Spatially Differentiated Trade of Permits for Multipollutant Electric Power Supply Chains

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Topics

- Paper Outline
- Motivation
- Literature
- Notation for Models
- Models
- Numerical Examples
- Conclusion

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- This support is gratefully acknowledged and appreciated.

Paper Outline

- Develop a modeling and computational framework that allows for the determination of optimal prices for the various permits applied to electric power plants in the context of electric power supply chain (generation/distribution/consumption) networks.

- The general framework that we develop allows for special cases of the model.

- Numerical Examples

Motivation

- The electrical industry growth and effect on air pollution
- Capture substitutability and complementarities of pollutants related to electricity generation
- Spatial Nature of Pollutants
- Importance of environmental-energy modeling which includes credit trading to address market failures in energy.

Electricity Production

- Electricity generation is the dominant industrial source of air emissions in the United States today. For example, fossil fuel-fired power plants are responsible for 67% of the nation's sulfur dioxide emissions, 23% of nitrogen oxide emissions, and 40% of man-made carbon dioxide emissions. (**EPA**)
- Electricity worldwide is produced predominantly by coal, which is responsible for 40% of the carbon dioxide pollution (hence global warming), and which is expected to maintain about a 36% share of the electricity generation market through 2020. (**IPCC**)
- The electricity industry is increasing, with the total consumption of electricity worldwide projected to reach 16.4 trillion kilowatt hours by 2010 and 23.1 by 2025. (**Business and Company Resource Center**)

	Nitrogen Oxides	Carbon Dioxide (greenhouse gas)	Sulfur Dioxide	Methane (greenhouse gas)	Mercury
Natural Gas	X	X		Emitted when not completely burned	
Coal	X	X	X		X
Oil	X	X	X	X	X
Municipal Solid Waste	X		X		X
Other (Nuclear, Hydropower, Solar, Geothermal, Biomass, Landfill Gas, Wind)	Biomass, Landfill Gas		Biomass		

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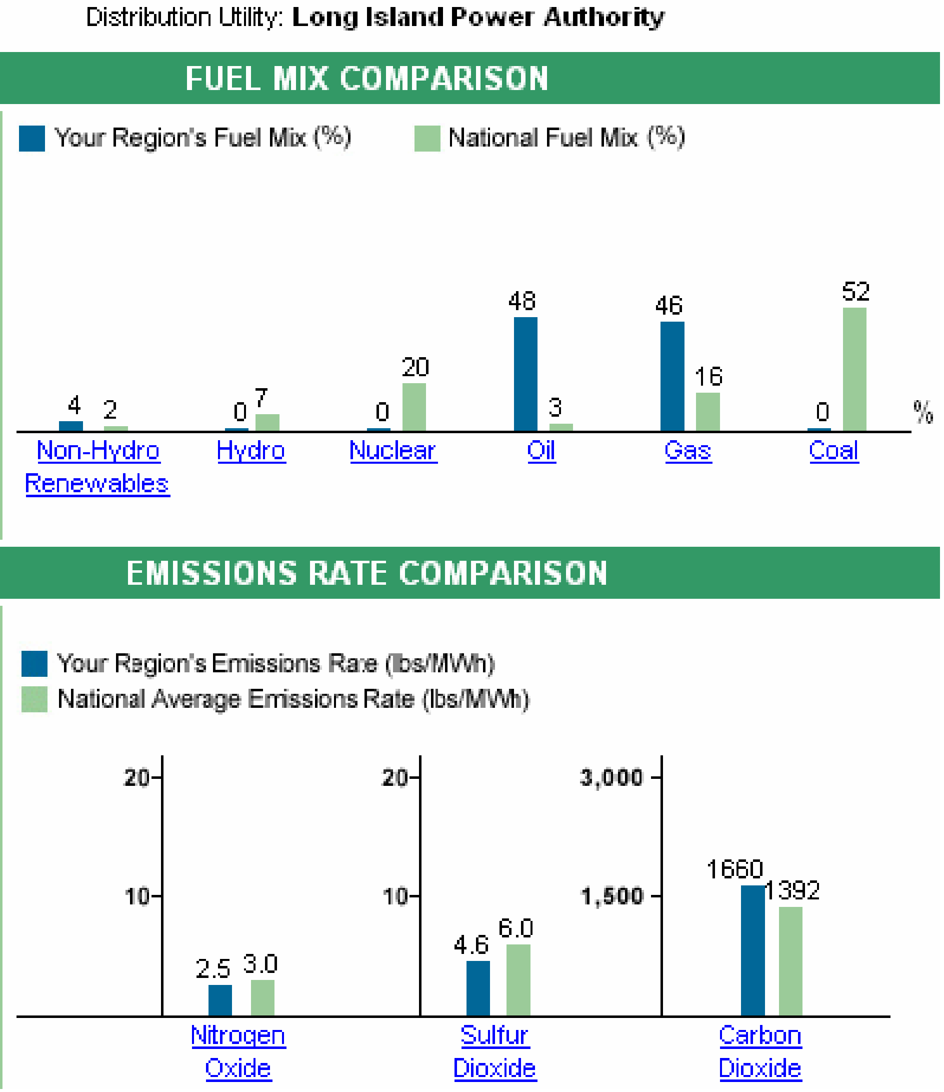
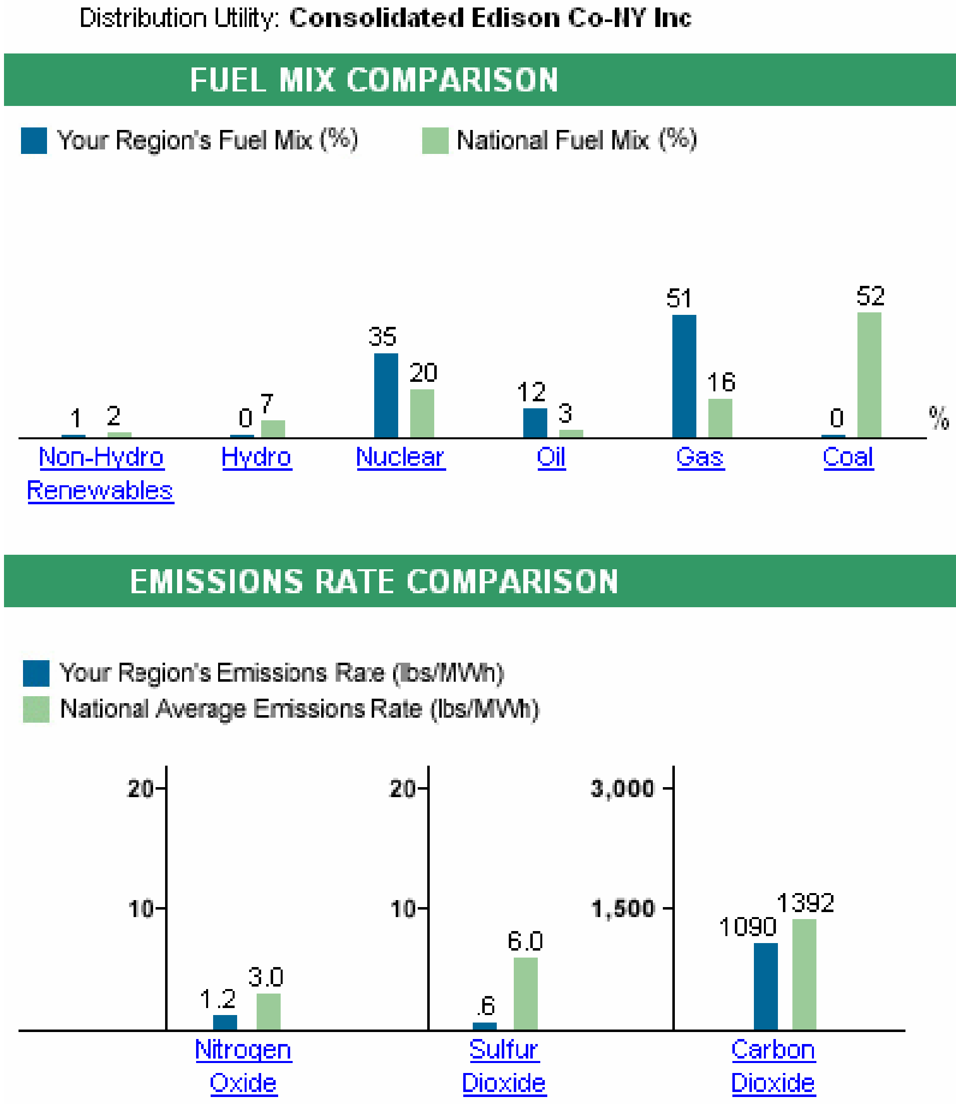


Distribution Utility: **Long Island Power Authority**



Distribution Utility: **Consolidated Edison Co-NY Inc**

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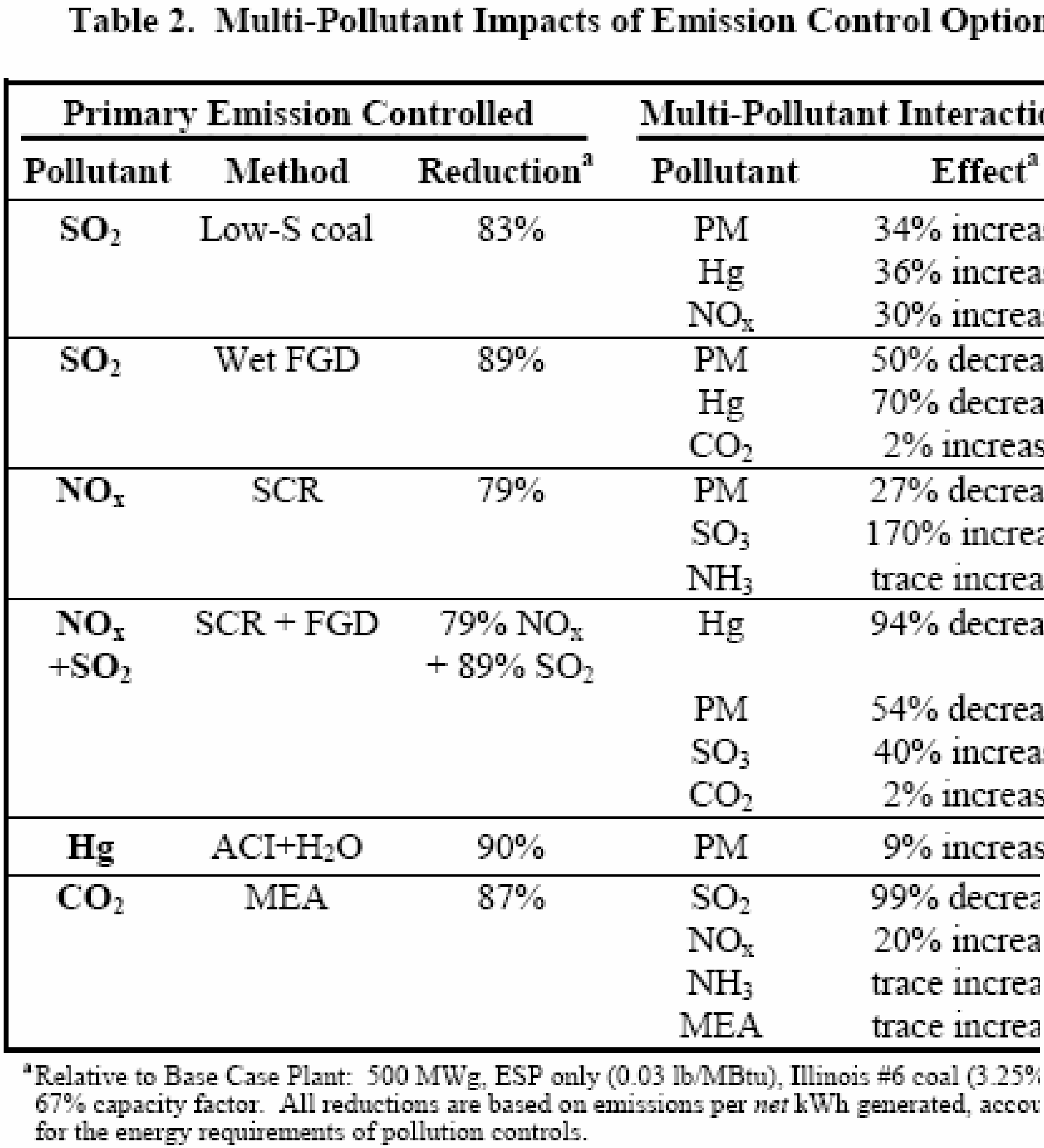
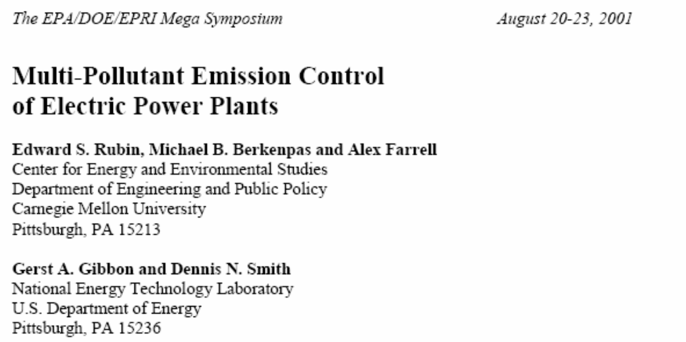


Table 2. Multi-Pollutant Impacts of Emission Control Options

Primary Emission Controlled			Multi-Pollutant Interactions	
Pollutant	Method	Reduction[a]	Pollutant	Effect[a]
SO_2	Low-S coal	83%	PM	34% increase
			Hg	36% increase
			NO_x	30% increase
SO_2	Wet FGD	89%	PM	50% decrease
			Hg	70% decrease
			CO_2	2% increase
NO_x	SCR	79%	PM	27% decrease
			SO_3	170% increase
			NH_3	trace increase
NO_x +SO_2	SCR + FGD	79% NO_x + 89% SO_2	Hg	94% decrease
			PM	54% decrease
			SO_3	40% increase
			CO_2	2% increase
Hg	ACI+H_2O	90%	PM	9% increase
CO_2	MEA	87%	SO_2	99% decrea
			NO_x	20% increa
			NH_3	trace increa
			MEA	trace increa

[a] Relative to Base Case Plant: 500 MWg, ESP only (0.03 lb/MBtu), Illinois #6 coal (3.25% 67% capacity factor. All reductions are based on emissions per *net* kWh generated, accou for the energy requirements of pollution controls.



The EPA/DOE/EPRI Mega Symposium *August 20-23, 2001*

Multi-Pollutant Emission Control of Electric Power Plants

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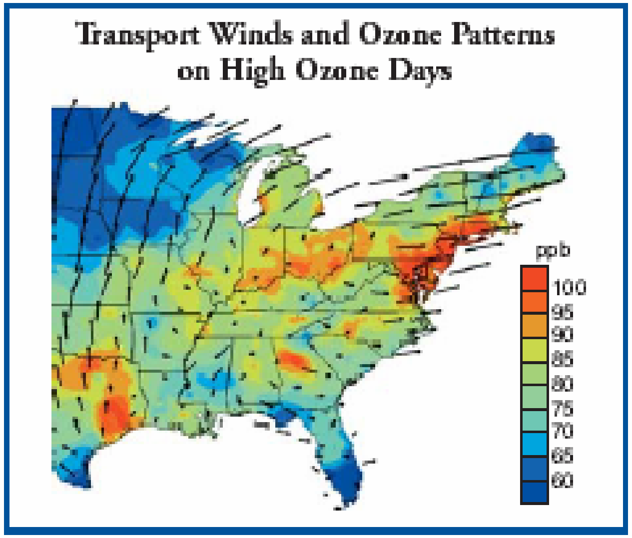
Spatial Nature of Pollutants

- The impacts of pollutants (such as SO_2, NO_x and Hg) depend critically on the location of their sources and where their impacts are realized, as in, for example, hot spots.

- There has been noted traffic volumes of trans-Pacific transport of pollutants from Asia to North America (most frequently) as well as trans-Atlantic transport from North America to Europe (**Akimoto (2003)**).


Transport Winds and Ozone Patterns on High Ozone Days

High ozone levels in the Northeast are typically associated with persistent transport from west to east. (Data represent high 90th percentile ozone conditions.)
Source: Ozone Transport Assessment Group

- Pollutants released from power plants located in the Midwestern U.S. travel by winds toward the East Coast of the U.S. and Canada. (**EPA**)

Existing and proposed market-based programs) to control multiple pollutants

- The Regional Clean Air Incentives Market (RECLAIM) program was implemented in California to control NOX and SOX pollutants.
- The proposed but not enacted Clear Skies was a national cap to reduce SO_2, NOx and Hg by 73%, 67%, and 69% by 2018, respectively.
- EPA's Clean Air Interstate Rule (CAIR) capped emissions of SO_2 and NOx in a large region covering more than 20 states, mostly east of the Mississippi, and the District of Columbia
- Together, the Clean Air Mercury Rule proposal and CAIR create a multi-pollutant strategy to improve air quality throughout the U.S. ***United Stated Environmental Protection Agency www.epa.gov***

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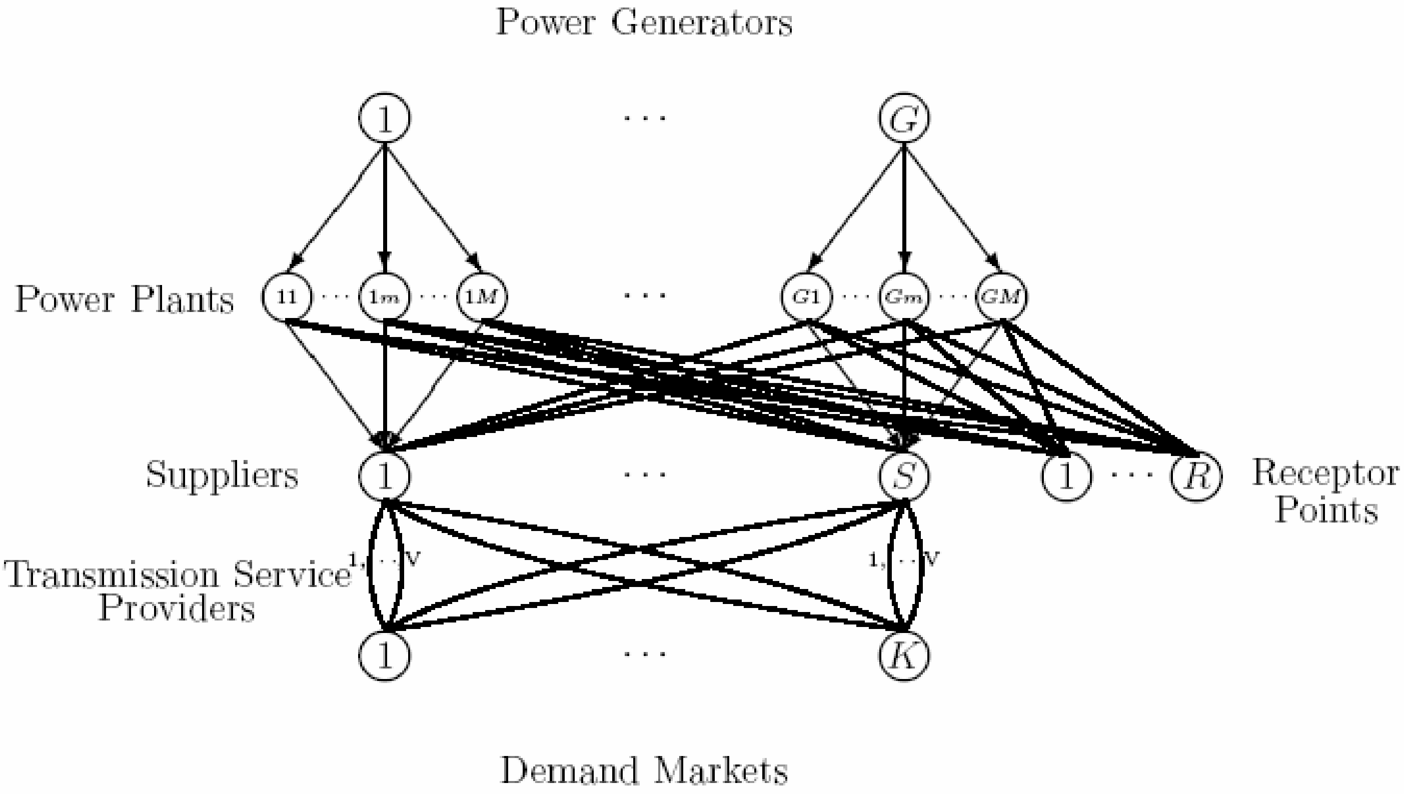
The Electric Power Supply Chain Network with Power Plants



A Multiple Permit Trading Scheme

Since we have assumed that each individual power generator is a profit-maximizer, the objective function of power generator, *g*, can be expressed as follows:

$$\text{Maximize} \quad \sum_{m=1}^{M}\sum_{s=1}^{S}\rho^*_{1gms}q_{gms} - \sum_{m=1}^{M} f_{gm}(q_m) - \sum_{m=1}^{M}\sum_{s=1}^{S} c_{gms}(q_{gms}) - \sum_{j=1}^{J}\sum_{m=1}^{M}\sum_{r=1}^{R}\tau_r^{j*}(l^j_{gmr} - l^{j0}_{gmr}).$$

Subject to:

$$\sum_{s=1}^{S} q_{gms} = q_{gm}, \quad m = 1, \ldots, M.$$

$$l^j_{gmr} \geq e^j_{gmr} q_{gm}, \quad j = 1 \ldots J; m = 1 \ldots, M; r = 1, \ldots, R.$$

$$q_{gms} \geq 0, \quad m = 1, \ldots, M; s = 1, \ldots, S,$$

$$l^j_{gmr} \geq 0, \quad j = 1, \ldots, J; m = 1, \ldots, M; r = 1, \ldots, R.$$

A Multiple Permit Trading Scheme

Since we have assumed that each individual retailer is a profit-maximizer, the objective function of each retailer can be expressed as follows:

$$\text{Maximize} \quad \sum_{k=1}^{K}\sum_{v=1}^{V}\rho_{2sk}^{v*}q_{sk}^{v} - c_s(Q^1) - \sum_{g=1}^{G}\sum_{m=1}^{M}\rho_{1gms}^{*}q_{gms} - \sum_{g=1}^{G}\sum_{m=1}^{M}\hat{c}_{gms}(q_{gms}) - \sum_{k=1}^{K}\sum_{v=1}^{V}c_{sk}^{v}(q_{sk}^{v})$$

subject to:

$$\sum_{k=1}^{K}\sum_{v=1}^{V}q_{sk}^{v} = \sum_{g=1}^{G}\sum_{m=1}^{M}q_{gms},$$

$$q_{gms} \geq 0, \quad g = 1, \ldots, G; m = 1, \ldots, M,$$

$$q_{sk}^{v} \geq 0; \quad k = 1, \ldots, K; v = 1, \ldots, V.$$

A Multiple Permit Trading Scheme

Market Equilibrium Conditions at Demand Market k

$$\rho_{2sk}^{v*} + \hat{c}_{sk}^{v}(Q^{2*}) \begin{cases} = \rho_{3k}(d^*), & \text{if} \quad q_{sk}^{v*} > 0, \\ \geq \rho_{3k}(d^*), & \text{if} \quad q_{sk}^{v*} = 0. \end{cases}$$

Equilibrium condition for the permits

$$\sum_{g=1}^{G} \sum_{m=1}^{M} [l_{gmr}^{j0} - l_{gmr}^{j*}] \begin{cases} = 0, & \text{if} \quad \tau_r^{j*} > 0, \\ \geq 0, & \text{if} \quad \tau_r^{j*} = 0. \end{cases}$$

The Equilibrium Conditions for the Electric Power Supply Chain Network with a Multipollutant Permit System

In equilibrium, the optimality conditions for all the power generators and for all the power suppliers, and the equilibrium conditions for all the demand markets as well as the multiple permit markets must be simultaneously satisfied so that no decision-maker has any incentive to alter his transactions.

Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Multiple Permit Markets

We assume that the cost functions of the generators are convex and continuously differentiable and that the generators compete with one another in a noncooperative manner (in the sense of Nash)

$$\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}}+\sum_{j=1}^{J}\sum_{r=1}^{R}\lambda_{gmr}^{j*}e_{gmr}^{j}\right]\times[q_{gm}-q_{gm}^*]+\sum_{s=1}^{S}\frac{\partial c_s(h^*)}{\partial h_s}\times[h_s-h_s^*]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}}+\frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}}\right]\times[q_{gms}-q_{gms}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\frac{\partial c_{sk}^{v}(q_{sk}^{v*})}{\partial q_{sk}^{v}}+\hat{c}_{sk}^{v}(Q^{2*})\right]\times[q_{sk}^{v}-q_{sk}^{v*}]+\sum_{k=1}^{K}\rho_{3k}(d^*)\times[d_k-d_k^*]$$

$$+\sum_{j=1}^{J}\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{r=1}^{R}\left[\tau_r^{j*}-\lambda_{gmr}^{j*}\right]\times[l_{gmr}^{j}-l_{gmr}^{j*}]$$

$$+\sum_{j=1}^{J}\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{r=1}^{R}\left[l_{gmr}^{j*}-e_{gmr}^{j}q_{gm}^*\right]\times[\lambda_{gmr}^{j}-\lambda_{gmr}^{j*}]+\sum_{j=1}^{J}\sum_{r=1}^{R}\left[\sum_{g=1}^{G}\sum_{m=1}^{M}(l_{gmr}^{j0}-l_{gmr}^{j*})\right]\times[\tau_r^{j}-\tau_r^{j*}]\geq 0,$$

$$\forall(q,h,Q^1,Q^2,d,l,\lambda,\tau)\in\mathcal{K}^6,$$

where

$$\mathcal{K}^6\equiv\{(q,h,Q^1,Q^2,d,l,\lambda,\tau)|(q,h,Q^1,Q^2,d,l,\lambda,\tau)\in R_+^{GM+S+GMS+SKV+K+2JGMR+JR}$$

and the constraints hold.

Special Cases of the Model

- **Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Multiple Permit Markets and Single Receptor Point**
- **Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Single Receptor Point and Single Pollutant**
- **Attainment of Environmental Standards**

Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Multiple Permit Markets and Single Receptor Point

$$\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}}+\sum_{j=1}^{J}\lambda_{gm}^{j*}e_{gm}^{j}\right]\times[q_{gm}-q_{gm}^*]+\sum_{s=1}^{S}\frac{\partial c_s(h^*)}{\partial h_s}\times[h_s-h_s^*]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}}+\frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}}\right]\times[q_{gms}-q_{gms}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\frac{\partial c_{sk}^{v}(q_{sk}^{v*})}{\partial q_{sk}^{v}}+\hat{c}_{sk}^{v}(Q^{2*})\right]\times[q_{sk}^{v}-q_{sk}^{v*}]-\sum_{k=1}^{K}\rho_{3k}(d^*)\times[d_k-d_k^*]$$

$$+\sum_{j=1}^{J}\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\tau^{j*}-\lambda_{gm}^{j*}\right]\times[l_{gm}^{j}-l_{gm}^{j*}]$$

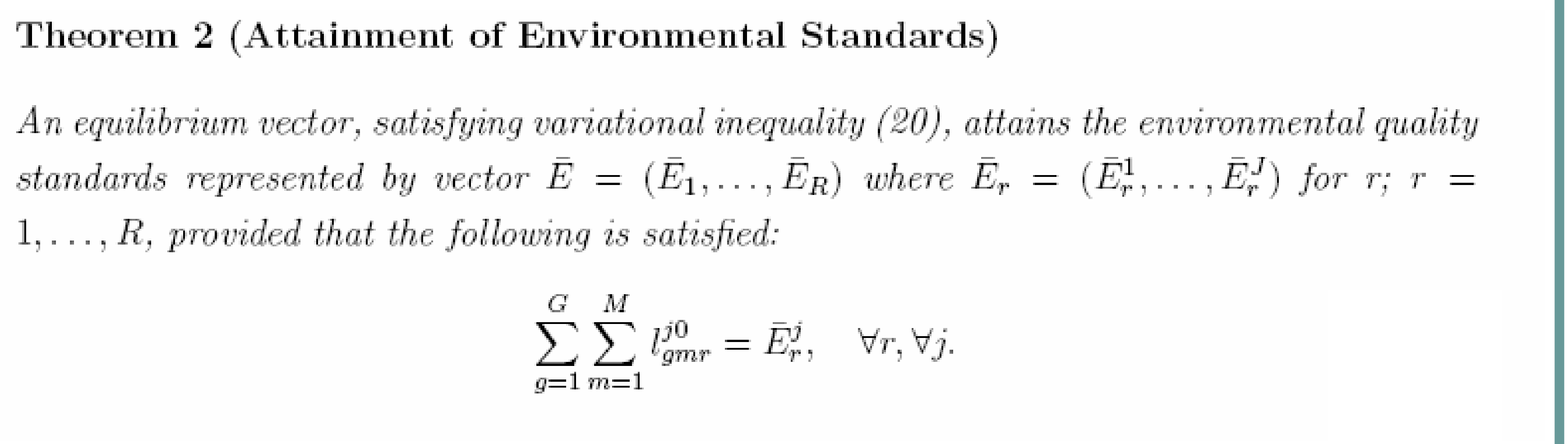
$$+\sum_{j=1}^{J}\sum_{g=1}^{G}\sum_{m=1}^{M}\left[l_{gm}^{*j}-e_{gm}^{j}q_{gm}^{*}\right]\times[\lambda_{gm}^{j}-\lambda_{gm}^{*j}]+\sum_{j=1}^{J}\left[\sum_{g=1}^{G}\sum_{m=1}^{M}(l_{gm}^{0j}-l_{gmr}^{*j})\right]\times[\tau^{j}-\tau^{j*}]\geq 0,$$

$$\forall(q,h,Q^1,Q^2,l,\lambda,\tau)\in\mathcal{K}^7$$

Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Single Receptor Point and Single Pollutant

$$
\begin{aligned}
&\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}}+\lambda^*_{gm}e_{gm}\right]\times\left[q_{gm}-q^*_{gm}\right]+\sum_{s=1}^{S}\frac{\partial c_s(h^*)}{\partial h_s}\times\left[h_s-h_s^*\right]\\
&+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}(q^*_{gms})}{\partial q_{gms}}+\frac{\partial \hat{c}_{gms}(q^*_{gms})}{\partial q_{gms}}\right]\times\left[q_{gms}-q^*_{gms}\right]\\
&+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{v=1}^{V}\left[\frac{\partial c^v_{sk}(q^{v*}_{sk})}{\partial q^v_{sk}}+\hat{c}^v_{sk}(Q^{2*})\right]\times\left[q^v_{sk}-q^{v*}_{sk}\right]-\sum_{k=1}^{K}\rho_{3k}(d^*)\times\left[d_k-d_k^*\right]\\
&+\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\tau^*-\lambda^*_{gm}\right]\times\left[l_{gm}-l^*_{gm}\right]+\sum_{g=1}^{G}\sum_{m=1}^{M}\left[l^*_{gm}-e_{gm}q^*_{gm}\right]\times\left[\lambda_{gm}-\lambda^*_{gm}\right]\\
&+\sum_{g=1}^{G}\sum_{m=1}^{M}\left(l^0_{gm}-l^*_{gm}\right)\times\left[\tau-\tau^*\right]\geq 0,\quad \forall(q,h,Q^1,Q^2,l,\lambda,\tau)\in\mathcal{K}^8
\end{aligned}
$$

Attainment of Environmental Standards

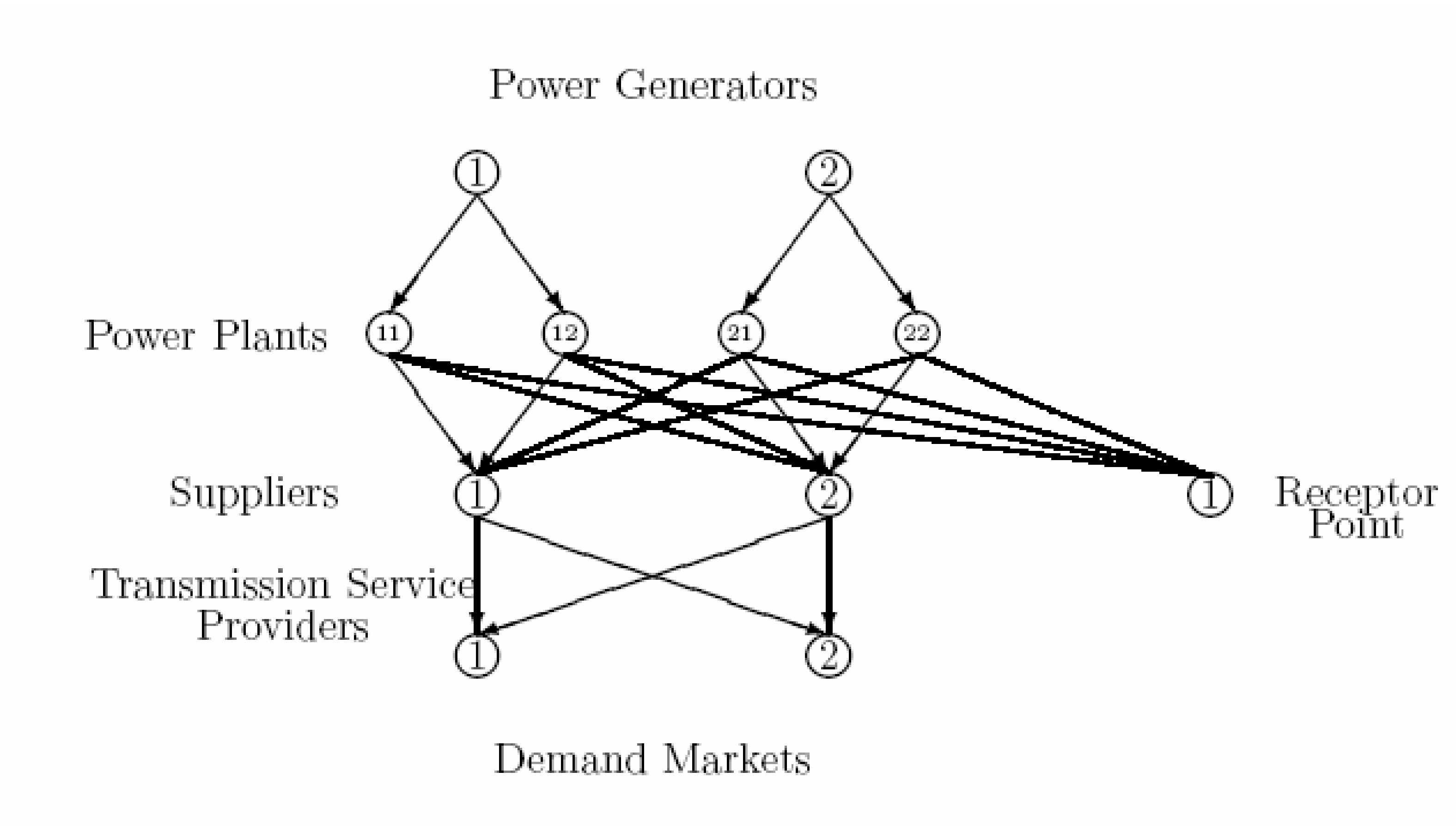


Theorem 2 (Attainment of Environmental Standards)

An equilibrium vector, satisfying variational inequality (20), attains the environmental quality standards represented by vector $\bar{E} = (\bar{E}_1, \ldots, \bar{E}_R)$ *where* $\bar{E}_r = (\bar{E}_r^1, \ldots, \bar{E}_r^J)$ *for* r; $r = 1, \ldots, R$, *provided that the following is satisfied:*

$$\sum_{g=1}^{G} \sum_{m=1}^{M} l_{gmr}^{j0} = \bar{E}_r^j, \quad \forall r, \forall j.$$

Numerical Examples



Numerical Examples

Example 1

The emission terms: e_{gm}; $g = 1, 2$; $m = 1, 2$ were all equal to 1.

The power generating cost functions for the power generators were given by:

$$f_{11}(q_1) = 2.5q_{11}^2 + q_{11}q_{21} + 2q_{11}, \quad f_{12}(q_2) = 2.5q_{12}^2 + q_{11}q_{12} + 2q_{22}, \quad f_{21}(q_1) = .5q_{21}^2 + .5q_{11}q_{21} + 2q_{21},$$

$$f_{22}(q_2) = .5q_{22}^2 + q_{12}q_{22} + 2q_{22}.$$

The transaction cost functions faced by the power generators and associated with transacting with the power suppliers were given by:

$$c_{111}(q_{111}) = .5q_{111}^2 + 3.5q_{111},\ c_{112}(q_{112}) = .5q_{112}^2 + 3.5q_{112},\ c_{121}(q_{121}) = .5q_{121}^2 + 3.5q_{121},$$

Numerical Examples

Example 1

$$c_{122}(q_{122}) = .5q_{122}^2 + 3.5q_{122},$$

$$c_{211}(q_{211}) = .5q_{211}^2 + 2q_{211},\ c_{212}(q_{212}) = .5q_{212}^2 + 2q_{212},\ c_{221}(q_{221}) = .5q_{221}^2 + 2q_{221},$$

$$c_{222}(q_{222}) = .5q_{222}^2 + 2q_{222}.$$

The operating costs of the power generators, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2} q_{i2})^2.$$

The demand market price functions at the demand markets were:

$$\rho_{31}(d) = -1.33d_1 + 366.6, \quad \rho_{32} = -1.33d_2 + 366.6,$$

and the transaction costs between the power suppliers and the consumers at the demand markets were given by: $\hat{c}_{sk}^1(q_{sk}^1) = q_{sk}^1 + 5, \quad s = 1,2; k = 1,2.$

All other transaction costs were assumed to be equal to zero.

Numerical Examples

Example 2

Example 2 had the same data as Example 1, but we now tightened the emissions standard so that $\bar{E} = 50$. The initial license allocation was now given by: $l^0_{11} = l^0_{12} = l^0_{21} = l^0_{22} = 12.5$.

Example 3

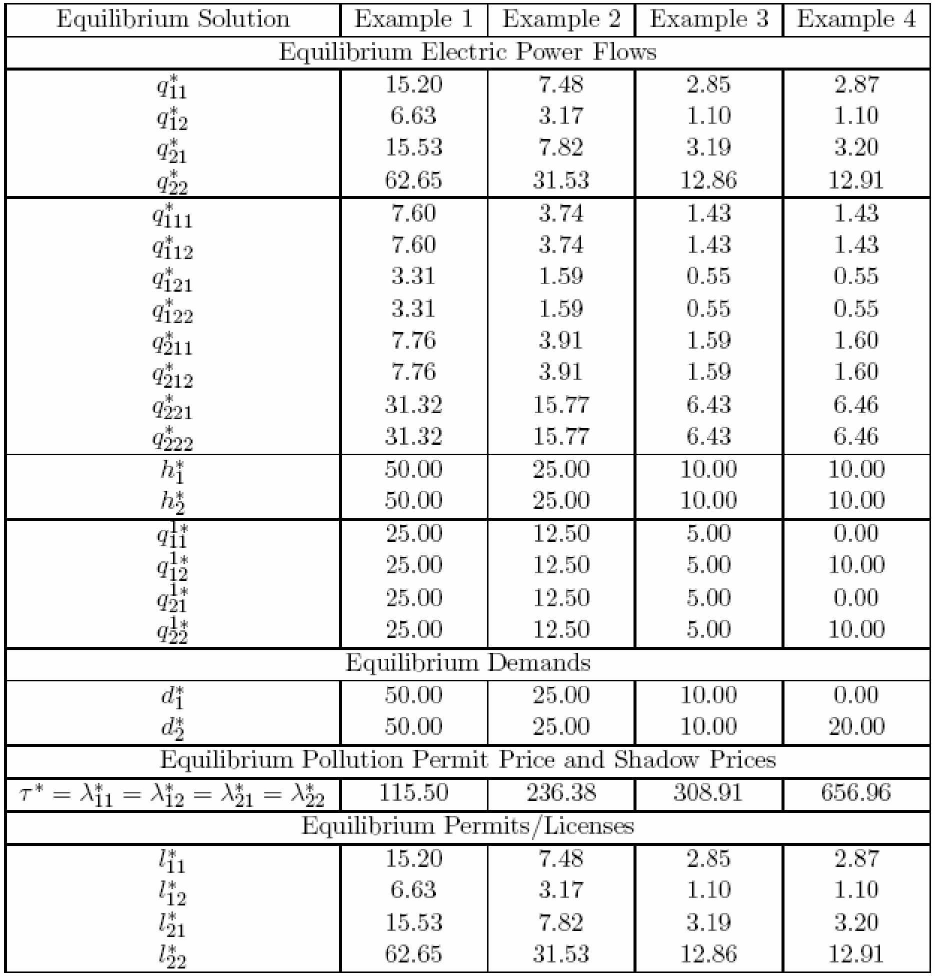
Example 3 had the identical data to that in Examples 1 and 2, except that the environmental standard was further tightened to $\bar{E} = 20$ with the new initial license allocation given by: $l^0_{11} = l^0_{12} = l^0_{21} = l^0_{22} = 5$.

Example 4

Example 4 had the same data as Example 3 except that we modified the second demand market price function for electric power to:

$$\rho_{32}(d) = -1.33 d_2 + 733.30.$$

Solutions to Numerical Examples



Equilibrium Solution	Example 1	Example 2	Example 3	Example 4
Equilibrium Electric Power Flows				
q_{11}^*	15.20	7.48	2.85	2.87
q_{12}^*	6.63	3.17	1.10	1.10
q_{21}^*	15.53	7.82	3.19	3.20
q_{22}^*	62.65	31.53	12.86	12.91
q_{111}^*	7.60	3.74	1.43	1.43
q_{112}^*	7.60	3.74	1.43	1.43
q_{121}^*	3.31	1.59	0.55	0.55
q_{122}^*	3.31	1.59	0.55	0.55
q_{211}^*	7.76	3.91	1.59	1.60
q_{212}^*	7.76	3.91	1.59	1.60
q_{221}^*	31.32	15.77	6.43	6.46
q_{222}^*	31.32	15.77	6.43	6.46
h_1^*	50.00	25.00	10.00	10.00
h_2^*	50.00	25.00	10.00	10.00
q_{11}^{1*}	25.00	12.50	5.00	0.00
q_{12}^{1*}	25.00	12.50	5.00	10.00
q_{21}^{1*}	25.00	12.50	5.00	0.00
q_{22}^{1*}	25.00	12.50	5.00	10.00
Equilibrium Demands				
d_1^*	50.00	25.00	10.00	0.00
d_2^*	50.00	25.00	10.00	20.00
Equilibrium Pollution Permit Price and Shadow Prices				
$\tau^* = \lambda_{11}^* = \lambda_{12}^* = \lambda_{21}^* = \lambda_{22}^*$	115.50	236.38	308.91	656.96
Equilibrium Permits/Licenses				
l_{11}^*	15.20	7.48	2.85	2.87
l_{12}^*	6.63	3.17	1.10	1.10
l_{21}^*	15.53	7.82	3.19	3.20
l_{22}^*	62.65	31.53	12.86	12.91

Conclusions

- The model presented in this paper may help policy-makers to determine an appropriate quantity of permits of each pollutant in the market and associated effect on the permit prices and power plants in the electric power generation industry, including flows and prices.

- The numerical results demonstrate, as the theory predicts, that the permit level achieves the desired goal, in that the imposed licenses on the emission of each pollutant is not exceeded.

- Moreover, it illustrates the spectrum of scenarios that can be explored in terms of changes in the amount of the permit allocation, its effects on emissions; changes in emission factors; changes in the demand functions, etc.

Thank you!

For more information, please see the

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