

A Mean-Variance Disaster Relief Supply Chain Network Model for Risk Reduction with Stochastic Link Costs, Time Targets, and Demand Uncertainty

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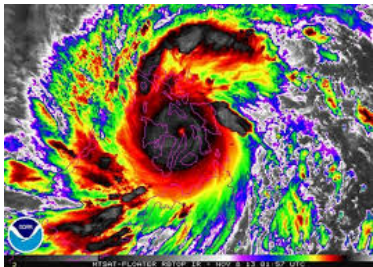
- ▶ Background and Motivation
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Background and Motivation

- Natural disasters, such as earthquakes, hurricanes, tsunamis, floods, tornadoes, fires, and droughts, invoke all phases of the disaster management cycle from preparedness and mitigation to response and recovery.
- Notable recent examples of disasters include: Hurricane Katrina in 2005 and Superstorm Sandy in 2012, the two costliest disasters to strike the U.S., the earthquake in Haiti in 2005, the triple disaster in Fukushima, Japan in 2011, Typhoon Haiyan that struck the Phillipines in 2013, and the devastating earthquake in Nepal in 2015.

Haiyan Typhoon in the Philippines in 2013

Typhoon Haiyan was a very powerful tropical cyclone that devastated portions of Southeast Asia, especially the Philippines, on November 8, 2013. It is the deadliest Philippine typhoon on record, killing at least 6,190 people in that country alone. Haiyan was also the strongest storm recorded at landfall. As of January 2014, bodies were still being found. The overall economic losses from Typhoon Haiyan totaled \$10 billion.



Nepal Earthquake in 2015

The 7.8 magnitude earthquake that struck Nepal on April 25, 2015, and the aftershocks that followed, killed nearly 9,000 people and injured 22,000 others. This disaster also pushed about 700,000 people below the poverty line in the Himalayan nation, which is one of the world's poorest. About 500,000 homes were made unlivable by the quakes, leaving about three million people homeless. Much infrastructure was also badly damaged and 1/3 of the healthcare facilities devastated. According to *The Wall Street Journal*, Nepal needs \$6.66 billion to rebuild.



The Ebola Crisis in West Africa



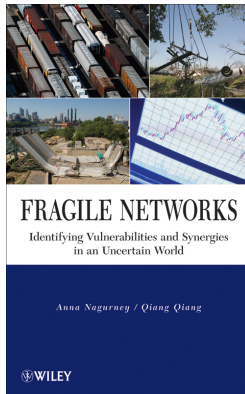
According to bbc.com and the World Health Organization, more than one year from the first confirmed case recorded on March 23, 2014, at least 11,178 people have been reported as having died from Ebola in six countries; Liberia, Guinea, Sierra Leone, Nigeria, the US and Mali.

Ms. Debbie Wilson of Doctors Without Borders



On February 4, 2015, the students in my Humanitarian Logistics and Healthcare class at the Isenberg School heard Debbie Wilson, a nurse, who has worked with Doctors Without Borders, speak on her 6 weeks of experiences battling Ebola in Liberia in September and October 2014.

As noted in Nagurney and Qiang (2009), the number of disasters is growing as well as the number of people affected by disasters.



Hence, the development of appropriate analytical tools that can assist humanitarian organizations and nongovernmental organizations as well as governments in the various disaster management phases has become a challenge to both researchers and practitioners.

Elements of Our Model

Recently, there has been growing interest in constructing **integrated frameworks** that can assist in multiple phases of disaster management. **Network-based models and tools**, which allow for a graphical depiction of disaster relief supply chains and provide the flexibility of adding nodes and links, coupled with effective computational procedures, in particular, offer promise.

Such models necessarily have to be **optimization-based** and must incorporate **stochastic elements** since in disaster situations there is uncertainty associated with the demand for relief supplies and also with various link costs.

In addition, as noted in Nagurney, Masoumi, and Yu (2015), **time plays a critical role in disaster relief supply chains** and, therefore, time must be a fundamental element in disaster relief models.

The Importance of Time in Disaster Relief

The U.S. Federal Emergency Management Agency (FEMA) has identified **key benchmarks to response and recovery, which emphasize time** and are: to meet the survivors' initial demands within 72 hours, to restore basic community functionality within 60 days, and to return to as normal of a situation within 5 years (Fugate (2012)).

Timely and efficient delivery of relief supplies to the affected population **not only decreases the fatality rate but may also prevent chaos**. In the case of Typhoon Haiyan, slow relief delivery efforts forced people to seek any possible means to survive. Several relief trucks were attacked and had food stolen, and some areas were reported to be on the brink of anarchy (Chicago Tribune (2013) and CBS News (2013)).

The Importance of Time in Disaster Relief

In Nepal, post the April 2015 earthquake, there was near chaos at the Katmandu airport with relief airplanes not able to land, with numerous Nepalese citizens seeking to leave while Nepalese expatriates attempted to return to help their families (Luke and McVicker (2015)). The BBC News (2015) reported that the slow distribution of aid led to clashes between protesters and riot police.

Moreover, humanitarian relief organizations, for the most part, receive their primary funding and support from donors. Hence, they are responsible to these and other stakeholders in terms of accountability of the use of their financial funds (see Toyasaki and Wakolbinger (2014)).

Presentation Based on Paper of the Same Title

This presentation is based on our paper, “A Mean-Variance Disaster Relief Supply Chain Network Model for Risk Reduction with Stochastic Link Costs, Time Targets, and Demand Uncertainty,” where many additional references can be found.

Inspiration for the Model

The model is inspired by the supply chain network integration model for risk reduction in the case of mergers and acquisitions developed by Liu and Nagurney, in the paper: “Risk Reduction and Cost Synergy in Mergers and Acquisitions via Supply Chain Network Integration. *Journal of Financial Decision-Making*, (2011), 7(2), 1-18, coupled with the integrated disaster relief framework of Nagurney, Masoumi, and Yu in the paper, “An Integrated Disaster Relief Supply Chain Network Model with Time Targets and Demand Uncertainty,” in *Regional Science Matters: Studies Dedicated to Walter Isard*, (2015), P. Nijkamp, A. Rose, and K. Kourtit, Editors, Springer, 287-318.

Risk Reduction Model of Liu and Nagurney (2011)

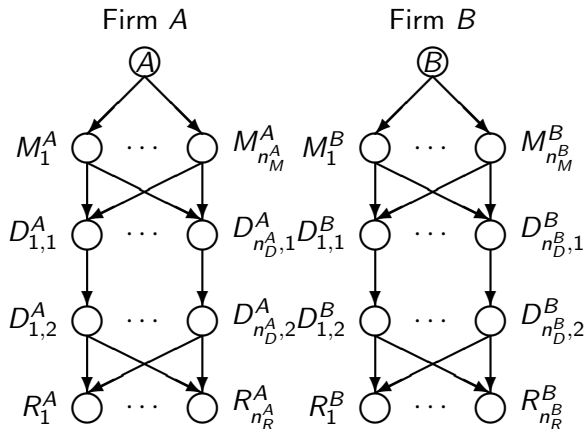


Figure 1: The Pre-Merger Supply Chain Network

Risk Reduction Model of Liu and Nagurney (2011)

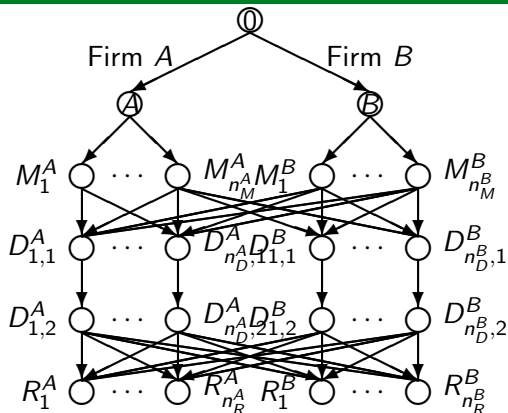


Figure 2: Firms A and B Merge: Demand Points of Either Firm Can Get the Product from Any Manufacturing Plant Via Any Distribution Center

Synergy measures are developed and the framework is also applicable to the teaming of organizations as in horizontal collaboration.

Integrated Disaster Relief model of Nagurney, Masoumi, and Yu (2015)

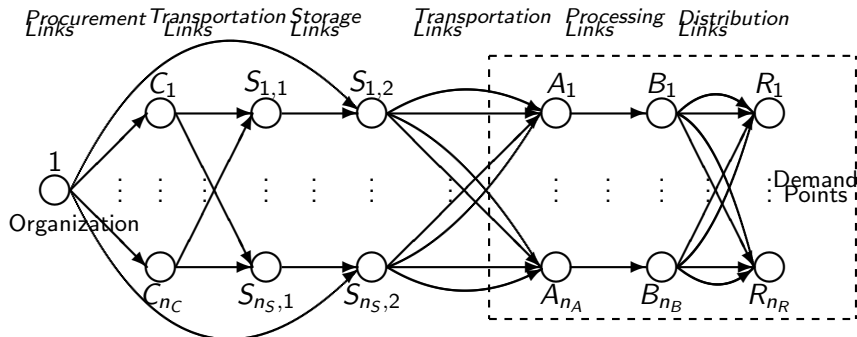


Figure 3: Network Topology of the Integrated Disaster Relief Supply Chain

Inspiration for the Model

The MV approach to risk reduction dates to the work of the Nobel laureate Harry Markowitz (1952, 1959) and is still relevant in finance (Schneeweis, Crowder, and Kazemi (2010)), in supply chains (Chen and Federgruen (2000) and Kim, Cohen, and Netessine (2007)), as well as in disaster relief and humanitarian operations, where the focus, to-date, has been on inventory management (Ozbay and Ozguven (2007) and Das (2014)).

Inspiration for the Model

The new model constructed here is the **first to integrate preparedness and response in a supply chain network framework** using a **Mean-Variance approach for risk reduction under demand and cost uncertainty** and **time targets plus penalties for shortages and surpluses.**

Bozorgi-Amiri et al. (2013) developed a model with uncertainty on the demand side and also in procurement and transportation using expected costs and variability with associated weights but **did not consider the critical time elements** as well as the **possibility of local versus nonlocal procurement post- or pre-disaster.**

Inspiration for the Model

In addition, Boyles and Waller (2009) developed a MV model for the minimum cost network flow problem with stochastic link costs and emphasized that an MV approach is especially relevant in logistics and distribution problems with critical implications for supply chains.

They noted that a solution that only minimizes expected cost and not variances may not be as reliable and robust as one that does.

What We Seek to Achieve with the Model

- In our model, the **humanitarian organization seeks to minimize its expected total operational costs and the total risk in operations with an individual weight assigned to its valuation of the risk, as well as the minimization of expected costs of shortages and surpluses and tardiness penalties associated with the target time goals at the demand points.**

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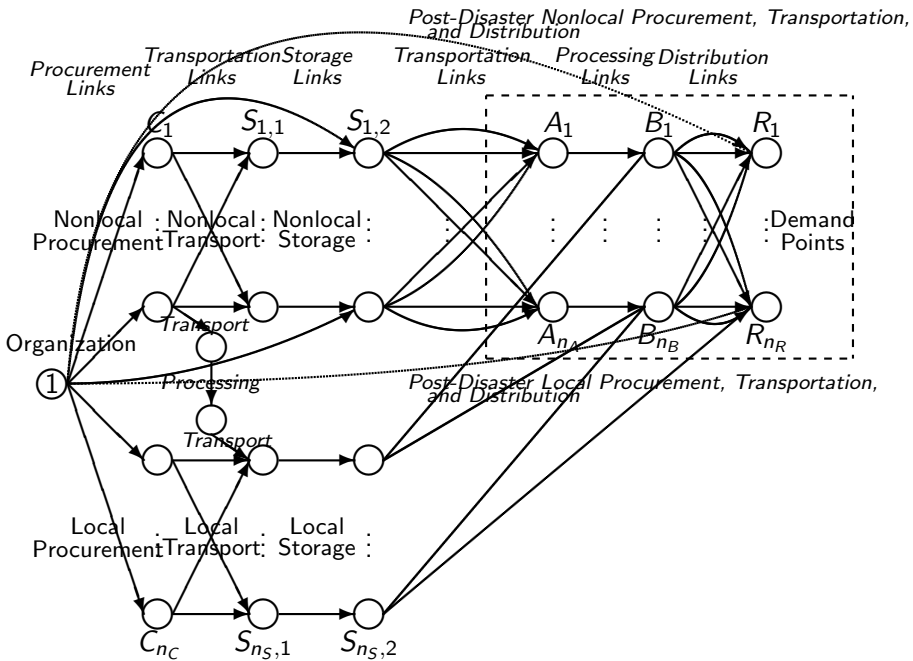
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- The risk is captured through the variance of the total operational costs, which is of relevance also to the reporting of the proper use of funds to stakeholders, including donors.
- The time goal targets associated with the demand points enable prioritization of demand points as to the timely delivery of relief supplies.

What We Seek to Achieve with the Model

- This framework handles **both the pre-positioning of relief supplies, whether local or nonlocal**, as well as **the procurement (local or nonlocal), transport, and distribution of supplies post-disaster**. **There is growing empirical evidence showing that the use of local resources in humanitarian supply chains can have positive impacts (see Matopoulos, Kovacs, and Hayes (2014))**. Earlier work on procurement with stochastic components did not distinguish between local or nonlocal procurement (see Falasca and Zobel (2011)).

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- The time element in our model is captured through **link time completion functions as the relief supplies progress along paths in the supply chain network**. Each path consists of a series of directed links, from the origin node, which represents the humanitarian organization, to the destination nodes, which are the demand points for the relief supplies.



Mean-Variance Disaster Relief Supply Chain Model

In the model, the demand is uncertain due to the unpredictability of the actual demand at the demand points. The probability distribution of demand might be derived using census data and/or information gathered during the disaster preparedness phase. Since d_k denotes the actual (uncertain) demand at destination point k , we have:

$$P_k(D_k) = P_k(d_k \leq D_k) = \int_0^{D_k} \mathcal{F}_k(u) du, \quad k = 1, \dots, n_R, \quad (1)$$

where P_k and \mathcal{F}_k denote the probability distribution function, and the probability density function of demand at point k , respectively. Here v_k is the “projected demand” for the disaster relief item at demand point k ; $k = 1, \dots, n_R$. The amounts of shortage and surplus at destination node k are calculated according to:

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \quad k = 1, \dots, n_R, \quad (2a)$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \quad k = 1, \dots, n_R. \quad (2b)$$

Mean-Variance Disaster Relief Supply Chain Model

The expected values of shortage and surplus at each demand point are, hence:

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (u - v_k) \mathcal{F}_k(u) du, \quad k = 1, \dots, n_R, \quad (3a)$$

$$E(\Delta_k^+) = \int_0^{v_k} (v_k - u) \mathcal{F}_k(u) du, \quad k = 1, \dots, n_R. \quad (3b)$$

The expected penalty incurred by the humanitarian organization due to the shortage and surplus of the relief item at each demand point is equal to:

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \quad k = 1, \dots, n_R. \quad (4)$$

Mean-Variance Disaster Relief Supply Chain Model

We have the following two sets of conservation of flow equations. The projected demand at destination node k , v_k , is equal to the sum of flows on all paths in the set \mathcal{P}_k , that is:

$$v_k \equiv \sum_{p \in \mathcal{P}_k} x_p, \quad k = 1, \dots, n_R. \quad (5)$$

The flow on link a , f_a , is equal to the sum of flows on paths that contain that link:

$$f_a = \sum_{p \in \mathcal{P}} x_p \delta_{ap}, \quad \forall a \in L, \quad (6)$$

where δ_{ap} is equal to 1 if link a is contained in path p and is 0, otherwise.

Mean-Variance Disaster Relief Supply Chain Model

Here we consider total operational link cost functions of the form:

$$\hat{c}_a = \hat{c}_a(f_a, \omega_a) = \omega_a \hat{g}_a f_a + g_a f_a, \quad \forall a \in L, \quad (7)$$

where \hat{g}_a and g_a are positive-valued for all links $a \in L$. We permit ω_a to follow any probability distribution and the ω s of different supply chain links can be correlated with one another.

The term $\hat{g}_a f_a$ in (8) represents the part of the total link operational cost that is subject to variation of ω_a with $g_a f_a$ denoting that part of the total cost that is independent of ω_a .

The random variables ω_a , $a \in L$ can capture various elements of uncertainty, due, for example, to disruptions because of the disaster, and price uncertainty for storage, procurements, transport, processing, and distribution services.

Mean-Variance Disaster Relief Supply Chain Model

The completion time function associated with the activities on link a is given by:

$$\tau_a(f_a) = \hat{t}_a f_a + t_a, \quad \forall a \in L,$$

where \hat{t}_a and t_a are ≥ 0 .

The target for completion of activities on paths corresponding to demand point k is given by T_k and is imposed for each demand point k by the humanitarian organization decision-maker.

The target for a path p to demand point k is then $T_{kp} = T_k - t_p$, where $t_p = \sum_{a \in L} t_a \delta_{ap}$, $\forall p \in \mathcal{P}_k$.

Mean-Variance Disaster Relief Supply Chain Model

The variable z_p is the amount of deviation with respect to the target time T_{kp} associated with the late delivery of relief items to k on path p , $\forall p \in \mathcal{P}_k$. We group the s_p s into the vector $z \in R_+^{n_P}$.

$\gamma_k(z)$ is the tardiness penalty function corresponding to demand point k ; $k = 1, \dots, n_R$.

Mean-Variance Disaster Relief Supply Chain Model

The objective function faced by the organization's decision-maker, which he seeks to minimize, is the following:

$$\begin{aligned} & E \left[\sum_{a \in L} \hat{c}_a(f_a, \omega_a) \right] + \alpha \text{Var} \left[\sum_{a \in L} \hat{c}_a(f_a, \omega_a) \right] + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) \\ & \quad + \sum_{k=1}^{n_R} \gamma_k(z) \\ = & \sum_{a \in L} E [\hat{c}_a(f_a, \omega_a)] + \alpha \text{Var} \left[\sum_{a \in L} \hat{c}_a(f_a, \omega_a) \right] + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) \\ & \quad + \sum_{k=1}^{n_R} \gamma_k(z), \end{aligned} \tag{8}$$

where E denotes the expected value, Var denotes the variance, and α represents the risk aversion factor (weight) for the organization that the organization's decision-maker places on the risk.

Mean-Variance Disaster Relief Supply Chain Model

The goal of the decision-maker is, thus, to minimize the following problem, with the objective function in (8), in lieu of (7), taking the form in (9) below:

$$\begin{aligned} \text{Minimize } & \sum_{a \in L} E(\omega_a) \hat{g}_a f_a + \sum_{a \in L} g_a f_a + \alpha \text{Var}\left(\sum_{a \in L} \omega_a \hat{g}_a f_a\right) \\ & + \sum_{k=1}^{n_R} (\lambda_k^+ - E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z) \end{aligned} \quad (9)$$

subject to constraint (6) and the following constraints:

$$x_p \geq 0, \quad \forall p \in \mathcal{P}, \quad (10)$$

$$z_p \geq 0, \quad \forall p \in \mathcal{P}, \quad (11)$$

$$\sum_{q \in \mathcal{P}} \sum_{a \in L} \hat{t}_a x_q \delta_{aq} \delta_{ap} - z_p \leq T_{kp}, \quad \forall p \in \mathcal{P}_k; k = 1, \dots, n_R. \quad (12)$$

Mean-Variance Disaster Relief Supply Chain Model

In view of constraint (6) we can reexpress the objective function in (9) in path flows (rather than in link flows and path flows) to obtain the following optimization problem:

$$\begin{aligned} & \text{Minimize } \sum_{a \in L} \left[E(\omega_a) \hat{g}_a \sum_{q \in \mathcal{P}} x_q \delta_{aq} + g_a \sum_{q \in \mathcal{P}} x_q \delta_{aq} \right] \\ & + \alpha \text{Var} \left(\sum_{a \in L} \omega_a \hat{g}_a \sum_{q \in \mathcal{P}} x_q \delta_{aq} \right) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z) \end{aligned} \quad (13)$$

subject to constraints: (10) – (12).

Let K denote the feasible set:

$$K \equiv \{(x, z, \mu) \mid x \in R_+^{n_P}, z \in R_+^{n_P}, \text{ and } \mu \in R_+^{n_P}\}, \quad (14)$$

where μ is the vector of Lagrange multipliers corresponding to the constraints in (12) with an individual element corresponding to path p denoted by μ_p .

Mean-Variance Disaster Relief Supply Chain Model

Before presenting the variational inequality formulation of the optimization problem immediately above, we review the respective partial derivatives of the expected values of shortage and surplus of the disaster relief item at each demand point with respect to the path flows, derived in Dong, Zhang, and Nagurney (2004), Nagurney, Yu, and Qiang (2011), and Nagurney, Masoumi, and Yu (2012). In particular, they are given by:

$$\frac{\partial E(\Delta_k^-)}{\partial x_p} = P_k \left(\sum_{q \in \mathcal{P}_k} x_q \right) - 1, \quad \forall p \in \mathcal{P}_k; \quad k = 1, \dots, n_R, \quad (15a)$$

and,

$$\frac{\partial E(\Delta_k^+)}{\partial x_p} = P_k \left(\sum_{q \in \mathcal{P}_k} x_q \right), \quad \forall p \in \mathcal{P}_k; \quad k = 1, \dots, n_R. \quad (15b)$$

Mean-Variance Disaster Relief Supply Chain Model

Theorem 1

The optimization problem (13), subject to its constraints (10) – (12), is equivalent to the variational inequality problem: determine $(x^*, z^*, \mu^*) \in K$, such that, $\forall (x, z, \mu) \in K$:

$$\begin{aligned} & \sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[\sum_{a \in L} (E(\omega_a) \hat{g}_a + g_a) \delta_{ap} + \alpha \frac{\partial \text{Var}(\sum_{a \in L} \omega_a \hat{g}_a \sum_{q \in \mathcal{P}} x_q^* \delta_{aq})}{\partial x_p} \right. \\ & \left. + \lambda_k^+ P_k \left(\sum_{q \in \mathcal{P}_k} x_q^* \right) - \lambda_k^- (1 - P_k \left(\sum_{q \in \mathcal{P}_k} x_q^* \right)) + \sum_{q \in \mathcal{P}} \sum_{a \in L} \mu_q^* g_a \delta_{aq} \delta_{ap} \right] \\ & \times [x_p - x_p^*] + \sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[\frac{\partial \gamma_k(z^*)}{\partial z_p} - \mu_p^* \right] \times [z_p - z_p^*] \\ & + \sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[T_{kp} + z_p^* - \sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q^* \delta_{aq} \delta_{ap} \right] \times [\mu_p - \mu_p^*] \geq 0. \quad (16) \end{aligned}$$

Mean-Variance Disaster Relief Supply Chain Model

Variational inequality (16) can be put into standard form: find $X^* \in \mathcal{K}$:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (17)$$

with the feasible set $\mathcal{K} \equiv K$, the column vectors $X \equiv (x, z, \mu)$, and $F(X) \equiv (F_1(X), F_2(X), F_3(X))$:

$$F_1(X) = \left[\sum_{a \in L} (E(\omega_a) \hat{g}_a + g_a) \delta_{ap} + \alpha \frac{\partial \text{Var}(\sum_{a \in L} \omega_a \hat{g}_a \sum_{q \in \mathcal{P}} x_q \delta_{aq})}{\partial x_p} \right. \\ \left. + \lambda_k^+ P_k \left(\sum_{q \in \mathcal{P}_k} x_q \right) - \lambda_k^- \left(1 - P_k \left(\sum_{q \in \mathcal{P}_k} x_q \right) \right) + \sum_{q \in \mathcal{P}} \sum_{a \in L} \mu_q g_a \delta_{aq} \delta_{ap}, p \in \mathcal{P}_k; \right. \\ \left. F_2(X) = \left[\frac{\partial \gamma_k(z)}{\partial z_p} - \mu_p, p \in \mathcal{P}_k; k = 1, \dots, n_R \right], \right. \\ \left. F_3(X) = \left[T_{kp} + z_p - \sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q \delta_{aq} \delta_{ap}, p \in \mathcal{P}_k; \forall k \right]. \quad (18) \right.$$

The Algorithm

At an iteration τ of the Euler method (cf. Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})), \quad (19)$$

where $P_{\mathcal{K}}$ is the projection on the feasible set \mathcal{K} and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \quad (20)$$

where $\langle \cdot, \cdot \rangle$ is the inner product in n -dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n -dimensional function from \mathcal{K} to R^n , with $F(X)$ being continuous.

Explicit Formulae for the Euler Method Applied to the Disaster Relief Supply Chain Network Variational Inequality

Closed form expressions for the product path flows, the time deviations, and the Lagrange multipliers, $\forall p \in \mathcal{P}_k; \forall k$:

$$\begin{aligned} x_p^{\tau+1} = & \max\{0, x_p^\tau + a_\tau(\lambda_k^-(1 - P_k(\sum_{q \in \mathcal{P}_k} x_q^\tau)) - \lambda_k^+ P_k(\sum_{q \in \mathcal{P}_k} x_q^\tau)) \\ & - \sum_{a \in L} (E(\omega_a)\hat{g}_a + g_a)\delta_{ap} - \alpha \frac{\partial \text{Var}(\sum_{a \in L} \omega_a \hat{g}_a \sum_{q \in \mathcal{P}} x_q^\tau \delta_{aq})}{\partial x_p} \\ & - \sum_{q \in \mathcal{P}} \sum_{a \in L} \mu_q^\tau g_a \delta_{aq} \delta_{ap}\}; \end{aligned} \quad (21)$$

$$z_p^{\tau+1} = \max\{0, z_p^\tau + a_\tau(\mu_p^\tau - \frac{\partial \gamma_k(z^\tau)}{\partial z_p})\}, \quad (22)$$

$$\mu_p^{\tau+1} = \max\{0, \mu_p^\tau + a_\tau(\sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q^\tau \delta_{aq} \delta_{ap} - T_{kp} - z_p^\tau)\}. \quad (23)$$

The Algorithm

In view of (21), we can define a generalized marginal total cost on path p ; $p \in \mathcal{P}$, denoted by $G\hat{C}'_p$, where

$$G\hat{C}'_p \equiv \sum_{a \in L} (E(\omega_a)\hat{g}_a + g_a)\delta_{ap} + \alpha \frac{\partial \text{Var}(\sum_{a \in L} \omega_a \hat{g}_a \sum_{q \in \mathcal{P}} x_q \delta_{aq})}{\partial x_p}. \quad (24)$$

Numerical Examples – Illustrative Examples

We first present a smaller example for clarity purposes, along with variants. We implemented the Euler method, as described above, in FORTRAN, using a Linux system at the University of Massachusetts Amherst. The convergence criterion was $\epsilon = 10^{-6}$; that is, the Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each variable (see (21), (22), and (23)) differed from its respective value at the preceding iteration by no more than ϵ . The sequence $\{a_\tau\}$ was: $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \dots)$. We initialized the algorithm by setting each variable equal to 0.00.

The Network Topology for the Supply Chain in Example 1 and its Variants

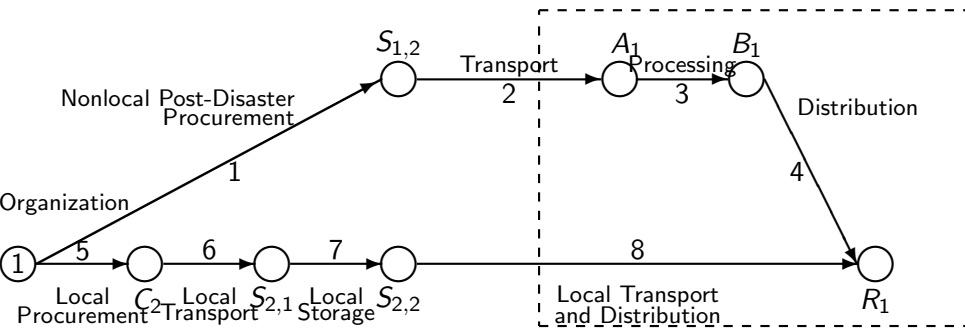


Figure 5: Disaster Relief Supply Chain Network Topology For Example 1 and its Variants

Example 1 and Variants

This example corresponds to an island location that is subject to major storms. The humanitarian relief organization is depicted by node 1 and there is a single demand point for the relief supplies denoted by R_1 , which is located on the island.

The organization is considering two options, that is, strategies, reflected by the two paths connecting node 1 with node R_1 with path p_1 consisting of the links: 1, 2, 3, and 4, and path p_2 consisting of the links: 5, 6, 7, and 8.

The local transport and distribution are done by ground transport. However, the transport on link 2 is done by air.

Example 1 and Variants

The covariance matrix associated with the link total cost functions $\hat{c}_a(f_a, \omega_a)$, $a \in L$, is the 8×8 matrix $\sigma^2 I$. In the variants of Example 1 we explore different values for σ^2 and also different values for α , the risk aversion factor (see (13)). The organization's risk aversion factor $\alpha = 1$ in Example 1 and its Variants 1, 2, and 3.

The demand for the relief item at the demand point R_1 (in thousands of units) is assumed to follow a uniform probability distribution on the interval $[10, 20]$. The path flows and the link flows are also in thousands of units. Therefore,

$$P_{R_1}\left(\sum_{p \in \mathcal{P}_1} x_p\right) = \frac{\sum_{p \in \mathcal{P}_1} x_p - 10}{20 - 10} = \frac{x_{p_1} + x_{p_2} - 10}{10}.$$

Example 1 and Variants

We now describe how we construct the marginalized total link costs for the numerical examples from which the marginalized total path costs as in (24) are then constructed.

For our numerical examples, we have that:

$$\sum_{a \in L} \sigma^2 \hat{g}_a^2 f_a^2 = \text{Var}\left(\sum_{a \in L} \omega_a \hat{g}_a f_a\right) = \text{Var}\left(\sum_{a \in L} \omega \hat{g}_a \sum_{q \in \mathcal{P}} x_q \delta_{aq}\right), \quad (25)$$

so that:

$$\frac{\partial \text{Var}\left(\sum_{a \in L} \omega_a \hat{g}_a \sum_{q \in \mathcal{P}} x_q \delta_{aq}\right)}{\partial x_p} = 2\sigma^2 \sum_{a \in L} \hat{g}_a^2 f_a \delta_{ap}. \quad (26)$$

In view of (26) and (24) we may define the generalized marginal total cost on a link a , \hat{g}'_a , as:

$$g \hat{c}'_a \equiv E(\omega_a) \hat{g}_a + g_a + \alpha 2\sigma^2 \hat{g}_a^2 f_a, \quad (27)$$

so that

$$G \hat{C}'_p = \sum_{a \in L} g \hat{c}'_a \delta_{ap}, \quad \forall p \in \mathcal{P}. \quad (28)$$

Example 1 and Variants

In Example 1, $\sigma^2 = .1$ and for Variant 1: $\sigma^2 = 1$. The time target at demand point R_1 , $T_1 = 48$ (in hours).

The link time completion functions for links: 5, 6, and 7 are 0.00 since these are completed prior to the disaster and the supplies on the path with these links are, hence, immediately available for local transport and distribution.

Also, we set $\lambda_1^- = 1000$ and $\lambda_1^+ = 100$.

The organization is significantly more concerned with a shortage of the relief item than with a surplus. The tardiness penalty function

$$\gamma_{R_1}(z) = 3\left(\sum_{p \in \mathcal{P}_{R_1}} z_p^2\right).$$

Results for Example 1 and Variants

Table 1: Link Total Cost, Expected Value of Random Link Cost, Marginal Generalized Link Total Cost, and Time Completion Functions for Example 1 and Variant 1 and Optimal Link Flows: $E(\omega_a) = 1, \forall a, \alpha = 1$

Link a	$\hat{c}_a(f_a, \omega_a)$	$g\hat{c}'_a$	$\tau_a(f_a)$	$f_a^*;$ $\sigma^2 = .1$	$f_a^*;$ $\sigma^2 = 1$
1	$\omega_1 3f_1 + f_1$	$\alpha 18\sigma^2 f_1 + 4$	$f_1 + 1$	4.70	4.90
2	$\omega_2 2f_2 + f_2$	$\alpha 8\sigma^2 f_2 + 3$	$f_2 + 2$	4.70	4.90
3	$\omega_3 .5f_3 + f_3$	$\alpha .5\sigma^2 f_3^2 + 1.5$	$f_3 + .5$	4.70	4.90
4	$\omega_4 .4f_4 + f_4$	$\alpha .32\sigma^2 f_4 + 1.4$	$f_4 + 1$	4.70	4.90
5	$\omega_5 2f_5 + f_5$	$\alpha 8\sigma^2 f_5 + 3$	0.00	14.18	12.84
6	$\omega_6 .1f_6 + f_6$	$\alpha .02\sigma^2 f_6 + 1.1$	0.00	14.18	12.84
7	$\omega_7 f_7 + f_7$	$\alpha 2\sigma^2 f_7 + 2$	0.00	14.18	12.84
8	$\omega_8 .5f_8 + f_8$	$\alpha .5\sigma^2 f_8 + 1.5$	$.2f_8 + 2$	14.18	12.84

Variants 2 and 3

Variants 2 and 3 of Example 1 are constructed as follows and the data are reported in Table 3. For Variant 2, we retain the data for Example 1 with $\sigma^2 = .1$ but now assume that **air transport, due to the expected storm damage of the island airport, is no longer possible. Maritime transport is, nevertheless, available, so link 2 in Figure 5 now corresponds to maritime transport** rather than air transport. All the data, hence, for Variant 2 are as for Example 1 except that the total operational cost data and the time completion data for link 2 change as reported in Table 2.

Variant 3 is constructed from Variant 2 but with $\sigma^2 = 1$ (as in Variant 1 of Example 1).

Results for Variants 2 and 3

Table 2: Link Total Cost, Expected Value of Random Link Cost, Marginal Generalized Link Total Cost, and Time Completion Functions for Example 1 Variants 2 and 3 and Optimal Link Flows: $E(\omega_a) = 1, \forall a, \alpha = 1$

Link a	$\hat{c}_a(f_a, \omega_a)$	$g\hat{c}'_a$	$\tau_a(f_a)$	$f_a^*;$ $\sigma^2 = .1$	$f_a^*;$ $\sigma^2 = 1$
1	$\omega_1 3f_1 + f_1$	$\alpha 18\sigma^2 f_1 + 4$	$f_1 + 1$	0.00	0.51
2	$\omega_2 12f_2 + 10f_2$	$\alpha 288\sigma^2 f_2 + 3$	$3f_2 + 10$	0.00	0.51
3	$\omega_3 .5f_3 + f_3$	$\alpha .5\sigma^2 f_3 + 1.5$	$f_3 + .5$	0.00	0.51
4	$\omega_4 .4f_4 + f_4$	$\alpha .32\sigma^2 f_4 + 1.4$	$f_4 + 1$	0.00	0.51
5	$\omega_5 2f_5 + f_5$	$\alpha 8\sigma^2 f_5 + 3$	0.00	18.84	16.90
6	$\omega_6 .1f_6 + f_6$	$\alpha .02\sigma^2 f_6 + 1.1$	0.00	18.84	16.90
7	$\omega_7 f_7 + f_7$	$\alpha 2\sigma^2 f_7 + 2$	0.00	18.84	16.90
8	$\omega_8 .5f_8 + f_8$	$\alpha .5\sigma^2 f_8 + 1.5$	$.2f_8 + 2$	18.84	16.90

Results for Variants 4 and 5

In Variants 4 and 5 we explore the impact on the strategies and on the optimal link flows of **increasing the risk aversion factor α** .

Specifically, in Variant 4 we utilize the Variant 1 data in Table 2 but we increase α to 10 and in Variant 5 we increase α even more to 100. We report the input data and results for $\alpha = 10$ and for $\alpha = 100$ in Table 3.

Results for Variants 4 and 5

Table 3: Link Total Cost, Expected Value of Random Link Cost, Marginal Generalized Link Total Cost, and Time Completion Functions for Example 1 Variants 4 and 5 and Optimal Link Flows: $E(\omega_a) = 1, \forall a, \sigma^2 = 1$

Link a	$\hat{c}_a(f_a, \omega_a)$	$g\hat{c}'_a$	$\tau_a(f_a)$	$f_a^*;$ $\alpha = 10$	$f_a^*;$ $\alpha = 100$
1	$\omega_1 3f_1 + f_1$	$\alpha 18\sigma^2 f_1 + 4$	$f_1 + 1$	3.17	.68
2	$\omega_2 2f_2 + f_2$	$\alpha 8\sigma^2 f_2 + 3$	$f_2 + 2$	3.17	.68
3	$\omega_3 .5f_3 + f_3$	$\alpha .5\sigma^2 f_3 + 1.5$	$f_3 + .5$	3.17	.68
4	$\omega_4 .4f_4 + f_4$	$\alpha .32\sigma^2 f_4 + 1.4$	$f_4 + 1$	3.17	.68
5	$\omega_5 2f_5 + f_5$	$\alpha 8\sigma^2 f_5 + 3$	0.00	8.10	1.74
6	$\omega_6 .1f_6 + f_6$	$\alpha .02\sigma^2 f_6 + 1.1$	0.00	8.10	1.74
7	$\omega_7 f_7 + f_7$	$\alpha 2\sigma^2 f_7 + 2$	0.00	8.10	1.74
8	$\omega_8 .5f_8 + f_8$	$\alpha .5\sigma^2 f_8 + 1.5$	$.2f_8 + 2$	8.10	1.74

Background on the Mexico Case Study

According to the United Nations (2011), **Mexico is ranked as one of the world's thirty most exposed countries to three or more types of natural disasters, notably, storms, hurricanes, floods, as well as earthquakes, and droughts.**

For example, as reported by The International Bank for Reconstruction and Development/The World Bank (2012), **41% of Mexico's national territory is exposed to storms, hurricanes, and floods; 27% to earthquakes, and 29% to droughts.**

Background on the Mexico Case Study

The hurricanes can come from the Atlantic or Pacific oceans or the Caribbean.

As noted by de la Fuente (2011), the single most costly disaster in Mexico were the 1985 earthquakes, followed by the floods in the southern state of Tabasco in 2007, with damages of more than 3.1 billion U.S. dollars.

Mexico Case Study

We consider a humanitarian organization such as the Mexican Red Cross, which is interested in preparing for another possible hurricane, and recalls **the devastation wrought by Hurricane Manuel and Hurricane Ingrid, which struck Mexico within a 24 hour period in September 2013.**

Ingrid caused 32 deaths, primarily, in eastern Mexico, whereas Manuel resulted in at least 123 deaths, primarily in western Mexico (NOAA (2014)). According to Pasch and Zelinsky (2014), the total economic impact of Manuel alone was estimated to be approximately \$4.2 billion (U.S.), with the biggest losses occurring in Guerrero.

Mexico Case Study

In Example 2, we assume that the Mexican Red Cross is mainly concerned about the delivery of relief supplies to the Mexico City area and the Acapulco area.

Ingrid affected Mexico City and Manuel affected the Acapulco area and also points northwest.



Photos of Acapulco post Manuel courtesy The Weather Channel.

Example 2 (Mexico Case Study) and Variant

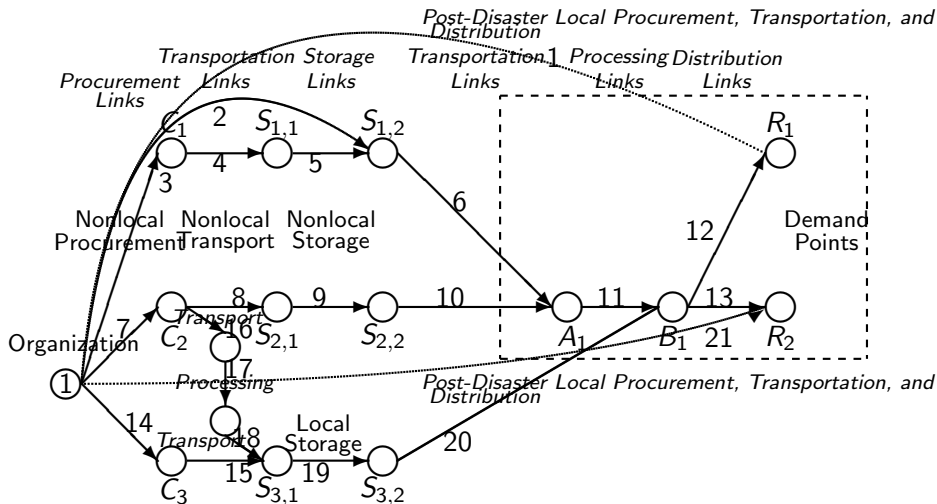


Figure 6: Disaster Relief Supply Chain Network Topology for Example 2 and its Variant

Mexico Case Study

The Mexican Red Cross represents the organization in Figure 6 and is denoted by node 1.

There are two demand points, R_1 and R_2 , for the ultimate delivery of the relief supplies. R_1 is situated closer to Mexico City and R_2 is closer to Acapulco.

Nonlocal procurement is done through two locations in Texas, C_1 and C_2 . Because of good relationships with the U.S. and the American Red Cross, there are two nonlocal storage facilities that the Mexican Red Cross can utilize, both located in Texas, and represented by links 5 and 9 emanating from $S_{1,1}$ and $S_{2,1}$, respectively.

Local storage, on the other hand, is depicted by the link emanating from node $S_{3,1}$, link 19.

The Mexican Red Cross can also procure locally (see C_3).

Mexico Case Study

Nonlocal procurement, post-disaster, is depicted by link 2, whereas procurement locally, post-disaster, and direct delivery to R_1 and R_2 are depicted by links 1 and 21, respectively.

Link 11 is a processing link to reflect processing of the arriving relief supplies from the U.S. and we assume one portal A_1 , which is in southcentral Mexico.

Link 17 is also a processing link but that processing is done prior to storage locally and pre-disaster. Such a link is needed if the goods are procured nonlocally (link 7). The transport is done via road in the disaster relief supply chain network in Figure 6.

Mexico Case Study

The demand for the relief items at the demand point R_1 (in thousands of units) is assumed to follow a uniform probability distribution on the interval $[20, 40]$. The path flows and the link flows are also in thousands of units. Therefore,

$$P_{R_1}\left(\sum_{p \in \mathcal{P}_1} x_p\right) = \frac{\sum_{p \in \mathcal{P}_1} x_p - 20}{40 - 20} = \frac{\sum_{i=1}^6 x_{p_i} - 20}{20}.$$

Also, the demand for the relief item at R_2 (in thousands of units) is assumed to follow a uniform probability distribution on the interval $[20, 40]$. Hence,

$$P_{R_2}\left(\sum_{p \in \mathcal{P}_2} x_p\right) = \frac{\sum_{p \in \mathcal{P}_2} x_p - 20}{40 - 20} = \frac{\sum_{i=7}^{12} x_{p_i} - 20}{20}.$$

Mexico Case Study

The time targets for the delivery of supplies at R_1 and R_2 , respectively, in hours, are: $T_1 = 48$ and $T_2 = 48$. The penalties at the two demand points for shortages are: $\lambda_1^- = 10,000$ and $\lambda_2^- = 10,000$ and for surpluses: $\lambda_1^+ = 100$ and $\lambda_2^+ = 100$. The tardiness penalty function $\gamma_{R_1}(z) = 3(\sum_{p \in \mathcal{P}_{R_1}} z_p^2)$ and the tardiness penalty function $\gamma_{R_2}(z) = 3(\sum_{p \in \mathcal{P}_{R_2}} z_p^2)$.

As in Example 1 and its variants, we assume that, for Example 2, the covariance matrix associated with the link total cost functions $\hat{c}_a(f_a, \omega_a)$, $a \in L$, is a 21×21 matrix $\sigma^2 I$.

In Example 2, $\sigma^2 = 1$ and the risk aversion factor $\alpha = 10$ since the humanitarian organization is risk-averse with respect to its costs associated with its operations.

The additional data for Example 2 are given in Tables 4 and 5, where we also report the computed optimal link flows via the Euler method, which are calculated from the computed path flows reported in Table 6.

Note that the time completion functions in Tables 4 and 5, $\tau_a(f_a)$, $\forall a \in L$, are 0.00 if the links correspond to procurement, transport, and storage, pre-disaster, since such supplies are immediately available for shipment once a disaster strikes.

Mexico Case Study

Table 4: Link Total Cost, Expected Value of Random Link Cost, Marginal Generalized Link Total Cost, and Time Completion Functions for Example 2 and Optimal Link Flows: $\alpha = 10$

Link a	$\hat{c}_a(f_a, \omega_a)$	$E(\omega_a)$	$g\hat{c}'_a$	$\tau_a(f_a)$	$f_a^*;$ $\sigma^2 = 1$
1	$\omega_1 6f_1 + f_1$	2	$\alpha 72\sigma^2 f_1 + 13$	$f_1 + 15$	9.07
2	$\omega_2 3f_2 + f_2$	2	$\alpha 18\sigma^2 f_2 + 7$	$f_2 + 7$	2.54
3	$\omega_3 2f_3 + f_3$	1	$\alpha 8\sigma^2 f_3 + 3$	0.00	2.57
4	$\omega_4 3f_4 + f_4$	1	$\alpha 18\sigma^2 f_4 + 4$	0.00	2.57
5	$\omega_5 2f_5 + f_5$	1	$\alpha 8\sigma^2 f_5 + 3$	0.00	2.57
6	$\omega_6 2f_6 + f_6$	2	$\alpha 8\sigma^2 f_6 + 5$	$2f_6 + 10$	5.11
7	$\omega_7 2f_7 + f_7$	1	$\alpha 8\sigma^2 f_7 + 3$	0.00	8.51
8	$\omega_8 3f_8 + f_8$	1	$\alpha 18\sigma^2 f_8 + 4$	0.00	4.36
9	$\omega_9 2f_9 + f_9$	1	$\alpha 8\sigma^2 f_9 + 3$	0.00	4.36

Table 5: Table 4 continued

10	$\omega_{10}2f_{10} + f_{10}$	1	$\alpha 8\sigma^2 f_{10} + 3$	$2f_{10} + 10$	4.36
11	$\omega_{11}f_{11} + f_{11}$	2	$\alpha 2\sigma^2 f_{11} + 3$	$f_{11} + 2$	9.47
12	$\omega_{12}f_{12} + f_{12}$	2	$\alpha 2\sigma^2 f_{12} + 3$	$f_{12} + 6$	17.78
13	$\omega_{13}f_{13} + f_{13}$	2	$\alpha 2\sigma^2 f_{13} + 3$	$f_{13} + 7$	17.64
14	$\omega_{14}f_{14} + f_{14}$	1	$\alpha 2\sigma^2 f_{14} + 2$	0.00	21.79
15	$\omega_{15}f_{15} + f_{15}$	1	$\alpha 2\sigma^2 f_{15} + 2$	0.00	21.79
16	$\omega_{16}f_{16} + f_{16}$	1	$\alpha 2\sigma^2 f_{16} + 2$	0.00	4.15
17	$\omega_{17}.5f_{17} + f_{17}$	1	$\alpha \sigma^2 .5f_{17} + 1.5$	0.00	4.15
18	$\omega_{18}f_{18} + f_{18}$	1	$\alpha 2\sigma^2 f_{18} + 2$	0.00	4.15
19	$\omega_{19}.5f_{19} + f_{19}$	2	$\alpha \sigma^2 .5f_{19} + 1.5$	0.00	25.94
20	$\omega_{20}f_{20} + f_{20}$	2	$\alpha 2\sigma^2 f_{20} + 2$	$2f_{20} + 5$	25.94
21	$\omega_{21}6f_{21} + f_{21}$	2	$\alpha 72\sigma^2 f_{21} + 13$	$f_{21} + 14$	9.13

Results for Example 2 - Mexico Case Study

Table 6: Path Definitions, Target Times, Optimal Path Flows, Optimal Path Time Deviations, and Optimal Lagrange Multipliers for Example 2

	Path Definition (Links)	x_p^*	z_p^*	μ_p^*
\mathcal{P}_{R_1} :	$p_1 = (1)$	9.07	0.00	0.00
	$p_2 = (2, 6, 11, 12)$	1.27	34.75	208.53
	$p_3 = (3, 4, 5, 6, 11, 12)$	1.29	25.26	151.56
	$p_4 = (7, 8, 9, 10, 11, 12)$	2.18	23.78	142.69
	$p_5 = (7, 16, 17, 18, 19, 20, 12)$	2.98	50.48	302.85
	$p_6 = (14, 15, 19, 20, 12)$	10.06	50.48	302.85
\mathcal{P}_{R_2} :	$p_7 = (2, 6, 11, 13)$	1.27	35.48	212.88
	$p_8 = (3, 4, 5, 6, 11, 13)$	1.29	25.99	155.91
	$p_9 = (7, 8, 9, 10, 11, 13)$	2.18	24.51	147.04
	$p_{10} = (7, 16, 17, 18, 19, 20, 13)$	1.17	51.20	307.19
	$p_{11} = (14, 15, 19, 20, 13)$	11.74	51.20	307.19
	$p_{12} = (21)$	9.13	0.00	0.00

Example 2 - Variant 1

In Variant 1 of Example 2, we kept the data as in Example 2, but now we assumed that the humanitarian organization has a better forecast for the demand at the two demand points. The demand for the relief items at the demand point R_1 again follows a uniform probability distribution but on the interval $[30, 40]$ so that:

$$P_{R_1}\left(\sum_{p \in \mathcal{P}_1} x_p\right) = \frac{\sum_{p \in \mathcal{P}_1} x_p - 30}{40 - 30} = \frac{\sum_{i=1}^6 x_{p_i} - 30}{10}.$$

Also, the demand for the relief item at R_2 follows a uniform probability distribution on the interval $[30, 40]$ so that:

$$P_{R_2}\left(\sum_{p \in \mathcal{P}_2} x_p\right) = \frac{\sum_{p \in \mathcal{P}_2} x_p - 30}{40 - 30} = \frac{\sum_{i=7}^{12} x_{p_i} - 30}{10}.$$

Results for Example 2 - Variant 1

Table 7: Path Definitions, Target Times, Optimal Path Flows, Optimal Path Time Deviations, and Optimal Lagrange Multipliers for Ex. 2 Var. 1

	Path Definition (Links)	x_p^*	z_p^*	μ_p^*
\mathcal{P}_{R_1} :	$p_1 = (1)$	11.30	0.00	0.00
	$p_2 = (2, 6, 11, 12)$	1.37	43.13	258.78
	$p_3 = (3, 4, 5, 6, 11, 12)$	1.49	33.42	200.49
	$p_4 = (7, 8, 9, 10, 11, 12)$	2.58	32.28	193.69
	$p_5 = (7, 16, 17, 18, 19, 20, 12)$	2.81	64.37	386.19
	$p_6 = (14, 15, 19, 20, 12)$	12.29	64.37	386.19
\mathcal{P}_{R_2} :	$p_7 = (2, 6, 11, 13)$	1.37	43.92	263.49
	$p_8 = (3, 4, 5, 6, 11, 13)$	1.49	34.20	205.20
	$p_9 = (7, 8, 9, 10, 11, 13)$	2.57	33.07	198.40
	$p_{10} = (7, 16, 17, 18, 19, 20, 13)$	1.96	65.15	390.90
	$p_{11} = (14, 15, 19, 20, 13)$	13.04	65.15	390.90
	$p_{12} = (21)$	11.36	0.00	0.00

Results for Example 2 - Variant 1

The projected demands are: $v_{R_1} = 31.84$ and $v_{R_2} = 31.79$. The greatest percentage increase in path flow volumes occurs on paths p_1 and p_6 for demand point R_1 and on paths p_{11} and p_{12} for demand point R_2 , reinforcing the results obtained for Example 2

For both Example 2 and its variant the time targets are met for paths p_1 and p_2 since $\mu_{p_1}^*$ and $\mu_{p_2}^* = 0.00$ for both examples. Hence, direct local procurement post-disaster is effective time-wise, and cost-wise. Mexico is a large country and this result is quite reasonable.

Summary and Conclusions

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- Each link has an associated time completion function and the decision-maker determines his risk-aversion.

Summary and Conclusions

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- ▶ 3. The supply chain network topology allows for the procurement and pre-positioning of supplies locally and is more general than that in earlier literature.
- ▶ 4. The generality of the framework allows for numerous sensitivity analysis exercises to evaluate risk-aversion, the assessment of the impacts of the size of penalties on shortages and supplies, as well as modifications to the cost and time completion functions.

Summary and Conclusions

The framework consolidates decision-making associated with two phases of disaster management: preparedness and response, incorporates uncertainty in costs and demands and includes the critical time element.

Future research may include extending this framework to assess synergies associated with horizontal cooperation among humanitarian organizations in relief operations.

Acknowledgments

This paper is dedicated to the students in Professor Anna Nagurney's Humanitarian Logistics and Healthcare class in 2015 at the Isenberg School of Management and to all the victims of natural disasters over the centuries as well as to humanitarian professionals.

THANK YOU!



The Virtual Center for Supernetworks

Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

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The Virtual Center for Supernetworks is an interdisciplinary center at the Isenberg School of Management that advances knowledge on large-scale networks and integrates operations research and management science, engineering, and economics. Its Director is Dr. Anna Nagurney, the John F. Smith Memorial Professor of Operations Management.

Mission: The Virtual Center for Supernetworks fosters the study and application of supernetworks and serves as a resource on networks ranging from transportation and logistics, including supply chains, and the Internet, to a spectrum of economic networks.

The Applications of Supernetworks Include: decision-making, optimization, and game theory; supply chain management; critical infrastructure from transportation to electric power networks; financial networks; knowledge and social networks; energy, the environment, and sustainability; cybersecurity; Future Internet Architectures; risk management; network vulnerability, resiliency, and performance metrics; humanitarian logistics and healthcare.

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