A Competitive Multiperiod Supply Chain Network Model with Freight Carriers and Green Technology Investment Option

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# Motivation

#### Increase environmental awareness and improve ecological footprint

- Walmart plan for CO2 reduction with its extended supply chain
- Siemens (2015) will spend nearly \$110 million to lower the company's emissions. To reduce carbon emissions in half by 2020 and save between \$ 20 to \$30 million annually.
- Dell plan to use packaging material made of wheat straw; This new material uses 40% less energy to produce, 90% less water, and costs less to make than traditional packaging
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- product shipping costs
- energy rating
- and initial technology investments
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- Consumer awareness of green technology and foot print outcomes in spatial price equilibrium conditions



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- Modeling supply chain decision making and management from operational, tactical, and strategic business perspectives (Brandenburg et al. (2014); Ding et al. (2016); Fahimnia et al. (2015); Ouardighi et al. (2016); Zhu and He (2017))
- Environmental decision making in supply chain management processes and associated optimization from a number of dimensions (Nagurney et al. (2007); Cruz (2008); Frota Neto et al. (2008))
- Utilizing regulatory policies related to internalizing externalities such as including emission taxes (Cruz and Liu (2011); Dhavale and Sarkis (2015); Diabat and Simchi-Levi (2009); Zakeri et al. (2015))
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Figure: The supply chain network with freight carriers

### Multiperiod Green Supply Chain-Freight Carrier Network



Figure: The supply chain network with freight carriers

Notation	Definition
$\delta_{mi}$	Energy rating of manufacturer <i>i</i>
$\delta_{co}$	Energy rating of carrier o
$\delta_{rj}$	Energy rating of retailer <i>j</i>
$\delta_{max}$	Maximum possible level of energy rating.

Ma

$$\text{ximize} \quad \sum_{t=1}^{T} \frac{1}{(1+r)^{t}} \left\{ \sum_{j=1}^{N} p_{ijt}^{1*} q_{ijt}^{1} - PC_{it}(S_{t}, \delta_{mi}) - \sum_{j=1}^{N} TC_{ijt}(q_{ijt}^{1}, \delta_{mi}) - WC_{it}(I_{it}, \delta_{mi}) - \sum_{j=1}^{N} \sum_{o=1}^{O} R_{ijot}(p_{t}^{2*}, \delta_{co}) p_{ijot}^{2*} \right\} - TSI_{i}(\delta_{mi})$$

$$(1)$$

$$S_{i1} - I_{i1} \ge \sum_{j=1}^{N} q_{ij1}^{1}$$
(2)

$$I_{i(t-1)} + S_{it} - I_{it} \ge \sum_{j=1}^{N} q_{ijt}^{1}, \qquad \forall t = 2, \dots, T$$
(3)

$$q_{ijt}^{1} = \sum_{o=1}^{O} R_{ijot}(p_{t}^{2}, \delta_{co}), \qquad \forall j, t$$

$$\tag{4}$$

$$\delta_{mi} \le \delta_{co}, \qquad \forall o \tag{5}$$

Ma

$$\begin{aligned} \text{aximize} \quad \sum_{t=1}^{T} \frac{1}{(1+r)^{t}} \left\{ \sum_{j=1}^{N} p_{ijt}^{1*} q_{ijt}^{1} - PC_{it}(S_{t}, \delta_{mi}) - \sum_{j=1}^{N} TC_{ijt}(q_{ijt}^{1}, \delta_{mi}) \right. \\ \left. - WC_{it}(I_{it}, \delta_{mi}) - \sum_{j=1}^{N} \sum_{o=1}^{O} R_{ijot}(p_{t}^{2*}, \delta_{co}) p_{ijot}^{2*} \right\} - TSI_{i}(\delta_{mi}) \end{aligned}$$

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and the nonnegativity constraints:  $q_{ijt}^1 \ge 0, \ S_{it} \ge 0, \ I_{it} \ge 0, \ 0 \le \delta_{mi} \le \delta_{max}, \ \forall j, t.$ 

$$\begin{aligned} \text{Maximize} \quad \sum_{t=1}^{T} \frac{1}{(1+r)^{t}} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{N} R_{ijot}(p_{t}^{2}, \delta_{co}) p_{ijot}^{2} - \sum_{i=1}^{M} \sum_{j=1}^{N} CC_{ijot}(q_{ijot}^{2}, \delta_{co}) q_{ijot}^{2} - \sum_{i=1}^{M} AC_{iot}(B_{iot}, \delta_{co}) \right\} - TSI_{o}(\delta_{co}) \end{aligned}$$
(6)

$$\sum_{j=1}^{N} R_{ijo1}(p_1^2, \delta_{co}) - B_{io1} \ge \sum_{j=1}^{N} q_{ijo1}^2$$
(7)

$$B_{io(t-1)} + \sum_{j=1}^{N} R_{ijot}(p_t^2, \delta_{co}) - B_{iot} \ge \sum_{j=1}^{N} q_{ijot}^2, \qquad \forall t = 2, \dots, T$$
(8)

$$\sum_{t=1}^{T} \sum_{o=1}^{O} q_{ijot}^2 = \sum_{t=1}^{T} q_{ijt}^1, \qquad \forall i, j$$
(9)

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$$p_{ijot}^2 \ge 0, \; B_{iot} \ge 0, \; q_{ijot}^2 \ge 0, \; 0 \le \delta_{co} \le \delta_{max}, \qquad orall i, j, t.$$

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We modify it to two inequality constraints as:

Generalized Nash equilibrium problem (GNEP)

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Τ

Maximize

$$\sum_{t=1}^{r} \frac{1}{(1+r)^{t}} \left\{ \rho_{jt}^{3*} \sum_{k=1}^{r} q_{jkt}^{3} - IC_{jt}(Z_{jt}, \delta_{rj}) - HC_{jt}(Y_{t}, \delta_{rj}) - \sum_{k=1}^{K} TC_{jkt}(q_{jkt}^{3}, \delta_{rj}) - \sum_{i=1}^{M} \rho_{ijt}^{1*} q_{ijt}^{1} \right\} - TSI_{j}(\delta_{rj})$$
(12)

subject to:

$$\sum_{t=1}^{T} Y_{jt} = \sum_{t=1}^{T} \sum_{i=1}^{M} \sum_{o=1}^{O} q_{ijot}^{2}$$
(13)

$$\sum_{t=1}^{T} Y_{jt} = \sum_{t=1}^{T} \sum_{i=1}^{M} q_{ijt}^{1}$$
(14)

$$Y_{j1} - Z_{j1} \ge \sum_{k=1}^{K} q_{jk1}^3$$
(15)

$$Z_{j(t-1)} + Y_{jt} - Z_{jt} \ge \sum_{k=1}^{K} q_{jkt}^3, \quad \forall t = 2, \dots, T$$
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### The Behavior of Consumers within the Demand Markets

$$\frac{1}{(1+r)^{t}} [p_{jt}^{3*} + SC_{jkt}(q_{jkt}^{3*})] \begin{cases} = \frac{1}{(1+r)^{t}} p_{kjt}^{4*}, & \text{if} \quad q_{jkt}^{3*} > 0, \\ \\ \ge \frac{1}{(1+r)^{t}} p_{kjt}^{4*}, & \text{if} \quad q_{jkt}^{3*} = 0 \end{cases}$$
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and

$$D_{kjt}(p^{4*}, \delta_{rj}^{*}) \begin{cases} = q_{jkt}^{3*}, & \text{if } p_{kjt}^{4*} > 0, \\ \\ \leq q_{jkt}^{3*}, & \text{if } p_{kjt}^{4*} = 0. \end{cases}$$
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Conditions (18) and (19) must hold simultaneously for all demand markets. These conditions correspond to the well-known spatial price equilibrium conditions (cf. Nagurney (1999); Takayama and Judge (1964)).

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### Theorem 1: Variational Inequality Formulation

The equilibrium conditions governing the multiperiod supply chain - freight carrier model are equivalent to the solution of the variational inequality problem given by: determine  $(q^{1*}, q^{2*}, q^{3*}, S^*, l^*, \delta_m^*, p^{2*}, B^*, \delta_c^*, Y^*, Z^*, \delta_r^*, p^{4*}, \mu^{1*}, \mu^{2*}, \mu^{3*}, \theta^*, \eta^{1*}, \eta^{2*}, \nu^{1*}, \nu^{2*}, \gamma^*) \in \mathcal{K}$ , satisfying

$$\langle F(X^*), X - X^* \rangle \ge 0, \quad \forall X \in \mathcal{K}$$
 (20)

where

 $X \equiv (q^{1}, q^{2}, q^{3}, S, I, \delta_{m}, p^{2}, B, \delta_{c}, Y, Z, \delta_{r}, p^{4}, \mu^{1}, \mu^{2}, \mu^{3}, \theta, \eta^{1}, \eta^{2}, \nu^{1}, \nu^{2}, \gamma)$ 

$$\begin{split} F(X) &\equiv (F_{q_{ijt}^{1}}, F_{q_{ijot}^{2}}, F_{q_{jkt}^{3}}, F_{S_{it}}, F_{I_{it}}, F_{\delta_{mi}}, F_{\rho_{ijot}^{2}}, F_{B_{iot}}, F_{\delta_{co}}, F_{Y_{jt}}, F_{Z_{jt}}, F_{\delta_{ij}}, F_{\rho_{jkt}^{4}}, \\ F_{\mu_{in}^{1}}, F_{\mu_{in}^{2}}, F_{\mu_{ij}^{3}}, F_{\theta_{iit}}, F_{\eta^{1}}, F_{\eta^{2}}, F_{\nu^{1}}, F_{\nu^{2}}, F_{\gamma}) \end{split}$$

. The term  $\langle\cdot,\cdot
angle$  denotes the inner product in N-dimensional Euclidean space.

### The Modified Projection Method

### Step 0: Initialization

Start with  $X^0 \in \mathcal{K}$ , as a feasible initial point, and let  $\tau = 1$ . Set  $\omega$  such that  $0 < \omega < \frac{1}{L}$ , where L is the Lipschitz constant for function F(X).

### Step 1: Computation

Compute  $\bar{X}^{\tau}$  by solving the variational inequality subproblem:

$$\langle \bar{X}^{\tau} + \omega F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^{\tau} \rangle \ge 0, \qquad \forall X \in \mathcal{K}.$$
 (21)

#### Step 2: Adaptation

Compute  $X^{\tau}$  by solving the variational inequality subproblem:

$$\langle X^{\tau} + \omega F(\bar{X}^{\tau}) - X^{\tau-1}, X - X^{\tau} \rangle \ge 0, \qquad \forall X \in \mathcal{K}.$$
 (22)

### Numerical Examples

### Example 1

Two manufacturers, M = 2; two retailers, N = 2; two carriers, O = 2; and two demand markets, K = 2; competing over five planning periods, T = 5.



Figure: Example 1 Supply Chain Network

The energy rating,  $\delta$ , can be zero and should not be more than 1,  $(\delta_{max} = 1)$ 

The cost functions are:

$$\begin{aligned} & PC_{it}(S_{it}, \delta_{mi}) = \alpha^{it}S_{1t} + 0.05(S_{it})^2 - \delta_{mi}S_{it}, \qquad i = 1, 2, t = 1, \dots, 5. \\ & \alpha^{1t} = [2, 2.5, 3, 3.5, 4], \qquad \alpha^{2t} = [3, 4, 4.5, 5, 5.5]. \\ & WC_{it}(I_{it}, \delta_{mi}) = 1.05I_{it} + 0.002(I_{it})^2 - \delta_{mi}I_{it} + 10, \qquad i = 1, 2; t = 1, \dots, 5. \\ & TC_{ijt}(q_{ijt}, \delta_{mi}) = 1.5q_{ijt} + 0.8(q_{ijt})^2 - \delta_{mi}q_{ijt}, \qquad i = 1, 2; j = 1, 2; t = 1, \dots, 5. \\ & HC_{jt}(Y_{jt}, \delta_{ij}) = 3Y_{jt} + 0.05(Y_{jt})^2 - \delta_{ij}Y_{jt}, \qquad j = 1, 2; t = 1, \dots, 5. \\ & HC_{jt}(Z_{jt}, \delta_{ij}) = 1.01Z_{jt} + 0.002(Z_{jt})^2 - \delta_{ij}Z_{jt}, \qquad t = 1, \dots, 5. \\ & IC_{jt}(Z_{jt}, \delta_{ij}) = 1.01Z_{jt} + 0.002(Z_{jt})^2 - \delta_{ij}Z_{jt}, \qquad t = 1, \dots, 5. \\ & R_{ijot}(\rho_t^2, \delta_{co}) = 20 - 1.5\rho_{ijot}^2 + 0.5\sum_{c \neq o} \rho_{ijct}^2 + 3\delta_{co}, \qquad i = 1, 2; j = 1, 2; o = 1, 2; t = 1, \dots, 5. \\ & CC_{ijot}(q_{ijot}^2, \delta_{co}) = 1.1q_{ijot}^2 + 0.003q_{ijot}^2 - \delta_{co}q_{ijot}^2, \qquad i = 1, 2; j = 1, 2; o = 1, 2; t = 1, \dots, 5. \\ & AC_{iot}(B_{iot}, \delta_{co}) = B_{iot} + 0.001(B_{iot})^2 - \delta_{co}B_{iot}, \qquad i = 1, 2; o = 1, 2; t = 1, \dots, 5. \end{aligned}$$

The investment cost functions for manufacturers, retailers, and carriers are defined, respectively, as:

$$TSI_{i}^{1} = 500 + 300(\delta_{mi})^{2}, \qquad i = 1, 2.$$
  
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$$TSI_i^1 = 500 + 300(\delta_{mi})^2, \qquad i = 1, 2.$$

$$\begin{split} TSI_{j}^{3} &= 500 + 200 (\delta_{ij})^{2}, \qquad j = 1, 2. \\ TSI_{o}^{2} &= 500 + 200 (\delta_{co})^{2}, \qquad o = 1, 2. \end{split}$$

The cost functions are:

$$\begin{split} & PC_{it}(S_{it}, \delta_{mi}) = \alpha^{it}S_{1t} + 0.05(S_{it})^2 - \delta_{mi}S_{it}, \qquad i = 1, 2, t = 1, \dots, 5. \\ & \alpha^{1t} = [2, 2.5, 3, 3.5, 4], \qquad \alpha^{2t} = [3, 4, 4.5, 5, 5.5]. \\ & WC_{it}(I_{it}, \delta_{mi}) = 1.05I_{it} + 0.002(I_{it})^2 - \delta_{mi}I_{it} + 10, \qquad i = 1, 2; t = 1, \dots, 5. \\ & TC_{ijt}(q_{ijt}, \delta_{mi}) = 1.5q_{ijt} + 0.8(q_{ijt})^2 - \delta_{mi}q_{ijt}, \qquad i = 1, 2; t = 1, \dots, 5. \\ & HC_{jt}(Y_{jt}, \delta_{rj}) = 3Y_{jt} + 0.05(Y_{jt})^2 - \delta_{rj}Y_{jt}, \qquad j = 1, 2; t = 1, \dots, 5. \\ & HC_{jt}(Z_{jt}, \delta_{rj}) = 1.01Z_{jt} + 0.002(Z_{jt})^2 - \delta_{rj}Z_{jt}, \qquad t = 1, \dots, 5. \\ & R_{ijot}(\rho_t^2, \delta_{co}) = 20 - 1.5\rho_{ijot}^2 + 0.5\sum_{c \neq o} \rho_{ijct}^2 + 3\delta_{co}, \qquad i = 1, 2; j = 1, 2; o = 1, 2; t = 1, \dots, 5. \\ & CC_{ijot}(q_{ijot}^2, \delta_{co}) = 1.1q_{ijot}^2 + 0.003q_{ijot}^2 - \delta_{co}q_{ijot}^2, \qquad i = 1, 2; j = 1, 2; o = 1, 2; t = 1, \dots, 5. \\ & AC_{iot}(B_{iot}, \delta_{co}) = B_{iot} + 0.001(B_{iot})^2 - \delta_{co}B_{iot}, \qquad i = 1, 2; o = 1, 2; t = 1, \dots, 5. \end{split}$$

The investment cost functions for manufacturers, retailers, and carriers are defined, respectively, as:

$$TSl_i^1 = 500 + 300(\delta_{mi})^2, \qquad i = 1, 2.$$
  
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$$TSl_o^2 = 500 + 200(\delta_{co})^2, \qquad o = 1, 2.$$

The demand functions for customers within demand market 1 are defined to be less sensitive to future product prices, while customers within demand market 2 are defined to be more sensitive to future product prices.

$$\begin{split} D_{1j1}(p^4, \delta_{ij}) &= 130 - 1.3p_{1j1}^4 + 2\delta_{ij}, \quad D_{1j2}(p^4, \delta_{ij}) = 110 - 1.1p_{1j2}^4 + 2\delta_{ij}, \\ D_{1j3}(p^4, \delta_{ij}) &= 80 - 0.9p_{1j3}^4 + 2\delta_{ij}, \quad D_{1j4}(p^4, \delta_{ij}) = 50 - 0.7p_{1j4}^4 + 2\delta_{ij}, \\ D_{1j5}(p^4, \delta_{ij}) &= 40 - 0.4p_{1j5}^4 + 2\delta_{ij}, \quad j = 1, 2. \end{split}$$

$$\begin{split} D_{2j1}(p^4,\delta_{ij}) &= 80 - 0.7p_{2j1}^4 + 2\delta_{ij}, \quad D_{2j2}(p^4,\delta_{ij}) = 120 - 1p_{2j2}^4 + 2\delta_{ij}, \\ D_{2j3}(p^4,\delta_{ij}) &= 150 - 1.2p_{2j3}^4 + 2\delta_{ij}, \quad D_{2j4}(p^4,\delta_{ij}) = 180 - 1.7p_{2j4}^4 + 2\delta_{ij}, \\ D_{2j5}(p^4,\delta_{ij}) &= 200 - 2p_{2j5}^4 + 2\delta_{ij}, \quad j = 1, 2. \end{split}$$

### **Example 1: Equilibrium Solution**



### (a) Supplies at manufacturers



(c) Carriers' service backlogs



### (b) Manufacturers' inventories



# (d) Carriers' orders and shipment services from manufacturers



Figure: Retailers' and customers' product flow

**Energy rating level** 

 $\delta_m = 1, \qquad \qquad \delta_c = 1, \qquad \qquad \delta_r = 0$ 

Baseline is Example 1, but the time periods have been extended (T = 10). The demand functions for periods 6 to 10 are:

$$\begin{split} D_{1j6}(p^4,\delta_{ij}) &= 40 - 0.4p_{1j6}^4 + 2\delta_{ij}, \quad D_{1j7}(p^4,\delta_{ij}) = 40 - 0.4p_{1j7}^4 + 2\delta_{ij}, \\ D_{1j8}(p^4,\delta_{ij}) &= 40 - 0.4p_{1j8}^4 + 2\delta_{ij}, \quad D_{1j9}(p^4,\delta_{ij}) = 40 - 0.4p_{1j9}^4 + 2\delta_{ij}, \\ D_{1j10}(p^4,\delta_{ij}) &= 40 - 0.4p_{1j10}^4 + 2\delta_{ij}, \quad j = 1, 2. \end{split}$$

$$\begin{aligned} D_{2j6}(p^4, \delta_{ij}) &= 200 - 2p_{2j6}^4 + 2\delta_{ij}, \quad D_{2j7}(p^4, \delta_{ij}) &= 160 - 1.7p_{2j7}^4 + 2\delta_{ij}, \\ D_{2j8}(p^4, \delta_{ij}) &= 130 - 1.5p_{2j8}^4 + 2\delta_{ij}, \quad D_{2j9}(p^4, \delta_{ij}) &= 130 - p_{2j9}^4 + 2\delta_{ij}, \\ D_{2j10}(p^4, \delta_{ij}) &= 100 - p_{2j10}^4 + 2\delta_{ij}, \quad j = 1, 2. \end{aligned}$$

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**Energy rating level** 

 $\delta_m = 1, \qquad \delta_c = 1, \qquad \delta_r = 1$ 

Follows the same network structure as Example 1 but with varying cost functions for all network parties in order to focus on constraints

$$\delta_{mi} \leq \delta_{co}, \quad \forall o \tag{5}$$
$$\delta_{rj} \leq \delta_{mi}, \quad \forall i \tag{17}$$

Here, we vary the coefficient of  $\delta$  in cost functions

$$TSI_i^1 = 500 + 360(\delta_{mi})^2, \qquad i = 1, 2.$$
  

$$TSI_o^2 = 500 + 360(\delta_{co})^2, \qquad o = 1, 2.$$
  

$$TSI_i^3 = 500 + 360(\delta_{rj})^2, \qquad j = 1, 2.$$

from 360 to 560 by increment of 20 and analyze the companies' capability in acquiring green technology.

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from 360 to 560 by increment of 20 and analyze the companies' capability in acquiring green technology.

### **Example 3: Equilibrium Solution**



(a) In an obliged network

Figure: Energy rating of all entities for different investment levels

### **Example 3: Equilibrium Solution**



(a) In an obliged network (b) In an uncommitted network

Figure: Energy rating of all entities for different investment levels

# • Sustainability and greenness in supply chain should be viewed holistically

- The decisions to manage supply chain must be conditioned by the structure of any game that underlies the determination of decisions by supply chain partners
- Time and the cost of investment affect firms' decisions, profitability, competitive advantage, and their environmental impact
- Governments can bring down the barrier of entry for green energy by taking steps to subsidize the green technology adoption and protect the posterity of our planet



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# Thank you!

The price that manufacturer i; i = 1, ..., M charges retailer j; j = 1, ..., N at time period t; t = 1, ..., T:

$$p_{ijt}^{1*} = (1+r)^{t} (\mu_{it}^{*} + \theta_{ijt}^{*}) + \frac{\partial T C_{ijt} (q_{ijt}^{1*}, \delta_{mi}^{*})}{\partial q_{ijt}^{1}},$$
(23)

The prices of products at the retailers:

$$p_{jt}^{3*}=(1+r)^t \mu_{jt}^*+rac{\partial \mathit{TC}_{jkt}(q_{jkt}^{3*},\delta_{rj}^*)}{\partial q_{jkt}},$$

The price that manufacturer i; i = 1, ..., M charges retailer j; j = 1, ..., N at time period t; t = 1, ..., T:

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The prices of products at the retailers:

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(24)

The feasible set underlying the variational inequality problem is not compact. However, by imposing a rather weak condition, we can guarantee the existence of a solution pattern. Let

$$\begin{split} \mathcal{K}_{b} &= \{ (q^{1}, q^{2}, q^{3}, S, I, \delta_{m}, p^{2}, B, \delta_{c}, Y, Z, \delta_{r}, p^{4}, \mu^{1}, \mu^{2}, \mu^{3}, \theta, \eta^{1}, \eta^{2}, \nu^{1}, \nu^{2}, \gamma) | 0 \leqslant q^{1} \leqslant b_{1}; \\ 0 \leqslant q^{2} \leqslant b_{2}; 0 \leqslant q^{3} \leqslant b_{3}; 0 \leqslant S \leqslant b_{4}; 0 \leqslant I \leqslant b_{5}; 0 \leqslant \delta_{m} \leqslant \delta_{max}^{b}; 0 \leqslant p^{2} \leqslant b_{6}; 0 \leqslant B \leqslant b_{7}; \\ 0 \leqslant \delta_{c} \leqslant \delta_{max}^{b}; 0 \leqslant Y \leqslant b_{8}; 0 \leqslant Z \leqslant b_{9}; 0 \leqslant \delta_{r} \leqslant \delta_{max}^{b}; 0 \leqslant p^{4} \leqslant b_{10}; 0 \leqslant \mu^{1} \leqslant b_{11}; 0 \leqslant \mu^{2} \leqslant b_{12}; \\ 0 \leqslant \mu^{3} \leqslant b_{13}; -b_{14} \leqslant \theta \leqslant b_{15}; 0 \leqslant \eta^{1} \leqslant b_{16}; 0 \leqslant \eta^{2} \leqslant b_{17}; 0 \leqslant \nu^{1} \leqslant b_{18}; -b_{19} \leqslant \nu^{2} \leqslant b_{20}, \\ &-b_{21} \leqslant \gamma \leqslant b_{22} \} \end{split}$$

Hence, the following variational inequality admits at least one solution  $X^b \in \mathcal{K}_b$  since  $\mathcal{K}_b$  is compact and F is continuous.

$$\langle F(X^b), X - X^b \rangle \ge 0, \qquad \forall X^b \in \mathcal{K}_b.$$
 (26)

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Hence, the following variational inequality admits at least one solution  $X^b \in \mathcal{K}_b$  since  $\mathcal{K}_b$  is compact and F is continuous.

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