# Spatially Differentiated Trade of Permits for Multipollutant Electric Power Supply Chains

**Trisha Woolley and Anna Nagurney**
Department of Finance
and Operations Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

**John Stranlund**
Department of Resource Economics
College of Natural Resources and the Environment
University of Massachusetts
Amherst, Massachusetts 01003

**INFORMS Annual Meeting, Pittsburgh, PA, November 5–8, 2006**

The Virtual Center for Supernetworks

Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

UMassAmherst ISENBERG SCHOOL OF MANAGEMENT

# Topics

- Paper Outline
- Motivation
- Literature
- Notation for Models
- Models
- Numerical Examples
- Conclusion
- References

# Acknowledgements

- This research was supported, in part, by NSF Grant. No.: IIS 00026471 and, in part, by the John F. Smith Memorial Fund at the Isenberg School of Management

- This support is gratefully acknowledged and appreciated.

# Paper Outline

- Develop a modeling and computational framework that allows for the determination of optimal prices for the various permits applied to electric power plants in the context of electric power supply chain (generation/distribution/consumption) networks.
- The general framework that we develop allows for special cases of the model.
- Numerical Examples

# Motivation

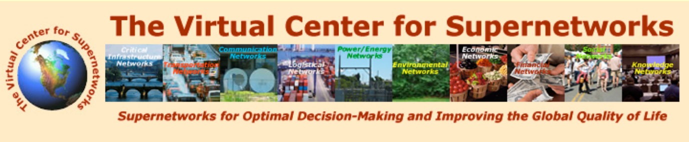
- The electrical industry growth and effect on air pollution
- Capture substitutability and complementarities of pollutants related to electricity generation
- Spatial Nature of Pollutants
- Importance of environmental-energy modeling which includes credit trading to address market failures in energy.

# Electricity Production

- Electricity generation is the dominant industrial source of air emissions in the United States today. For example, fossil fuel-fired power plants are responsible for 67% of the nation's sulfur dioxide emissions, 23% of nitrogen oxide emissions, and 40% of man-made carbon dioxide emissions. (**EPA**)

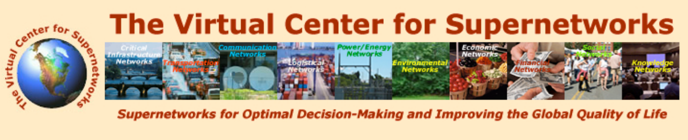
- Electricity worldwide is produced predominantly by coal, which is responsible for 40% of the carbon dioxide pollution (hence global warming), and which is expected to maintain about a 36% share of the electricity generation market through 2020. (**IPCC**)

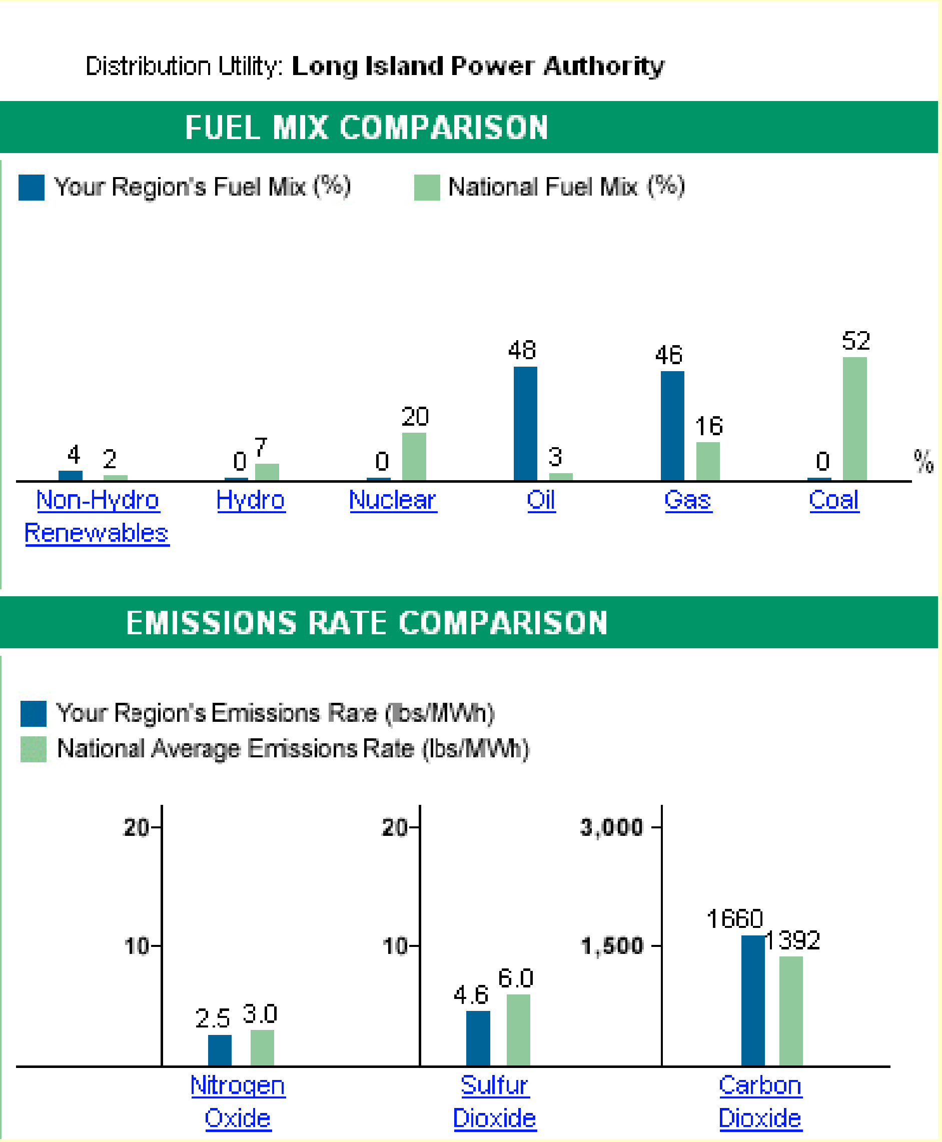
- The electricity industry is increasing, with the total consumption of electricity worldwide projected to reach 16.4 trillion kilowatt hours by 2010 and 23.1 by 2025. (**Business and Company Resource Center**)



| | Nitrogen Oxides | Carbon Dioxide (greenhouse gas) | Sulfur Dioxide | Methane (greenhouse gas) | Mercury |
|---|---|---|---|---|---|
| **Natural Gas** | X | X | | Emitted when not completely burned | |
| **Coal** | X | X | X | | X |
| **Oil** | X | X | X | X | X |
| **Municipal Solid Waste** | X | | X | | X |
| **Other (Nuclear, Hydropower, Solar, Geothermal, Biomass, Landfill Gas, Wind)** | Biomass, Landfill Gas | | Biomass | | |

***United Stated Environmental Protection Agency  www.epa.gov***

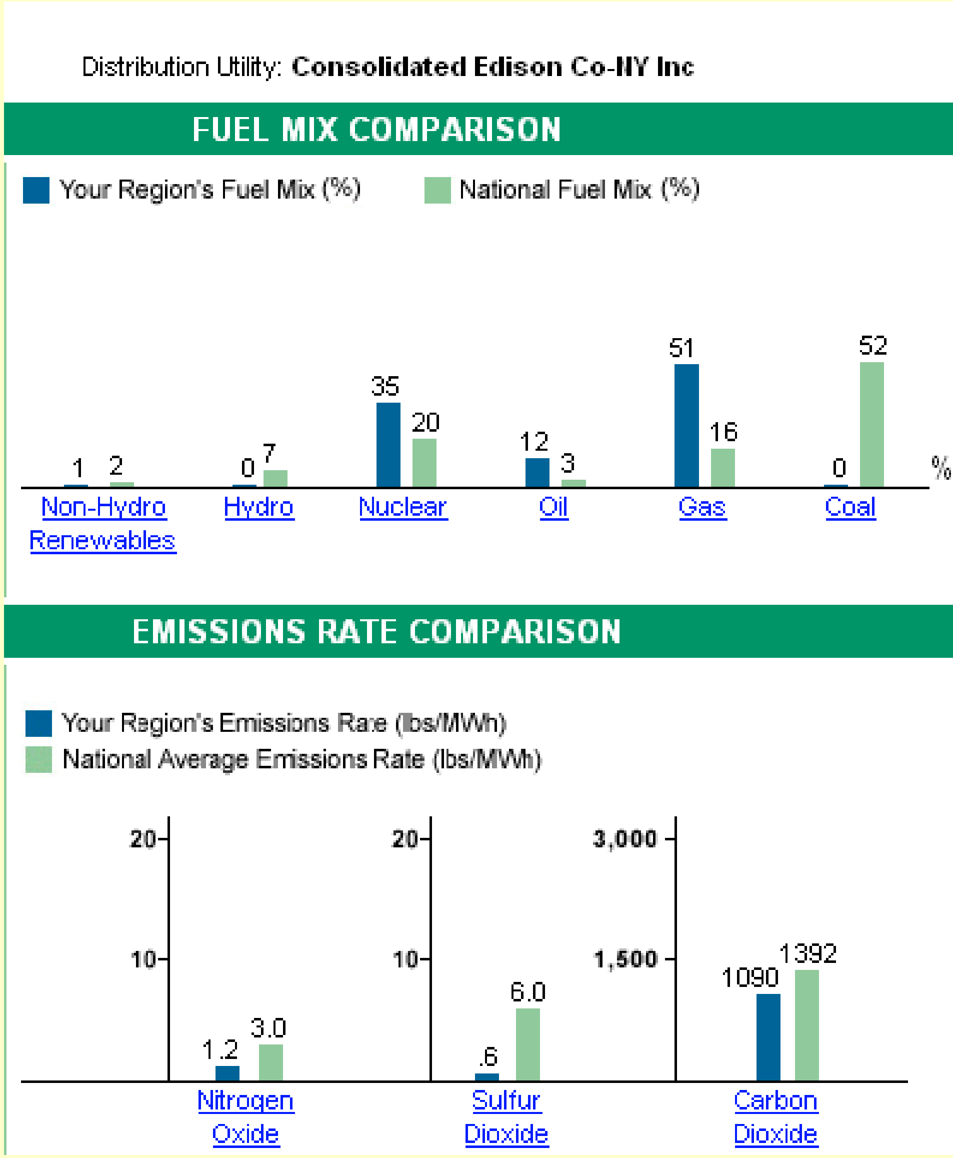




Distribution Utility: **Long Island Power Authority**

FUEL MIX COMPARISON

EMISSIONS RATE COMPARISON

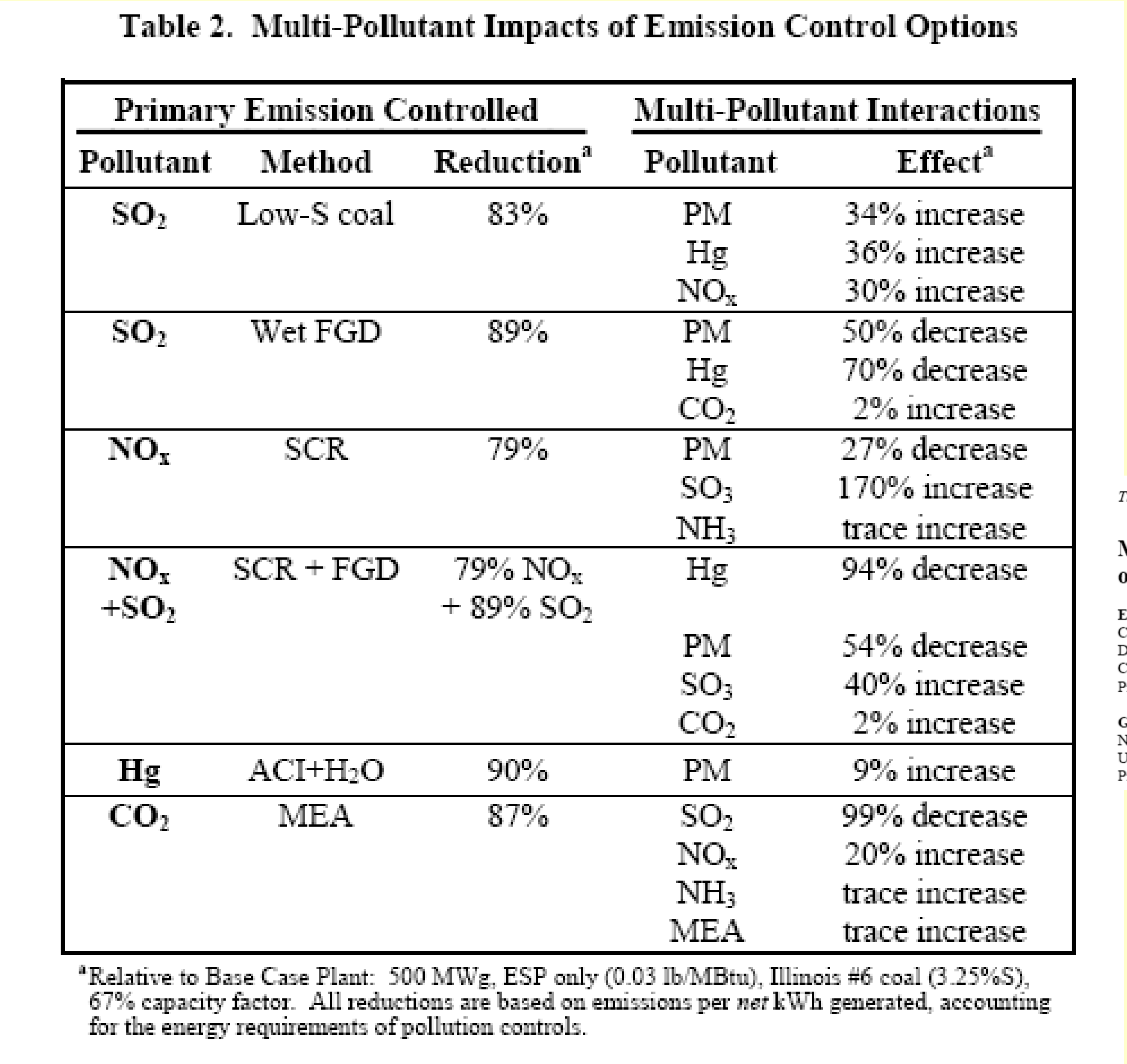


Distribution Utility: **Consolidated Edison Co-NY Inc**

FUEL MIX COMPARISON

EMISSIONS RATE COMPARISON

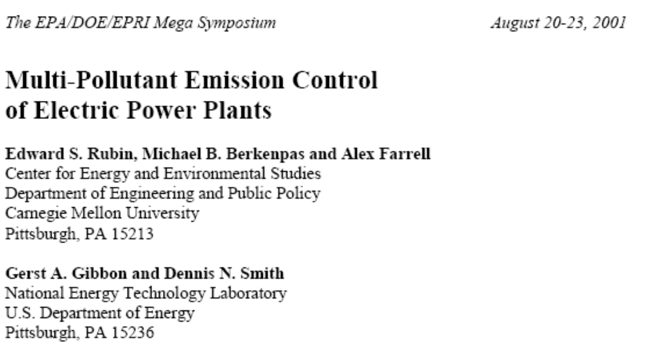
***United Stated Environmental Protection Agency  www.epa.gov***



## Table 2. Multi-Pollutant Impacts of Emission Control Options

| Primary Emission Controlled | | | Multi-Pollutant Interactions | |
|---|---|---|---|---|
| **Pollutant** | **Method** | **Reduction**[a] | **Pollutant** | **Effect**[a] |
| **$SO_2$** | Low-S coal | 83% | PM | 34% increase |
| | | | Hg | 36% increase |
| | | | $NO_x$ | 30% increase |
| **$SO_2$** | Wet FGD | 89% | PM | 50% decrease |
| | | | Hg | 70% decrease |
| | | | $CO_2$ | 2% increase |
| **$NO_x$** | SCR | 79% | PM | 27% decrease |
| | | | $SO_3$ | 170% increase |
| | | | $NH_3$ | trace increase |
| **$NO_x$ +$SO_2$** | SCR + FGD | 79% $NO_x$ + 89% $SO_2$ | Hg | 94% decrease |
| | | | PM | 54% decrease |
| | | | $SO_3$ | 40% increase |
| | | | $CO_2$ | 2% increase |
| **Hg** | ACI+$H_2O$ | 90% | PM | 9% increase |
| **$CO_2$** | MEA | 87% | $SO_2$ | 99% decrease |
| | | | $NO_x$ | 20% increase |
| | | | $NH_3$ | trace increase |
| | | | MEA | trace increase |

[a] Relative to Base Case Plant: 500 MWg, ESP only (0.03 lb/MBtu), Illinois #6 coal (3.25%S), 67% capacity factor. All reductions are based on emissions per *net* kWh generated, accounting for the energy requirements of pollution controls.



*The EPA/DOE/EPRI Mega Symposium* *August 20-23, 2001*

**Multi-Pollutant Emission Control**
**of Electric Power Plants**

**Edward S. Rubin, Michael B. Berkenpas and Alex Farrell**
Center for Energy and Environmental Studies
Department of Engineering and Public Policy
Carnegie Mellon University
Pittsburgh, PA 15213

**Gerst A. Gibbon and Dennis N. Smith**
National Energy Technology Laboratory
U.S. Department of Energy
Pittsburgh, PA 15236

# Spatial Nature of Pollutants

- The impacts of pollutants (such as SO2, NOx and Hg) depend critically on the location of their sources and where their impacts are realized, as in, for example, hot spots.

- There has been noted traffic volumes of trans-Pacific transport of pollutants from Asia to North America (most frequently) as well as trans-Atlantic transport from North America to Europe (**Akimoto (2003)**).

- Pollutants released from power plants located in the Midwestern U.S. travel by winds toward the East Coast of the U.S. and Canada.  (**EPA**)

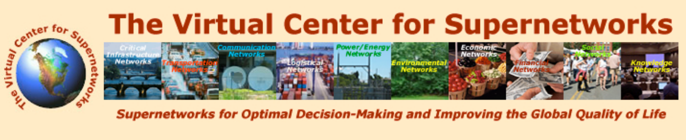
# Existing and proposed (market-based programs) to control multiple pollutants

- The Regional Clean Air Incentives Market (RECLAIM) program was implemented in California to control NOX and SOX pollutants.
- The proposed but not enacted Clear Skies was a national cap to reduce SO2, NOx and Hg by 73%, 67%, and 69% by 2018, respectively.
- EPA's Clean Air Interstate Rule (CAIR) capped emissions of SO2 and NOx in a large region covering more than 20 states, mostly east of the Mississippi, and the District of Columbia
- Together, the Clean Air Mercury Rule proposal and CAIR create a multi-pollutant strategy to improve air quality throughout the U.S.

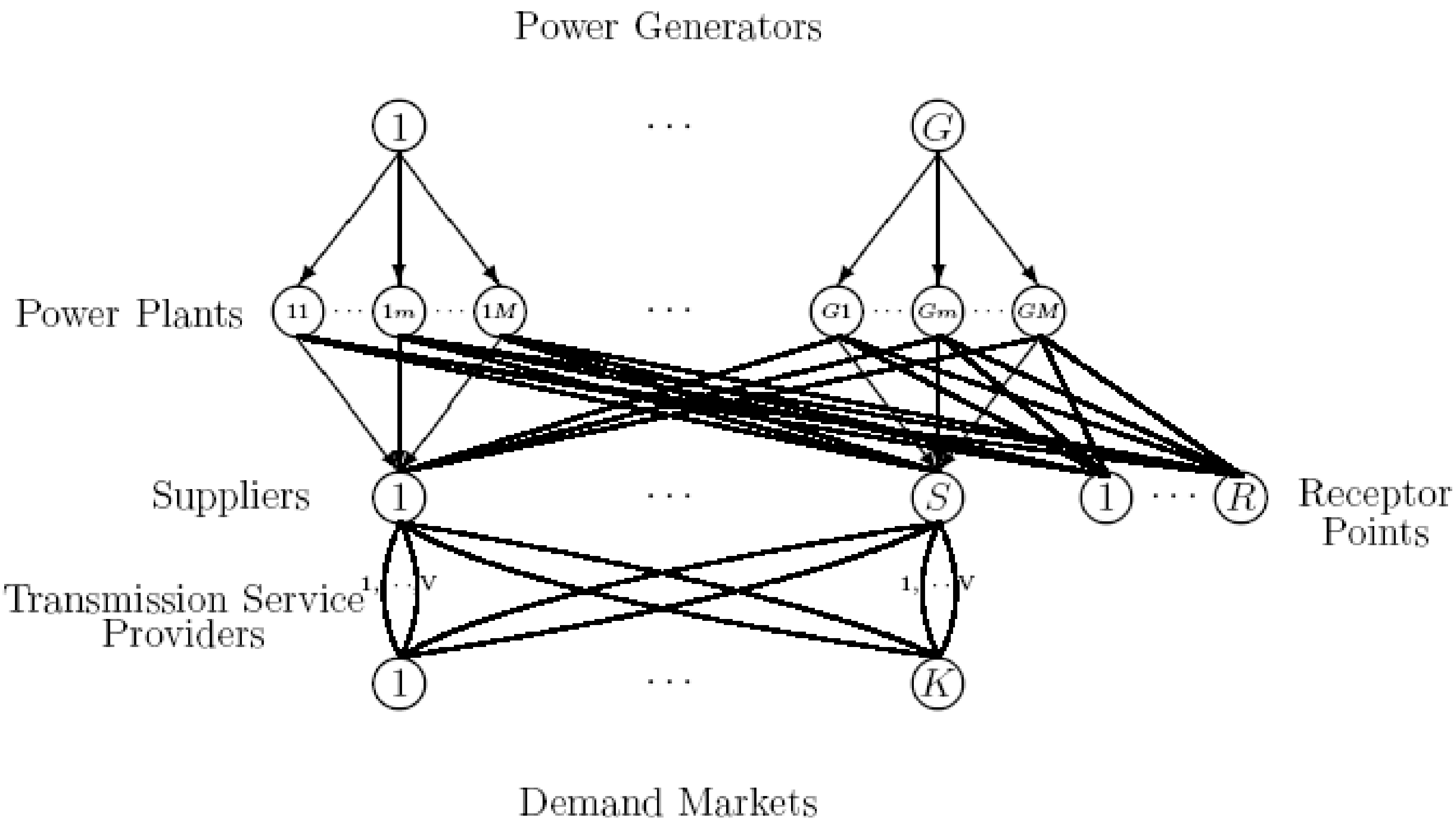
***United Stated Environmental Protection Agency  www.epa.gov***

# Literature

- K. Wu, A. Nagurney, Z. Liu and J. Stranlund, Modeling generator power plant portfolios and pollution taxes in electric power supply chain networks: A transportation network equilibrium transformation. (2006) (To appear in *Transportation Research D*; see also http://supernet.som.umass.edu)
- A. Nagurney and D. Matsypura, A supply chain network perspective for electric power generation, supply, transmission, and consumption. In: *Advances in Computational Economics, Finance and Management Science*, E. J. Kontoghiorghes and C. Gatu, (Eds.), Springer, Berlin, Germany, in press, (2005); appears in abbreviated and condensed form in: *Proceedings of the International Conference in Computing, Communications and Control Technologies*, Austin, Texas, Volume VI, pp. 127-134, (2004).
- Hogan, W. W. (1992) Contract networks for electric power transmission. Journal of Regulatory Economics 4, 211-242.
- K. K. Dhanda, A. Nagurney and P. Ramanujam, *Environmental Networks: A Framework for Economic Decision-Making and Policy Analysis*. Edward Elgar Publishers, Cheltenham, England, (1999).
- Nagurney, A., and Dhanda, K. K. (2000) Marketable pollution permits in oligopolistic markets with transaction costs. Operations Research, 48, 424-435.
- Chen, Y. H., and Hobbs, B. F. (2005) An oligopolistic electricity market model with tradable NOx permits. IEEE Transactions on Power Systemms, 20, 119-129.
- Tietenberg, T. H. (1985) Emissions trading: an exercise in reforming pollution policy. Resources for the Future, Washington DC.
- Rubin, E. S., Berkenpas, M. B., Farrel, A., Gibbon, G. A., and Smith, D. N. (2001) Multi-pollutant emission control of electric power plants. Proceedings of EPA-DOE-EPRI Mega Symposium, Chicago, Illinois, EPRI, Palo Alto, California.
- Schwarz, P. (2005) Multipollutant efficiency standards for electricity production. Contemporary Economic Policy, 23, 341-356.



# The Electric Power Supply Chain Network with Power Plants



# Notation for the Electric Power Supply Chain Network Model

| Notation | Definition |
|---|---|
| $q_{gm}$ | quantity of electricity produced by generator $g$ using power plant $m$, where $g = 1, \ldots, G; m = 1, \ldots, M$ |
| $q_m$ | $G$-dimensional vector of electric power generated by the gencos using power plant technology $m$ with components: $q_{1m}, \ldots, q_{Gm}$ |
| $q$ | $GM$-dimensional vector of all the electric power outputs generated by the gencos at the power plants |
| $Q^1$ | $GMS$-dimensional vector of electric power flows between the power plants of the power generators and the power suppliers with component $gms$ denoted by $q_{gms}$ |
| $Q^2$ | $SVK$-dimensional vector of power flows between suppliers and demand markets with component $svk$ denoted by $q^v_{sk}$ and denoting the flow between supplier $s$ and demand market $k$ via transmission provider $v$ |
| $d$ | $K$-dimensional vector of market demands with component $k$ denoted by $d_k$ |
| $f_{gm}(q_m)$ | power generating cost function of power generator $g$ using power plant $m$ with marginal power generating cost with respect to $q_{gm}$ denoted by $\frac{\partial f_{gm}}{\partial q_{gm}}$ |
| $c_{gms}(q_{gms})$ | transaction cost incurred by power generator $g$ using power plant $m$ in transacting with power supplier $s$ with marginal transaction cost denoted by $\frac{\partial c_{gms}(q_{gms})}{\partial q_{gms}}$ |

# Notation for the Electric Power Supply Chain Network Model

| | |
|---|---|
| $h$ | $S$-dimensional vector of the power suppliers' supplies of the electric power with component $s$ denoted by $h_s$, with $h_s \equiv \sum_{g=1}^{G} \sum_{m=1}^{M} q_{gms}$ |
| $c_s(h) \equiv c_s(Q^1)$ | operating cost of power supplier $s$ with marginal operating cost with respect to $h_s$ denoted by $\frac{\partial c_s}{\partial h_s}$ and the marginal operating cost with respect to $q_{gms}$ denoted by $\frac{\partial c_s(Q^1)}{\partial q_{gms}}$ |
| $c^v_{sk}(q^v_{sk})$ | transaction cost incurred by power supplier $s$ in transacting with demand market $k$ via transmission provider $v$ with marginal transaction cost with respect to $q^v_{sk}$ denoted by $\frac{\partial c^v_{sk}(q^v_{sk})}{\partial q^v_{sk}}$ |
| $\hat{c}_{gms}(q_{gms})$ | transaction cost incurred by power supplier $s$ in transacting with power generator $g$ for power generated by plant $m$ with marginal transaction cost denoted by $\frac{\partial \hat{c}_{gms}(q_{gms})}{\partial q_{gms}}$ |
| $\hat{c}^v_{sk}(Q^2)$ | unit transaction cost incurred by consumers at demand market $k$ in transacting with power supplier $s$ via transmission provider $v$ |
| $\rho_{3k}(d)$ | demand market price function at demand market $k$ |

# A Multiple Permit Trading Scheme

**The Optimization problem of the power generator can be expressed as follows:**

$$\text{Maximize} \quad \sum_{m=1}^{M}\sum_{s=1}^{S}\rho_{1gms}^{*}q_{gms} - \sum_{m=1}^{M} f_{gm}(q_m) - \sum_{m=1}^{M}\sum_{s=1}^{S} c_{gms}(q_{gms}) - \sum_{j=1}^{J}\sum_{m=1}^{M}\sum_{r=1}^{R}\tau_r^{j*}(l_{gmr}^{j} - l_{gmr}^{j0}).$$

**Subject to:**

$$\sum_{s=1}^{S} q_{gms} = q_{gm}, \quad m = 1, \ldots, M.$$

$$l_{gmr}^{j} \geq e_{gmr}^{j} q_{gm}, \quad j = 1 \ldots J; m = 1 \ldots, M; r = 1, \ldots, R.$$

$$q_{gms} \geq 0, \quad m = 1, \ldots, M; s = 1, \ldots, S,$$

$$l_{gmr}^{j} \geq 0, \quad j = 1, \ldots, J; m = 1, \ldots, M; r = 1, \ldots, R.$$

# A Multiple Permit Trading Scheme

**The Optimization problem faced by the supplier can be expressed as follows:**

$$\text{Maximize} \quad \sum_{k=1}^{K}\sum_{v=1}^{V}\rho_{2sk}^{v*}q_{sk}^{v} - c_s(Q^1) - \sum_{g=1}^{G}\sum_{m=1}^{M}\rho_{1gms}^{*}q_{gms} - \sum_{g=1}^{G}\sum_{m=1}^{M}\hat{c}_{gms}(q_{gms}) - \sum_{k=1}^{K}\sum_{v=1}^{V}c_{sk}^{v}(q_{sk}^{v})$$

subject to:

$$\sum_{k=1}^{K}\sum_{v=1}^{V}q_{sk}^{v} = \sum_{g=1}^{G}\sum_{m=1}^{M}q_{gms},$$

$$q_{gms} \geq 0, \quad g = 1,\ldots,G; m = 1,\ldots,M,$$

$$q_{sk}^{v} \geq 0; \quad k = 1,\ldots,K; v = 1,\ldots,V.$$

# A Multiple Permit Trading Scheme

**Market Equilibrium Conditions at Demand Market k**

$$\rho_{2sk}^{v*} + \hat{c}_{sk}^{v}(Q^{2*}) \begin{cases} = \rho_{3k}(d^*), & \text{if} \quad q_{sk}^{v*} > 0, \\ \geq \rho_{3k}(d^*), & \text{if} \quad q_{sk}^{v*} = 0. \end{cases}$$

**Equilibrium condition for the permits**

$$\sum_{g=1}^{G} \sum_{m=1}^{M} [l_{gmr}^{j0} - l_{gmr}^{j*}] \begin{cases} = 0, & \text{if} \quad \tau_r^{j*} > 0, \\ \geq 0, & \text{if} \quad \tau_r^{j*} = 0. \end{cases}$$

# Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Multiple Permit Markets

$$\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}}+\sum_{j=1}^{J}\sum_{r=1}^{R}\lambda_{gmr}^{j*}e_{gmr}^{j}\right]\times[q_{gm}-q_{gm}^*]+\sum_{s=1}^{S}\frac{\partial c_s(h^*)}{\partial h_s}\times[h_s-h_s^*]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}}+\frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}}\right]\times[q_{gms}-q_{gms}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\frac{\partial c_{sk}^{v}(q_{sk}^{v*})}{\partial q_{sk}^{v}}+\hat{c}_{sk}^{v}(Q^{2*})\right]\times[q_{sk}^{v}-q_{sk}^{v*}]+\sum_{k=1}^{K}\rho_{3k}(d^*)\times[d_k-d_k^*]$$

$$+\sum_{j=1}^{J}\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{r=1}^{R}\left[\tau_r^{j*}-\lambda_{gmr}^{j*}\right]\times[l_{gmr}^{j}-l_{gmr}^{j*}]$$

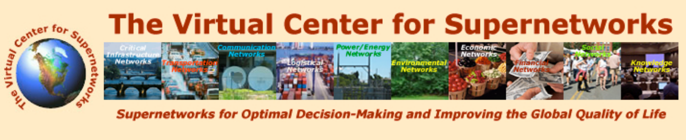
$$+\sum_{j=1}^{J}\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{r=1}^{R}\left[l_{gmr}^{j*}-e_{gmr}^{j}q_{gm}^*\right]\times[\lambda_{gmr}^{j}-\lambda_{gmr}^{j*}]+\sum_{j=1}^{J}\sum_{r=1}^{R}\left[\sum_{g=1}^{G}\sum_{m=1}^{M}(l_{gmr}^{j0}-l_{gmr}^{j*})\right]\times[\tau_r^{j}-\tau_r^{j*}]\geq 0,$$

$$\forall (q,h,Q^1,Q^2,d,l,\lambda,\tau)\in\mathcal{K}^6,$$

*where*

$$\mathcal{K}^6\equiv\{(q,h,Q^1,Q^2,d,l,\lambda,\tau)|(q,h,Q^1,Q^2,d,l,\lambda,\tau)\in R^{GM+S+GMS+SKV+K+2JGMR+JR}$$

*and the constraints hold.*



# Special Cases of the Model

- **Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Multiple Permit Markets and Single Receptor Point**

- **Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Single Receptor Point and Single Pollutant**

- **Attainment of Environmental Standards**

# Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Multiple Permit Markets and Single Receptor Point

$$
\begin{aligned}
&\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}}+\sum_{j=1}^{J}\lambda_{gm}^{j*}e_{gm}^{j}\right]\times\left[q_{gm}-q_{gm}^*\right]+\sum_{s=1}^{S}\frac{\partial c_s(h^*)}{\partial h_s}\times\left[h_s-h_s^*\right]\\
&+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}}+\frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}}\right]\times\left[q_{gms}-q_{gms}^*\right]\\
&+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\frac{\partial c_{sk}^{v}(q_{sk}^{v*})}{\partial q_{sk}^{v}}+\hat{c}_{sk}^{v}(Q^{2*})\right]\times\left[q_{sk}^{v}-q_{sk}^{v*}\right]-\sum_{k=1}^{K}\rho_{3k}(d^*)\times\left[d_k-d_k^*\right]\\
&+\sum_{j=1}^{J}\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\tau^{j*}-\lambda_{gm}^{j*}\right]\times\left[l_{gm}^{j}-l_{gm}^{j*}\right]\\
&+\sum_{j=1}^{J}\sum_{g=1}^{G}\sum_{m=1}^{M}\left[l_{gm}^{*j}-e_{gm}^{j}q_{gm}^*\right]\times\left[\lambda_{gm}^{j}-\lambda_{gm}^{*j}\right]+\sum_{j=1}^{J}\left[\sum_{g=1}^{G}\sum_{m=1}^{M}(l_{gm}^{0j}-l_{gmr}^{*j})\right]\times\left[\tau^{j}-\tau^{j*}\right]\geq 0,\\
&\forall (q,h,Q^1,Q^2,l,\lambda,\tau)\in\mathcal{K}^7,
\end{aligned}
$$

The Virtual Center for Supernetworks

Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

UMassAmherst ISENBERG SCHOOL OF MANAGEMENT

# Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Single Receptor Point and Single Pollutant

$$\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}}+\lambda_{gm}^* e_{gm}\right]\times\left[q_{gm}-q_{gm}^*\right]+\sum_{s=1}^{S}\frac{\partial c_s(h^*)}{\partial h_s}\times\left[h_s-h_s^*\right]$$

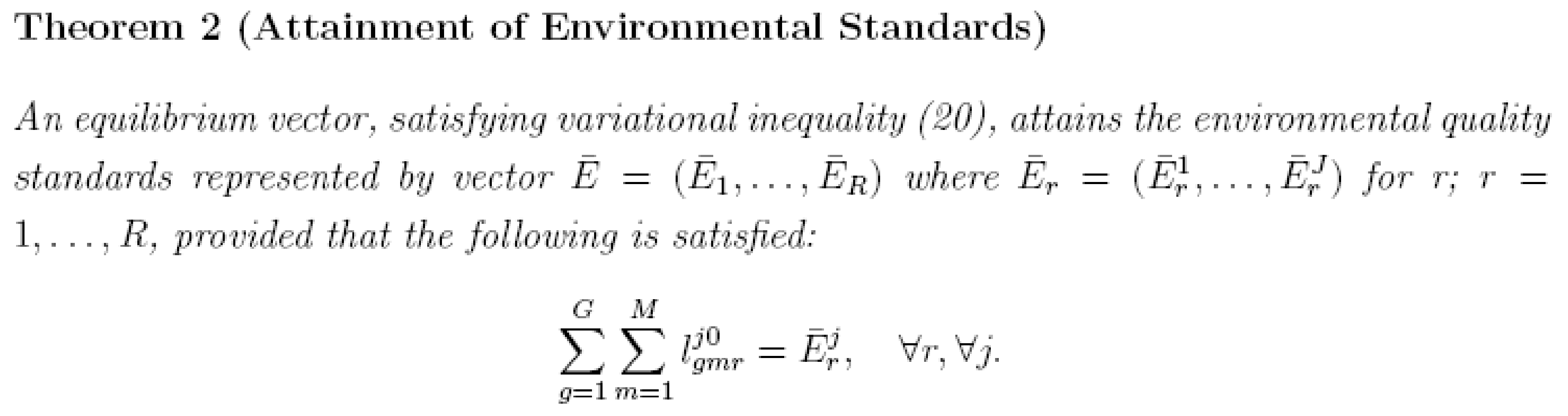
$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}}+\frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}}\right]\times\left[q_{gms}-q_{gms}^*\right]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\frac{\partial c_{sk}^{v}(q_{sk}^{v*})}{\partial q_{sk}^{v}}+\hat{c}_{sk}^{v}(Q^{2*})\right]\times\left[q_{sk}^{v}-q_{sk}^{v*}\right]-\sum_{k=1}^{K}\rho_{3k}(d^*)\times\left[d_k-d_k^*\right]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\tau^*-\lambda_{gm}^*\right]\times\left[l_{gm}-l_{gm}^*\right]+\sum_{g=1}^{G}\sum_{m=1}^{M}\left[l_{gm}^*-e_{gm}q_{gm}^*\right]\times\left[\lambda_{gm}-\lambda_{gm}^*\right]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}(l_{gm}^0-l_{gm}^*)\times\left[\tau-\tau^*\right]\geq 0,\quad \forall (q,h,Q^1,Q^2,l,\lambda,\tau)\in\mathcal{K}^8,$$

# Attainment of Environmental Standards

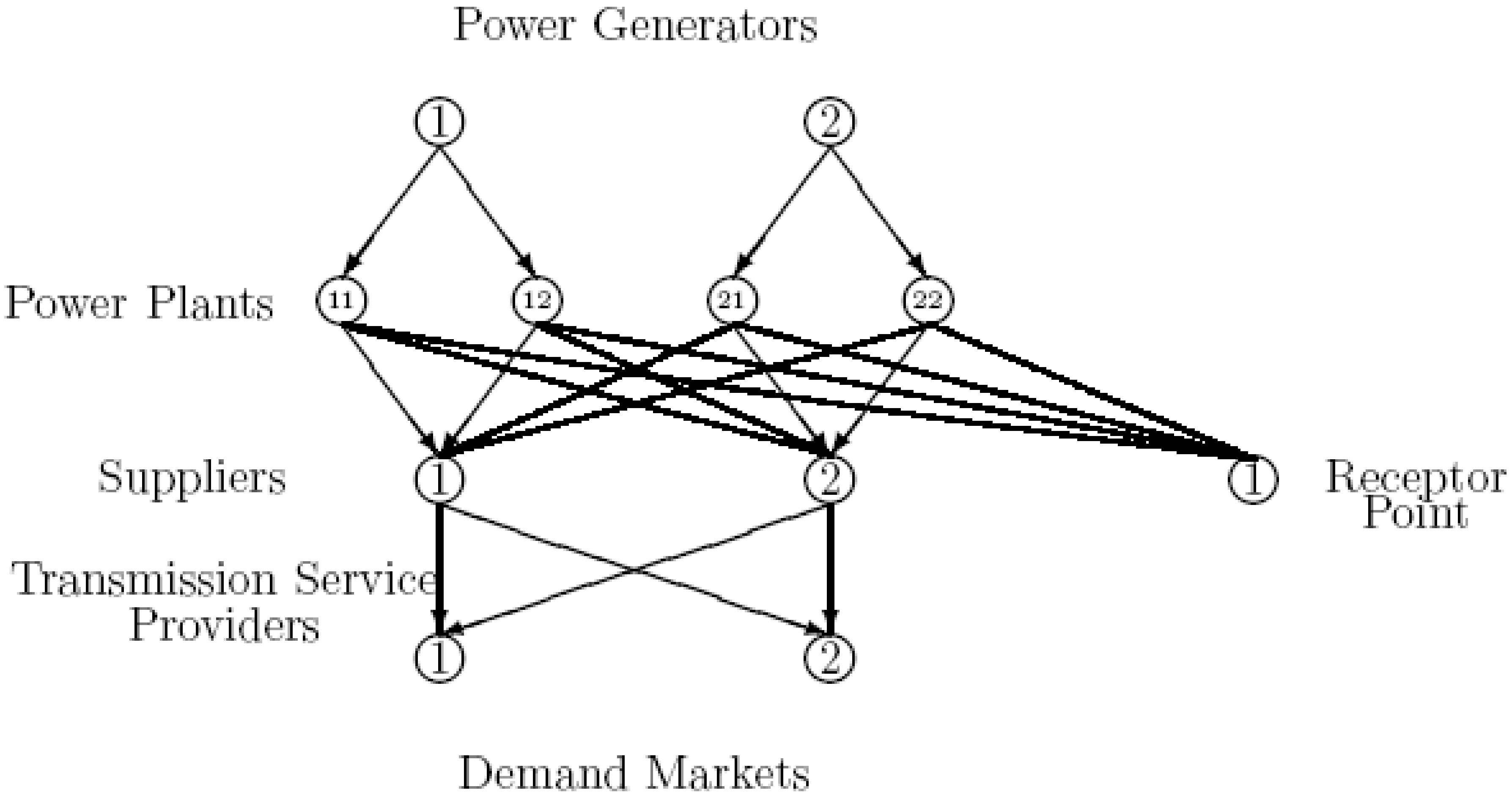


**Theorem 2 (Attainment of Environmental Standards)**

*An equilibrium vector, satisfying variational inequality (20), attains the environmental quality standards represented by vector* $\bar{E} = (\bar{E}_1, \ldots, \bar{E}_R)$ *where* $\bar{E}_r = (\bar{E}_r^1, \ldots, \bar{E}_r^J)$ *for* $r$*;* $r = 1, \ldots, R$*, provided that the following is satisfied:*

$$\sum_{g=1}^{G} \sum_{m=1}^{M} l_{gmr}^{j0} = \bar{E}_r^j, \quad \forall r, \forall j.$$

# Numerical Examples



# Numerical Examples

## Example 1

The emission terms: $e_{gm}$; $g = 1, 2$; $m = 1, 2$ were all equal to 1.

The power generating cost functions for the power generators were given by:

$$f_{11}(q_1) = 2.5q_{11}^2+q_{11}q_{21}+2q_{11}, \quad f_{12}(q_2) = 2.5q_{12}^2+q_{11}q_{12}+2q_{22}, \quad f_{21}(q_1) = .5q_{21}^2+.5q_{11}q_{21}+2q_{21},$$

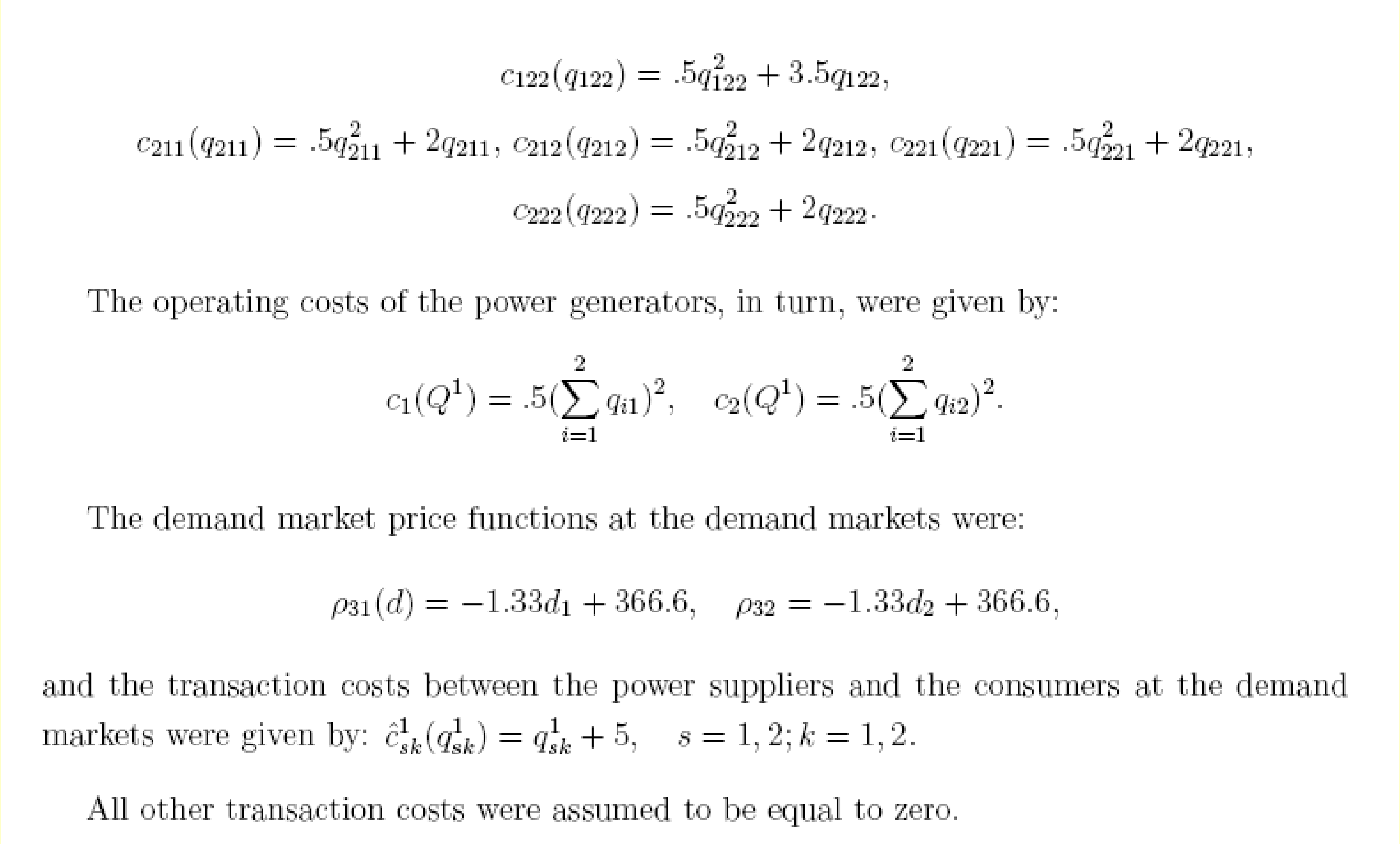
$$f_{22}(q_2) = .5q_{22}^2 + q_{12}q_{22} + 2q_{22}.$$

The transaction cost functions faced by the power generators and associated with transacting with the power suppliers were given by:

$$c_{111}(q_{111}) = .5q_{111}^2 + 3.5q_{111}, \; c_{112}(q_{112}) = .5q_{112}^2 + 3.5q_{112}, \; c_{121}(q_{121}) = .5q_{121}^2 + 3.5q_{121},$$

# Numerical Examples

## Example 1



$$c_{122}(q_{122}) = .5q_{122}^2 + 3.5q_{122},$$

$$c_{211}(q_{211}) = .5q_{211}^2 + 2q_{211},\ c_{212}(q_{212}) = .5q_{212}^2 + 2q_{212},\ c_{221}(q_{221}) = .5q_{221}^2 + 2q_{221},$$

$$c_{222}(q_{222}) = .5q_{222}^2 + 2q_{222}.$$

The operating costs of the power generators, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2} q_{i2})^2.$$

The demand market price functions at the demand markets were:

$$\rho_{31}(d) = -1.33d_1 + 366.6, \quad \rho_{32} = -1.33d_2 + 366.6,$$

and the transaction costs between the power suppliers and the consumers at the demand markets were given by: $\hat{c}_{sk}^1(q_{sk}^1) = q_{sk}^1 + 5, \quad s = 1, 2; k = 1, 2.$

All other transaction costs were assumed to be equal to zero.

# Numerical Examples

## Example 2

Example 2 had the same data as Example 1, but we now tightened the emissions standard so that $\bar{E} = 50$. The initial license allocation was now given by: $l^0_{11} = l^0_{12} = l^0_{21} = l^0_{22} = 12.5$.

## Example 3

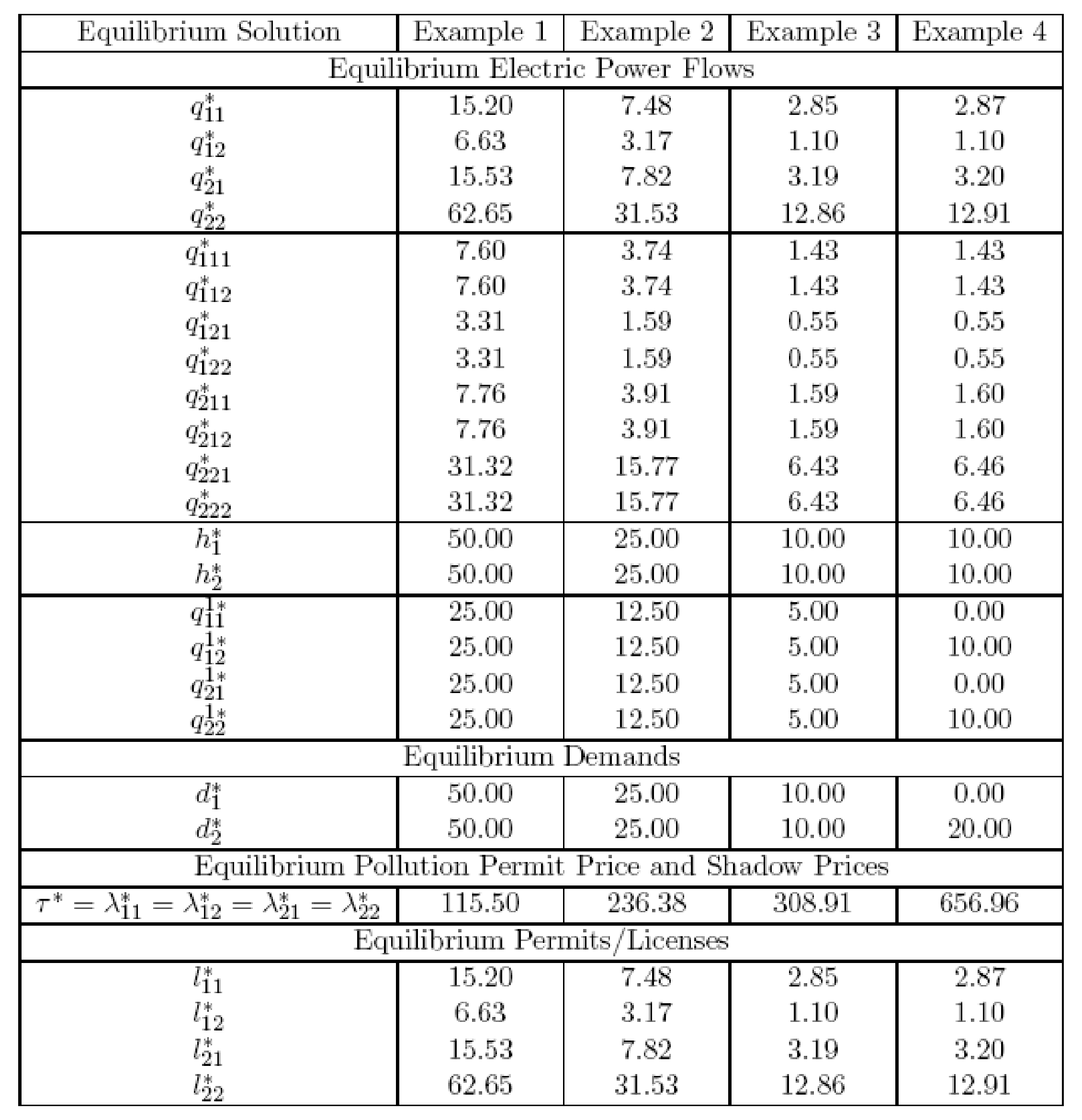
Example 3 had the identical data to that in Examples 1 and 2, except that the environmental standard was further tightened to $\bar{E} = 20$ with the new initial license allocation given by: $l^0_{11} = l^0_{12} = l^0_{21} = l^0_{22} = 5$.

## Example 4

Example 4 had the same data as Example 3 except that we modified the second demand market price function for electric power to:

$$\rho_{32}(d) = -1.33d_2 + 733.30.$$

# Solutions to Numerical Examples



| Equilibrium Solution | Example 1 | Example 2 | Example 3 | Example 4 |
|---|---|---|---|---|
| Equilibrium Electric Power Flows | | | | |
| $q_{11}^*$ | 15.20 | 7.48 | 2.85 | 2.87 |
| $q_{12}^*$ | 6.63 | 3.17 | 1.10 | 1.10 |
| $q_{21}^*$ | 15.53 | 7.82 | 3.19 | 3.20 |
| $q_{22}^*$ | 62.65 | 31.53 | 12.86 | 12.91 |
| $q_{111}^*$ | 7.60 | 3.74 | 1.43 | 1.43 |
| $q_{112}^*$ | 7.60 | 3.74 | 1.43 | 1.43 |
| $q_{121}^*$ | 3.31 | 1.59 | 0.55 | 0.55 |
| $q_{122}^*$ | 3.31 | 1.59 | 0.55 | 0.55 |
| $q_{211}^*$ | 7.76 | 3.91 | 1.59 | 1.60 |
| $q_{212}^*$ | 7.76 | 3.91 | 1.59 | 1.60 |
| $q_{221}^*$ | 31.32 | 15.77 | 6.43 | 6.46 |
| $q_{222}^*$ | 31.32 | 15.77 | 6.43 | 6.46 |
| $h_1^*$ | 50.00 | 25.00 | 10.00 | 10.00 |
| $h_2^*$ | 50.00 | 25.00 | 10.00 | 10.00 |
| $q_{11}^{1*}$ | 25.00 | 12.50 | 5.00 | 0.00 |
| $q_{12}^{1*}$ | 25.00 | 12.50 | 5.00 | 10.00 |
| $q_{21}^{1*}$ | 25.00 | 12.50 | 5.00 | 0.00 |
| $q_{22}^{1*}$ | 25.00 | 12.50 | 5.00 | 10.00 |
| Equilibrium Demands | | | | |
| $d_1^*$ | 50.00 | 25.00 | 10.00 | 0.00 |
| $d_2^*$ | 50.00 | 25.00 | 10.00 | 20.00 |
| Equilibrium Pollution Permit Price and Shadow Prices | | | | |
| $\tau^* = \lambda_{11}^* = \lambda_{12}^* = \lambda_{21}^* = \lambda_{22}^*$ | 115.50 | 236.38 | 308.91 | 656.96 |
| Equilibrium Permits/Licenses | | | | |
| $l_{11}^*$ | 15.20 | 7.48 | 2.85 | 2.87 |
| $l_{12}^*$ | 6.63 | 3.17 | 1.10 | 1.10 |
| $l_{21}^*$ | 15.53 | 7.82 | 3.19 | 3.20 |
| $l_{22}^*$ | 62.65 | 31.53 | 12.86 | 12.91 |

# Conclusions

- The model presented in this paper may help policy-makers to determine an appropriate quantity of permits of each pollutant in the market and associated effect on the permit prices and power plants in the electric power generation industry, including flows and prices.

- The numerical results demonstrate, as the theory predicts, that the permit level achieves the desired goal, in that the imposed licenses on the emission of each pollutant is not exceeded.

- Moreover, it illustrates the spectrum of scenarios that can be explored in terms of changes in the amount of the permit allocation, its effects on emissions; changes in emission factors; changes in the demand functions, etc.

# *Thank you!*

***For more information, please see the***

## The Virtual Center for Supernetworks

*Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life*

Home About Background Activities Publications Media Links What's New Search

Supernetworks Laboratory for Computation and Visualization

# http://supernet.som.umass.edu/lab.htm