A Multiperiod, Multicommodity, Capacitated International Agricultural Trade Network Equilibrium Model with Applications to Ukraine in Wartime

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Acknowledgment and Dedication

This presentation is dedicated to farmers in Ukraine and worldwide.



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The War in Ukraine

 The full-scale invasion of Ukraine by Russia on February 24, 2022 has resulted in immense losses of lives and an increase in human suffering. It has severely impacted the economy of Ukraine with repercussions globally.



Hassani et al.

The Impacts on Ukraine's Agricultural Sector

 Between 20 to 30% of the arable land in Ukraine has remained idle due to mining and other damages because of the full-scale invasion, resulting in around a 40% decrease in the production of grains in Ukraine.



SOURCE: USDA

The Impacts on Ukraine's Agricultural Sector

- The blockade of the Ukrainian Black Sea ports, which used to handle around 90% of the grain exports from Ukraine, caused a global shortage of grains.
- The full-scale invasion has cost Ukraine around 15% of its grain storage capacity.





The Impacts on Global Food Security and Economy

- Reports has indicated a rise of around 17% in the population facing food insecurity worldwide due to the full-scale invasion.
- The disruptions in the exports of Ukrainian grain have cost the global economy more than 1.6 billion dollars.
- Many countries heavily rely on Ukrainian grains, especially vulnerable countries in the Middle East and North Africa (MENA) region.



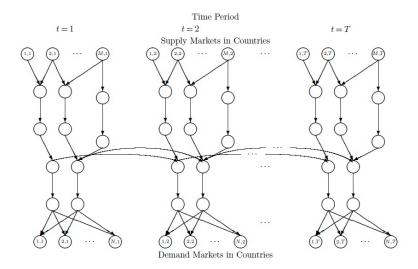


Literature Review

- The theory of variational inequalities (cf. Nagurney (1999, 2006)) is the methodology used to develop the modeling and algorithmic framework.
- Spatial price equilibrium (SPE) models were first introduced in the groundbreaking contributions of Samuelson (1952) and Takayama and Judge (1964, 1971) and are partial equilibrium models under perfect competition.
- SPE models have been widely used in modeling disaster scenarios and relevant food security issues, the trade of agricultural products, and the impacts of different policy instruments.
- Nagurney et al. (2023) developed a multicommodity international SPE model with exchange rates in which transportation through multiple intermediate countries was possible; however, the model did not include storage and imposed no capacities.
- Nagurney et al. (2024) constructed a multicommodity international agricultural SPE model with bounds on the production and transportation of commodities, but the model included only one period, and transportation through multiple countries was not considered.

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The Multiperiod International Trade Model



Notation for The Multiperiod International Trade Model

Notation	Parameter Definition			
$u_{i,t}$	supply capacity at supply node $i; i \in \mathcal{I}$, in time period $t; t \in \mathcal{T}$.			
$u_a^{ au}$	transportation capacity on transportation link $a; a \in \mathcal{L}^{\tau}$.			
u_a^{σ}	inventory capacity on inventory link $a; a \in L^{\sigma}$.			
Notation				
Q_p^h	the flow of commodity $h; h \in \mathcal{H}$, on path $p; p \in \mathcal{P}$. We group all the commodity path flows into the vector $Q \in \mathcal{R}_+^{Hn_{\mathcal{P}}}$, where $n_{\mathcal{P}}$ is the number of paths in the network.			
f_a^h	the flow of commodity $h; h \in \mathcal{H}$, on link $a; a \in \mathcal{L}$. We group all the commodity link flows into the vector $f \in \mathcal{R}_{+}^{Hn_{\mathcal{L}}}$, where $n_{\mathcal{L}}$ is the number of links in the network.			
$s_{i,t}^h$	the supply of the commodity h ; $h \in \mathcal{H}$, at supply node i ; $i \in \mathcal{I}$, in time period t ; $t \in \mathcal{T}$. We group all the commodity supplies into the vector $s \in \mathcal{R}_{+}^{HMT}$.			
$d_{j,t'}^h$	the demand for the commodity $h; h \in \mathcal{H}$, at demand node $j; j \in \mathcal{J}$, in time period $t'; t' \in \mathcal{T}$. We group all the commodity demands into the vector $d \in \mathcal{R}^{HNT}$.			
$\lambda_{i,t}$	the Lagrange multiplier associated with the supply capacity constraint at supply node $i, i \in \mathcal{I}$, in time period $t, i \in \mathcal{T}$. We group all such Lagrange multipliers into the vector $\lambda \in \mathcal{R}_t^{MT}$.			
μ_a	the Lagrange multiplier associated with the transportation capacity constraint on transportation link $a_: a \in \mathcal{L}^{\tau}$. We group all such Lagrange multipliers into the vector $\mu \in \mathcal{R}_{-\mathcal{L}^{\tau}}^{n,\mathcal{L}^{\tau}}$, where $n_{\mathcal{L}^{\tau}}$ is the number of transportation links in the trade network.			
γ_a	the Lagrange multiplier associated with the inventory capacity constraint of inventory link a ; $a \in \mathcal{L}^{\sigma}$. We group all such Lagrange multipliers into the vector $\gamma \in \mathcal{R}_{+}^{n_{c}\sigma}$, where $n_{\mathcal{L}^{\sigma}}$ is the number of inventory links in the tradintework.			
Notation	Function Definition			
$\pi_{i,t}^h(Q)$	the supply price function for commodity $h; h \in \mathcal{H}$, at supply node $i; i \in \mathcal{I}$, in time period $t; t \in \mathcal{T}$.			
$\rho_{j,t'}^h(Q)$	the demand price function for commodity $h; h \in \mathcal{H}$, at demand node $j; j \in \mathcal{J}$, in time period $t; t \in \mathcal{T}$.			
$c_a^h(Q)$	the unit link cost associated with the commodity $h; h \in \mathcal{H}$, on link $a; a \in \mathcal{L}$.			

The Multiperiod International Trade Model

The commodity path flows must be nonnegative:

$$Q_p^h \ge 0, \quad \forall h \in \mathcal{H}, p \in \mathcal{P}.$$
 (1)

The cost on a path p for commodity h is given by the following expression:

$$C_p^h = \sum_{a \in \mathcal{L}} \delta_{a,p} c_a^h(Q), \quad \forall h \in \mathcal{H}, p \in \mathcal{P},$$
 (2)

where $\delta_{a,p}=1$, if link a is contained in path p, and is 0, otherwise; i.e., the cost on a path for a commodity is equal to the sum of the costs on the links that make up the path for the commodity.

The Equilibrium Conditions

Definition 1: The Equilibrium Conditions

A path flow and Lagrange multiplier pattern $(Q^*, \lambda^*, \mu^*, \gamma^*) \in \mathcal{K}^1 \equiv \{(Q, \lambda, \mu, \gamma) | Q \in \mathcal{R}_+^{\mathsf{Hn}_\mathcal{P}}, \lambda \in \mathcal{R}_+^{\mathsf{MT}}, \mu \in \mathcal{R}_+^{\mathsf{n}_\mathcal{L}^\sigma}, \gamma \in \mathcal{R}_+^{\mathsf{n}_\mathcal{L}^\sigma}\}$ is an equilibrium under capacities if the following conditions hold:

$$\pi_{i,t}^{h}(Q^{*}) + C_{p}^{h}(Q^{*}) + \lambda_{i,t}^{*} + \sum_{a \in \mathcal{L}^{\tau}} \delta_{a,p} \mu_{a}^{*} + \sum_{a \in \mathcal{L}^{\sigma}} \delta_{a,p} \gamma_{a}^{*} - \rho_{j,t'}^{h}(Q^{*}) \geq 0 \ \perp \ Q_{p}^{h*} \geq 0$$

$$\forall p \in \mathcal{P}_{j,t'}^{i,t}, h \in \mathcal{H} \tag{3}$$

$$u_{i,t} - \sum_{h \in \mathcal{H}} \sum_{p \in \mathcal{P}^{i,t}} Q_p^{h*} \ge 0 \perp \lambda_{i,t}^* \ge 0 \ \forall i \in \mathcal{I}, t \in \mathcal{T}$$
 (4)

$$u_a^{\tau} - \sum_{h \in \mathcal{H}} \sum_{p \in \mathcal{P}} \delta_{a,p} Q_p^{h*} \ge 0 \perp \mu_a^* \ge 0 \ \forall a \in \mathcal{L}^{\tau}$$
 (5)

$$u_a^{\sigma} - \sum_{h \in \mathcal{H}} \sum_{p \in \mathcal{P}} \delta_{a,p} Q_p^{h*} \ge 0 \perp \gamma_a^* \ge 0 \ \forall a \in \mathcal{L}^{\sigma}. \tag{6}$$

Variational Inequality Formulation

The Variational Inequality Formulation in Path Flows and Lagrange Multipliers

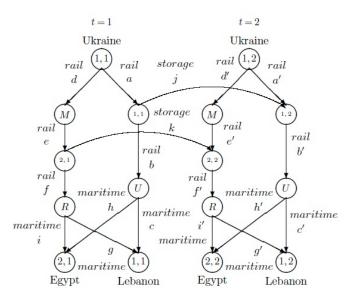
A path flow and Lagrange multiplier pattern $(Q^*, \lambda^*, \mu^*, \gamma^*) \in \mathcal{K}^1$ is an equilibrium under capacities according to Definition 1 if and only if it satisfies the variational inequality:

$$\sum_{h \in \mathcal{H}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \sum_{t' \in \mathcal{T}} \sum_{p \in \mathcal{P}_{j,t'}^{i,t}} \left[\pi_{i,t}^{h}(Q^{*}) + C_{p}^{h}(Q^{*}) + \lambda_{i,t}^{*} + \sum_{a \in \mathcal{L}^{\mathcal{T}}} \delta_{a,p} \mu_{a}^{*} + \sum_{a \in \mathcal{L}^{\sigma}} \delta_{a,p} \gamma_{a}^{*} - \rho_{j,t'}^{h}(Q^{*}) \right] \\
\times \left[Q_{p}^{h} - Q_{p}^{h*} \right] + \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left[u_{i,t} - \sum_{h \in \mathcal{H}} \sum_{p \in \mathcal{P}} Q_{p}^{h*} \right] \times \left[\lambda_{i,t} - \lambda_{i,t}^{*} \right] \\
+ \sum_{a \in \mathcal{L}^{\mathcal{T}}} \left[u_{a}^{\tau} - \sum_{h \in \mathcal{H}} \sum_{p \in \mathcal{P}} \delta_{a,p} Q_{p}^{h*} \right] \times \left[\mu_{a} - \mu_{a}^{*} \right] \\
+ \sum_{a \in \mathcal{L}^{\sigma}} \left[u_{a}^{\sigma} - \sum_{h \in \mathcal{H}} \sum_{p \in \mathcal{P}} \delta_{a,p} Q_{p}^{h*} \right] \times \left[\gamma_{a} - \gamma_{a}^{*} \right] \geq 0, \quad \forall (Q, \lambda, \mu, \gamma) \in \mathcal{K}^{1}. \tag{7}$$

Numerical Examples

- A series of numerical examples inspired by the disruptions to the international trade of agricultural commodities caused by Russia's full-scale invasion of Ukraine are presented. **All data is contained in our paper**.
- The examples focus on the export of agricultural commodities of wheat and corn from Ukraine to Lebanon and Egypt.
- All examples consist of two periods corresponding to the projected yearly commodity shipments in metric tons.
- Lebanon and Egypt are two representative MENA countries significantly affected by the full-scale invasion in terms of the food security of their populations.
- The algorithmic framework used to solve the examples is that of the Modified Projection Method of Korpelevich (1977).

Network Topology



Path Descriptions

To Lebanon, t=1

- $p_1 = (a, b, c)$: Rail transport from farms to a storage facility in Ukraine and to a Black Sea port in Ukraine, then maritime transport to Lebanon.
- $p_2 = (d, e, f, g)$: Rail transport from farms to Moldova, to a storage facility in Moldova, and to a Black Sea port in Romania, then maritime transport to Lebanon.

To Egypt, t=1

- $p_3 = (a, b, h)$: Rail transport from farms to a storage facility in Ukraine and to a Black Sea port in Ukraine, then maritime transport to Egypt.
- $p_4 = (d, e, f, i)$: Rail transport from farms to Moldova, to a storage facility in Moldova, and to a Black Sea port in Romania, then maritime transport to Egypt.

To Lebanon and Egypt, t = 2

Paths p_5 , p_6 , p_7 , and p_8 are the same as the first four paths, respectively, but all in the second period and with links denoted with a'.

Paths with Storage Links Joining the Two Periods

- $p_9 = (a, j, b', c')$: Rail transport from farms to a storage facility in Ukraine in the first period, storage to the second period, rail transport from the storage facility to a Black Sea port in Ukraine, and maritime transport to Lebanon.
- $p_{10} = (d, e, k, f', g')$: Rail transport from farms to Moldova and to a storage facility in Moldova in the first period, storage to the second period, rail transport from the storage facility to a Black Sea port in Romania, and maritime transport to Lebanon.
- $p_{11} = (a, j, b', h')$: Rail transport from farms to a storage facility in Ukraine in the first period, storage to the second period, rail transport from the storage facility to a Black Sea port in Ukraine, and maritime transport to Egypt.
- $p_{12} = (d, e, k, f', i)$: Rail transport from farms to Moldova and to a storage facility in Moldova in the first period, storage to the second period, rail transport from the storage facility to a Black Sea port in Romania, and maritime transport to Egypt.

Numerical Example Scenarios

Example 1: Single Commodity of Wheat, Prior to Full-Scale Invasion

Example 2: Two Commodities of Wheat and Corn, Prior to Full-Scale Invasion

Similar capacities to those in Example 1 are imposed on two commodities.

Example 3: Two Commodities, First Period Prior to the Full-Scale Invasion, Second Period After

The second period corresponds to when prices were significantly affected, maritime transportation from Ukrainian Black Sea ports was blockaded, and production was severely disrupted due to damages to arable land.

Example 4: Two Commodities, First Period is Before the Black Sea Grain Initiative, Second Period is After It

The transportation capacity on Ukrainian Black Sea ports are restored to those before the invasion. The damages to storage facilities in Ukraine have resulted in a decrease in the storage capacity in Ukraine.

Results for Commodity 1 (Wheat)

	Example 1	Example 2	Example 3	Example 4
$Q_{p_1}^{1*}$	571,868	532,483	512,066	0
$Q_{p_2}^{1*}$	0	0	0	0
$Q_{p_3}^{1*}$	1,917,419	1,799,439	1,730,628	0
$Q_{p_4}^{1*}$	0	0	0	0
$Q_{p_5}^{1*}$	571,870	532,484	0	185,660
$Q_{p_6}^{1*}$	0	0	0	0
$Q_{p_7}^{1*}$	1,917,410	1,799,431	0	518,442
$Q_{p_8}^{1*}$	0	0	0	0
$Q_{p_0}^{1*}$	0	0	0	27,213
$Q_{p_{10}}^{1*}$	0	0	91,669	118,989
$Q_{p_{11}}^{1*}$	0	0	0	359,996
$Q_{p_{12}}^{1*}$	0	0	303,271	180,273
$s_{1.1}^{1*}$	2,489,287	2,331,922	2,637,635	686,470
$s_{1,2}^{1*}$	2,489,280	2,331,916	0	704, 102
$d_{1,1}^{1*}$	571,868	532,483	512,066	0
$d_{1,2}^{1*}$	571,870	532,484	91,669	331,861
$d_{2,1}^{1*}$	1,917,419	1,799,439	1,730,628	0
$d_{2,2}^{1*}$	1,917,410	1,799,431	303,271	1,058,711
$\pi^{1}_{1,1}$	\$262.45	\$262.56	\$264.16	\$103.75
$\pi_{1,2}^{1}$	\$262.45	\$262.56	\$100	\$103.82
$\rho_{1,1}^{1}$	\$344.27	\$346.04	\$346.96	\$530
$\rho_{1,2}^{1}$	\$344.27	\$346.04	\$525.87	\$515.07
$\rho_{2,1}^{1}$	\$341.24	\$343.01	\$344.04	\$530
$\rho_{2,2}^{1}$	\$341.24	\$343.01	\$525.45	\$514.12

Results for Commodity 2 (Corn)

	Example 2	Example 3	Example 4
$Q_{p_1}^{2*}$	162, 194	157,295	0
$Q_{p_2}^{2*}$	0	0	0
$Q_{p_3}^{2*}$	733,834	710,866	0
$Q_{p_4}^{2*}$	0	0	0
$Q_{p_5}^{2*}$	162,194	0	72,929
$Q_{p_6}^{2*}$	0	0	0
$Q_{p_7}^{2*}$	733,836	0	223,055
$Q_{p_8}^{2*}$	0	0	0
$Q_{p_0}^{2*}$	0	0	6,938
$Q_{p_{10}}^{2*}$	0	17,862	40,491
$Q_{p_{11}}^{2*}$	0	0	105,854
$Q_{p_{12}}^{2*}$	0	87,217	160,248
$s_{1,1}^{2*}$	896,028	973, 241	313,530
$s_{1,2}^{2*}$	896,030	0	295,983
$d_{1,1}^{2*}$	162, 194	157,295	0
$d_{1,2}^{2*}$	162,194	17,862	120,358
$d_{2,1}^{2*}$	733,834	710,866	0
$d_{2,1}^{2*}$ $d_{2,2}^{2*}$ $\pi_{1,1}^{2}$	733,836	87,217	489,156
$\pi_{1,1}^2$	\$253.62	\$255.01	\$94.51
$\pi_{1,2}^2$	\$253.62	\$90	\$94.37
$\rho_{1,1}^{2}$	\$345.41	\$345.84	\$520
$\rho_{1,2}^{2}$	\$345.41	\$518.39	\$509.17
$\rho_{2,1}^{2}$	\$341.65	\$342.23	\$520
$\rho_{2,2}^{2}$	\$341.65	\$517.82	\$507.77

- A zero path flow means that no trade is happening on that specific path. Blockades and damages to transportation infrastructure or high transportation and storage costs due to risks result in a path not being used.
- Before the invasion, more than 90% of Ukraine's grain was exported through the country's Black Sea ports. The commodity path flow patterns in the examples reiterate the importance of efficient maritime routes for the export of grain from Ukraine.
- In actuality, Lebanon imports more than 70% of its wheat demand from Ukraine. Egypt used to import around 25% of its wheat demand from Ukraine. The results replicate these trade patterns.
- In reality, Ukrainian farmers used to earn around \$270 per ton of wheat before the full-scale invasion, which is reflected in the results. The resultant demand prices in both Lebanon and Egypt are also quite close to the reported prices.

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- On top of 70% of its wheat demand, Lebanon imports around 20% of its demand for corn from Ukraine, and, also, Egypt procures about 5% of its corn from Ukraine.
- With the addition of corn as the second commodity, the flow of wheat to both Lebanon and Egypt decreases, since the commodities are competing over production and transportation.
- With the added competition from corn in production and transportation, less wheat is shipped at essentially the same supply price, and the demand price of wheat increases in both Lebanon and Egypt.
- In Example 1 and 2, no storage is observed since the demand in both demand nodes can be met more cheaply from the supply of the same period.

- Due to the blockade of the Ukrainian Black Sea ports in the second period, commodities are stored in Moldova to be exported from Romanian Black Sea ports in the second period. Production and storage in the first period and transport in the second period are preferred since the full-scale invasion has affected transportation costs.
- With transportation significantly disrupted in the second period due to the full-scale invasion, the shipment flows of both commodities to Lebanon and Egypt decrease in the first period, since then a part of the supply of the first period is stored to serve the demand of the second period.
- In normal times, almost no Ukrainian grain is shipped via land transport through the western borders of Ukraine, and these links are naturally mostly used for the transport of commodities made locally.

- With the initiative in place in the second period, all the wheat and corn harvests in the first period are stored inside Ukraine and in Moldova to be carried to the second period.
- During the blockade of its Black Sea ports, Ukraine was dealing with storage issues since the undamaged storage facilities were nearly full, which is reflected in the results when both inventorying links are at their capacity.
- Egypt imports the stored wheat in the first period with a lower transportation cost, while Lebanon is forced into paying for the high transportation cost of the alternative route. In the case of corn, Egypt is forced into importing more corn via the more expensive route than the more efficient one since Lebanon is more price-sensitive for this commodity.
- Even with the initiative in place, the transportation costs remained high due to the war-related risks and slow inspections of the ships.

Summary

- The examples shed light on the importance of efficient maritime transportation of grains from the Black Sea ports of Ukraine, and the impacts of the full-scale invasion on the production and storage of agricultural commodities.
- The results reveal the impacts of the significantly decreasing earnings of Ukrainian farmers and the increasing consumer prices.
- The examples indicate that Lebanon and Egypt compete over Ukraine's severely limited production, transportation, and storage capacity for grains.
- The results show the priority of wheat over corn in both country demand markets since wheat is an essential part of most staple foods in MENA countries.
- One of the examples highlights the importance of keeping maritime routes from Ukraine for the export of agricultural products operational.

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