1$ $f_2(q) = 2.5q_2^2 + q_1 q_2 + 2q_2$ $f_3(q) = .5q_3^2 + .5q_1 q_3 + 2q_3$
Transaction cost functions	$c_{11}(Q^1) = .5q_{11}^2 + 3.5q_{11}$ $c_{12}(Q^1) = .5q_{12}^2 + 3.5q_{12}$ $c_{21}(Q^1) = .5q_{21}^2 + 3.5q_{21}$ $c_{22}(Q^1) = .5q_{22}^2 + 3.5q_{22}$ $c_{31}(Q^1) = .5q_{31}^2 + 2q_{31}$ $c_{32}(Q^1) = .5q_{32}^2 + 2q_{32}$
Operating costs of power suppliers	$c_1(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i1})^2$ $c_2(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i2})^2$
Expected demand functions	$d_k(\rho_{3k}) = E(\hat{d}_k) = \frac{1}{2}\frac{1000}{(\rho_{3k}+1)}$
Risk function for generator 1 Associated weight	$r_1 = (\sum_{s=1}^2 q_{1s} - 2)^2$ $\alpha_1 = 1$
Risk function for supplier 2 Associated weight	$r_2 = \sum_{g=1}^3 q_{g2}$ $\beta_2 = 0.5$

Solutions to Example 3

Computed Equilibrium Values

	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Number of Iterations	1120	936	841	2738
q_{11}^*	1.87	1.87	1.87	1.87
q_{12}^*	1.77	1.77	1.77	1.77
q_{21}^*	2.24	2.24	2.24	2.24
q_{22}^*	2.14	2.14	2.14	2.14
q_{31}^*	9.19	9.18	9.18	9.20
q_{32}^*	9.09	9.08	9.08	9.10
q_{11}^{*1}	4.43	4.43	4.43	4.43
q_{12}^{*1}	4.43	4.43	4.43	4.43
q_{13}^{*1}	4.43	4.43	4.43	4.43
q_{21}^{*1}	4.33	4.33	4.33	4.33
q_{22}^{*1}	4.33	4.33	4.33	4.33
q_{23}^{*1}	4.33	4.33	4.33	4.33
γ_1^*	46.61	46.56	46.56	46.66
γ_2^*	46.71	46.66	46.66	46.76
ρ_{31}^*	56.04	55.99	55.99	56.10
ρ_{32}^*	56.04	55.99	55.99	56.10
ρ_{33}^*	56.04	55.99	55.99	56.10

Example 4

Power generating cost functions	$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1$ $f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2$ $f_3(q) = .5q_3^2 + .5q_1q_3 + 2q_3$
Transaction cost functions	$c_{11}(Q^1) = .5q_{11}^2 + 3.5q_{11}$ $c_{12}(Q^1) = .5q_{12}^2 + 3.5q_{12}$ $c_{21}(Q^1) = .5q_{21}^2 + 3.5q_{21}$ $c_{22}(Q^1) = .5q_{22}^2 + 3.5q_{22}$ $c_{31}(Q^1) = .5q_{31}^2 + 2q_{31}$ $c_{32}(Q^1) = .5q_{32}^2 + 2q_{32}$
Operating costs of power suppliers	$c_1(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i1})^2$ $c_2(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i2})^2$
Expected demand functions	$d_1(\rho_{31}) = E(\hat{d}_1) = \frac{1}{2}\frac{2000}{(\rho_{31}+1)}$ $d_2(\rho_{32}) = E(\hat{d}_2) = \frac{1}{2}\frac{1000}{(\rho_{32}+1)}$ $d_3(\rho_{33}) = E(\hat{d}_3) = \frac{1}{2}\frac{1000}{(\rho_{33}+1)}$
Risk function for generator 1 Associated weight	$r_1 = (\sum_{s=1}^2 q_{1s} - 2)^2$ $\alpha_1 = 1$
Risk function for supplier 2 Associated weight	$r_2 = \sum_{g=1}^3 q_{g2}$ $\beta_2 = 0.5$

Solutions to Example 4

Computed Equilibrium Values

	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Number of Iterations	1186	706	323	2705
q_{11}^*	2.14	2.14	2.14	2.14
q_{12}^*	2.04	2.04	2.04	2.04
q_{21}^*	2.63	2.63	2.63	2.63
q_{22}^*	2.53	2.53	2.53	2.53
q_{31}^*	10.71	10.70	10.70	10.72
q_{32}^*	10.61	10.60	10.60	10.62
q_{11}^{*1}	7.50	7.50	7.50	7.50
q_{12}^{*1}	3.98	3.98	3.98	3.98
q_{13}^{*1}	3.98	3.98	3.98	3.98
q_{21}^{*1}	7.40	7.40	7.40	7.40
q_{22}^{*1}	3.88	3.88	3.88	3.88
q_{23}^{*1}	3.88	3.88	3.88	3.88
γ_1^*	53.59	53.54	53.56	53.64
γ_2^*	53.69	53.64	53.66	53.74
ρ_{31}^*	66.09	66.04	66.06	66.14
ρ_{32}^*	62.57	62.52	62.55	62.63
ρ_{33}^*	62.57	62.52	62.55	62.63

Example 5

Power generating cost functions	$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1$ $f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2$ $f_3(q) = .5q_3^2 + .5q_1q_3 + 2q_3$
Transaction cost functions	$c_{11}(Q^1) = .5q_{11}^2 + 3.5q_{11}$ $c_{12}(Q^1) = .5q_{12}^2 + 3.5q_{12}$ $c_{21}(Q^1) = .5q_{21}^2 + 3.5q_{21}$ $c_{22}(Q^1) = .5q_{22}^2 + 3.5q_{22}$ $c_{31}(Q^1) = .5q_{31}^2 + 2q_{31}$ $c_{32}(Q^1) = .5q_{32}^2 + 2q_{32}$
Operating costs of power suppliers	$c_1(Q^1, Q^2) = .5(\sum_{i=1}^{3} q_{i1})^2$ $c_2(Q^1, Q^2) = .5(\sum_{i=1}^{3} q_{i2})^2$
Expected demand functions	$d_k(\rho_{3k}) = E(\hat{d}_k) = \frac{1}{2}\frac{100000}{(\rho_{3k}+1)}$

Example 6

Power generating cost functions	$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1$ $f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2$ $f_3(q) = .5q_3^2 + .5q_1q_3 + 2q_3$
Transaction cost functions	$c_{11}(Q^1) = .5q_{11}^2 + 3.5q_{11}$ $c_{12}(Q^1) = .5q_{12}^2 + 3.5q_{12}$ $c_{21}(Q^1) = .5q_{21}^2 + 3.5q_{21}$ $c_{22}(Q^1) = .5q_{22}^2 + 3.5q_{22}$ $c_{31}(Q^1) = .5q_{31}^2 + 2q_{31}$ $c_{32}(Q^1) = .5q_{32}^2 + 2q_{32}$
Operating costs of power suppliers	$c_1(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i1})^2$ $c_2(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i2})^2$
Expected demand functions	$d_k(\rho_{3k}) = E(\hat{d}_k) = \frac{1}{2}\frac{100000}{(\rho_{3k}+1)}$
Risk function for generator 1 Associated weight	$r_1 = (\sum_{s=1}^2 q_{1s} - 2)^2$ $\alpha_1 = 1$

Example 7

Power generating cost functions	$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1$ $f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2$ $f_3(q) = .5q_3^2 + .5q_1q_3 + 2q_3$
Transaction cost functions	$c_{11}(Q^1) = .5q_{11}^2 + 3.5q_{11}$ $c_{12}(Q^1) = .5q_{12}^2 + 3.5q_{12}$ $c_{21}(Q^1) = .5q_{21}^2 + 3.5q_{21}$ $c_{22}(Q^1) = .5q_{22}^2 + 3.5q_{22}$ $c_{31}(Q^1) = .5q_{31}^2 + 2q_{31}$ $c_{32}(Q^1) = .5q_{32}^2 + 2q_{32}$
Operating costs of power suppliers	$c_1(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i1})^2$ $c_2(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i2})^2$
Expected demand functions	$d_k(\rho_{3k}) = E(\hat{d}_k) = \frac{1}{2}\frac{100000}{(\rho_{3k}+1)}$
Risk function for generator 1 Associated weight	$r_1 = (\sum_{s=1}^2 q_{1s} - 2)^2$ $\alpha_1 = 1$
Risk function for supplier 2 Associated weight	$r_2 = \sum_{g=1}^3 q_{g2}$ $\beta_2 = 0.5$

Example 8

Power generating cost functions	$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1$ $f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2$ $f_3(q) = .5q_3^2 + .5q_1q_3 + 2q_3$
Transaction cost functions	$c_{11}(Q^1) = .5q_{11}^2 + 3.5q_{11}$ $c_{12}(Q^1) = .5q_{12}^2 + 3.5q_{12}$ $c_{21}(Q^1) = .5q_{21}^2 + 3.5q_{21}$ $c_{22}(Q^1) = .5q_{22}^2 + 3.5q_{22}$ $c_{31}(Q^1) = .5q_{31}^2 + 2q_{31}$ $c_{32}(Q^1) = .5q_{32}^2 + 2q_{32}$
Operating costs of power suppliers	$c_1(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i1})^2$ $c_2(Q^1, Q^2) = .5(\sum_{i=1}^3 q_{i2})^2$
Expected demand functions	$d_1(\rho_{31}) = E(\hat{d}_1) = \frac{1}{2}\frac{200000}{(\rho_{31}+1)}$ $d_2(\rho_{32}) = E(\hat{d}_2) = \frac{1}{2}\frac{100000}{(\rho_{32}+1)}$ $d_3(\rho_{33}) = E(\hat{d}_3) = \frac{1}{2}\frac{100000}{(\rho_{33}+1)}$
Risk function for generator 1 Associated weight	$r_1 = (\sum_{s=1}^2 q_{1s} - 2)^2$ $\alpha_1 = 1$
Risk function for supplier 2 Associated weight	$r_2 = \sum_{g=1}^3 q_{g2}$ $\beta_2 = 0.5$

Solutions to Examples 5–8

Computed Equilibrium Values

	Example 5	Example 6	Example 7	Example 8
Number of Iterations	2513	2566	2566	2693
q_{11}^*	24.07	18.02	18.07	20.76
q_{12}^*	24.07	18.02	17.97	20.66
q_{21}^*	24.07	25.59	25.64	29.52
q_{22}^*	24.07	25.59	25.54	29.42
q_{31}^*	96.78	100.35	100.37	115.50
q_{32}^*	96.78	100.35	100.27	115.40
q_{11}^{*1}	48.30	47.99	48.02	80.29
q_{12}^{*1}	48.30	47.99	48.02	42.74
q_{13}^{*1}	48.30	47.99	48.02	42.74
q_{21}^{*1}	48.30	47.99	47.92	80.19
q_{22}^{*1}	48.30	47.99	47.92	42.64
q_{23}^{*1}	48.30	47.99	47.92	42.64
γ_1^*	463.31	467.03	467.12	536.89
γ_2^*	463.31	467.03	467.22	536.99
ρ_{31}^*	516.61	520.02	520.15	622.18
ρ_{32}^*	516.61	520.02	520.15	584.63
ρ_{33}^*	516.61	520.02	520.15	584.63

Conclusions

- We proposed a novel, supply chain network framework for rigorous:
  - modeling,
  - qualitative analysis,
  - computation of trajectories

  of dynamic evolutions of the electricity transactions between tiers of the electric power supply chain network as well as prices associated with the electric power at different distribution levels
- We have generalized the recent work of Nagurney and Matsypura (2004) to include supply-side risk and demand uncertainty
- We have established and theoretically proven that the set of equilibria are *independent* of the speeds of adjustment

Thank you!

The full paper is available upon request

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