An Integrated Multitiered Supply Chain Network Model of Competing Agricultural Firms and Processing Firms

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5th International Conference on Dynamics of Disasters
July 16-18, 2021

Agriculture Industry and the Covid-19 Pandemic

- The agriculture industry was worth 5 trillion dollars globally in 2015 and accounted for 4 percent of global domestic product (GDP) in 2018.
- The Covid-19 pandemic has resulted in many disruptions to the agriculture industry with **shortages of labor, bottlenecks in distribution, and changes in demand.**
- Covid-19 outbreaks in certain U.S. meat processing facilities led to a meat shortage in many supermarkets, and caused the meat prices to rise 16 percent in May 2020.

Agricultural Supply Chain Networks

- The dynamics in the agriculture industry are very complex, including tight profit margins and competition among stakeholders.
- **The system for creating and sustaining the connections from the agricultural product sources all the way to the consumers is called an agricultural supply chain network.**
- Agricultural supply chains are very intricate local, regional, and global networks, **creating pathways from farms to consumers**.
- The sets of activities in the supply chain include farming/production, processing, storage, transportation, and distribution.

Agricultural Supply Chain Networks

- The agricultural supply chain (ASC) network worldwide is predominantly **multitiered** with various stakeholders such as **the agricultural firms, processing firms, distribution firms, food-service firms, and the hotels and restaurants, grocers, and retail organizations.**
- Much of the value added in the agricultural supply chain occurs at the processing stage where margins range between 10-20 percent usually.
- The food processing sector can be categorized based on the product; for example, whether meat, dairy, beverages, sugar, snacks, etc.

Our Research

1. Develops a multitiered competitive agricultural supply chain network model with agricultural firms (AFs) and processing firms (PFs).
2. Captures competition among agricultural firms and processing firms via game theory where the governing Cournot-Nash equilibrium conditions correspond to a variational inequality problem.
3. Applies an algorithmic scheme, which yields closed-form expressions at each iteration for the strategic variables.
4. Our modeling framework is illustrated through a numerical study consisting of several supply chain disruption scenarios.

Literature Review

- **Agricultural Supply Chains:**

  Alocilja and Ritchie (1990), Apaiah and Hendrix (2005), Biswas and Pal (2005), Jiao et al. (2005), Ahumada and Villalobos (2009), Borodin et al. (2016), Banasik et al (2017), Routroy and Behera (2017), Banasik et al. (2019), Liu et al (2021).

- **Challenges: Disruptions, Quality, and Perishability:**

  Rong, Akkerman, and Grunow (2011), Yu and Nagurney (2013), Aung and Chang (2014), Besik and Nagurney (2017), Behzadi et al. (2018), Hobbs (2020), Sharma et al. (2020), Lejarza and Baldea (2021), Nagurney (2021a), Nagurney (2021b).

- **Multitiered Supply Chain Models:**

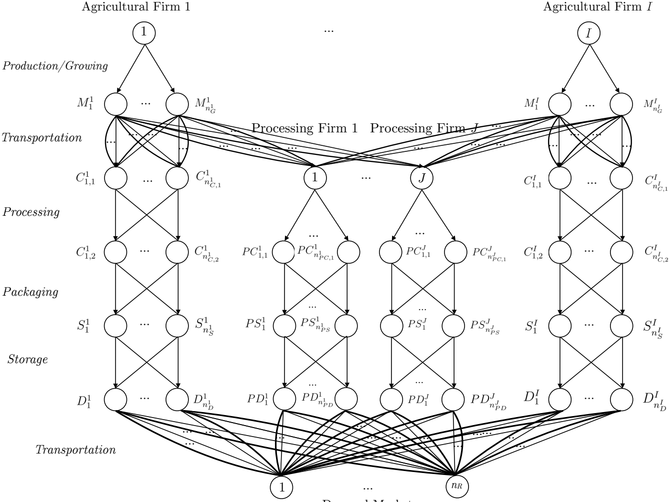
  Nagurney, Dong, and Zhang (2002), Dong et al. (2005), Yamada et al. (2011), Nagurney (2018), Saberi et al. (2018), Gossler et al. (2019), Dutta and Nagurney (2020), Taghikhah et al. (2021).

Our Contributions

- We study the economic implications of the integrated multitiered ASC networks.
- We have demand price functions that include the product flows from processing firms and the product flows from the agricultural firms.
- The mathematical models, and the computational study also address:
  1. Implications of having an integrated multitiered ASC network system;
  2. Ramifications of supply chain disruptions in the Covid-19 pandemic;
  3. Trade-offs for agricultural firms.

The Integrated Multitiered ASC Network Topology

Figure 1: Multitiered Food Supply Chain Network Topology



The Integrated Multitiered ASC Network Model

- Agricultural firms also have the option to sell their unprocessed agricultural products to processing firms as well as directly at the demand markets.
- We capture competition in both vertically integrated supply chain networks of agricultural firms as well as the multitiered ones.
- We look at the behavior of processing firms who purchase unprocessed agricultural products from agricultural firms to sell at demand markets while competing noncooperatively with agricultural firms and other processing firms.
- We construct the mathematical model for agricultural products that are in the category of minimally processed foods, which require little alteration during the processing operation.

Preliminary Notation

Notation	Definition
$x_{p_{AF}}$	The nonnegative flow of the agricultural product on a path p_{AF} sent from an agricultural firm i to a demand market k.
q_{ij}	The nonnegative amount of agricultural firm i's product shipment from its production site to processing firm j through the link set $\hat{L}_1^j$.
ρ^*_{1ij}	The price that the processing firm j is willing to pay for the agricultural product shipment of agricultural firm i.
$x_{p_{PF}}$	The nonnegative flow of agricultural product on a path p_{PF} sent from a processing firm j to a demand market k.
d^{AF}_{ik}	The demand for agricultural firm i's product at demand market k. We group all d^{AF}_{ik} elements into the vector $d^{AF} \in R^{In_R}_+$.
d^{PF}_{jk}	The demand for processing firm j's product at demand market k. We group all d_{jk} elements into the vector $d^{PF} \in R^{Jn_R}_+$.
$\rho^{AF}_{ik}(d^{AF}, d^{PF})$	The demand price function for agricultural firm i's product at demand market k.
$\rho^{PF}_{jk}(d^{PF}, d^{AF})$	The demand price function for processing firm j's product at demand market k.

Link Sets

- Two main link sets for agricultural firms and processing firms: L and $\hat{L}$.
- For each agricultural firm i, we have $L^i = L_1^i \cup L_2^i$

  L_1^i: set of links associated with the production/growing links of agricultural firm i.

  L_2^i: contains all the remaining supply chain operations of agricultural firm i excluding the shipment links of agricultural firm i's agricultural products from its production/growing facilities to processing firms.
- For each processing firm j, we have $\hat{L}^j = \hat{L}_1^j \cup \hat{L}_2^j$

  $\hat{L}_1^j$: set of links associated with the shipment of agricultural products from the agricultural firms' production sites to processing firm j.

  $\hat{L}_2^j$: contains all the remaining supply chain operations of processing firm j.

Behavior of the Agricultural Firms

Nonnegative Path Flows

For each path, p_{AF}, corresponding to an agricultural firm, the following nonnegativity condition must hold:

$$x_{p_{AF}} \geq 0, \quad \forall p_{AF} \in P_k^i;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R. \tag{1}$$

Demand

The demand at the demand market k for the agricultural product of agricultural firm $i; i = 1, \ldots, I$ is given by:

$$d_{ik}^{AF} = \sum_{p_{AF} \in P_k^i} x_{p_{AF}}, \quad k = 1, \ldots, n_R. \tag{2}$$

Nonnegative Product Shipments

The amount of product shipment, q_{ij}, from an agricultural firm i to a processing firm j through shipment links in $\hat{L}_1^j$ must be nonnegative, that is:

$$q_{ij} \geq 0, \quad i = 1, \ldots, I;\ j = 1, \ldots, J. \tag{3}$$

Conservation of Flow Equations I

The following conservation of flow equations must hold for each agricultural firm i; $i = 1, \ldots, I$:

$$f_a = \sum_{k=1}^{n_R} \sum_{p_{AF} \in P_k^i} x_{p_{AF}} \delta_{ap_{AF}} + \sum_{j=1}^{J} q_{ij}, \quad \forall a \in L_1^i;\ i = 1, \ldots, I. \tag{4}$$

Here $\delta_{ap_{AF}}$ is equal to 1 if the link a is included in the path p_{AF}, and 0, otherwise.

Behavior of the Agricultural Firms

Total Operational Link Cost Functions for L_1^i

The total operational cost on link a, in general, is a function of all the flows in vector $f^1 \in R_+^{n_{L_1}}$, and we have that:

$$\hat{h}_a = \hat{h}_a(f^1), \quad \forall a \in L_1^i;\ i = 1, \ldots, I. \tag{5}$$

Conservation of Flow Equations II

Let f_b denote the flow of agricultural product on link b, $\forall b \in L_2^i$, in which the following conservation of flow equations must hold for each agricultural firm i; $i = 1, \ldots, I$:

$$f_b = \sum_{k=1}^{n_R} \sum_{p_{AF} \in P_k^i} x_{p_{AF}} \delta_{bp_{AF}}, \quad \forall b \in L_2^i;\ i = 1, \ldots, I. \tag{6}$$

where $\delta_{bp_{AF}}$ is equal to 1 if the link b is included in the path p_{AF}, and 0, otherwise.

Total Operational Link Cost Functions for L_2^i

The operational cost functions associated with the remaining links in the supply chain of the agricultural firm i are as follows:

$$\hat{c}_b = \hat{c}_b(f^2), \quad \forall b \in L_2^i;\ i = 1, \ldots, I. \tag{7}$$

The Profit Functions of the Agricultural Firms

The Profit Functions of the Agricultural Firms

The profit function of agricultural firm i; $i = 1, \ldots, I$, is the difference between its revenue and its total costs, where the total costs are the total operational costs over L^i. The profit/utility function of agricultural firm i, denoted by U_i^{AF}, is given by:

$$U_i^{AF} = \sum_{k=1}^{n_R} \rho_{ik}^{AF}(d^{AF}, d^{PF}) d_{ik}^{AF} + \sum_{j=1}^{J} \rho_{1ij}^{*} q_{ij} - \sum_{a \in L_1^i} \hat{h}_a(f^1) - \sum_{b \in L_2^i} \hat{c}_b(f^2). \quad (8)$$

Game Theory Framework: Cournot-Nash Equilibrium

Definition 1: A Cournot-Nash Equilibrium of the Agricultural Firms

Agricultural product path flows from agricultural firms to the demand markets, and agricultural product shipments from agricultural firms to processing firms $(X^{AF*}, q^*) \in R_+^{n_P+IJ}$ *are said to constitute a Cournot-Nash equilibrium if for each agricultural firm* i; $i = 1, \ldots, I$,

$$U_i^{AF}(X_i^{AF*}, \hat{X}_i^{AF*}, q_i^*, \hat{q}_i^*, X^{PF*}) \geq U_i^{AF}(X_i^{AF}, \hat{X}_i^{AF*}, q_i, \hat{q}_i^*, X^{PF*}), \quad \forall (X^{AF}, q) \in R_+^{n_P+IJ}, \tag{9}$$

where

$$\hat{X}_i^{AF*} \equiv (X_1^{AF*}, \ldots, X_{i-1}^{AF*}, X_{i+1}^{AF*}, \ldots, X_I^{AF*}) \quad \textit{and} \quad \hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_I^*).$$

A Cournot-Nash equilibrium is established if no agricultural firm can unilaterally improve upon its profit by selecting an alternative vector of agricultural product path flows to demand markets, and product shipments from agricultural firms to processing firms.

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation of Cournot-Nash Equilibrium Conditions

Assume that for each agricultural firm i; $i = 1, \ldots, I$, the utility function $U_i^{AF}(X^{AF}, q)$ is concave with respect to its variables X_i^{AF} and q_i, and is continuously differentiable. Then $(X^{AF*}, q^*) \in R_+^{n_P+IJ}$ is a Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I}\langle \nabla_{X_i^{AF}} \hat{U}_i^{AF}(X^{AF*}, q^*), X_i^{AF} - X_i^{AF*}\rangle - \sum_{i=1}^{I}\langle \nabla_{q_i} \hat{U}_i^{AF}(X^{AF*}, q^*), q_i - q_i^*\rangle \geq 0,$$

$$\forall (X^{AF}, q) \in R_+^{n_P+IJ}. \tag{10}$$

Here, $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space where $\nabla_{X_i^{AF}} \hat{U}_i^{AF}$ (X^{AF}, q) denotes the gradient of $\hat{U}_i^{AF}(X^{AF}, q)$ with respect to X_i^{AF}, and $\nabla_{q_i} \hat{U}_i^{AF}(X^{AF}, q)$ denotes the gradient of $\hat{U}_i^{AF}(X^{AF}, q)$ with respect to q_i.

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation of Cournot-Nash Equilibrium Conditions

Variational inequality (10) in turn, is equivalent to the variational inequality that determines the vector of equilibrium agricultural product path flows, and product shipments from agricultural firms to processing firms, $(X^{AF*}, q^*) \in R_+^{n_P+IJ}$, such that:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(x^{AF*},q^*)}{\partial x_{p_{AF}}}+\frac{\partial\hat{C}_{p_{AF}}(x^{AF*})}{\partial x_{p_{AF}}}-\hat{\rho}_{ik}^{AF}(x^{AF*},x^{PF*})-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}_{il}^{AF}(x^{AF*},x^{PF*})}{\partial x_{p_{AF}}}\sum_{r_{AF}\in P_k^i}x_{r_{AF}}^*\right]\times[x_{p_{AF}}-x_{p_{AF}}^*]$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(x^{AF*},q^*)}{\partial q_{ij}}-\rho_{1ij}^*\right]\times[q_{ij}-q_{ij}^*]\geq 0,\quad \forall (X^{AF},q)\in R_+^{n_P+IJ}. \tag{11}$$

Furthermore, for each p_{AF}; $p_F \in P_k^i$; $i = 1, \ldots, I$; $k = 1, \ldots, n_R$; $l = 1, \ldots, n_R$,

$$\frac{\partial\hat{C}_{p_{AF}}(x)}{\partial x_{p_{AF}}}\equiv\sum_{b\in L_2^i}\sum_{d\in L_2^i}\frac{\partial\hat{c}_d(f^2)}{\partial f_b}\delta_{bp_{AF}}, \text{and } \frac{\partial\hat{\rho}_{il}^{AF}(x^{AF},x^{PF})}{\partial x_{p_{AF}}}\equiv\frac{\partial\rho_{il}^{AF}(d^{AF},d^{PF})}{\partial d_{ik}^{AF}}. \tag{12}$$

Behavior of the Processing Firms

Nonnegative Path Flows

We have the nonnegativity condition for the path flows from processing firms to the demand markets, where the path p_{PF} consists of a sequence of links that originates from the node corresponding to the processing firm $j; j = 1, \ldots, J$ and terminates in a demand market node:

$$x_{p_{PF}} \geq 0, \quad \forall p_{PF} \in P_k^j;\ j = 1, \ldots, J;\ k = 1, \ldots, n_R. \tag{13}$$

Demand

The demand at the demand market k for the agricultural product of processing firm j must satisfy:

$$d_{jk}^{PF} = \sum_{p_{PF} \in P_k^j} x_{p_{PF}}, \quad j = 1, \ldots, J;\ k = 1, \ldots, n_R. \tag{14}$$

Relationship between Product Shipments and Path Flows

The following condition must be satisfied for each $j; j = 1, \ldots, J$:

$$\sum_{i=1}^{I} q_{ij} \geq \sum_{k=1}^{n_R} \sum_{p_{PF} \in P_k^j} x_{p_{PF}}. \tag{15}$$

Behavior of the Agricultural Firms

Total Operational Link Cost Functions for $\hat{L}_1^j$

The operational cost function related to shipment of agricultural products from the agricultural firms to processing firm j in the supply chain network are:

$$\hat{z}_b = \hat{z}_b(q), \quad \forall b \in \hat{L}_1^j;\ j = 1, \ldots, J. \tag{16}$$

Conservation of Flow Equations III

The following conservation of flow equations must hold for each processing firm j; $j = 1, \ldots, , J$:

$$f_e = \sum_{k=1}^{n_R} \sum_{p_{PF} \in P_k^j} x_{p_{PF}} \delta_{e_{p_{PF}}}, \quad \forall e \in \hat{L}_2^j;\ j = 1, \ldots, J, \tag{17}$$

where $\delta_{bp_{AF}}$ is equal to 1 if the link b is included in the path p_{AF}, and 0, otherwise.

Total Operational Link Cost Functions for $\hat{L}_2^j$

The remaining total operational cost functions for the supply supply chain operations of the processing firms, in general, are a function of all the flows in the vector $f^3 \in R_+^{n_{\hat{L}_2}}$. Hence, we have that:

$$\hat{c}_e = \hat{c}_e(f^3), \quad \forall e \in \hat{L}_2^j;\ j = 1, \ldots, J. \tag{18}$$

The Profit Functions of the Processing Firms

The Profit Functions of the Processing Firms

The utility/profit, U_j^{PF}, of processing firm $j; j = 1, \dots, J$, is the difference between its revenue and its costs, including the payouts to the AFs, where each processing firm j pays a price of ρ^*_{1ij} to agricultural firm i for its agricultural product shipment q_{ij}, that is:

$$U_j^{PF} = \sum_{k=1}^{n_R} \hat{\rho}_{jk}^{PF}(x^{PF}, x^{AF}) \sum_{p_{PF} \in P_k^j} x_{p_{PF}} - \sum_{i=1}^{I} \rho^*_{1ij} q_{ij} - \sum_{b \in \hat{L}_1^j} \hat{z}_b(q) - \sum_{e \in \hat{L}_2^j} \bar{\hat{c}}_e(x^{PF}). \quad (19)$$

Game Theory Framework: Cournot-Nash Equilibrium

Definition 2: A Cournot-Nash Equilibrium of Processing Firms

Agricultural product path flows from processing firms to demand markets, and product shipments from agricultural firms to processing firms, $(X^{PF*}, q^*) \in R_+^{n_{\hat{P}}+IJ}$, are said to constitute a Cournot-Nash equilibrium if for each processing firm j; $j = 1, \ldots, J$,

$$U_j^{PF}(X_j^{PF*}, \hat{X}_j^{PF*}, q_j^*, \hat{q}_j^*, X^{AF*}) \geq U_j^{PF}(X_j^{PF}, \hat{X}_j^{PF*}, q_j, \hat{q}_j^*, X^{AF*}), \quad \forall (X^{PF}, q) \in R_+^{n_{\hat{P}}+IJ}, \tag{20}$$

where

$$\hat{X}_j^{PF*} \equiv (X_1^{PF*}, \ldots, X_{j-1}^{PF*}, X_{j+1}^{PF*}, \ldots, X_J^{PF*}) \quad \textit{and} \quad \hat{q}_j^* \equiv (q_1^*, \ldots, q_{j-1}^*, q_{j+1}^*, \ldots, q_J^*).$$

A Cournot-Nash equilibrium is established if no processing firm can unilaterally improve upon its profit by selecting an alternative vector of agricultural product path flows to demand markets, and agricultural product shipments from agricultural firms to processing firms.

Variational Inequality Formulation of Processing Firms

Theorem 2: Variational Inequality Formulation of Cournot-Nash Equilibrium Conditions

Assume that for each processing firm j; $j = 1, \ldots, J$, the utility function $U_j^{PF}(X^{PF}, q)$ is concave with respect to its variables Q_j^{PF} and q_j, and is continuously differentiable. Then $(x^{PF*}, q^*) \in R_+^{n_{\hat{P}}+IJ}$ is a Cournot-Nash equilibrium according to Definition 2 if and only if it satisfies the variational inequality:

$$-\sum_{j=1}^{J} \langle \nabla_{X_j^{PF}} \hat{U}_j^{PF}(X^{PF*}, q^*), X^{PF} - X_i^{PF*} \rangle - \sum_{j=1}^{J} \langle \nabla_{q_j} \hat{U}_j^{PF}(X^{PF*}, q^*), q_j - q_j^* \rangle \geq 0,$$

$$\forall (X^{PF}, q) \in R_+^{n_{\hat{P}}+IJ}, \tag{21}$$

where $\nabla_{X_j^{PF}} \hat{U}_j^{PF}(X^{PF}, q)$ denotes the gradient of $\hat{U}_j^{PF}(X^{PF}, q)$ with respect to X_j^{PF}, and $\nabla_{q_j} \hat{U}_j^{PF}\ (X^{PF*}, q)$ denotes the gradient of $\hat{U}_j^{PF}(X^{PF}, q)$ with respect to q_j.

Variational Inequality Formulation of Processing Firms

Theorem 2: Variational Inequality Formulation of Cournot-Nash Equilibrium Conditions

We can reformulate variational inequality (21) in terms of agricultural product path flows, product shipments from agricultural firms to processing firms, and the Lagrange multipliers as:

$$\sum_{j=1}^{J}\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}\left[\frac{\partial\hat{C}_{p_{PF}}(x^{PF*})}{\partial x_{p_{PF}}}-\bar{\hat{\rho}}_{jk}^{PF}(x^{PF*},x^{AF*})-\sum_{h=1}^{n_R}\frac{\partial\bar{\hat{\rho}}_{jh}^{PF}(x^{PF*},x^{AF*})}{\partial x_{p_{PF}}}\sum_{s_{PF}\in P_k^j}x_{s_{PF}}^*+\eta_j^*\right]\times[x_{p_{PF}}-x_{p_{PF}}^*]$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{I}\left[\frac{\sum_{b\in\hat{L}_1^j}\partial\hat{z}_b(q^*)}{\partial q_{ij}}+\rho_{1ij}^*-\eta_j^*\right]\times[q_{ij}-q_{ij}^*]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}q_{ij}^*-\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}x_{p_{PF}}^*\right]\times[\eta_j-\eta_j^*]\geq 0,\quad\forall(x^{PF},q,\eta)\in R_+^{n_{\hat{P}}+IJ+J}. \tag{22}$$

Further, for each p_{PF}; $p_{PF}\in P_k^j$; $j=1,\ldots,J$; $k=1,\ldots,n_R$; $h=1,\ldots,n_R$:

$$\frac{\partial\hat{C}_{p_{PF}}(x^{PF})}{\partial x_{p_{PF}}}\equiv\sum_{e\in\hat{L}_2^j}\sum_{g\in\hat{L}_2^j}\frac{\partial\hat{c}_g(f^3)}{\partial f_e}\delta_{ep_{PF}},\ \text{and}\ \frac{\partial\bar{\hat{\rho}}_{jh}^{PF}(x^{PF},x^{AF})}{\partial x_{p_{PF}}}\equiv\frac{\partial\rho_{jh}^{PF}(d^{PF},d^{AF})}{\partial d_{jk}^{PF}}. \tag{23}$$

The Equilibrium Conditions for the Integrated Multitiered Supply Chain Network with AFs and PFs

Definition 3: Integrated Multitiered Supply Chain Network with AFs and PFs

The equilibrium state of the integrated multitiered agricultural supply chain network with agricultural firms and processing firms is one where both variational inequalities (13) and (22) hold simultaneously.

Variational Inequality Formulation of the Equilibrium Conditions

Theorem 3: Variational Inequality Formulation of the Equilibrium Conditions

The equilibrium conditions governing the integrated multitiered agricultural supply chain network model with agricultural firms and processing firms are equivalent to the solution of the variational inequality problem: determine $(x^{AF*}, q^*, x^{PF*}, \eta^*) \in R_+^{n_P+IJ+n_{\hat{P}}+J}$*, such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(x^{AF*},q^*)}{\partial x_{p_{AF}}}+\frac{\partial\hat{C}_{p_{AF}}(x^{AF*})}{\partial x_{p_{AF}}}-\hat{\rho}_{ik}^{AF}(x^{AF*},x^{PF*})-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}_{il}^{AF}(x^{AF*},x^{PF*})}{\partial x_{p_{AF}}}\sum_{r_{AF}\in P_k^i}x_{r_{AF}}^*\right]\times[x_{p_{AF}}-x_{p_{AF}}^*]$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(x^{AF*},q^*)}{\partial q_{ij}}+\frac{\sum_{b\in\hat{L}_2^j}\partial\bar{\hat{z}}_b(q^*)}{\partial q_{ij}}-\eta_j^*\right]\times[q_{ij}-q_{ij}^*]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}\left[\frac{\partial\hat{C}_{p_{PF}}(x^{PF*})}{\partial x_{p_{PF}}}-\bar{\hat{\rho}}_{jk}^{PF}(x^{PF*},x^{AF*})-\sum_{h=1}^{n_R}\frac{\partial\bar{\hat{\rho}}_{jh}^{PF}(x^{PF*},x^{AF*})}{\partial x_{p_{PF}}}\sum_{s_{PF}\in P_k^j}x_{s_{PF}}^*+\eta_j^*\right]\times[x_{p_{PF}}-x_{p_{PF}}^*]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}q_{ij}^*-\sum_{k=1}^{n_R}\sum_{p_{PF}\in P_k^j}x_{p_{PF}}^*\right]\times[\eta_j-\eta_j^*]\geq 0,\quad\forall(x^{AF},q,x^{PF},\eta)\in R_+^{n_P+IJ+n_{\hat{P}}+J}. \qquad (24)$$

The Algorithm: The Euler Method

- The Euler method is a discrete-time algorithm that captures the dynamics of the integrated multitiered model by employing an iterative scheme of Dupuis and Nagurney(1993).
- Specifically, at an iteration $t+1$ of the Euler method (see also Nagurney and Zhang (1996)) one computes:

$$X^{t+1} = P_{\mathcal{K}}(X^t - \alpha^t F(X^t)), \tag{25}$$

  where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (24).
- As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{\alpha^t\}$ must satisfy: $\sum_{t=0}^{\infty} \alpha^t = \infty$, $\alpha^t > 0$, $\alpha^t \to 0$, as $t \to \infty$.
- Specific conditions for convergence of this scheme as well as various applications to the solutions of network oligopolies can be found in Nagurney, Dupuis, and Zhang (1994), Nagurney and Zhang (1996), Nagurney (2010), Nagurney and Yu (2012), and Masoumi, Yu, and Nagurney (2012).

Explicit Formulae for the Euler Method

Path Flows of AFs

For each path $p_{AF} \in P_k^i$ compute:

$$x_{p_{AF}}^{t+1} = max\{0, x_{p_{AF}}^t + \alpha^t(\hat{\rho}_{ik}^{AF}(x^{AF^t}, x^{PF^t}) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}^{AF}(x^{AF^t}, x^{PF^t})}{\partial x_{p_{AF}}} \sum_{r_{AF} \in P_k^i} x_{r_{AF}}^t -$$

$$\frac{\sum_{a \in L_1^i} \partial \bar{\hat{h}}_a(x^{AF^t}, q^t)}{\partial x_{p_{AF}}} - \frac{\partial \hat{C}_{p_{AF}}(x^{AF^t})}{\partial x_{p_{AF}}})\}, \ \forall p_{AF} \in P_k^i; \ i = 1, \ldots, I; \ k = 1, \ldots, n_R. \quad (26)$$

Product Shipments

For each product shipment from the production facility of agricultural firm i to the processing firm j compute:

$$q_{ij}^{t+1} = max\{0, q_{ij}^{t+1} + \alpha^t(\eta_j - \frac{\sum_{a \in L_1^i} \partial \bar{\hat{h}}_a(x^{AF^t}, q^t)}{\partial q_{ij}} - \frac{\sum_{b \in \hat{L}_2^j} \partial \bar{\hat{z}}_b(q^t)}{\partial q_{ij}})\},$$

$$\forall i, j; \ i = 1, \ldots, I; \ j = 1, \ldots, J. \quad (27)$$

Explicit Formulae for the Euler Method

Path Flows of PFs

We also have the following closed form expressions for each path from the processing firms to the demand markets $p_{PF} \in P_k^j$: compute:

$$x_{p_{PF}}^{t+1} = max\{0, x_{p_{PF}}^t + \alpha^t(\hat{\rho}_{jk}^{PF}(x^{PF^t}, x^{AF^t}) + \sum_{h=1}^{n_R} \frac{\partial \bar{\hat{\rho}}_{jh}^{PF}(x^{PF^t}, x^{AF^t})}{\partial x_{p_{PF}}} \sum_{s_{PF} \in P_k^j} x_{s_{PF}}^t - \frac{\partial \hat{C}_{p_{PF}}(x^{PF^t})}{\partial x_{p_{PF}}}$$

$$-\eta_j^t)\}, \ \forall p_{PF} \in P_k^j; \ j = 1, \ldots, J; \ k = 1, \ldots, n_R. \quad (28)$$

Lagrange Multipliers

Finally, the closed form expressions for the Lagrange multipliers:

$$\eta_j^{t+1} = max\{0, \eta_j^t + \alpha^t(\sum_{k=1}^{n_R} \sum_{p_{PF} \in P_k^j} x_{p_{PF}}^t - \sum_{i=1}^{I} q_{ij}^t)\}, \ \forall j; \ j = 1, \ldots, J. \quad (29)$$

Numerical Study

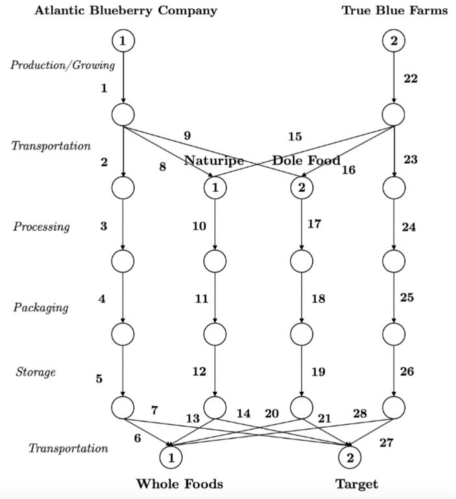
- We focus on frozen blueberry supply chains in the United States.
- Blueberries are available at various demand markets in the U.S. because of their popularity and high demand.
- The U.S. is the largest producer of blueberries, with a total of 309 thousand tons of production volume in 2019.

Numerical Study

We look at the interplay between blueberry farms, and processing firms selling their frozen blueberries at demand markets.

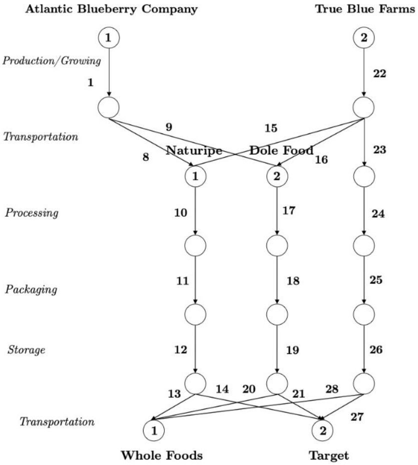
1. **Scenario 1:** Baseline Case Without any Supply Chain Disruptions
2. **Scenario 2:** Supply Chain Disruption I
3. **Scenario 3:** Supply Chain Disruption II
4. **Scenario 4:** Supply Chain Disruption III

Scenario 1: Baseline Case Without any Supply Chain Disruptions



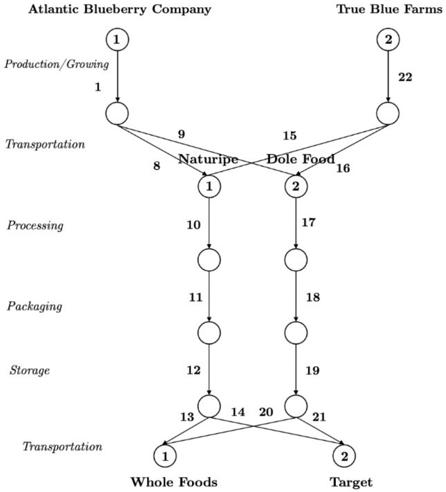
Scenario 2: Supply Chain Disruption I

Atlantic Blueberry Company is selling only to the processing firms, due to a supply chain disruption such as labor shortages as a consequence of the Covid-19 pandemic.



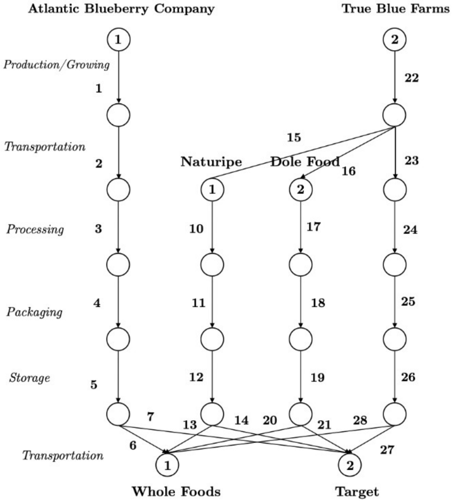
Scenario 3: Supply Chain Disruption II

Our goal is to explore the possible consequences of the Covid-19 pandemic, such as the shift in demand markets due to supply chain disruptions, for all the agricultural firms.



Scenario 4: Supply Chain Disruption III

This scenario is generated to demonstrate the impacts of transportation disruptions on agricultural firms and processing firms as a consequence of the Covid-19 pandemic.



Computed Equilibrium Solutions for Scenarios 1, 2, 3 and 4

Strategic Variables	**Scenario 1**	**Scenario 2**	**Scenario 3**	**Scenario 4**
$x^*_{p_{1AF}}$	30.23	0.00	0.00	31.07
$x^*_{p_{2AF}}$	20.75	0.00	0.00	21.85
$x^*_{p_{3AF}}$	32.29	33.55	0.00	31.86
$x^*_{p_{4AF}}$	23.49	23.28	0.00	22.91
$x^*_{p_{1PF}}$	85.29	87.94	91.05	80.10
$x^*_{p_{2PF}}$	73.76	75.26	76.90	68.56
$x^*_{p_{3PF}}$	87.93	90.58	93.58	84.41
$x^*_{p_{4PF}}$	76.40	77.90	79.44	72.86
q^*_{11}	80.01	88.31	83.77	0.00
q^*_{12}	82.75	91.19	86.91	0.00
q^*_{21}	78.94	74.67	83.78	151.58
q^*_{22}	81.68	77.55	86.91	154.71
η^*_1	0.10	0.08	0.76	1.61
η^*_2	2.48	2.34	0.67	1.04

Computed Demand Prices and Profits for Scenarios 1, 2, 3 and 4

Dem. Pri. & Profits	Scenario 1	Scenario 2	Scenario 3	Scenario 4
ρ_{11}^{AF}	12.19	-	-	12.26
ρ_{12}^{AF}	11.84	-	-	11.92
ρ_{21}^{AF}	13.16	13.37	-	13.25
ρ_{22}^{AF}	12.82	13.00	-	12.91
ρ_{11}^{PF}	8.94	9.12	9.33	9.12
ρ_{12}^{PF}	8.58	8.73	8.90	8.76
ρ_{21}^{PF}	8.88	9.07	9.28	9.04
ρ_{22}^{PF}	8.52	8.67	8.84	8.68
$U_1^{AF}(X^{AF*})$	256.55	18.38	29.13	234.66
$U_2^{AF}(X^{AF*})$	306.12	313.60	29.14	380.38
$U_1^{PF}(X^{PF*})$	779.51	820.97	869.22	717.54
$U_2^{PF}(X^{PF*})$	833.12	877.11	927.94	774.68

Discussion of the Numerical Results

Scenario 1: Without any Supply Chain Disruptions

- Lower demand prices for the frozen blueberries of the processing firms;
- Highest profit earned by Dole Food.

Scenario 2: Supply Chain Disruption I - Atlantic Blueberry Company

- Significant decrease in profits enjoyed by Atlantic Blueberry Company;
- Demand prices increase
- Profit increase for True Blue Farms', Naturipe's, and Dole Food.

Discussion of Numerical Results

Scenario 3: Supply Chain Disruption II - Atlantic Blueberry Company & True Blue Farms

- Increase in demand prices of Naturipe and Dole Food;
- Slight increase in profits for Naturipe and Dole Food;
- Decrease in profits for True Blue Farms.

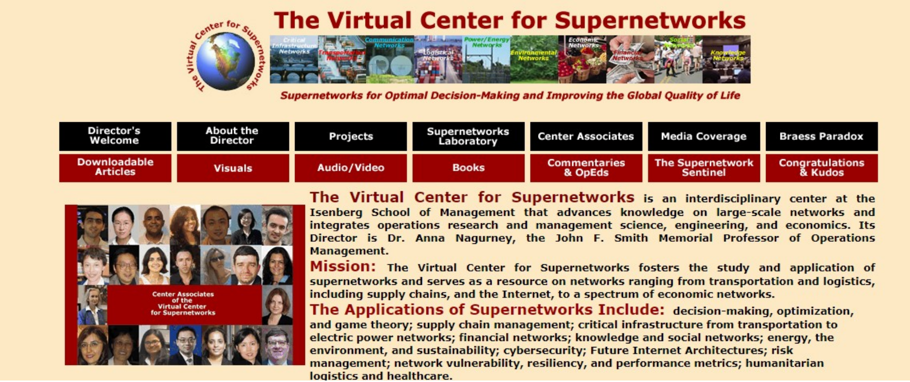
Scenario 4: Supply Chain Disruption III - Transportation Disruption

- True Blue Farms takes advantage of the situation and achieves higher earnings;
- The profits of the processing firms decrease.

Conclusions

- With the Covid-19 pandemic the fragility of the agricultural supply chain became evident.
- **We develop the first integrated multitiered agricultural supply chain network model in which agricultural firms and processing firms compete noncooperatively.**
- We present the variational inequality formulation of the equilibrium conditions and use an algorithmic scheme to obtain equilibrium solutions.
- We test our modeling framework through a numerical study based on frozen blueberry supply chains and analyze several disruption scenarios, including ones related to the Covid-19 pandemic.
- **Our findings show that, when agricultural firms sell their produce only to the processing firms and do not compete at the demand markets, their profits decrease significantly.**

THANK YOU!


The Virtual Center for Supernetworks
Center for Supernetworks
The Virtual
Critical Infrastructure Networks
Transportation Networks
Communication Networks
Logistical Networks
Power/Energy Networks
Environmental Networks
Economic Networks
Financial Networks
Social Networks
Knowledge Networks
Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life
Director's Welcome
About the Director
Projects
Supernetworks Laboratory
Center Associates
Media Coverage
Braess Paradox
Downloadable Articles
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Books
Commentaries & OpEds
The Supernetwork Sentinel
Congratulations & Kudos
Center Associates of the Virtual Center for Supernetworks
The Virtual Center for Supernetworks is an interdisciplinary center at the Isenberg School of Management that advances knowledge on large-scale networks and integrates operations research and management science, engineering, and economics. Its Director is Dr. Anna Nagurney, the John F. Smith Memorial Professor of Operations Management.
Mission: The Virtual Center for Supernetworks fosters the study and application of supernetworks and serves as a resource on networks ranging from transportation and logistics, including supply chains, and the Internet, to a spectrum of economic networks.
The Applications of Supernetworks Include: decision-making, optimization, and game theory; supply chain management; critical infrastructure from transportation to electric power networks; financial networks; knowledge and social networks; energy, the environment, and sustainability; cybersecurity; Future Internet Architectures; risk management; network vulnerability, resiliency, and performance metrics; humanitarian logistics and healthcare.

APPENDIX

Cost Functions

Operations	**Links**	$\hat{h}_a(f^1)$	$\hat{c}_b(f^2)$	$\hat{z}_a(q)$	$\hat{c}_e(f^3)$
production	1	$0.001f_1^2 + 0.02f_1$	-	-	-
transportation	2	-	$0.03f_2^2 + 0.32_2$	-	-
processing	3	-	$0.015f_3^2 + 0.3f_3$	-	-
packaging	4	-	$0.015f_4^2 + 0.3f_4$	-	-
storage	5	-	$0.015f_5^2 + 0.3f_5$	-	-
transportation	6	-	$0.02f_6^2 + 0.2f_6$	-	-
transportation	7	-	$0.02f_7^2 + 0.2f_7$	-	-
transportation	8	-	-	$0.002q_{11}^2 + 0.1q_{11}$	-
transportation	9	-	-	$0.002q_{12}^2 + 0.1q_{12}$	-
processing	10	-	-	-	$0.005f_{10}^2 + 0.01f_{10}$
packaging	11	-	-	-	$0.005f_{11}^2 + 0.03f_{11}$
storage	12	-	-	-	$0.005f_{12}^2 + 0.03f_{12}$
transportation	13	-	-	-	$0.002f_{13}^2 + 0.04_{13}$
transportation	14	-	-	-	$0.002f_{14}^2 + 0.03f_{14}$
transportation	15	-	-	$0.002q_{15}^2 + 0.1q_{15}$	-
transportation	16	-	-	$0.002q_{16}^2 + 0.1q_{16}$	-
processing	17	-	-	-	$0.005f_{17}^2 + 0.01f_{17}$
packaging	18	-	-	-	$0.005f_{18}^2 + 0.03f_{18}$
storage	19	-	-	-	$0.005f_{19}^2 + 0.03f_{19}$
transportation	20	-	-	-	$0.001f_{20}^2 + 0.04f_{20}$
transportation	21	-	-	-	$0.001f_{21}^2 + 0.03f_{21}$
production	22	$0.001f_{22}^2 + 0.02f_{22}$	-	-	-
transportation	23	-	$0.03f_{23}^2 + 0.2f_{23}$	-	-
processing	24	-	$0.015f_{24}^2 + 0.3f_{24}$	-	-
packaging	25	-	$0.015f_{25}^2 + 0.3f_{25}$	-	-
storage	26	-	$0.015f_{26}^2 + 0.3f_{26}$	-	-
transportation	27	-	$0.02f_{27}^2 + 0.2f_{27}$	-	-
transportation	28	-	$0.02f_{28}^2 + 0.2f_{28}$	-	-

Demand Price Functions

Atlantic Blueberry Company:

$$\rho_{11}^{AF}(d^{AF}, d^{PF}) = -0.025d_{11}^{AF} - 0.01d_{21}^{AF} - 0.01d_{11}^{PF} - 0.01d_{21}^{PF} + 15,$$

$$\rho_{12}^{AF}(d^{AF}, d^{PF}) = -0.02d_{12}^{AF} - 0.01d_{22}^{AF} - 0.01d_{12}^{PF} - 0.01d_{22}^{PF} + 14,$$

True Blue Farms:

$$\rho_{21}^{AF}(d^{AF}, d^{PF}) = -0.025d_{21}^{AF} - 0.01d_{11}^{AF} - 0.01d_{11}^{PF} - 0.01d_{21}^{PF} + 16,$$

$$\rho_{22}^{AF}(d^{AF}, d^{PF}) = -0.02d_{22}^{AF} - 0.01d_{12}^{AF} - 0.01d_{12}^{PF} - 0.01d_{22}^{PF} + 15,$$

Naturipe:

$$\rho_{11}^{PF}(d^{PF}, d^{AF}) = -0.03d_{11}^{PF} - 0.01d_{21}^{PF} - 0.01d_{11}^{AF} - 0.01d_{21}^{AF} + 13,$$

$$\rho_{12}^{PF}(d^{PF}, d^{AF}) = -0.03d_{12}^{PF} - 0.01d_{22}^{PF} - 0.01d_{12}^{AF} - 0.01d_{22}^{AF} + 12,$$

Dole Food:

$$\rho_{21}^{PF}(d^{PF}, d^{AF}) = -0.03d_{21}^{PF} - 0.01d_{11}^{PF} - 0.01d_{11}^{AF} - 0.01d_{21}^{AF} + 13,$$

$$\rho_{22}^{PF}(d^{PF}, d^{AF}) = -0.03d_{22}^{PF} - 0.01d_{12}^{PF} - 0.01d_{12}^{AF} - 0.01d_{22}^{AF} + 12.$$

Equilibrium Solutions in Scenario 1

Path p	$x^*_{p_{AF}}$	$x^*_{p_{PF}}$	q^*	η^*
$x^*_{p_{1AF}}$	30.23	-	-	-
$x^*_{p_{2AF}}$	20.75	-	-	-
$x^*_{p_{3AF}}$	32.29	-	-	-
$x^*_{p_{4AF}}$	23.49	-	-	-
$x^*_{p_{1PF}}$	-	85.29	-	-
$x^*_{p_{2PF}}$	-	73.76	-	-
$x^*_{p_{3PF}}$	-	87.93	-	-
$x^*_{p_{4PF}}$	-	76.40	-	-
q^*_{11}	-	-	80.01	-
q^*_{12}	-	-	82.75	-
q^*_{21}	-	-	78.94	-
q^*_{22}	-	-	81.68	-
η^*_1	-	-	-	0.10
η^*_2	-	-	-	2.48

Demand Prices in Scenario 1

- The demand prices of the agricultural firms, and processing firms, in dollars per frozen 2 pounds blueberry pack at the demand markets are:

Atlantic Blueberry Company: $\rho_{11}^{AF} = 12.19, \quad \rho_{12}^{AF} = 11.84,$

True Blue Farms: $\rho_{21}^{AF} = 13.16, \quad \rho_{22}^{AF} = 12.82,$

Naturipe: $\rho_{11}^{PF} = 8.94, \quad \rho_{12}^{PF} = 8.58,$

Dole Food: $\rho_{21}^{PF} = 8.88, \quad \rho_{22}^{PF} = 8.52.$

- Our results show that the demand prices for the frozen blueberries of the processing firms are lower than the agricultural firms' demand prices.

Profits in Scenario 1

- We have $\rho^*_{111} = 0.44$, $\rho^*_{112} = 0.44$, $\rho^*_{121} = 0.45$, and $\rho^*_{122} = 0.45$.
- The profits of the agricultural firms and processing firms are:

$$U_1^{AF}(X^{AF*}) = 256.55, \quad U_2^{AF}(X^{AF*}) = 306.12,$$

$$U_1^{PF}(X^{PF*}) = 779.51, \quad U_2^{PF}(X^{PF*}) = 833.12.$$

- The highest profit is enjoyed by Dole Food in this scenario.
- In contrast, the agricultural firm, Atlantic Blueberry Company, has the lowest profit.

Equilibrium Solutions in Scenario 2

Path p	$x^*_{p_{AF}}$	$x^*_{p_{PF}}$	q^*	η^*
$x^*_{p_{1AF}}$	0.00	-	-	-
$x^*_{p_{2AF}}$	0.00	-	-	-
$x^*_{p_{3AF}}$	33.55	-	-	-
$x^*_{p_{4AF}}$	23.28	-	-	-
$x^*_{p_{1PF}}$	-	87.94	-	-
$x^*_{p_{2PF}}$	-	75.26	-	-
$x^*_{p_{3PF}}$	-	90.58	-	-
$x^*_{p_{4PF}}$	-	77.90	-	-
q^*_{11}	-	-	88.31	-
q^*_{12}	-	-	91.19	-
q^*_{21}	-	-	74.67	-
q^*_{22}	-	-	77.55	-
η^*_1	-	-	-	0.08
η^*_2	-	-	-	2.34

Demand Prices in Scenario 2

- The demand prices of the agricultural firms, and processing firms, in dollars per frozen blueberry pack, at the demand markets are:

$$\textbf{True Blue Farms: } \rho_{21}^{AF} = 13.37, \quad \rho_{22}^{AF} = 13.00,$$

$$\textbf{Naturipe: } \rho_{11}^{PF} = 9.12, \quad \rho_{12}^{PF} = 8.73,$$

$$\textbf{Dole Food: } \rho_{21}^{PF} = 9.07, \quad \rho_{22}^{PF} = 8.67.$$

- The demand prices of True Blue Farms', Naturipe's, and Dole Food's frozen blueberries at Whole Foods and Target increase from their values in Scenario 1.

Profits in Scenario 2

- We find, $\rho^*_{111} = 0.38$, $\rho^*_{112} = 0.38$, $\rho^*_{121} = 0.44$, and $\rho^*_{122} = 0.44$.

- The profits of the agricultural firms and processing firms are:

$$U_1^{AF}(X^{AF*}) = 18.38, \quad U_2^{AF}(X^{AF*}) = 313.60,$$

$$U_1^{PF}(X^{PF*}) = 820.97, \quad U_2^{PF}(X^{PF*}) = 877.11.$$

Equilibrium Solutions in Scenario 2

Path p	$x^*_{p_{AF}}$	$x^*_{p_{PF}}$	q^*	η^*
$x^*_{p_{1AF}}$	0.00	-	-	-
$x^*_{p_{2AF}}$	0.00	-	-	-
$x^*_{p_{3AF}}$	0.00	-	-	-
$x^*_{p_{4AF}}$	0.00	-	-	-
$x^*_{p_{1PF}}$	-	91.05	-	-
$x^*_{p_{2PF}}$	-	76.90	-	-
$x^*_{p_{3PF}}$	-	93.58	-	-
$x^*_{p_{4PF}}$	-	79.44	-	-
q^*_{11}	-	-	83.77	-
q^*_{12}	-	-	86.91	-
q^*_{21}	-	-	83.78	-
q^*_{22}	-	-	86.91	-
η^*_1	-	-	-	0.76
η^*_2	-	-	-	0.67

Demand Prices and Profits in Scenario 3

- the demand prices of processing firms, in dollars per frozen blueberry pack, for the demand markets are computed as:

$$\textbf{Naturipe: } \rho_{11}^{PF} = 9.33, \quad \rho_{12}^{PF} = 8.90,$$

$$\textbf{Dole Food: } \rho_{21}^{PF} = 9.28, \quad \rho_{22}^{PF} = 8.84.$$

- We have, $\rho_{111}^{*} = 0.36$, $\rho_{112}^{*} = 0.36$, $\rho_{121}^{*} = 0.36$, and $\rho_{122}^{*} = 0.36$.
- The profits of the agricultural firms and processing firms are computed as:

$$U_1^{AF}(X^{AF*}) = 29.13, \quad U_2^{AF}(X^{AF*}) = 29.14,$$

$$U_1^{PF}(X^{PF*}) = 869.22, \quad U_2^{PF}(X^{PF*}) = 927.94.$$

Equilibrium Solutions in Scenario 4

Path p	$x^*_{p_{AF}}$	$x^*_{p_{PF}}$	q^*	η^*
$x^*_{p_{1AF}}$	31.07	-	-	-
$x^*_{p_{2AF}}$	21.85	-	-	-
$x^*_{p_{3AF}}$	31.86	-	-	-
$x^*_{p_{4AF}}$	22.91	-	-	-
$x^*_{p_{1PF}}$	-	80.10	-	-
$x^*_{p_{2PF}}$	-	68.56	-	-
$x^*_{p_{3PF}}$	-	84.41	-	-
$x^*_{p_{4PF}}$	-	72.86	-	-
q^*_{11}	-	-	0.00	-
q^*_{12}	-	-	0.00	-
q^*_{21}	-	-	151.58	-
q^*_{22}	-	-	154.71	-
η^*_1	-	-	-	1.61
η^*_2	-	-	-	1.04

Equilibrium Solutions in Scenario 4

- The demand prices of the agricultural firms, and processing firms, in dollars per frozen blueberry pack, for the demand markets are computed as:

$$\textbf{Atlantic Blueberry Company: } \rho_{11}^{AF} = 12.26, \quad \rho_{12}^{AF} = 11.92,$$

$$\textbf{True Blue Farms: } \rho_{21}^{AF} = 13.25, \quad \rho_{22}^{AF} = 12.91,$$

$$\textbf{Naturipe: } \rho_{11}^{PF} = 9.12, \quad \rho_{12}^{PF} = 8.76,$$

$$\textbf{Dole Food: } \rho_{21}^{PF} = 9.04, \quad \rho_{22}^{PF} = 8.68.$$

- We obtained $\rho_{121}^{*} = 0.74$, and $\rho_{122}^{*} = 0.74$. The profits enjoyed by agricultural firms and processing firms are reported as:

$$U_1^{AF}(X^{AF*}) = 234.66 \quad U_2^{AF}(X^{AF*}) = 380.38,$$

$$U_1^{PF}(X^{PF*}) = 717.54, \quad U_2^{PF}(X^{PF*}) = 774.68.$$