# Global Supply Chain Dynamics with Multicriteria Decision-Making Under Risk and Uncertainty

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# Motivation

- Challenging intellectual questions
- Importance of supply chain decision-making
- Increasing globalization
- E-commerce

# Some of the Related Literature


- Beckmann, M. J., McGuire, C. B., and Winsten, C. B. (1956), **Studies in the Economics of Transportation.** Yale University Press, New Haven, Connecticut
- Nagurney, A (1999), **Network Economics: A Variational Inequality Approach**, Second and Revised Edition, Kluwer Academic Publishers, Dordrecht, The Netherlands
- Nagurney, A., Dong, J., and Zhang, D. (2002), *A Supply Chain Network Equilibrium Model*, Transportation Research E 38, 281-303

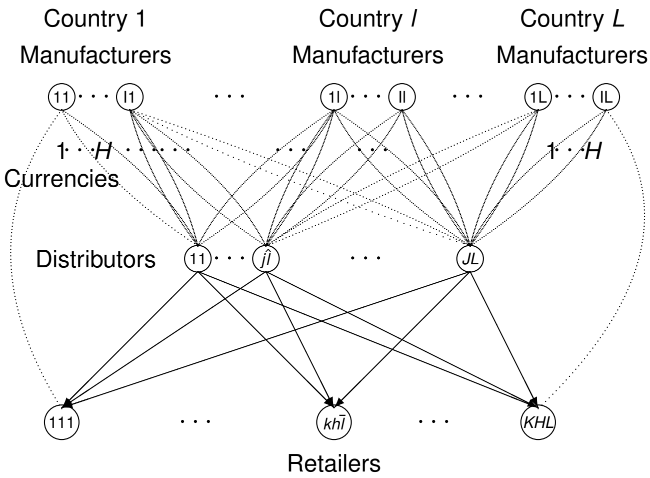
# Global Supply Chain

- Network equilibrium problem
- Several classes of decision-makers
- Unique optimization problem for every ‘player’ in the system
- Dynamic adjustment process

# Characteristics of the Model

- Risk
- Time
- E-commerce

# Supernetwork Structure



# Dynamics of the Prices at Retailers

## Assumption

The rate of change of the price is proportional to the difference between the *expected* demand and the total amount transacted with the retailer.

$$\dot{\rho}_{3kh\bar{l}} = \begin{cases} \phi_{kh\bar{l}}\left[d_{kh\bar{l}} - \sum_{i=1}^{I}\sum_{l=1}^{L} q_{kh\bar{l}}^{il} - \sum_{j=1}^{J}\sum_{\hat{l}=1}^{L} q_{kh\bar{l}}^{j\hat{l}}\right], & \text{if} \quad \rho_{3kh\bar{l}} > 0 \\ \max\left\{0, \phi_{kh\bar{l}}\left[d_{kh\bar{l}} - \sum_{i=1}^{I}\sum_{l=1}^{L} q_{kh\bar{l}}^{il} - \sum_{j=1}^{J}\sum_{\hat{l}=1}^{L} q_{kh\bar{l}}^{j\hat{l}}\right]\right\}, & \text{if} \quad \rho_{3kh\bar{l}} = 0. \end{cases}$$

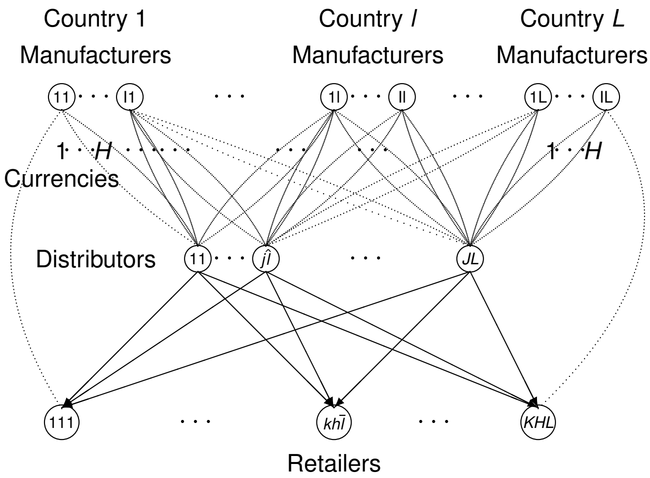
# Dynamics of the Prices at Distributors

## Assumption

The rate of change of the price is proportional to the difference between the total amount transacted with the retailers and the amount acquired from the manufacturers.

$$\dot{\gamma}_{j\hat{l}} = \begin{cases} \phi_{j\hat{l}} \left[ \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\bar{l}=1}^{L} q_{kh\bar{l}}^{j\hat{l}} - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} q_{jh\hat{l}}^{il} \right], & \text{if} \quad \gamma_{j\hat{l}} > 0 \\ \max \left\{ 0, \phi_{j\hat{l}} \left[ \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\bar{l}=1}^{L} q_{kh\bar{l}}^{j\hat{l}} - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} q_{jh\hat{l}}^{il} \right] \right\}, & \text{if} \quad \gamma_{j\hat{l}} = 0. \end{cases}$$

# Supernetwork Structure



# Multicriteria Decision-Making Behavior of Manufacturers

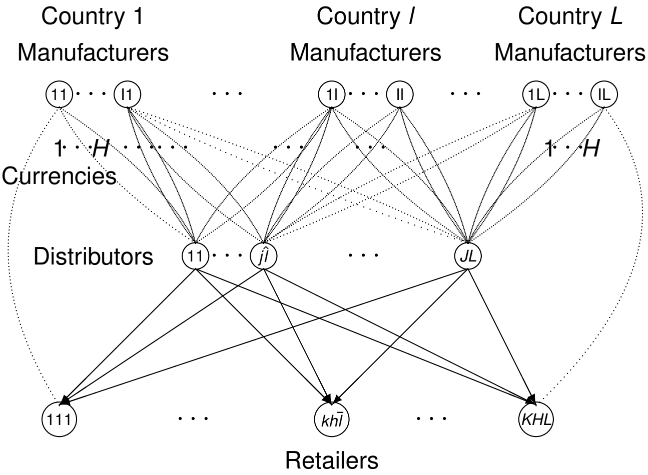
## Maximize Profit

$$\text{Maximize} \quad \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}(\rho^{il}_{1jh\hat{l}} \times e_h)q^{il}_{jh\hat{l}} + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\bar{l}=1}^{L}(\rho^{il}_{1kh\bar{l}} \times e_h)q^{il}_{kh\bar{l}}$$
$$- f^{il}(Q^1, Q^2) - \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} c^{il}_{jh\hat{l}}(q^{il}_{jh\hat{l}}) - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\bar{l}=1}^{L} c^{il}_{kh\bar{l}}(q^{il}_{kh\bar{l}})$$

## Minimize Risk

$$\text{Minimize} \quad r^{il}(Q^1, Q^2)$$

# Supernetwork Structure



# Multicriteria Decision-Making Behavior of Distributors

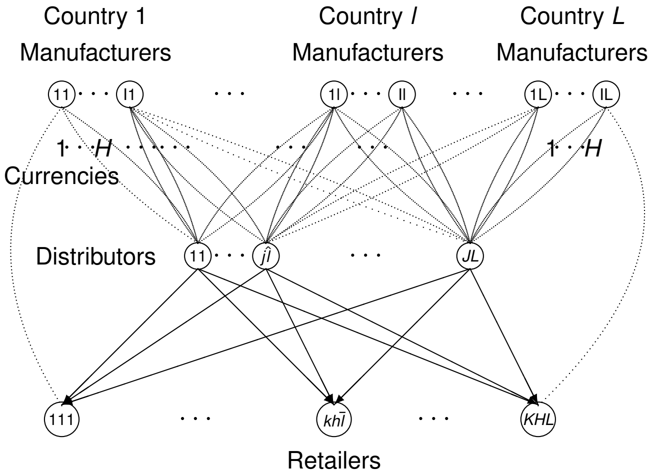
## Maximize Profit

Maximize $\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\bar{l}=1}^{L}(\rho_{2kh\bar{l}}^{j\hat{l}} \times e_h) q_{kh\bar{l}}^{j\hat{l}} - c_{j\hat{l}}(Q^1, Q^3) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}(\rho_{1jh\hat{l}}^{il} \times e_h) q_{jh\hat{l}}^{il}$

## Minimize Risk

Minimize $r^{j\hat{l}}(Q^1, Q^3)$

# Supernetwork Structure



# Optimizing Behavior of Retailers

## Maximize Profit

$$\text{Maximize} \quad E((\rho_{3kh\bar{l}} \times e_h) \cdot \min\{s_{kh\bar{l}}, \hat{d}_{kh\bar{l}}\}) - E(\lambda^+_{kh\bar{l}}\Delta^+_{kh\bar{l}} + \lambda^-_{kh\bar{l}}\Delta^-_{kh\bar{l}})$$
$$- c_{kh\bar{l}}(Q^2, Q^3) - \sum_{i=1}^{I}\sum_{l=1}^{L}(\rho^{il}_{1kh\bar{l}} \times e_h)q^{il}_{kh\bar{l}} - \sum_{j=1}^{J}\sum_{\hat{l}=1}^{L}(\rho^{j\hat{l}}_{2kh\bar{l}} \times e_h)q^{j\hat{l}}_{kh\bar{l}}$$

# Dynamics of Product Transactions between Manufacturer and Distributor

$$\dot{q}^{il}_{jh\hat{l}} = \begin{cases} \phi^{il}_{jh\hat{l}} \Bigg( \gamma_{j\hat{l}} - \dfrac{\partial f^{il}(Q^1,Q^2)}{\partial q^{il}_{jh\hat{l}}} - \dfrac{\partial c^{il}_{jh\hat{l}}(q^{il}_{jh\hat{l}})}{\partial q^{il}_{jh\hat{l}}} - \dfrac{\partial c_{j\hat{l}}(Q^1,Q^3)}{\partial q^{il}_{jh\hat{l}}} \\ -\alpha^{il} \dfrac{\partial r^{il}(Q^1,Q^2)}{\partial q^{il}_{jh\hat{l}}} - \beta^{j\hat{l}} \dfrac{\partial r^{j\hat{l}}(Q^1,Q^3)}{\partial q^{il}_{jh\hat{l}}} \Bigg), & \text{if} \quad q^{il}_{jh\hat{l}} > 0 \\ \max\Bigg\{ 0, \phi^{il}_{jh\hat{l}} \Bigg( \gamma_{j\hat{l}} - \dfrac{\partial f^{il}(Q^1,Q^2)}{\partial q^{il}_{jh\hat{l}}} - \dfrac{\partial c^{il}_{jh\hat{l}}(q^{il}_{jh\hat{l}})}{\partial q^{il}_{jh\hat{l}}} - \dfrac{\partial c_{j\hat{l}}(Q^1,Q^3)}{\partial q^{il}_{jh\hat{l}}} \\ -\alpha^{il} \dfrac{\partial r^{il}(Q^1,Q^2)}{\partial q^{il}_{jh\hat{l}}} - \beta^{j\hat{l}} \dfrac{\partial r^{j\hat{l}}(Q^1,Q^3)}{\partial q^{il}_{jh\hat{l}}} \Bigg) \Bigg\}, & \text{if} \quad q^{il}_{jh\hat{l}} = 0. \end{cases}$$

# Dynamics of Product Transactions between Manufacturer and Retailer

$$\dot{q}^{il}_{kh\bar{l}} = \begin{cases} \phi^{il}_{kh\bar{l}}\Bigg( (\lambda^{-}_{kh\bar{l}} + \rho_{3kh\bar{l}} \times e_h)(1 - P_{kh\bar{l}}(s_{kh\bar{l}}, \rho_{3kh\bar{l}})) - \frac{\partial f^{il}}{\partial q^{il}_{kh\bar{l}}} \\ - \frac{\partial c^{il}_{kh\bar{l}}}{\partial q^{il}_{kh\bar{l}}} - \frac{\partial c_{kh\bar{l}}}{\partial q^{il}_{kh\bar{l}}} - \alpha^{il} \frac{\partial r^{il}}{\partial q^{il}_{kh\bar{l}}} - \lambda^{+}_{kh\bar{l}} P_{kh\bar{l}}(s_{kh\bar{l}}, \rho_{3kh\bar{l}}) \Bigg), & \text{if} \quad q^{il}_{kh\bar{l}} > 0 \\ \max\Bigg\{0, \phi^{il}_{kh\bar{l}}\Bigg( (\lambda^{-}_{kh\bar{l}} + \rho_{3kh\bar{l}} \times e_h)(1 - P_{kh\bar{l}}(s_{kh\bar{l}}, \rho_{3kh\bar{l}})) - \frac{\partial f^{il}}{\partial q^{il}_{kh\bar{l}}} \\ - \frac{\partial c^{il}_{kh\bar{l}}}{\partial q^{il}_{kh\bar{l}}} - \frac{\partial c_{kh\bar{l}}}{\partial q^{il}_{kh\bar{l}}} - \alpha^{il} \frac{\partial r^{il}}{\partial q^{il}_{kh\bar{l}}} - \lambda^{+}_{kh\bar{l}} P_{kh\bar{l}}(s_{kh\bar{l}}, \rho_{3kh\bar{l}}) \Bigg) \Bigg\}, & \text{if} \quad q^{il}_{kh\bar{l}} = 0. \end{cases}$$

# Dynamics of Product Transactions between Distributor and Retailer

$$\dot{q}^{j\hat{l}}_{kh\bar{l}} = \begin{cases} \phi^{j\hat{l}}_{kh\bar{l}} \Big( (\lambda^{-}_{kh\bar{l}} + \rho_{3kh\bar{l}} \times e_h)(1 - P_{kh\bar{l}}(s_{kh\bar{l}}, \rho_{3kh\bar{l}})) \\ -\lambda^{+}_{kh\bar{l}} P_{kh\bar{l}}(s_{kh\bar{l}}, \rho_{3kh\bar{l}}) - \frac{\partial c_{j\hat{l}}}{\partial q^{j\hat{l}}_{kh\bar{l}}} - \frac{\partial c_{kh\bar{l}}}{\partial q^{j\hat{l}}_{kh\bar{l}}} - \beta^{j\hat{l}} \frac{\partial r^{j\hat{l}}}{\partial q^{j\hat{l}}_{kh\bar{l}}} - \gamma_{j\hat{l}} \Big), & \text{if} \quad q^{j\hat{l}}_{kh\bar{l}} > 0 \\ \max\Big\{ 0, \phi^{j\hat{l}}_{kh\bar{l}} \Big( (\lambda^{-}_{kh\bar{l}} + \rho_{3kh\bar{l}} \times e_h)(1 - P_{kh\bar{l}}(s_{kh\bar{l}}, \rho_{3kh\bar{l}})) \\ -\lambda^{+}_{kh\bar{l}} P_{kh\bar{l}}(s_{kh\bar{l}}, \rho_{3kh\bar{l}}) - \frac{\partial c_{j\hat{l}}}{\partial q^{j\hat{l}}_{kh\bar{l}}} - \frac{\partial c_{kh\bar{l}}}{\partial q^{j\hat{l}}_{kh\bar{l}}} - \beta^{j\hat{l}} \frac{\partial r^{j\hat{l}}}{\partial q^{j\hat{l}}_{kh\bar{l}}} - \gamma_{j\hat{l}} \Big) \Big\}, & \text{if} \quad q^{j\hat{l}}_{kh\bar{l}} = 0. \end{cases}$$

# Projected Dynamical System (PDS)

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0$$

# Stationary Points

**Theorem**

Since the feasible set is a convex polyhedron, the set of stationary points of the described PDS coincides with the set of solutions to the appropriately constructed VI problem: determine $X^* \in \mathcal{K}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}$$

# The Euler Method (Dupuis and Nagurney (1993))

Step 0: Initialization Set $X^0 \in \mathcal{K}$. Let $\mathcal{T}$ denote an iteration counter. Let $\mathcal{T} = 1$ and set the sequence $\{\alpha_{\mathcal{T}}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} \alpha_{\mathcal{T}} = \infty$, $\alpha_{\mathcal{T}} > 0$, $\alpha_{\mathcal{T}} \to 0$, as $\mathcal{T} \to \infty$.

Step 1: Computation Compute $X^{\mathcal{T}} \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + \alpha_{\mathcal{T}} F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - X^{\mathcal{T}} \rangle \geq 0, \qquad \forall X \in \mathcal{K}.$$

Step 2: Convergence Verification If $|X^{\mathcal{T}} - X^{\mathcal{T}-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

# Additional Results

## Theorem

If the feasible set is $R_+^N$ and $\phi \equiv (\phi_1, \ldots, \phi_N)^T$ is a vector of positive terms. Then:

$$\mathrm{VI}(F, \mathcal{K}) \equiv \mathrm{VI}(F', \mathcal{K})$$

where:

$$F' \equiv (F'_1, \ldots, F'_N)^T \quad \text{and} \quad F \equiv (\phi_1 F'_1, \ldots, \phi_N F'_N)^T$$

# Importance of Speed of Adjustment: $\phi$

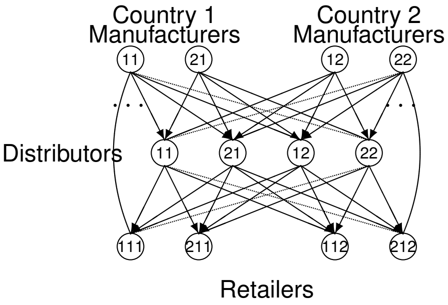
## Interpretation for *prices*

A speed of adjustment of a price may also be interpreted as the sensitivity of the specific demand market or distributor to the changes in the supply and/or demand of the product.

## Interpretation for *flows*

A speed of adjustment of a flow may also be interpreted as the sensitivity of the relationship between a pair of decision-makers to the changes in their marginal profits.

# Examples: Network Structure



# Examples: Data

## Production cost functions

$$f^{11}(q) = 2.5(q^{11})^2 + q^{11}q^{21} + 2q^{11}, \quad f^{21}(q) = 2.5(q^{21})^2 + q^{11}q^{21} + 2q^{21},$$
$$f^{12}(q) = 2.5(q^{12})^2 + q^{12}q^{22} + 2q^{12}, \quad f^{22}(q) = 2.5(q^{12})^2 + q^{12}q^{22} + 2q^{22}.$$

## Transaction cost functions

$$c^{il}_{jh\hat{l}} = .5(q^{il}_{jh\hat{l}})^2 + 3.5q^{il}_{jh\hat{l}}, \quad \forall i = 1,2; l = 1,2; j = 1,2; h = 1; \hat{l} = 1,2.$$
$$c^{il}_{kh\bar{l}} = .5(q^{il}_{kh\bar{l}})^2 + 5q^{il}_{kh\bar{l}}, \quad \forall i = 1,2; l = 1,2; k = 1,2; h = 1; \bar{l} = 1,2.$$

# Examples: Data

## Handling cost functions

$$c_{j\hat{l}} = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{jh\hat{l}}^{il})^2, \quad \forall j = 1,2; \hat{l} = 1,2$$
$$c_{kh\bar{l}} = .5(\sum_{j=1}^{2}\sum_{\hat{l}=1}^{2} q_{kh\bar{l}}^{j\hat{l}})^2, \quad \forall k = 1,2; h = 1; \bar{l} = 1,2$$

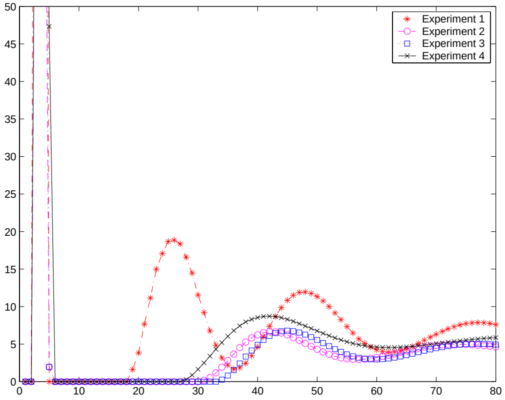
## Risk function

$$r^{11} = (\sum_{kh\bar{l}} q_{kh\bar{l}}^{11} - 2)^2$$

# Examples: Results

| | Example 1 | Example 2 | Example 3 |
|---|---|---|---|
| **Electronic Transactions** | | | |
| $q_{kh\bar{l}}^{11*}$ | 0.175 | 1.07 | 0.632 |
| $q_{kh\bar{l}}^{21*}$ | 0.175 | 1.07 | 1.185 |
| $q_{kh\bar{l}}^{12*}$ | 0.175 | 1.07 | 1.182 |
| $q_{kh\bar{l}}^{22*}$ | 0.175 | 1.07 | 1.182 |
| **Physical Transactions** | | | |
| $q_{jh\hat{l}}^{11*}$ | 0.186 | 0.286 | 0.700 |
| $q_{jh\hat{l}}^{21*}$ | 0.186 | 0.286 | 0.185 |
| $q_{jh\hat{l}}^{12*}$ | 0.186 | 0.286 | 0.182 |
| $q_{jh\hat{l}}^{22*}$ | 0.186 | 0.286 | 0.182 |
| $q_{kh\bar{l}}^{\hat{j}\hat{l}*}$ | 0.186 | 0.286 | 0.314 |
| **Prices** | | | |
| $\gamma_{\hat{j}\hat{l}}^{*}$ | 15.097 | 39.46 | 39.50 |
| $\rho_{3kh\bar{l}}^{*}$ | 32.88 | 90.31 | 90.53 |

# Dynamic Trajectory: Flow



# Conclusions

- We developed a dynamic, multitiered global supply chain supernetwork model which allows for electronic commerce and handles decision-making under risk and uncertainty
- The model includes distinct speeds of adjustment
- The new theoretical results show that the speeds of adjustment do not alter the set of stationary points of the dynamical model nor the equilibria of the corresponding variational inequality formulation

# Questions? Comments?

Thank you!

The full paper appears in *Transportation Research: Part E*, Special Issue on Global Logistics, 41(6), 585-612.

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