Fresh Produce Spatial Price Equilibrium on General Networks: Capturing Commodity Quality Deterioration Through Endogenous Transportation Time Delay Functions with Capacities

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Outline of Presentation

- **Introduction**
- **Literature Review and Contributions**
- **The Multicommodity Fresh Produce Trade Equilibrium Model on a General Network**
- **Commodity Quality Trade Network Performance Measures**
- **Numerical Examples**
- **Insights and Summary**

Introduction

Fresh Produce Trade Network

- **Fresh produce trade networks help to support food security and provide consumers with nutrition.**
- **The global revenue in 2022 from fresh vegetables and fruits was expected to be 691.20 and 622.80 billion US dollars with the expected volume growth for 2023 being 3.2% and 2.9%, respectively.**


Fresh Produce Quality Deterioration

- **Fresh produce is perishable, and, hence, even under the best conditions, its quality deteriorates over time.**
- **Major challenges associated with transportation include:**
  - **Congestion on roads and at seaports.**
  - **Reduced capacity of major links such as the Panama Canal, because of a drought.**
  - **Man-made attacks in the Red Sea and the Suez Canal.**
  - **Maritime route capacity reductions on the Black Sea due to Russia's full-scale invasion of Ukraine and Russia's pulling out of the Black Sea Grain Initiative.**

Two Major Shipping Routes: The Panama Canal

- **Panama Canal Authority decreased the number of daily cargo ship slots from 32 to 24 at the beginning of November 2023, with the number expected to drop to 18 by February 2024.**
- **Freight service providers that focus on refrigerated products were considering traveling around South America, resulting in a week's delay in the arrival of the fruit.**

Two Major Shipping Routes: The Suez Canal

- **Attacks by Houthi rebels in Yemen, since mid December 2023, on the Red Sea and the Suez Canal have disrupted global trade, including the export of Egyptian citrus fruit.**
- **Multiple container shipping companies are diverting their ships around the Cape of Good Hope, which has increased the travel time by approximately two weeks from Europe to Asia.**

Literature Review and Contributions

Literature Review

• Spatial price equilibrium models, have significantly advanced since the pioneering work of **Samuelson (1952)** and **Takayama and Judge (1964, 1971).**

• The theory of variational inequalities **(cf. Nagurney (1999, 2006))** is the methodology used to develop the modeling and algorithmic framework.

• The network quantifies link travel times with an extended Bureau of Public Roads cost function **(Special Report 209: Highway Capacity Manual (1985))** that embeds link capacities endogenously.

• **Nagurney and Li (2016)**: Discuss both imperfectly competitive supply chain models and perfectly competitive spatial price equilibrium frameworks that incorporate product quality, though not for fresh produce.

• **Nagurney and Besik (2022)**: A single commodity spatial price network equilibrium model in which the arc multipliers are flow-dependent but the authors assumed a single route between each pair of supply and demand markets.

Literature Review

• **Nagurney et al. (2023)**: International trade network with multiple paths and exchange rates, but with commodity-specific shipment bounds and no quality considerations.

• **Nagurney et al. (2024a, 2024b)**: Multicommodity equilibrium models with upper bounds at supply markets and on routes with a single link, considered even disaster scenarios and trade network performance measures, yet still lacking explicit quality deterioration factors.

• **Besik and Nagurney (2025))**: Multicommodity fresh produce trade network model that captures quality deterioration and congestion effects, both without and with minimum quality standards, based on single-link routes with linear congestion and exogenous capacity limits.

• **The multicommodity trade network equilibrium model in this paper is most closely influenced by the fresh produce trade model with quality deterioration developed by Besik and Nagurney (2025). However, the model in this paper differs from the earlier one in significant ways.**

Contributions

• Each supply–demand pair can be connected via **multiple routes**, each route composed of one or more links, reflecting **multi-modal transport** scenarios.

• Introduces **nonlinear** congestion functions that capture the **time delay** effects of all commodities sharing a link, directly impacting **quality deterioration**, whereas in Besik and Nagurney (2025) the influence on congestion was assumed to be linear.

• Shifts from quantity-only models by adopting a **price** and **quantity** approach with fixed, known initial quality levels, avoiding the need for estimating opportunity cost functions. Whereas the earlier model was in quantity variables, with the initial quality of each commodity as a variable at each supply markets.

• **Upper bounds** on shipments are embedded within time delay functions, enabling a direct assessment of how capacity constraints affect product quality.

• Proposes innovative **quality trade network performance metrics** (supply-based, demand-based, network-based, and commodity-specific) to evaluate market performance from a quality standpoint.

The Multicommodity Fresh Produce Trade Equilibrium Model on a General Network

The Multicommodity Fresh Produce Trade Equilibrium Model

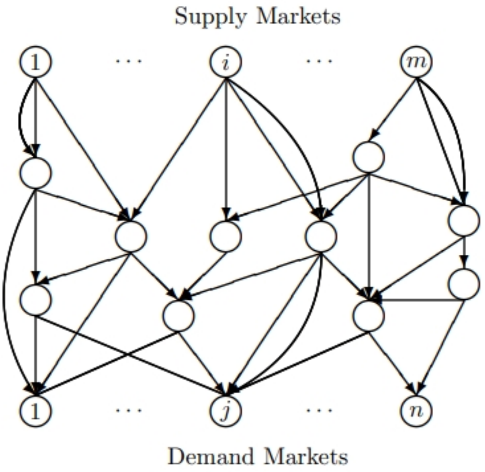

Figure 1: A Trade Network Topology

Expansion of the Bureau of Public Road (BPR) Functions

We let f_a^k denote the flow of commodity k on link a, for all commodities k and all links a.

$$t_a = t_a^0\Big(1 + \alpha_a\Big(\frac{\sum_{k=1}^{K} \eta_a^k f_a^k}{u_a}\Big)^{\gamma_a}\Big), \quad \forall a \in L, \tag{1}$$

η_a^k is a positive parameter for all k and for all a, and α_a is a positive parameter for all a since we assume congestion effects. γ_a represents how the travel time on a link is affected by the commodity flow on the link. We group all the link times into the vector $t \in R^{n_L}$.

The quality of commodity k on path $p \in P^i$ takes the form:

$$q_p^k = q_i^{0k} - \sum_{a \in p} \kappa_a t_a^0\Big(1 + \alpha_a\Big(\frac{\sum_{k=1}^{K} \eta_a^k f_a^k}{u_a}\Big)^{\gamma_a}\Big), \quad \forall k, \forall p \in P^i. \tag{2}$$

We group all the commodity quality levels for all the paths into the vector $q \in R^{Kn_P}$.

Notation

Variables	Definition
x_p^k	the flow of commodity k on path p. We group the commodity path flows into the vector $x \in R_+^{Kn_P}$.
f_a^k	the flow of commodity k on link a. We group the commodity link flows into the vector $f \in R_+^{Kn_L}$.
π_i^k	the supply price of commodity k at supply market i. We group all the commodity supply market prices into the vector $\pi \in R^{Km}$.
ρ_p^k	the demand price of commodity k transported on path p. We group all the commodity demand market prices into the vector $\rho \in R^{Kn_P}$.
Functions	**Definition**
$c_a^k(f,t,T)$	the unit cost of transportation of commodity k on link a.
$C_p^k(x,t,T)$	the unit cost of transportation of commodity k on path p. We group all the path costs into the vector $C \in R^{Kn_P}$.
$s_i^k(\pi)$	the supply of commodity k at supply market i. We group all the supply functions into the vector $s(\pi) \in R^{Km}$.
$d_p^k(\rho,q)$	the demand for commodity k that has been transported on path p. We group all the demand functions into the vector $d(\rho,q) \in R^{Kn_P}$.

Conservation of Flow Equations

The commodity path flows must be nonnegative, that is:

$$x_p^k \geq 0, \quad \forall k, \forall p \in P. \tag{3}$$

The commodity link flows are related to the commodity path flows thus:

$$f_a^k = \sum_{p \in P} x_p^k \delta_{ap}, \quad \forall k, \forall a \in L, \tag{4}$$

and, therefore, the commodity flow on a link is equal to the sum of flows of that commodity on paths that contain that link.
The unit transportation cost of a commodity on a path is equal to the sum of the unit transportation costs for that commodity on the links that comprise the path; that is:

$$C_p^k(x, t, T) = \sum_{a \in L} c_a^k(f, t, T) \delta_{ap}, \quad \forall p \in P, \tag{5}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.

Conservation of Flow Equations

In view of the conservation of flow equations (4), we may re-express the path quality levels (2) as:

$$q_p^k = q_i^{0k} - \sum_{a \in p} \kappa_a^k t_a^0 (1 + \alpha_a (\frac{\sum_{k=1}^K \eta_a^k \sum_{p \in P} x_p^k \delta_{ap}}{u_a})^{\gamma_a}), \quad \forall k, \forall p \in P^i. \qquad (6)$$

In view of (6), we can define new demand price functions $\tilde{d}_p^k(\rho, x) \equiv d_p^k(\rho, q)$, for all k and for all p.

Also, in view of (2) and (4), we can define new unit path cost functions: $\tilde{C}_p^k(x, T) \equiv C_p^k(x, t, T)$, for all commodities k and all paths p.

Equilibrium Conditions

Definition 1: Multicommodity Fresh Produce Trade Network Equilibrium Under Quality Deterioration on a General Network with Time Delays

A multicommodity path flow, supply price, and demand price pattern $(x^*, \pi^*, \rho^*) \in \mathcal{K}$ is a fresh produce trade network equilibrium under quality deterioration on a general network with time delays, if the following conditions hold: For all commodities k; $k = 1, \ldots, K$, and for all paths p connecting pairs of supply and demand markets (i, j); $i = 1, \ldots, m$ and $j = 1, \ldots, n$:

$$\pi_i^{k*} + \tilde{C}_p^k(x^*, T) \begin{cases} = \rho_p^{k*}, & \text{if} \quad x_p^{k*} > 0, \\ \geq \rho_p^{k*}, & \text{if} \quad x_p^{k*} = 0. \end{cases} \tag{7}$$

Equilibrium Conditions

Definition 1 (continued): Multicommodity Fresh Produce Trade Network Equilibrium Under Quality Deterioration on a General Network with Time Delays

For all commodities k and all supply markets i:

$$s_i^k(\pi^*) \begin{cases} = \sum_{p\in P^i} x_p^{k*}, & \text{if} \quad \pi_i^{k*} > 0, \\ \geq \sum_{p\in P^i} x_p^{k*}, & \text{if} \quad \pi_i^{k*} = 0, \end{cases} \tag{8}$$

plus, for all commodities k and for all paths p terminating at the demand markets:

$$\tilde{d}_p^k(\rho^*, x^*) \begin{cases} = x_p^{k*}, & \text{if} \quad \rho_p^{k*} > 0, \\ \leq x_p^{k*}, & \text{if} \quad \rho_p^{k*} = 0. \end{cases} \tag{9}$$

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation of the Multicommodity Fresh Produce Trade Network Equilibrium Under Quality Deterioration on a General Network with Time Delays

A multicommodity path flow, supply price, and demand price pattern $(x^*, \pi^*, \rho^*) \in \mathcal{K}$ *is a fresh produce trade network equilibrium under quality deterioration on a general network with time delays, according to Definition 1, if and only if it satisfies the variational inequality problem:*

$$\sum_{k=1}^{K}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{p\in P_j^i}\left[\pi_i^{k*}+\tilde{C}_p^k(x^*,T)-\rho_p^{k*}\right]\times\left[x_p^k-x_p^{k*}\right]$$

$$+\sum_{k=1}^{K}\sum_{i=1}^{m}\left[s_i^k(\pi^*)-\sum_{p\in P^i}x_p^{k*}\right]\times\left[\pi_i^k-\pi_i^{k*}\right]$$

$$+\sum_{k=1}^{K}\sum_{p\in P}\left[x_p^{k*}-\tilde{d}_p^k(\rho^*,x^*)\right]\times\left[\rho_p^k-\rho_p^{k*}\right]\geq 0,\quad \forall(x,\pi,\rho)\in\mathcal{K}. \qquad (10)$$

Commodity Quality Trade Network Performance Measures

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Supply-Based Quality Measures

In particular, we have the following individual commodity supply-based quality measure, $\mathcal{E}_S^{k,i}$, for commodity k and supply market i:

$$\mathcal{E}_S^{k,i} = \frac{1}{n_{P^i}} \sum_{p \in P^i} \frac{x_p^{k*} q_p^{k*}}{\rho_p^{k*}}, \quad k = 1, \ldots, K; i = 1, \ldots, m. \tag{11}$$

Expanding the individual supply-based quality measure in (11) to all commodities, we obtain the full set of commodities supply-based quality measure, $\mathcal{E}_S^i$, for supply market i:

$$\mathcal{E}_S^i = \frac{1}{K n_{P^i}} \sum_{k=1}^{K} \sum_{p \in P^i} \frac{x_p^{k*} q_p^{k*}}{\rho_p^{k*}}, \quad i = 1, \ldots, m. \tag{12}$$

Commodity Quality Trade Network Performance Measures

Demand-Based Quality Measures

Specifically, the individual commodity demand-based quality measure, $\mathcal{E}_D^{k,j}$, for commodity k and demand market j is given by:

$$\mathcal{E}_D^{k,j} = \frac{1}{n_{P_j}} \sum_{p \in P_j} \frac{x_p^{k*} q_p^{k*}}{\rho_p^{k*}}, \quad k = 1, \ldots, K; j = 1, \ldots, n. \tag{13}$$

Expanding the measure in (13) overall commodities yields the following measure, $\mathcal{E}_D^j$, for demand market j:

$$\mathcal{E}_D^{j} = \frac{1}{Kn_{P_j}} \sum_{k=1}^{K} \sum_{p \in P_j} \frac{x_p^{k*} q_p^{k*}}{\rho_p^{k*}}, \quad j = 1, \ldots, n. \tag{14}$$

Commodity Quality Trade Network Performance Measures

Network-Based Quality Measures

For each individual commodity k we define the network-based measure, $\mathcal{E}^k_{Network}$, as:

$$\mathcal{E}^k_{Network} = \frac{1}{n_P} \sum_{p \in P} \frac{x_p^{k*} q_p^{k*}}{\rho_p^{k*}}, \quad k = 1, \ldots, K. \tag{15}$$

The assessment of the full trade network over all the commodities can be captured in the network-based measure, $\mathcal{E}_{Network}$, as:

$$\mathcal{E}_{Network} = \frac{1}{K n_P} \sum_{k=1}^{K} \sum_{p \in P} \frac{x_p^{k*} q_p^{k*}}{\rho_p^{k*}}. \tag{16}$$

Numerical Examples

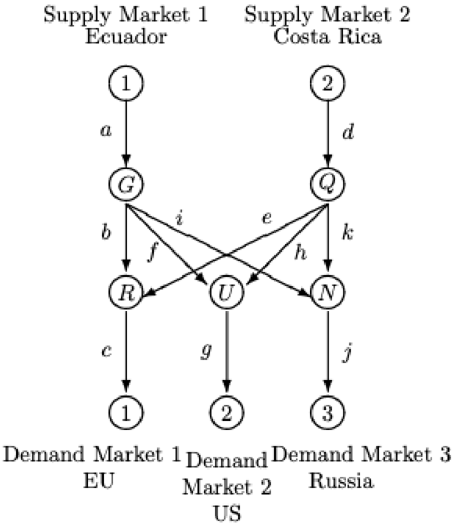
Numerical Examples

• A series of numerical examples are presented focusing on the banana shipments from Ecuador and Costa Rica to the EU, the US, and Russia via the Panama Canal. Due to time limitations, only two examples are discussed here.

• The functions in these examples mainly derive from 2022 data in the FAOSTAT (2024). **All data is contained in our paper.**

• All examples are solved using the Modified Projection Method of Korpelevich (1977), with a convergence tolerance of 10^{-7}.

Network Topology: Examples 1 and 2



Path Descriptions

- $p_1 = (a, b, c)$: Rail transport from Ecuador to port of Guayaquil, maritime transport via the Panama Canal to port of Rotterdam, then rail transport to the demand market in the EU.
- $p_2 = (d, e, c)$: Rail transport from Costa Rica to port of Quepos, maritime transport via the Panama Canal to port of Rotterdam, then rail transport to the deamand market in the EU.
- $p_3 = (a, f, g)$: Rail transport from Ecuador to port of Guayaquil, maritime transport via the Panama Canal to a port in the US, then rail transport to the demand market in the US.
- $p_4 = (d, h, g)$: Rail transport from Costa Rica to port of Quepos, maritime transport via the Panama Canal to a port in the US, then rail transport to the demand market in the US.
- $p_5 = (a, i, j)$: Rail transport from Ecuador to port of Guayaquil, maritime transport through the Panama Canal to port of Novorossiysk via the Atlantic, Mediterranean, and Black Seas. Then rail transport to the demand market inside Russia.
- $p_6 = (d, k, j)$: Rail transport from Costa Rica to port of Quepos, maritime transport via the Panama Canal to the port of Novorossiysk, then rail transport to the demand market inside Russia.

Example Scenarios

Example 1: Baseline Scenario

• Link capacities:

$$u_b = u_e = u_d = u_c = u_i = u_g = 3,000,000, \quad u_a = 5,000,000,$$
$$u_f = u_h = u_j = 2,000,000, \quad u_k = 20,000.$$

• Free-flow times (in hours):

$$t_a^0 = t_d^0 = t_c^0 = t_j^0 = t_g^0 = 15.00, \quad t_b^0 = t_e^0 = 140.00,$$
$$t_f^0 = t_h^0 = t_c^0 = 80.00, \quad t_i^0 = t_k^0 = 200.00.$$

Example 2: Full Network Under Tighter Upper Bounds on the Maritime Routes via the Panama Canal

• The effect of drought conditions in the Panama Canal is examined by reducing maritime link capacities by 40% and increasing thier free-flow times to 500.00 hours. All other data remain as in Example 1.

Results for Path Flows, Final Quality, and Transportation Times

Variables	Example 5	Example 6
$x_{p_1}^{1*}$	1,574,096.84	1,428,165.30
$x_{p_2}^{1*}$	981,068.25	936,088.82
$x_{p_3}^{1*}$	1,158,022.00	976,824.38
$x_{p_4}^{1*}$	641,004.83	582,006.05
$x_{p_5}^{1*}$	1,609,030.06	1,467,533.17
$x_{p_6}^{1*}$	1,301.10	25.52
$q_{p_1}^{1*}$	94.98	91.01
$q_{p_2}^{1*}$	96.73	94.26
$q_{p_3}^{1*}$	96.76	91.76
$q_{p_4}^{1*}$	98.49	95.43
$q_{p_5}^{1*}$	94.83	91.01
$q_{p_6}^{1*}$	96.68	95.09
$t_{p_1}^{1*}$	716.58 (30 Days)	1,284.17 (53 Days)
$t_{p_2}^{1*}$	466.95 (19 Days)	819.77 (34 Days)
$t_{p_3}^{1*}$	462.81 (19 Days)	1,176.78 (49 Days)
$t_{p_4}^{1*}$	214.91 (9 Days)	651.79 (27 Days)
$t_{p_5}^{1*}$	737.59 (31 Days)	1,283.71 (53 Days)
$t_{p_6}^{1*}$	472.89 (20 Days)	700.63 (29 Days)

Results for Supply Prices and Demand Prices

Variables	Example 5	Example 6
π_1^{1*}	266.19	212.91
π_2^{1*}	385.30	349.20
$\rho_{p_1}^{1*}$	671.14	710.89
$\rho_{p_2}^{1*}$	651.31	677.60
$\rho_{p_3}^{1*}$	584.46	650.73
$\rho_{p_4}^{1*}$	568.78	610.19
$\rho_{p_5}^{1*}$	658.19	694.83
$\rho_{p_6}^{1*}$	584.61	589.04

Results for Supply, Demand, and Transportation Costs Functions

Functions	Example 5	Example 6
s_1^1	4,341,148.90	3,872,522.85
s_2^1	1,623,374.18	1,518,120.39
$\tilde{d}_{p_1}^1$	1,574,096.84	1,428,165.30
$\tilde{d}_{p_2}^1$	981,068.25	936,088.82
$\tilde{d}_{p_3}^1$	1,158,022.00	976,824.38
$\tilde{d}_{p_4}^1$	641,004.83	582,006.05
$\tilde{d}_{p_5}^1$	1,609,030.06	1,467,533.17
$\tilde{d}_{p_6}^1$	1,301.10	25.52
$\tilde{C}_{p_1}^1$	404.95	497.98
$\tilde{C}_{p_2}^1$	266.01	328.40
$\tilde{C}_{p_3}^1$	318.27	437.82
$\tilde{C}_{p_4}^1$	183.48	260.99
$\tilde{C}_{p_5}^1$	392.00	481.92
$\tilde{C}_{p_6}^1$	199.31	239.84

Results for Quality Trade Performance Measures

Performance Measures	Example 5	Example 6
$\mathcal{E}_S^{1,1}$	215,442.66	170,937.19
$\mathcal{E}_S^{1,2}$	85,641.01	73,750.44
$\mathcal{E}_D^{1,1}$	184,239.94	156,529.02
$\mathcal{E}_D^{1,2}$	151,358.79	114,386.56
$\mathcal{E}_D^{1,3}$	116,026.77	96,115.86
$\mathcal{E}_{Network}^1$	150,541.84	122,343.81

Insights and Summary

Insights

- The study underscores the vulnerability of perishable commodities when critical transit routes are disrupted.

- Monitoring flow-dependent transportation times and quality deterioration across individual links is essential for mitigating supply chain risks.

- The proposed quality performance measures enable decision-makers to evaluate which markets excel in maintaining fresh produce quality and to assess overall trade network performance.

- Future research could extend the model to include other vital routes (e.g., the Suez Canal), incorporate uncertainty, additional commodity types, and policy instruments like tariffs and exchange rates.

- Overall, the framework offers valuable insights for policymakers and supply chain planners aiming to enhance fresh produce quality and improve trade network resilience.

Summary

• In this paper, we constructed a multicommodity trade network equilibrium model for fresh produce that explicitly incorporates quality deterioration across multi-link transportation routes.

• The model supports multiple transportation modes and captures bottlenecks and capacity reductions on individual links.

• Nonlinear time delay functions, adapted from the BPR formulation, account for flow-dependent transportation times. This framework reflects how reduced or enhanced upper bounds on each link can significantly influence both transportation times and the quality of the fresh produce.

• Equilibrium conditions were formulated as a variational inequality problem, and new quality performance measures (supply-based, demand-based, network-based) were introduced.

• The model was applied to banana shipments from Ecuador and Costa Rica to the EU, US, and Russia, integrating drought-induced congestion in the Panama Canal.

Thank You Very Much!