Refugee Migration Networks and Regulations: A Multiclass, Multipath Variational Inequality Framework

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Outline of Presentation

- **Motivation and Some Background**
- **Literature Review with a Focus on Networks and Migration**
- **The Refugee Migration Models and Variational Inequality Formulations**
- **Illustrative Examples**
- **Computation of Solutions to Larger Examples**
- **Summary and Conclusions**

Motivation and Some Background

Motivation and Some Background

• **Reasons for human migration are numerous**, from individuals seeking better economic opportunities and enhanced prosperity for themselves and their families, to those fleeing conflict, violence, and persecution. With climate change and the increasing number and severity of natural disasters, including hurricanes, floods, tornados, earthquakes, etc., some migrants are seeking locations of greater expected safety and security.

• **In 2017, the number of international migrants was an estimated 258 million persons (3.4% of the global population)**, with the total number of international migrants increasing by almost 50% since 2000 (United Nations (2017)).

• **The number of international migrants is growing faster than the global population.** The number of refugees and asylum seekers increased from 16 to 26 million, comprising about 10% of the international migrants.

Motivation and Some Background

The United Nations Convention Relating to the Status of Refugees in 1951 defined a refugee as an individual living outside his or her country of nationality, who is unable or unwilling to return because of a well-substantiated fear of persecution due to race, religion, nationality, membership in a political social group, etc.

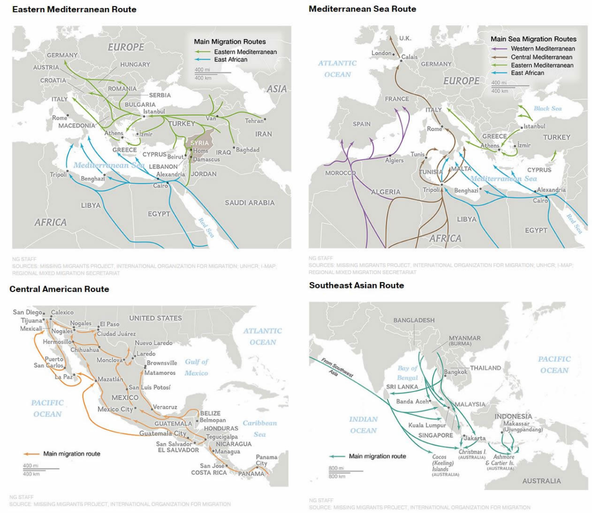
Here we also consider humans adversely affected by climate change, as refugees, and note that, as emphasized by Hebert, Perez, and Harati (2018), among the most studied causes of human migration are climate issues and conflicts, as well as economic reasons.

Motivation and Some Background

Refugees have historically always been part of human migration, seeking locations of greater safety and security for themselves and their families.

Now, with the COVID-19 pandemic, declared by the World Health Organization on March 11, 2020, Dolmans et al. (2020) report that COVID-19 is likely to exacerbate what is already a humanitarian emergency in terms of a global refugee crisis. The authors argue that refugees may encounter increased difficulty in seeking asylum due to measures imposed by governments in response to the pandemic.

Migration Routes



Source: National Geographic via IOM UN Migration Blog - 2015 data

Motivation and Some Background

Vivid depictions of people fleeing their origin locations permeate the news, whether attempting to escape the great strife and suffering in Syria; the violence in parts of Central America, the economic collapse of Venezuela, and even flooding in parts of Asia as well as droughts in parts of Africa.

Motivation and Some Background

At times, refugees will travel in extremely dangerous conditions to escape the dire circumstances at their origin nodes.

In 2015, the UN Refugee Agency reported a maritime refugee crisis with, in the first half of that year, 137,000 refugees crossing the Mediterranean Sea to Europe, via very risky transport modes, and with many more unsuccessfully attempting such a passage. 800 died in the largest refugee shipwreck on record that April.

Motivation and Some Background

Governments of various nations, hence, are increasingly being faced with multiple challenges associated with human migration flows. In response to challenges, they are adopting different regulations.

According to the United Nations (2013), **migration policies in both origin and destination countries play an important role in determining the migratory flows**. In managing international migration flows, governments usually focus on different types of migrants, of which the most salient are highly skilled workers, dependents of migrant workers, irregular migrants, and refugees and asylum seekers (cf. Karagiannis (2016)).

Between 11 March 2020, when the WHO declared COVID-19 a pandemic, and 22 February 2021, nearly 105,000 movement restrictions were implemented around the world, according to the International Organization for Migration.

Motivation and Some Background

We can expect refugee migratory flows to continue to increase, posing a critical need for the provision of rigorous tools for policy-makers and decision-makers for the quantification of refugee migratory flows and the impacts of various regulations. That is the goal of this paper.

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Refugee migration networks and regulations: a multiclass, multipath variational inequality framework

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Abstract
In this paper, we take up the timely topic of the modeling, analysis, and solution of refugee migration networks. We construct a general, multiclass, multipath model, determine the governing equilibrium conditions, and provide alternative variational inequality formulations in path flows and in link flows. We also demonstrate how governmental imposed regulations associated with refugees can be captured via constraints. We provide qualitative properties and then establish, via a supernetwork transformation, that the model(s) are isomorphic to traffic network equilibrium models with fixed demands. Illustrative examples are given, along with numerical examples, inspired by a refugee crisis from Mexico to the United States, which are solved using the Euler method embedded with exact equilibration. The work sets the foundation for the development of additional models and algorithms and also provides insights as to who wins and who loses under certain refugee regulations.

Literature Review with a Focus on Networks and Migration

Literature Review with a Focus on Networks and Migration

- Nagurney (1989) introduced a multiclass migration equilibrium model, which did not include migration/movement costs, and **was isomorphic to a traffic network equilibrium with special structure**. The model was then extended to include flow-dependent migration costs and an expanded set of equilibrium conditions in Nagurney (1990).

- Nagurney, Pan, and Zhao (1992a) proposed **a multiclass human migration model**, which further generalized to include class transformations in Nagurney, Pan, and Zhao (1992b).

- Pan and Nagurney (1994), in turn, considered **chain migration (unlike the earlier work) and introduced a multi-stage (but single class) Markov chain model**. The authors established a connection between a sequence of variational inequalities and a non-homogeneous Markov chain. They also proved that, under certain assumptions, the stability of the one-step transition matrix guarantees the stability of the n-step transition matrix.

Literature Review with a Focus on Networks and Migration

- Pan and Nagurney (2006) utilized the methodology of **evolution variational inequalities for the first time to model the dynamic adjustment of a socio-economic process in the context of human migration**. The question of convergence of algorithms in this framework, which is infinite-dimensional, was also addressed (see also Daniele (2006)).

- Interestingly, many of the network equilibrium models of human migration, as above, have also **found application to the migration of animals in ecology with a focus on fish and maritime ecosystems** (see Mullon and Nagurney (2012), Mullon (2014), Mariani et al. (2016)).

- Kalashnikov et al. (2008) constructed a human migration model with a **conjectural variations equilibrium (CVE)**.

- Capello and Daniele (2019) developed **a Nash equilibrium model of human migration** with features of conjectural variations. The authors also provided a numerical example with sensitivity analysis focusing on the flow of migrants from Africa through the Mediterranean sea to Italy in 2018.

Literature Review with a Focus on Networks and Migration

Nagurney and Daniele (2020) was the first paper to include regulations within a human migration network framework.

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International human migration networks under regulations

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Abstract

International human migration has transformed economies and societies. The new millennium, with climate change and its impacts, and increasing conflicts and displacements, has experienced a great increase in international migrants, with associated challenges faced by governments. In this paper, we advance the modeling, analysis, and solution of international human migration problems by developing a network model with regulations. The formalism uses the theory of variational inequalities, coupled with Lagrange analysis, in order to gain insights as to the impacts of the regulations on utilities of multiple classes of migrants, and on the equilibrium flows. Our results add to the literature on operations research for societal impact, inspired by the real world.

Contributions in Our New Paper

1. An international human migration network model is constructed, which allows for route choices by the migrants, which are refugees.
2. The routes consist of one or more links, with cost functions that capture congestion, a factor that has been seen in practice.
3. The model is then extended to include regulations that can be imposed by distinct multiple countries. In previous work (cf. Nagurney and Daniele (2020)), it was assumed that a single country imposes the regulations on migrants.
4. A supernetwork transformation into a traffic network equilibrium problem with fixed demands is constructed. This identification enables the transfer of algorithmic schemes for the TNE problem, which has had a long history, to the novel application domain of refugee/migration networks.
5. Theoretical results are presented, along with an algorithm, with nice features for computations.
6. Numerical examples illustrate the modeling and algorithmic framework and enable insights for policy-makers and decision-makers.

The Refugee Migration Models and Variational Inequality Formulations

The Multiclass, Multipath Refugee Migration Models

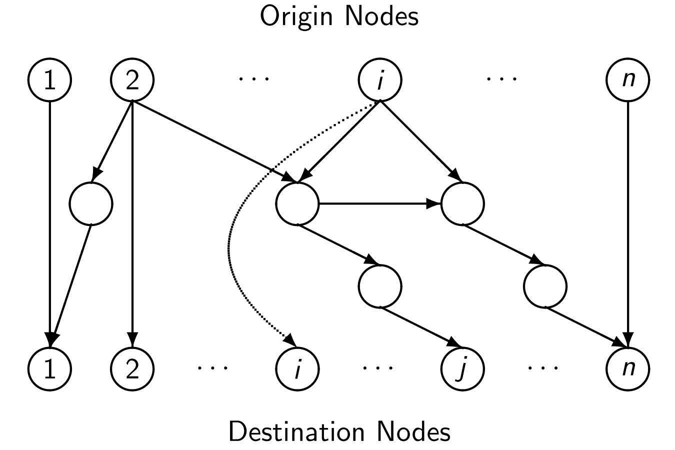


Figure: Sample Refugee Network Topology

Some Notation

Table: Common Notation for the Refugee Migration Models

Notation	Definition
x_r^k	flow of refugees of class k on route/path r. The $\{x_r^k\}$ elements are grouped into vector $x^k \in R_+^{n_P}$, where n_P denotes the number of paths in the migration network. We further group the x^k vectors; $k = 1, \ldots, J$, into vector $x \in R_+^{Jn_P}$.
f_a^k	flow of refugees of class k on link a. We group the link flows for class k for all links $a \in L$ into vector $f^k \in R^{n_L}$ where n_L is the number of links. We then group the link flows for all classes into vector $f \in R^{Jn_P}$.
p_i^k	nonnegative population of refugee class k at origin node i. We group the populations of class k; $k = 1, \ldots, J$, into vector $p^k \in R_+^n$. We further group all such vectors into vector $p \in R_+^{Jn}$.
$\bar{p}_i^k$	initial fixed population of class k at origin node i; $i = 1, \ldots, n$; $k = 1, \ldots, J$.
$u_i^k(p)$	utility perceived by refugee class k at node i; $i = 1, \ldots, n$; $k = 1, \ldots, J$. We group the utility functions for each k into vector $u^k \in R^n$ and then group all such vectors for all k into vector $u \in R^{Jn}$.
$c_a^k(f)$	migration cost associated with traversing link a by refugees of class k. Here we interpret the migration cost as a travel cost. We group link costs for each k into vector $c^k \in R^{n_L}$ and then group all such vectors into vector $c \in R^{Jn_L}$.
$C_r^k(x)$	cost of migration, that is, the travel cost, encumbered by class k in migrating on route r associated with an O/D pair w_{ij}; $i, j = 1, \ldots, n$; $k = 1, \ldots, J$.

Conservation of Flow

Since the route flows must be nonnegative, we have that

$$x_r^k \geq 0, \quad \forall r \in P, \forall k. \tag{1}$$

Furthermore, the refugee flows out of an origin node i must satisfy:

$$\bar{p}_i^k = \sum_{j=1}^{n} \sum_{r \in P_{w_{ij}}} x_r^k, \quad \forall i, \forall k. \tag{2}$$

The volume of population of each class k at each destination node j, after migration takes place, must satisfy the following equation:

$$p_j^k = \sum_{i=1}^{n} \sum_{r \in P_{w_{ij}}} x_r^k, \quad \forall j, \forall k. \tag{3}$$

The link flows are related to the route flows according to:

$$f_a^k = \sum_{r \in P} x_r^k \delta_{ar}, \quad \forall a, \forall k, \tag{4}$$

where $\delta_{ar} = 1$, if link a is contained in route r and 0, otherwise.

Additional Constructs

In view of the conservation of flow equations (4), we may define link cost functions in route/path flows, such that $\hat{c}_a^k = \hat{c}_a^k(x) \equiv c_a^k(f)$, for all links a and for all classes of refugees k.

The cost on a route r is equal to the sum of costs on the links that make up the route, that is,

$$C_r^k(x) = \sum_{a \in L} \hat{c}_a^k(x)\delta_{ar}, \quad \forall k, \forall r. \tag{5}$$

We define the feasible set $K^1 \equiv \{(p,x)|x \geq 0, \text{ and } (2) \text{ and } (3) \text{ hold}\}$.

Equilibrium Conditions for the Multiclass, Multipath Refugee Migration Model without Regulations

Definition 1: Multiclass, Multipath Refugee Migration Equilibrium without Regulations

A vector of populations and refugee migration flows $(p^*, x^*) \in K^1$ is in equilibrium if it satisfies the following conditions: For each class k; $k = 1, \dots, J$, and each pair of origin/destination nodes i, j; $i, j = 1, \dots, n$, and all routes $r \in P_{w_{ij}}$ we have that

$$u_i^k(p^*) + C_r^k(x^*) \begin{cases} = u_j^k(p^*) - \lambda_i^{k*}, & \text{if} \quad x_r^{k*} > 0, \\ \geq u_j^k(p^*) - \lambda_i^{k*}, & \text{if} \quad x_r^{k*} = 0, \end{cases} \tag{6}$$

and

$$\lambda_i^{k*} \begin{cases} \geq 0, & \text{if} \quad \sum_{j=1}^{n} \sum_{r \in P_{w_{ij}}, j \neq i} x_r^{k*} = \bar{p}_i^k, \\ = 0, & \text{if} \quad \sum_{j=1}^{n} \sum_{r \in P_{w_{ij}}, j \neq i} x_r^{k*} < \bar{p}_i^k. \end{cases} \tag{7}$$

Variational Inequality Formlations

Theorem 1: Variational Inequality Formulation of the Refugee Migration Model without Regulations in Path Flows

A population and refugee flow pattern $(p^*, x^*) \in K^1$ *is a refugee migration equilibrium without regulations according to Definition 1, if and only if it satisfies the variational inequality problem in path flows*

$$-\langle u(p^*), p - p^* \rangle + \langle C(x^*), x - x^* \rangle \geq 0, \quad \forall (p, x) \in K^1, \tag{8}$$

where $\langle \cdot, \cdot \rangle$ *denotes the inner product in the appropriately dimensioned Euclidean space.*

Existence of at least one solution to variational inequality (8) follows from the standard theory of variational inequalities (see Kinderlehrer and Stampacchia (1980) Theorem 3.1) under the assumption of continuity of the utility functions u and the migration cost functions c, since the feasible convex set K^1 is compact.

Variational Inequality Formulations

Alternative variational inequality formulations can induce distinct algorithmic schemes. Hence, for completeness, we now provide a link flow variational inequality formulation equivalent to the path flow one in (8). We first define the feasible set $K^2 \equiv \{(p, f) | \exists x \, \text{such that} \, (1) - (4) \, \text{hold}\}$.

Corollary 1: Variational Inequality Formulation of the Refugee Migration Model without Regulations in Link Flows

A population and refugee link flow pattern $(p^*, f^*) \in K^2$ is a refugee migration equilibrium without regulations according to Definition 1, if and only if it satisfies the variational inequality problem in link flows

$$-\langle u(p^*), p - p^* \rangle + \langle c(f^*), f - f^* \rangle \geq 0, \quad \forall (p, f) \in K^2. \tag{9}$$

Variational Inequality Formulation of the Refugee Migration Model with Regulations

We denote a specific country by h, where $h = 1, \ldots, H$ and define the set O^h consisting of origin nodes of refugees from countries/locations that the country h imposes a regulation on, and let D^h denote the set of destination nodes, which lie in country h. C^h denotes the set of refugee classes that country h imposes the regulations on and U^h is the nonnegative upper bound imposed by country h on refugee migratory flows.
The constraints can then be stated as follows:

$$\sum_{i \in O^h} \sum_{j \in D^h} \sum_{k \in C^h} \sum_{r \in P_{w_{ij}}} x_r^k \leq U^h, \quad h = 1, \ldots, H. \tag{10}$$

The set of constraints (10) is sufficiently general to capture specific, distinct migration regulations in practice. such as: an upper bound (which may be zero) of all classes from a certain country or countries; or an upper bound on a single class or several classes from a specific country or countries.

Variational Inequality Formulation of the Refugee Migration Model with Regulations

For the refugee migration model with regulations, the equilibrium conditions (6) and (7) are still relevant but with a new feasible set K^3 defined as below to include the constraints (10):

$$K^3 \equiv K^1 \cap \{x | (10) \text{ is satisfied}\}. \tag{11}$$

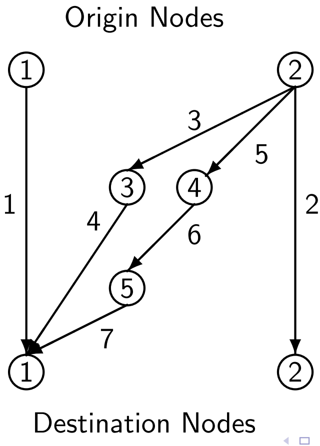
Theorem 2: Variational Inequality Formulation of the Refugee Migration Model with Regulations in Path Flows

A population and refugee migration flow pattern $(p^*, x^*) \in K^3$ is a refugee migration equilibrium with regulations in path flows, if and only if it satisfies the variational inequality problem

$$-\langle u(p^*), p - p^* \rangle + \langle C(x^*), x - x^* \rangle \geq 0, \quad \forall (p, x) \in K^3. \tag{12}$$

Illustrative Examples

According to the network in the Figure below, refugees residing in country 1 are not interested in migrating to country 2. On the other hand, refugees residing in country 2 are interested in the possibility of migrating to country 1. There are two available paths joining country 2 with country 1.



Illustrative Examples

The routes are comprised of links and are enumerated as follows:

$$r_1 = (1), \quad r_2 = (2), \quad r_3 = (3,4), \quad r_4 = (5,6,7).$$

We consider a single class of migrant. Hence, we suppress the superscript 1 in the notation. The data are: $\bar{p}_1 = 100$ and $\bar{p}_2 = 200$ with the utility functions: $u_1(p) = -p_1 + 1000$ and $u_2(p) = -p_2 + 500$. According to these utility functions, country 1 is more attractive to the refugees than country 2, under no population of refugees, because of the term of 1000 in the utility function associated with country 1 as opposed to the fixed term of 500 in the utility function associated with country 2.
The link migration cost functions are:

$$c_1 = c_2 = 0,$$

$$c_3(f) = f_3 + 200, \quad c_4(f) = f_4 + 100,$$

$$c_5(f) = f_5 + 30, \quad c_6(f) = .5f_6 + 40, \quad c_7(f) = f_7 + 30.$$

Illustrative Examples

We first consider the case without regulations. It is easy to compute the equilibrium solution, using simple algebra. Indeed, we find that:

$$x^*_{r_1} = 100, \quad x^*_{r_2} = 75, \quad x^*_{r_3} = 25, \quad x^*_{r_4} = 100;$$

hence,

$$p^*_1 = 225, \quad p^*_2 = 75,$$

with associated utilities being:

$$\hat{u}_1(x^*) = u_1(p^*) = 775, \quad \hat{u}_2(x^*) = u_2(p^*) = 425,$$

and the incurred migration costs on the routes at equilibrium: $C_{r_1} = 0$, $C_{r_2} = 0$, $C_{r_3} = C_{r_4} = 350$. Moreover, $\lambda^*_1 = \lambda^*_2 = 0$.

It is clear that the equilibrium conditions (6) and (7) hold.

Illustrative Examples

Observe that, initially, before the refugee migration takes place, $u(\bar{p}_1) = 900$ and $u_2(\bar{p}_2) = 300$.

Once equilibrium is achieved, those who have migrated from country 2 to country 1 more than double their utility, whereas those who remain in country 2 experience a gain in utility of over 33%.

Those in country 1, because of the increase in the number of refugees and that they are not migrating, suffer a reduction in utility of approximately 14%.

Illustrative Examples

We now suppose that a regulation is imposed on destination node 1 by country 1 of the following form:

$$x_{r_3} + x_{r_4} \leq U^1 = 25.$$

The new equilibrium solution is:

$$x^*_{r_1} = 100, \quad x^*_{r_2} = 175, \quad x^*_{r_3} = 0, \quad x^*_{r_4} = 25;$$

hence,

$$p^*_1 = 125, \quad p^*_2 = 175,$$

with associated utilities being:

$$\hat{u}_1(x^*) = u_1(p^*) = 875, \quad \hat{u}_2(x^*) = u_2(p^*) = 325,$$

and the migration costs on the routes: $C_{r_1} = 0$, $C_{r_2} = 0$, $C_{r_3} = 300$, $C_{r_4} = 162.50$. Moreover, $\lambda^*_1 = \lambda^*_2 = 0$. The optimal Lagrange multiplier associated with the regulation constraint is: $\mu^*_1 = 387.50$. One can see that route r_3 is too expensive and will not be used under the refugee migratory flow pattern.

Illustrative Examples

Now, refugees in country 1, at equilibrium, enjoy a higher utility than before the regulation was imposed of 875 versus 775, an increase of about 13%. On the other hand, refugees in country 2 now experience a lower utility, than before the regulation was imposed. They are no longer all free to migrate because of the imposed regulatory upper bound of 25 limiting the migration from country 2 to country 1.

Under the regulation, those who manage to migrate enjoy a higher utility than before of 425, but those who are left behind in country 2 **experience a lower utility than in the case without regulations – of 325, revealing a drop of about 30%.**

Computation of Solutions to Larger Examples

Computation of Solutions to Larger Examples

Our numerical examples are inspired by the refugee flows from Mexico to the United States, an issue that has been receiving a lot of attention in the press.

The baseline network for the numerical examples is depicted in the next Figure.

The top nodes correspond to origin nodes, representing cities/towns, with the first four nodes being locations in the southwestern United States and the next three top nodes corresponding to locations in Mexico, where refugee origin nodes have been identified.

The bottom nodes in the network in the next Figure are the destination node locations.

We consider a single class of refugee.

Computation of Solutions to Larger Examples

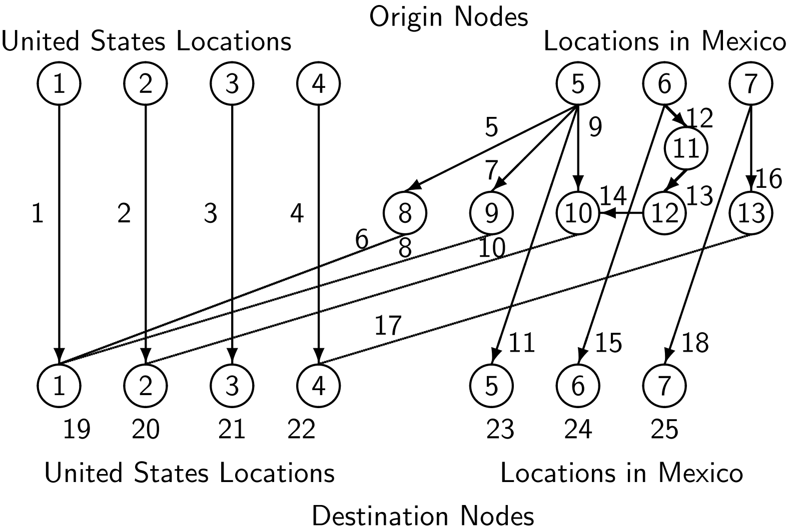


Figure: The Baseline Refugee Network Topology for the Larger Examples

Computation of Solutions to Larger Examples

We consider four examples: Example 1 through Example 4.

Example 1: No Regulations

The initial populations at the origin nodes are:

$$\bar{p}_1 = 1,400,000, \quad \bar{p}_2 = 20,000, \quad \bar{p}_3 = 70,000,$$

$$\bar{p}_4 = 260,000, \quad \bar{p}_5 = 50,000, \quad \bar{p}_6 = 225,000, \quad \bar{p}_7 = 30,000.$$

The utility functions associated with these locations are:

$$u_1(p) = -p_1 + 3,000,000, \; u_2(p) = -2p_2 + 200,000, \; u_3(p) = -p_3 + 1,500,000$$

$$u_4(p) = 3p_4 + 900,000, \; u_5(p) = -p_5 + 100,000, \; u_6(p) = -p_6 + 300,000,$$

$$u_7(p) = -p_7 + 100,000.$$

From the above utility functions, one can see that the locations in the United States are more attractive than those in Mexico, due to the significantly larger fixed utility term in the corresponding utility functions.

Computation of Solutions to Larger Examples

The link costs associated with remaining at one's location at nodes 1 through 7, respectively, are, all equal to 0.00:

$$c_1(f) = c_2(f) = c_3(f) = c_4(f) = c_{11}(f) = c_{15}(f) = c_{18}(f) = 0.00.$$

The costs associated with the refugee migrations are, in turn, as follows:

$$c_5(f) = .00006f_5^4 + 6f_5 + 4f_6 + 200, \; c_6(f) = 7f_6 + 3f_8 + 300,$$

$$c_7(f) = .00008f_7^4 + 8f_7 + 2f_8 + 400,$$

$$c_8(f) = .00004f_8^4 + 5f_8 + 2f_{10} + 450, \; c_9(f) = .00001f_9^4 + 6f_9 + 2f_{10} + 300,$$

$$c_{10}(f) = 4f_{10} + f_{12} + 400,$$

$$c_{12}(f) = 8f_{12} + 2f_{13} + 100, \; c_{13}(f) = .00001f_{13}^4 + 7f_{13} + 3f_9 + 50,$$

$$c_{14}(f) = 8f_{14} + 3f_9 + 100,$$

$$c_{16}(f) = 3f_{16} + f_{12} + 100, \; c_{17}(f) = .00003f_{17} + 3f_{17} + 50.$$

Computation of Solutions to Larger Examples

The routes are enumerated as follows:

$$r_1 = (1), \quad r_2 = (2), \quad r_3 = (3), \quad r_4 = (4),$$

$$r_5 = (5, 6), \quad r_6 = (7, 8), \quad r_7 = (9, 10), \quad r_8 = (11),$$

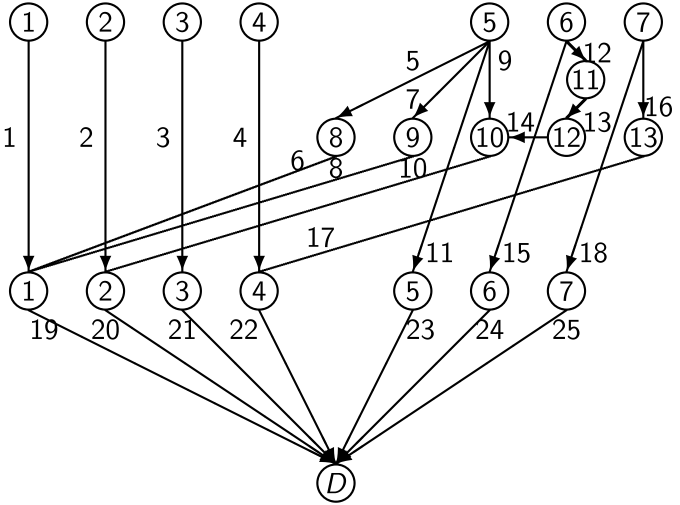
$$r_9 = (12, 13, 14, 10), \quad r_{10} = (15),$$

$$r_{11} = (16, 17), \quad r_{12} = (18).$$

For the solution of the problem, we first construct the supernetwork equivalence with the supernetwork topology as in the Figure and the O/D pairs, the links and paths, the link costs on the new links, the demands, and the path costs, defined accordingly.

Supernetwork Transformation of Examples

Origin Nodes



Super Destination Node

Computation of Solutions to Larger Examples

We implemented the Euler method, embedded with the exact equilibration algorithm as described in Nagurney and Zhang (1997); see also the book by Nagurney and Zhang (1996). We initialized the algorithm as follows. The initial populations were equally distributed among all the paths. The sequence $\{a_\tau\}$ in the Euler method satisfied the conditions required for convergence and was set to: $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$.

The algorithm was deemed to have converged if the absolute value of the difference between each successively computed path flow differed by no more than 10^{-7}. The computer system utilized was a Linux system at the University of Massachusetts Amherst.

Solution to Example 1

The computed equilibrium path flow pattern for Example 1 is:

$$x_{r_1}^* = 1,400,000.00, \; x_{r_2}^* = 20,000.00, \; x_{r_3}^* = 700,000.00, \; x_{r_4}^* = 260,000.00,$$

$$x_{r_5}^* = 400.25, \quad x_{r_6}^* = 336.64, \quad x_{r_7}^* = 317.17, \quad x_{r_8}^* = 48,945.95,$$

$$x_{r_9}^* = 290.25, \quad x_{r_{10}}^* = 224,709.75, \quad x_{r_{11}}^* = 199.54, \quad x_{r_{12}}^* = 29,800.46,$$

which corresponds to the following equilibrium link flow pattern:

$$f_1^* = 1,400,000.00, \; f_2^* = 20,000.00, \; f_3^* = 700,000.00, \; f_4^* = 260,000.00,$$

$$f_5^* = 400.25, \quad f_6^* = 400.25, \quad f_7^* = 336.64, \quad f_8^* = 336.64,$$

$$f_9^* = 317.17, \quad f_{10}^* = 607.41, \quad f_{11}^* = 48,945.95, \quad f_{12}^* = 290.25,$$

$$f_{13}^* = 290.25, \quad f_{14}^* = 290.25, \quad f_{15}^* = 224,709.75, \quad f_{16}^* = 199.54,$$

$$f_{17}^* = 199.54, \quad f_{18}^* = 29,800.46.$$

The computed equilibrium populations at the four locations in the United States and at the three locations in Mexico are:

$$p_1^* = 1,400,736.88, \quad p_2^* = 20,607.42, \quad p_3^* = 700,000.00,$$

Solution to Example 1

The associated incurred utilities at the equilibrium at the locations are:

$$u_1(p^*) = 1,599,263.13, \ u_2(p^*) = 158,785.17, \ u_3(p^*) = 800,000.00,$$

$$u_4(p^*) = 119,401.38, \ u_5(p^*) = 51,054.05, \ u_6(p^*) = 75,290.25,$$

$$u_7(p^*) = 70,199.55.$$

We also, for completeness, report the path costs at the computed equilibrium flows since it is easy to then verify that the equilibrium conditions, as stated in Definition 1, hold. Specifically, the incurred path costs at the equilibrium are:

$$C_{r_1}(x^*) = C_{r_2}(x^*) = C_{r_3}(x^*) = C_{r_4}(x^*) = 0.00,$$

$$C_{r_5}(x^*) = 1,548,207.50, \quad C_{r_6}(x^*) = 1,548,196.38, \quad C_{r_7}(x^*) = 107,729.39,$$

$$C_{r_8}(x^*) = 0.00, \quad C_{r_9}(x^*) = 83,500.69, \quad C_{r_{10}}(x^*) = 0.00,$$

$$C_{r_{11}}(x^*) = 49,200.66, \quad C_{r_{12}}(x^*) = 0.00.$$

Solutions to Examples 1 Through 4

Examples 2 through 4 are analogues of Example 1, but with regulations.

• In Example 2, we considered the following scenario: The US government has told the Mexican government that it is restricting the flow on route $r_5 = (5, 6)$ to zero; essentially resulting in the elimination of this path, since its processing facilities are experiencing delays due to congestion. The rest of the data remain as in Example 1.

• In Example 3, we studied the following scenario: Route $r_6 = (7, 8)$ is now unavailable to refugees, but the other routes and data remain as in Example 1.

• And, in Example 4, we investigate the scenario that the United States is concerned about the influx of refugees and both routes r_5 and r_6 from Mexico are, in effect, banned/eliminated.

Solutions to Examples 1 Through 4

In order to enable cross comparisons with the examples with regulations and with the baseline Example 1, in the following Tables, we report, respectively, the computed equilibrium path flows, the associated path costs for all four examples, the associated computed equilibrium populations, whereas the incurred utilities at the locations are reported in the final Table.

Solutions to Examples 1 Through 4

Path	Equilibrium Path Flow			
	Ex. 1	Ex. 2	Ex. 3	Ex. 4
r_1	1,400,00.00	1,400,00.00	1,400,00.00	1,400,00.00
r_2	20,000.00	20,000.00	20,000.00	20,000.00
r_3	700,000.00	700,000.00	700,000.00	700,000.00
r_4	260,000.00	260,000.00	260,000.00	260,000.00
r_5	400.25	–	400.36	–
r_6	336.64	336.68	–	–
r_7	317.17	317.48	317.43	317.74
r_8	48,945.95	49,345.84	49,282.21	49,682.27
r_9	290.25	290.25	290.25	290.25
r_{10}	224,709.75	224,709.75	224,709.75	224,709.75
r_{11}	199.54	199.54	199.54	199.54
r_{12}	29,800.46	29,800.46	29,800.46	29,800.46

Table: Equilibrium Path Flows for Examples 1-4

Solutions to Examples 1 Through 4

Path	**Equilibrium Path Cost**			
	Ex. 1	Ex. 2	Ex. 3	Ex. 4
r_1	0.00	0.00	0.00	0.00
r_2	0.00	0.00	0.00	0.00
r_3	0.00	0.00	0.00	0.00
r_4	0.00	0.00	0.00	0.00
r_5	1,548,207.50	–	1,548,880.63	–
r_6	1,548,196.38	1,549,024.00	–	–
r_7	107,729.39	108,128.58	108,065.58	108,465.59
r_8	0.00	0.00	0.00	0.00
r_9	83,500.69	83,500.22	83,502.10	83,504.66
r_{10}	0.00	0.00	0.00	0.00
r_{11}	49,200.66	49,200.63	49,201.31	49,200.66
r_{12}	0.00	0.00	0.00	0.00

Table: Equilibrium Path Costs for Examples 1-4

Solutions to Examples 1 Through 4

Node	Equilibrium Population			
	Ex. 1	Ex. 2	Ex. 3	Ex. 4
1	1,400,736.88	1,400,336.63	1,400,400.38	1,400,000.00
2	20,607.42	20,607.72	20,607.67	20,607.67
3	700,000.00	700,000.00	700,000.00	700,000.00
4	260,199.55	260,199.55	260,199.55	260,199.55
5	48,945.95	49,345.84	49,282.21	49,682.27
6	224,709.75	224,709.75	224,709.75	224,709.75
7	29,800.46	224,709.75	224,709.75	29,800.46

Table: Equilibrium Populations at the Locations for Examples 1-4

Solutions to Examples 1 Through 4

Node	Utility at Equilibrium			
	Ex. 1	Ex. 2	Ex. 3	Ex. 4
1	1,599,263.13	1,599.663.38	1,599,599.63	1,600,000.00
2	158,785.17	158,784.56	158,784.66	158,784.00
3	800,000.00	800,000.00	800,000.00	800,000.00
4	119,401.38	119,401.38	119,401.38	119,401.38
5	51,054.05	50,654.16	50,717.79	50,317.73
6	75,290.25	75,290.25	75,290.25	75,290.25
7	70,199.55	70,199.55	70,199.55	70,199.55

Table: Equilibrium Utilities at the Locations for Examples 1-4

Solutions to Examples 1 Through 4

One can see, from the above Table, that (as occurred also in the illustrative examples), under the regulations (as in Examples 2 through 4), the utility of those subject to regulations, as for those in node 5, is reduced.

On the other hand, those in location 1, which now has a lower flow of refugees, experience a higher utility.

Also, with both refugee routes blocked to the US, the population that remains at node 5 is the highest in Example 4, as compared to the value in Examples 2 and 3.

Summary and Conclusions

Summary and Conclusions

- This paper constructs a novel multiclass, multipath refugee migration network model, without and with regulations.
- The models are formulated, analyzed, and solved using the theory of variational inequalities, where a supernetwork transformation is also applied.
- Our examples show who may benefit from the imposition of certain regulations and who may lose.
- We expect that research in the future may investigate additional types of regulations of refugee networks, for which other computational schemes may be utilized, with further exploitation of network structure.

Thank You!

For more information: https://supernet.isenberg.umass.edu/