Grand Challenges and Opportunities in Supply Chain Networks: From Analysis to Design

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I would like to thank the organizers of this seminar series for inviting me to speak.

Special acknowledgments and thanks to my students and collaborators who have made research and teaching always stimulating and rewarding.
Outline

- Background and Motivation
- Why User Behavior Must be Captured in Network Design
- Methodologies for Formulation, Analysis, and Computations
- An Empirical Application to Electric Power Supply Chains
- Network Design Through Mergers and Acquisitions
- A Challenging Network Design Problem and Model for Critical Needs with Outsourcing
- Applications to Vaccine Production and Emergencies
- Extensions to Perishable Product Supply Chains without and with Competition
- Summary, Conclusions, and Suggestions for Future Research
Background and Motivation
Supply chains are the *critical infrastructure and backbones* for the production, distribution, and consumption of goods as well as services in our globalized *Network Economy*.

Supply chains, in their most fundamental realization, *consist of manufacturers and suppliers, distributors, retailers, and consumers at the demand markets*.

Today, supply chains may span thousands of miles across the globe, involve numerous suppliers, retailers, and consumers, and be underpinned by multimodal transportation and telecommunication networks.
A General Supply Chain

Suppliers

Manufacturers

Distribution Centers

Demand Markets

Domestic Manufacturer

Information

International Manufacturer

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Grand Challenges in Supply Chain Networks
Examples of Supply Chains

- food and food products
- high tech products
- automotive
- energy (oil, electric power, etc.)
- clothing and toys
- humanitarian relief
- healthcare supply chains
- supply chains in nature.
Food Supply Chains

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High Tech Products

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Energy Supply Chains

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Clothing and Toys

[Images of clothing and toys]
Healthcare Supply Chains

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Humanitarian Relief

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Grand Challenges in Supply Chain Networks
Supply Chains in Nature
Supply chains may be characterized by *decentralized decision-making* associated with the different economic agents or by *centralized* decision-making.

Supply chains are, in fact, *Complex Network Systems*.

Hence, *any formalism that seeks to model supply chains and to provide quantifiable insights and measures must be a system-wide one and network-based*.

Indeed, such crucial issues as the stability and resiliency of supply chains, as well as their adaptability and responsiveness to events in a *global environment of increasing risk and uncertainty* can only be rigorously examined from the view of supply chains as network systems.
Characteristics of Supply Chains and Networks Today

- **Large-scale nature** and complexity of network topology;

- Congestion, which leads to nonlinearities;

- Alternative behavior of users of the networks, which may lead to paradoxical phenomena;

- Possibly conflicting criteria associated with optimization;

- Interactions among the underlying networks themselves, such as the Internet with electric power networks, financial networks, and transportation and logistical networks;

- Recognition of their fragility and vulnerability;

- Policies surrounding networks today may have major impacts not only economically, but also socially, politically, and security-wise.
Representation of Supply Chains as Networks
By depicting supply chains as networks, consisting of nodes, links, flows (and also associated functions and behavior) we can:
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- build powerful extensions using the graphical/network conceptualization.
The equivalence between supply chains and transportation networks established in Nagurney, *Transportation Research E* 42 (2006), 293-316.
Multilevel supply chain established by Nagurney, Ke, Cruz, Hancock, and Southworth in *Environment & Planning B* 29 (2002), 795-818.
In 1952, Copeland in his book, *A Study of Moneyflows in the United States*, NBER, NY, asked whether money flows lie water or electricity?

In 1956, Beckmann, McGuire, and Winsten in their classic book, *Studies in the Economics of Transportation*, Yale University Press, hypothesized that electric power generation and distribution networks could be transformed into transportation network equilibrium problems.
Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation

Hence, we have shown that both electricity as well as money flow like transportation flows.
Our Approach to Supply Chain Network Analysis and Design

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Why User Behavior Must be Captured in Network Design
Supply Chain Network Design Must Capture the Behavior of Users
Behavior on Congested Networks

*Decision-makers select their cost-minimizing routes.*

- Decentralized
- Selfish
- Centralized
- Unselfish

**Centralized Unselfish S–O**

**User-Optimized U–O**

- Decentralized vs. Selfish vs. Centralized vs. Unselfish

*Flows are routed so as to minimize the total cost to society.*
Two fundamental principles of travel behavior, due to Wardrop (1952), with terms coined by Dafermos and Sparrow (1969).

**User-optimized (U-O) (network equilibrium) Problem** – each user determines his/her cost minimizing route of travel between an origin/destination, until an equilibrium is reached, in which no user can decrease his/her cost of travel by unilateral action (in the sense of Nash).

**System-optimized (S-O) Problem** – users are allocated among the routes so as to minimize the total cost in the system, where the total cost is equal to the sum over all the links of the link’s user cost times its flow.

The U-O problems, under certain simplifying assumptions, possesses optimization reformulations. But now we can handle cost asymmetries, multiple modes of transport, and different classes of travelers, without such assumptions.
We Can State These Conditions Mathematically!
The U-O and S-O Conditions

**Definition: U-O or Network Equilibrium – Fixed Demands**

A path flow pattern \( x^* \), with nonnegative path flows and O/D pair demand satisfaction, is said to be U-O or in equilibrium, if the following condition holds for each O/D pair \( w \in W \) and each path \( p \in P_w \):

\[
C_p(x^*) \begin{cases} 
= \lambda_w, & \text{if } x^*_p > 0, \\
\geq \lambda_w, & \text{if } x^*_p = 0.
\end{cases}
\]

**Definition: S-O Conditions**

A path flow pattern \( x \) with nonnegative path flows and O/D pair demand satisfaction, is said to be S-O, if for each O/D pair \( w \in W \) and each path \( p \in P_w \):

\[
\hat{C}'_p(x) \begin{cases} 
= \mu_w, & \text{if } x_p > 0, \\
\geq \mu_w, & \text{if } x_p = 0,
\end{cases}
\]

where \( \hat{C}'_p(x) = \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap} \), and \( \mu_w \) is a Lagrange multiplier.
The importance of behavior will now be illustrated through a famous example known as the Braess paradox which demonstrates what can happen under $U$-$O$ as opposed to $S$-$O$ behavior.

Although the paradox was presented in the context of transportation networks, it is relevant to other network systems in which decision-makers act in a noncooperative (competitive) manner.
The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand of 6, the equilibrium path flows are $x_{p_1}^* = x_{p_2}^* = 3$ and

The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83$.

$c_a(f_a) = 10f_a$, $c_b(f_b) = f_b + 50,$
$c_c(f_c) = f_c + 50$, $c_d(f_d) = 10f_d$. 
Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path \( p_3 = (a, e, d) \).

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path \( p_3, C_{p_3} = 70 \).

The new equilibrium flow pattern network is

\[
x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2.
\]

The equilibrium path travel cost: \( C_{p_1} = C_{p_2} = C_{p_3} = 92 \).

\[ c_e(f_e) = f_e + 10 \]
The 1968 Braess article has been translated from German to English and appears as:

"On a Paradox of Traffic Planning,"

Transportation Science 39, 446-450.
The Braess Paradox Around the World

1969 - Stuttgart, Germany - The traffic worsened until a newly built road was closed.

1990 - Earth Day - New York City - 42\textsuperscript{nd} Street was closed and traffic flow improved.

2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.
Interview on Broadway for America Revealed on March 15, 2011
Under S-O behavior, the total cost in the network is minimized, and the new route $p_3$, under the same demand, would not be used.

The Braess paradox never occurs in S-O networks.
Methodologies for Formulation, Analysis, and Computations
We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized supply chain network problems.

**Definition: The Variational Inequality Problem**

The finite-dimensional variational inequality problem, \( \text{VI}(F, \mathcal{K}) \), is to determine a vector \( X^* \in \mathcal{K} \), such that:

\[
\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},
\]

where \( F \) is a given continuous function from \( \mathcal{K} \) to \( \mathbb{R}^N \), \( \mathcal{K} \) is a given closed convex set, and \( \langle \cdot, \cdot \rangle \) denotes the inner product in \( \mathbb{R}^N \).
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of supply chain network problems from centralized to decentralized ones as well as to design problems.
Geometric Interpretation of VI\((F, \mathcal{K})\) and a Projected Dynamical System (Dupuis and Nagurney (1993), Nagurney and Zhang (1996))

In particular, \(F(X^*)\) is “orthogonal” to the feasible set \(\mathcal{K}\) at the point \(X^*\).

Associated with a VI is a Projected Dynamical System, which provides a natural underlying dynamics associated with travel (and other) behavior to the equilibrium.
To model the *dynamic behavior of complex networks*, including supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in *evolutionary games* (Sandholm (2005, 2011)), *ecological predator-prey networks* (Nagurney and Nagurney (2011a, b)), and *even neuroscience* (Girard et al. (2008)).
Recall the Braess network with the added link e.

What happens as the demand increases?
For Networks with Time-Dependent Demands
We Use Evolutionary Variational Inequalities
The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands Solved as an *Evolutionary Variational Inequality* (Nagurney, Daniele, and Parkes, *Computational Management Science* (2007)).
In Demand Regime I, Only the New Path is Used.
In Demand Regime II, the travel demand lies in the range \([2.58, 8.89]\), and the Addition of a New Link (Path) Makes Everyone Worse Off!
In Demand Regime III, when the travel demand exceeds 8.89, Only the Original Paths are Used!
The new path is never used, under U-O behavior, when the demand exceeds 8.89, even out to infinity!
Other Networks that Behave like Traffic Networks

The Internet and electric power networks
The U.S. electric power industry: Half a trillion dollars of net assets, $220 billion annual sales, 40% of domestic primary energy (Energy Information Administration (2000, 2005)).

Deregulation:
- Wholesale market
- Bilateral contracts
- Power pool.

Electric power supply chain networks:
- Generation technologies
- Insensitive demands
- Transmission congestion.

In 2007, the total transmission congestion cost in New England was about $130 million (ISO New England Annual Market Report, 2007).
We have developed an empirical, large-scale electric supply chain network equilibrium model, formulated it as a VI problem, and were able to solve it by exploiting the connection between electric power supply chain networks and transportation networks using our proof of a hypothesis posed in the classic book, Studies in the Economics of Transportation, by Beckmann, McGuire, and Winsten (1956).

Features of the Model

The model captures both economic transactions and physical transmission constraints.

The model considers the behaviors of all major decision-makers including gencos, consumers and the independent system operator (ISO).

The model considers multiple fuel markets, electricity wholesale markets, and operating reserve markets.
There are 82 generating companies who own and operate 573 generating units. We considered 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal. The whole area was divided into 10 regions:

1. Maine,
2. New Hampshire,
3. Vermont,
4. Connecticut (excluding Southwest Connecticut),
5. Southwestern Connecticut (excluding the Norwalk-Stamford area),
6. Norwalk-Stamford area,
7. Rhode Island,
8. Southeastern Massachusetts,
9. Western and Central Massachusetts,
Graphic of New England

1. Maine
2. New Hampshire
3. Vermont
4. Connecticut (excluding Southwestern Connecticut)
5. Southwestern Connecticut (excluding the Norwalk-Stamford area)
6. Norwalk-Stamford area
7. Rhode Island
8. Southeastern Massachusetts
9. Western and Central Massachusetts
10. Boston/Northeastern Massachusetts
The Electric Power Supply Chain Network with Fuel Supply Markets

Fuel Markets for Fuel Type 1

Generating Units of Gencos in Regions (genco, region, unit)

Power Pool

Fuel Markets for Fuel Type a

Demand Market Sectors Region 1

Fuel Markets for Fuel Type A

Demand Market Sectors Region r

Demand Market Sectors Region R
We tested the model on the data of July 2006 which included \(24 \times 31 = 744\) hourly demand/price scenarios. We sorted the scenarios based on the total hourly demand, and constructed the load duration curve. We divided the duration curve into 6 blocks (\(L_1 = 94\) hours, and \(L_w = 130\) hours; \(w = 2, \ldots, 6\)) and calculated the average regional demands and the average weighted regional prices for each block.

*The empirical model had on the order of 20,000 variables.*
Actual Prices Vs. Simulated Prices ($/Mwh)
Sensitivity Analysis

We used the same demand data, and then varied the prices of natural gas and residual fuel oil. We assumed that the percentage change of distillate fuel oil and jet fuel prices were the same as that of the residual fuel oil price.

The next figure presents the average electricity price for the two peak blocks under oil/gas price variations.

The surface in the figure represents the average peak electricity prices under different natural gas and oil price combinations.
Sensitivity Analysis
If the price of one type of fuel is fixed, the electricity price changes less percentage-wise than the other fuel price does. This is mainly because fuel diversity can mitigate fuel price shocks.

Additional simulation results can be found in our *Naval Research Logistics* paper, including:

- How natural gas prices can be significantly influenced by oil prices through electric power networks and markets.
- How changes in the demand for electricity influence the electric power and fuel markets.

The model and results are useful in determining and quantifying the interactions between electric power flows and prices and the various fuel supply markets.

*Such information is important to policy-makers who need to ensure system reliability as well as for the energy asset owners and investors who need to manage risk and to evaluate their assets.*
Supply chain network design (and redesign) can be accomplished through link and node additions (as well as their removals).
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It can also be accomplished through the integration of networks as in mergers and acquisitions.
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It can also be accomplished through the integration of networks as in mergers and acquisitions.

It can be accomplished through the design of the network from scratch as we shall demonstrate.
Supply Chain Network Design
Through
Mergers and Acquisitions
M&As totaled over $2 trillion in 2009, down 32% from full-year 2008 and down 53% from the record high in 2007, according to data from Thomson Reuters.

Mergers announced in October 2010 include Bain Capital / Gymboree, at $1.789 billion and Dynamex / Greenbriar Equity Group ($207 million).

Some of the most visible recent mergers have occurred in the airline industry with Delta and Northwest completing their merger in October 2008 and United and Continental closing on the formation of United Continental Holdings Oct. 1, 2010.
In the third quarter of 2012, mergers and acquisition activity was $455 billion worldwide.

Successful mergers can add tremendous value; however, the failure rate is estimated to be between 74% and 83% (Devero (2004)).

*It is worthwhile to develop tools to better predict the associated strategic gains, which include, among others, cost savings (Eccles, Lanes, and Wilson (1999)).*
A successful merger depends on the ability to measure the anticipated synergy of the proposed merger (cf. Chang (1988)).

Figure 1: Case 0: Firms A and B Prior to Horizontal Merger
Figure 2: Case 1: Firms A and B Merge
Figure 3: Case 2: Firms A and B Merge
Figure 4: Case 3: Firms A and B Merge
The measure that we utilized in Nagurney (2009) to capture the gains, if any, associated with a horizontal merger Case $i; i = 1, 2, 3$ is as follows:

$$S^i = \left[ \frac{TC^0 - TC^i}{TC^0} \right] \times 100\%,$$

where $TC^i$ is the total cost associated with the value of the objective function $\sum_{a \in L^i} \hat{c}_a(f_a)$ for $i = 0, 1, 2, 3$ evaluated at the optimal solution for Case $i$. Note that $S^i; i = 1, 2, 3$ may also be interpreted as synergy.
This model can also be applied to the teaming of organizations in the case of humanitarian operations.
Humanitarian Logistics: Networks for Africa

Rockefeller Foundation Bellagio Center Conference, Bellagio, Lake Como, Italy
May 5-9, 2008
Conference Organizer: Anna Nagurney, John F. Smith Memorial Professor
University of Massachusetts at Amherst

See: http://hlogistics.isenberg.umass.edu/
Some Examples of Oligopolies

- airlines
- freight carriers
- automobile manufacturers
- oil companies
- beer / beverage companies
- wireless communications
- fast fashion brands
- certain financial institutions.
Figure 5: Supply Chain Network Structure of the Oligopoly

Figure 6: Mergers of the First $n_1'$ Firms and the Next $n_2'$ Firms
In addition, supply chain network design can be accomplished through the evolution and integration of disparate network systems, including social networks.

Two References:


Flows are Relationship Levels

Flows are Product Transactions

Social Network

The Supernetwork

Supply Chain Network

Figure 7: The Multilevel Supernetwork Structure of the Integrated Supply Chain / Social Network System
Flows are Relationship Levels

The Supernetwork

Social Network

Financial Network with Intermediation

Figure 8: The Multilevel Supernetwork Structure of the Integrated Financial Network / Social Network System
A Challenging Network Design Problem
and
Model for Critical Needs with Outsourcing
The number of disasters is increasing globally, as is the number of people affected by disasters. At the same time, with the advent of increasing globalization, viruses are spreading more quickly and creating new challenges for medical and health professionals, researchers, and government officials.

Between 2000 and 2004, the average annual number of disasters was 55% higher than in the period 1994 through 1999, with 33% more humans affected in the former period than in the latter (cf. Balcik and Beamon (2008) and Nagurney and Qiang (2009)).
However, although the average number of disasters has been increasing annually over the past decade the average percentage of needs met by different sectors in the period 2000 through 2005 identifies significant shortfalls.

According to Development Initiatives (2006), based on data in the Financial Tracking System of the Office for the Coordination of Humanitarian Affairs, from 2000-2005, the average needs met by different sectors in the case of disasters were:

- 79% by the food sector;
- 37% of the health needs;
- 35% of the water and sanitation needs;
- 28% of the shelter and non-food items, and
- 24% of the economic recovery and infrastructure needs.
Hurricane Katrina in 2005

Hurricane Katrina has been called an “American tragedy,” in which essential services failed completely (Guidotti (2006)).
Delivering the humanitarian relief supplies (water, food, medicines, etc.) to the victims was a major logistical challenge.
The Triple Disaster in Japan on March 11, 2011

Now the world is reeling from the aftereffects of the triple disaster in Japan with disruptions in the high tech, automotive, and even food industries with potential additional ramifications because of the radiation.

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H1N1 (Swine) Flu

As of May 2, 2010, worldwide, more than 214 countries and overseas territories or communities have reported laboratory confirmed cases of pandemic influenza H1N1 2009, including over 18,001 deaths (www.who.int).

Parts of the globe experienced serious flu vaccine shortages, both seasonal and H1N1 (swine) ones, in late 2009.
Fragile Networks

We are living in a world of *Fragile Networks.*
Background and Motivation

Underlying the delivery of goods and services in times of crises, such as in the case of disasters, pandemics, and life-threatening major disruptions, are supply chains, without which essential products do not get delivered in a timely manner, with possible increased disease, injuries, and casualties.

It is clear that better-designed supply chain networks would have facilitated and enhanced various emergency preparedness and relief efforts and would have resulted in less suffering and lives lost.
Supply chain networks provide the logistical backbones for the provision of products as well as services both in corporate as well as in emergency and humanitarian operations.

Here we focus on supply chains in the case of

**Critical Needs Products.**
Critical Needs Products

Critical needs products are those that are essential to the survival of the population, and can include, for example, vaccines, medicine, food, water, etc., depending upon the particular application.

The demand for the product should be met as nearly as possible since otherwise there may be additional loss of life.

In times of crises, a system-optimization approach is mandated since the demands for critical supplies should be met (as nearly as possible) at minimal total cost.
An Overview of Some of the Relevant Literature


This part of the presentation is based on the paper:

“Supply Chain Network Design for Critical Needs with Outsourcing,”

where additional background as well as references can be found.
We assume that the organization (government, humanitarian one, socially responsible firm, etc.) is considering $n_M$ manufacturing facilities/plants; $n_D$ distribution centers, but must serve the $n_R$ demand points.

The supply chain network is modeled as a network $G = [N, L]$, consisting of the set of nodes $N$ and the set of links $L$. Let $L^1$ and $L^2$ denote the links associated with “in house” supply chain activities and the outsourcing activities, respectively. The paths joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the product is produced and, ultimately, delivered to those in need at the demand points.

The optimization model can handle both design (from scratch) and redesign scenarios.
Supply Chain Network Topology with Outsourcing

The Organization

Manufacturing at the Plants

Distribution Center Storage

Shipping

Demand Points

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The possible manufacturing links from the top-tiered node 1 are connected to the possible manufacturing nodes of the organization, which are denoted, respectively, by: $M_1, \ldots, M_{n_M}$.

The possible shipment links from the manufacturing nodes, are connected to the possible distribution center nodes of the organization, denoted by $D_{1,1}, \ldots, D_{n_D,1}$.

The links joining nodes $D_{1,1}, \ldots, D_{n_D,1}$ with nodes $D_{1,2}, \ldots, D_{n_D,2}$ correspond to the possible storage links.

There are possible shipment links joining the nodes $D_{1,2}, \ldots, D_{n_D,2}$ with the demand nodes: $R_1, \ldots, R_{n_R}$. 

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There are also outsourcing links, which may join the top node to each bottom node (or the relevant nodes for which the outsourcing activity is feasible, as in production, storage, or distribution, or a combination thereof). The organization does not control the capacities on these links since they have been established by the particular firm that corresponds to the outsource link.

The ability to outsource supply chain network activities for critical needs products provides alternative pathways for the production and delivery of products during times of crises such as disasters.
Demands, Path Flows, and Link Flows

Let $d_k$ denote the demand at demand point $k; k = 1, \ldots, n_R$, which is a random variable with probability density function given by $F_k(t)$. Let $x_p$ represent the nonnegative flow of the product on path $p$; $f_a$ denote the flow of the product on link $a$.

Conservation of Flow Between Path Flows and Link Flows

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L,$$  \hspace{1cm} (1)

that is, the total amount of a product on a link is equal to the sum of the flows of the product on all paths that utilize that link. $\delta_{ap} = 1$ if link $a$ is contained in path $p$, and $\delta_{ap} = 0$, otherwise.
Supply Shortage and Surplus

Let

\[ v_k \equiv \sum_{p \in P_w_k} x_p, \quad k = 1, \ldots, n_R, \]  

(2)

where \( v_k \) can be interpreted as the *projected demand* at demand market \( k; k = 1, \ldots, n_R \). Then,

\[ \Delta_k^- \equiv \max\{0, d_k - v_k\}, \quad k = 1, \ldots, n_R, \]  

(3)

\[ \Delta_k^+ \equiv \max\{0, v_k - d_k\}, \quad k = 1, \ldots, n_R, \]  

(4)

where \( \Delta_k^- \) and \( \Delta_k^+ \) represent the supply shortage and surplus at demand point \( k \), respectively. The expected values of \( \Delta_k^- \) and \( \Delta_k^+ \) are given by:

\[ E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k) F_k(t) d(t), \quad k = 1, \ldots, n_R, \]  

(5)

\[ E(\Delta_k^+) = \int_0^{v_k} (v_k - t) F_k(t) d(t), \quad k = 1, \ldots, n_R. \]  

(6)
The Operation Costs, Investment Costs and Penalty Costs

The total cost on a link is assumed to be a function of the flow of the product on the link. We have, thus, that

$$\hat{c}_a = \hat{c}_a(f_a), \quad \forall a \in L.$$  \hspace{1cm} (7)

We denote the nonnegative existing capacity on a link $a$ by $\bar{u}_a$, $\forall a \in L$. Note that the organization can add capacity to the “in house” link $a$; $\forall a \in L^1$. We assume that

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \quad \forall a \in L^1.$$  \hspace{1cm} (8)

The expected total penalty at demand point $k$; $k = 1, \ldots, n_R$, is,

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+),$$  \hspace{1cm} (9)

where $\lambda_k^-$ is the unit penalty of supply shortage at demand point $k$ and $\lambda_k^+$ is that of supply surplus. Note that $\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)$ is a function of the path flow vector $x$. 

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The organization seeks to determine the optimal levels of product processed on each supply chain network link (including the outsourcing links) coupled with the optimal levels of capacity investments in its supply chain network activities subject to the minimization of the total cost.

The total cost includes the total cost of operating the various links, the total cost of capacity investments, and the expected total supply shortage/surplus penalty.
The Supply Chain Network Design Optimization Problem

Minimize

\[ \sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L^1} \hat{\pi}_a(u_a) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) \]  

subject to: constraints (1), (2) and

\[ f_a \leq \bar{u}_a + u_a, \quad \forall a \in L^1, \]  

\[ f_a \leq \bar{u}_a, \quad \forall a \in L^2, \]  

\[ u_a \geq 0, \quad \forall a \in L^1, \]  

\[ x_p \geq 0, \quad \forall p \in P. \]
The Feasible Set

We associate the Lagrange multiplier $\omega_a$ with constraint (11) for link $a \in L^1$ and we denote the associated optimal Lagrange multiplier by $\omega_a^*$. Similarly, Lagrange multiplier $\gamma_a$ is associated with constraint (12) for link $a \in L^2$ with the optimal multiplier denoted by $\gamma_a^*$. These two terms may also be interpreted as the price or value of an additional unit of capacity on link $a$. We group these Lagrange multipliers into the vectors $\omega$ and $\gamma$, respectively. Let $K$ denote the feasible set such that

$$K \equiv \{(x, u, \omega, \gamma) | x \in R^{np}_+, u \in R^{nL^1}_+, \omega \in R^{nL^1}_+, \text{ and } \gamma \in R^{nL^2}_+\}.$$
Theorem

The optimization problem is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal link capacity enhancements, and the vectors of optimal Lagrange multipliers \((x^*, u^*, \omega^*, \gamma^*) \in K\), such that:

\[
\sum_{k=1}^{n_R} \sum_{p \in P_{w_k}} \left[ \frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^1} \omega^*_a \delta_{ap} + \sum_{a \in L^2} \gamma^*_a \delta_{ap} + \lambda^+_k P_k \left( \sum_{p \in P_{w_k}} x^*_p \right) \right] \\
- \lambda^-_k \left( 1 - P_k \left( \sum_{p \in P_{w_k}} x^*_p \right) \right) \times [x_p - x^*_p] \\
+ \sum_{a \in L^1} \left[ \frac{\partial \hat{\pi}_a(u^*_a)}{\partial u_a} - \omega^*_a \right] \times [u_a - u^*_a] + \sum_{a \in L^1} \left[ \bar{u}_a + u^*_a - \sum_{p \in P} x^*_p \delta_{ap} \right] \times [\omega_a - \omega^*_a] \\
+ \sum_{a \in L^2} \left[ \bar{u}_a - \sum_{p \in P} x^*_p \delta_{ap} \right] \times [\gamma_a - \gamma^*_a] \geq 0, \quad \forall (x, u, \omega, \gamma) \in K. \quad (15)
\]
In addition, (15) can be reexpressed in terms of links flows as: determine the vector of optimal link flows, the vectors of optimal projected demands and link capacity enhancements, and the vectors of optimal Lagrange multipliers \((f^*, v^*, u^*, \omega^*, \gamma^*) \in K^1\), such that:

\[
\sum_{a \in L^1} \left[ \frac{\partial \hat{c}_a(f^*_a)}{\partial f_a} + \omega_a^* \right] \times [f_a - f_a^*] + \sum_{a \in L^2} \left[ \frac{\partial \hat{c}_a(f^*_a)}{\partial f_a} + \gamma_a^* \right] \times [f_a - f_a^*] \\
+ \sum_{a \in L^1} \left[ \frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \omega_a^* \right] \times [u_a - u_a^*] \\
+ \sum_{k=1}^{n_R} [\lambda_k^+ P_k(v_k^*) - \lambda_k^-(1 - P_k(v_k^*))] \times [v_k - v_k^*] + \sum_{a \in L^1} [\bar{u}_a + u_a^* - f_a^*] \times [\omega_a - \omega_a^*] \\
+ \sum_{a \in L^2} [\bar{u}_a - f_a^*] \times [\gamma_a - \gamma_a^*] \geq 0, \quad \forall (f, v, u, \omega, \gamma) \in K^1, \quad (16)
\]

where \(K^1 \equiv \{(f, v, u, \omega, \gamma) | \exists x \geq 0, \text{ and (1), (2), (13), and (14) hold, and } \omega \geq 0, \gamma \geq 0\} \).
Applications to Vaccine Production
and
Emergencies
By applying the general theoretical model to the company’s data, the firm can determine whether it needs to expand its facilities (or not), how much of the vaccine to produce where, how much to store where, and how much to have shipped to the various demand points. Also, it can determine whether it should outsource any of its vaccine production and at what level.

The firm by solving the model with its company-relevant data can then ensure *that the price that it receives for its vaccine production and delivery is appropriate* and that it recovers its incurred costs and obtains, if negotiated correctly, an equitable profit.
A company can, using the model, prepare and plan for an emergency such as a natural disaster in the form of a hurricane and identify where to store a necessary product (such as food packets, for example) so that the items can be delivered to the demand points in a timely manner and at minimal total cost.
The Algorithm, Explicit Formulae, and Numerical Examples
The Algorithm

At an iteration $\tau$ of the Euler method (see Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_K(X^{\tau} - a_\tau F(X^{\tau})),$$  \hspace{1cm} (17)

where $P_K$ is the projection on the feasible set $K$ and $F$ is the function that enters the variational inequality problem: determine $X^* \in K$ such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in K,$$  \hspace{1cm} (18)

where $\langle \cdot, \cdot \rangle$ is the inner product in $n$-dimensional Euclidean space, $X \in \mathbb{R}^n$, and $F(X)$ is an $n$-dimensional function from $K$ to $\mathbb{R}^n$, with $F(X)$ being continuous.

The sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$. 
Explicit Formulae for (17) to the Supply Chain Network Design Variational Inequality (15)

\[ x_p^{\tau+1} = \max\{0, x_p^{\tau} + a_\tau(\lambda_k^- (1 - P_k(\sum_{p \in P_{w_k}} x_p^{\tau})) - \lambda_k^+ P_k(\sum_{p \in P_{w_k}} x_p^{\tau})) \} \]

\[-\frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \sum_{a \in L^1} \omega_a^{\tau} \delta_{ap} - \sum_{a \in L^2} \gamma_a^{\tau} \delta_{ap} \}, \forall p \in P; \quad (19)\]

\[ u_a^{\tau+1} = \max\{0, u_a^{\tau} + a_\tau(\omega_a^{\tau} - \frac{\partial \hat{\pi}_a(u_a^{\tau})}{\partial u_a}) \}, \quad \forall a \in L^1; \quad (20)\]

\[ \omega_a^{\tau+1} = \max\{0, \omega_a^{\tau} + a_\tau(\sum_{p \in P} x_p^{\tau} \delta_{ap} - \bar{u}_a - u_a^{\tau}) \}, \quad \forall a \in L^1; \quad (21)\]

\[ \gamma_a^{\tau+1} = \max\{0, \gamma_a^{\tau} + a_\tau(\sum_{p \in P} x_p^{\tau} \delta_{ap} - \bar{u}_a) \}, \quad \forall a \in L^2. \quad (22)\]
Numerical Examples

The Organization

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Anna Nagurney
Grand Challenges in Supply Chain Networks
Example 1

The demands at the three demand points followed a uniform probability distribution on the intervals $[0, 10]$, $[0, 20]$, and $[0, 30]$, respectively:

\[
P_1\left( \sum_{p \in P_{w_1}} x_p \right) = \frac{\sum_{p \in P_{w_1}} x_p}{10}, \quad P_2\left( \sum_{p \in P_{w_2}} x_p \right) = \frac{\sum_{p \in P_{w_2}} x_p}{20},
\]

\[
P_3\left( \sum_{p \in P_{w_3}} x_p \right) = \frac{\sum_{p \in P_{w_3}} x_p}{30},
\]

where $w_1 = (1, R_1)$, $w_2 = (1, R_2)$, and $w_3 = (1, R_3)$.

The penalties were:

\[
\lambda_1^- = 50, \quad \lambda_1^+ = 0; \quad \lambda_2^- = 50, \quad \lambda_2^+ = 0; \quad \lambda_3^- = 50, \quad \lambda_3^+ = 0.
\]

The capacities associated with the three outsourcing links were:

\[
\bar{u}_{18} = 5, \quad \bar{u}_{19} = 10, \quad \bar{u}_{20} = 5.
\]

We set $\bar{u}_a = 0$ for all links $a \in L^1$. 

Table 1: Total Cost Functions and Solution for Example 1

<table>
<thead>
<tr>
<th>Link a</th>
<th>$\hat{c}_a(f_a)$</th>
<th>$\hat{\pi}_a(u_a)$</th>
<th>$f_a^*$</th>
<th>$u_a^*$</th>
<th>$\omega_a^*$</th>
<th>$\gamma_a^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$f_1^2 + 2f_1$</td>
<td>$.5u_1^2 + u_1$</td>
<td>1.34</td>
<td>1.34</td>
<td>2.34</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>$.5f_2^2 + f_2$</td>
<td>$.5u_2^2 + u_2$</td>
<td>2.47</td>
<td>2.47</td>
<td>3.47</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>$.5f_3^2 + f_3$</td>
<td>$.5u_3^2 + u_3$</td>
<td>2.05</td>
<td>2.05</td>
<td>3.05</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>$1.5f_4^2 + 2f_4$</td>
<td>$.5u_4^2 + u_4$</td>
<td>0.61</td>
<td>0.61</td>
<td>1.61</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>$f_5^2 + 3f_5$</td>
<td>$.5u_5^2 + u_5$</td>
<td>0.73</td>
<td>0.73</td>
<td>1.73</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>$f_6^2 + 2f_6$</td>
<td>$.5u_6^2 + u_6$</td>
<td>0.83</td>
<td>0.83</td>
<td>1.83</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>$.5f_7^2 + 2f_7$</td>
<td>$.5u_7^2 + u_7$</td>
<td>1.64</td>
<td>1.64</td>
<td>2.64</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>$.5f_8^2 + 2f_8$</td>
<td>$.5u_8^2 + u_8$</td>
<td>1.67</td>
<td>1.67</td>
<td>2.67</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>$f_9^2 + 5f_9$</td>
<td>$.5u_9^2 + u_9$</td>
<td>0.37</td>
<td>0.37</td>
<td>1.37</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>$.5f_{10}^2 + 2f_{10}$</td>
<td>$.5u_{10}^2 + u_{10}$</td>
<td>3.11</td>
<td>3.11</td>
<td>4.11</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>$f_{11}^2 + f_{11}$</td>
<td>$.5u_{11}^2 + u_{11}$</td>
<td>2.75</td>
<td>2.75</td>
<td>3.75</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>$.5f_{12}^2 + 2f_{12}$</td>
<td>$.5u_{12}^2 + u_{12}$</td>
<td>0.04</td>
<td>0.04</td>
<td>1.04</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>$.5f_{13}^2 + 5f_{13}$</td>
<td>$.5u_{13}^2 + u_{13}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.45</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 2: Total Cost Functions and Solution for Example 1 (continued)

<table>
<thead>
<tr>
<th>Link $a$</th>
<th>$\hat{c}_a(f_a)$</th>
<th>$\hat{\pi}_a(u_a)$</th>
<th>$f_a^*$</th>
<th>$u_a^*$</th>
<th>$\omega_a^*$</th>
<th>$\gamma_a^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>$f_{14}^2$</td>
<td>$.5u_{14}^2 + u_{14}$</td>
<td>3.07</td>
<td>3.07</td>
<td>4.07</td>
<td>–</td>
</tr>
<tr>
<td>15</td>
<td>$f_{15}^2 + 2f_{15}$</td>
<td>$.5u_{15}^2 + u_{15}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.45</td>
<td>–</td>
</tr>
<tr>
<td>16</td>
<td>$.5f_{16}^2 + 3f_{16}$</td>
<td>$.5u_{16}^2 + u_{16}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.45</td>
<td>–</td>
</tr>
<tr>
<td>17</td>
<td>$.5f_{17}^2 + 2f_{17}$</td>
<td>$.5u_{17}^2 + u_{17}$</td>
<td>2.75</td>
<td>2.75</td>
<td>3.75</td>
<td>–</td>
</tr>
<tr>
<td>18</td>
<td>$10f_{18}$</td>
<td>–</td>
<td>5.00</td>
<td>–</td>
<td>–</td>
<td>14.77</td>
</tr>
<tr>
<td>19</td>
<td>$12f_{19}$</td>
<td>–</td>
<td>10.00</td>
<td>–</td>
<td>–</td>
<td>13.00</td>
</tr>
<tr>
<td>20</td>
<td>$15f_{20}$</td>
<td>–</td>
<td>5.00</td>
<td>–</td>
<td>–</td>
<td>16.96</td>
</tr>
</tbody>
</table>

Note that the optimal supply chain network design for Example 1 is, hence, as the initial topology but with links 13, 15, and 16 removed since those links have zero capacities and associated flows. Note that the organization took advantage of outsourcing to the full capacity available.
Figure 9: The Optimal Supply Chain Network Design for Example 1
Example 2 had the identical data to that in Example 1 except that we now assumed that the organization had capacities on its supply chain network activities where $\bar{u}_a = 10$, for all $a \in L^1$.

Table 3: Total Cost Functions and Solution for Example 2

<table>
<thead>
<tr>
<th>Link $a$</th>
<th>$\hat{c}_a(f_a)$</th>
<th>$\hat{\pi}_a(u_a)$</th>
<th>$f_a^*$</th>
<th>$u_a^*$</th>
<th>$\omega_a^*$</th>
<th>$\gamma_a^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$f_1^2 + 2f_1$</td>
<td>$.5u_1^2 + u_1$</td>
<td>1.84</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>$.5f_2^2 + f_2$</td>
<td>$.5u_2^2 + u_2$</td>
<td>4.51</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>$.5f_3^2 + f_3$</td>
<td>$.5u_3^2 + u_3$</td>
<td>3.85</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>4</td>
<td>$1.5f_4^2 + 2f_4$</td>
<td>$.5u_4^2 + u_4$</td>
<td>0.88</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>5</td>
<td>$f_5^2 + 3f_5$</td>
<td>$.5u_5^2 + u_5$</td>
<td>0.97</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>6</td>
<td>$f_6^2 + 2f_6$</td>
<td>$.5u_6^2 + u_6$</td>
<td>1.40</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>7</td>
<td>$.5f_7^2 + 2f_7$</td>
<td>$.5u_7^2 + u_7$</td>
<td>3.11</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>8</td>
<td>$.5f_8^2 + 2f_8$</td>
<td>$.5u_8^2 + u_8$</td>
<td>3.47</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>9</td>
<td>$f_9^2 + 5f_9$</td>
<td>$.5u_9^2 + u_9$</td>
<td>0.38</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
</tbody>
</table>
Table 4: Total Cost Functions and Solution for Example 2 (continued)

<table>
<thead>
<tr>
<th>Link a</th>
<th>( \hat{c}_a(f_a) )</th>
<th>( \hat{\pi}_a(u_a) )</th>
<th>( f_a^* )</th>
<th>( u_a^* )</th>
<th>( \omega_a^* )</th>
<th>( \gamma_a^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>( 0.5f_{10}^2 + 2f_{10} )</td>
<td>( 0.5u_{10}^2 + u_{10} )</td>
<td>5.75</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>( f_{11}^2 + f_{11} )</td>
<td>( 0.5u_{11}^2 + u_{11} )</td>
<td>4.46</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>( 0.5f_{12}^2 + 2f_{12} )</td>
<td>( 0.5u_{12}^2 + u_{12} )</td>
<td>0.82</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>( 0.5f_{13}^2 + 5f_{13} )</td>
<td>( 0.5u_{13}^2 + u_{13} )</td>
<td>0.52</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>( f_{14}^2 )</td>
<td>( 0.5u_{14}^2 + u_{14} )</td>
<td>4.41</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>( f_{15}^2 + 2f_{15} )</td>
<td>( 0.5u_{15}^2 + u_{15} )</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>16</td>
<td>( 0.5f_{16}^2 + 3f_{16} )</td>
<td>( 0.5u_{16}^2 + u_{16} )</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>17</td>
<td>( 0.5f_{17}^2 + 2f_{17} )</td>
<td>( 0.5u_{17}^2 + u_{17} )</td>
<td>4.41</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>18</td>
<td>( 10f_{18} )</td>
<td>-</td>
<td>5.00</td>
<td>-</td>
<td>-</td>
<td>10.89</td>
</tr>
<tr>
<td>19</td>
<td>( 12f_{19} )</td>
<td>-</td>
<td>10.00</td>
<td>-</td>
<td>-</td>
<td>11.59</td>
</tr>
<tr>
<td>20</td>
<td>( 15f_{20} )</td>
<td>-</td>
<td>5.00</td>
<td>-</td>
<td>-</td>
<td>11.96</td>
</tr>
</tbody>
</table>

Note that links 13 and 16 now have positive associated flows although at very low levels.
Figure 10: The Optimal Supply Chain Network Design for Example 2
Example 3

Example 3 had the same data as Example 2 except that we changed the probability distributions so that we now had:

\[
P_1\left(\sum_{p \in P_{w_1}} x_p\right) = \frac{\sum_{p \in P_{w_1}} x_p}{110},
\]

\[
P_2\left(\sum_{p \in P_{w_2}} x_p\right) = \frac{\sum_{p \in P_{w_2}} x_p}{120},
\]

\[
P_3\left(\sum_{p \in P_{w_3}} x_p\right) = \frac{\sum_{p \in P_{w_3}} x_p}{130}.
\]


Table 5: Total Cost Functions and Solution for Example 3

<table>
<thead>
<tr>
<th>Link</th>
<th>( \hat{c}_a(f_a) )</th>
<th>( \hat{\pi}_a(u_a) )</th>
<th>( f_a^* )</th>
<th>( u_a^* )</th>
<th>( \omega_a^* )</th>
<th>( \gamma_a^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( f_1^2 + 2f_1 )</td>
<td>( .5u_1^2 + u_1 )</td>
<td>4.23</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>( .5f_2^2 + f_2 )</td>
<td>( .5u_2^2 + u_2 )</td>
<td>9.06</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>( .5f_3^2 + f_3 )</td>
<td>( .5u_3^2 + u_3 )</td>
<td>8.61</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>( 1.5f_4^2 + 2f_4 )</td>
<td>( .5u_4^2 + u_4 )</td>
<td>2.05</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>( f_5^2 + 3f_5 )</td>
<td>( .5u_5^2 + u_5 )</td>
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<td>0.00</td>
<td>-</td>
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<td>( f_6^2 + 2f_6 )</td>
<td>( .5u_6^2 + u_6 )</td>
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<td>-</td>
</tr>
<tr>
<td>7</td>
<td>( .5f_7^2 + 2f_7 )</td>
<td>( .5u_7^2 + u_7 )</td>
<td>5.77</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>( .5f_8^2 + 2f_8 )</td>
<td>( .5u_8^2 + u_8 )</td>
<td>7.01</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>( f_9^2 + 5f_9 )</td>
<td>( .5u_9^2 + u_9 )</td>
<td>1.61</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>( .5f_{10}^2 + 2f_{10} )</td>
<td>( .5u_{10}^2 + u_{10} )</td>
<td>12.34</td>
<td>2.34</td>
<td>3.34</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>( f_{11}^2 + f_{11} )</td>
<td>( .5u_{11}^2 + u_{11} )</td>
<td>9.56</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>( .5f_{12}^2 + 2f_{12} )</td>
<td>( .5u_{12}^2 + u_{12} )</td>
<td>5.82</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>13</td>
<td>( .5f_{13}^2 + 5f_{13} )</td>
<td>( .5u_{13}^2 + u_{13} )</td>
<td>2.38</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
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Table 6: Total Cost Functions and Solution for Example 3 (continued)

<table>
<thead>
<tr>
<th>Link</th>
<th>$\hat{c}_a(f_a)$</th>
<th>$\hat{\pi}_a(u_a)$</th>
<th>$f_a^*$</th>
<th>$u_a^*$</th>
<th>$\omega_a^*$</th>
<th>$\gamma_a^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>$f_{14}^2$</td>
<td>$.5u_{14}^2 + u_{14}$</td>
<td>4.14</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>15</td>
<td>$f_{15}^2 + 2f_{15}$</td>
<td>$.5u_{15}^2 + u_{15}$</td>
<td>2.09</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>16</td>
<td>$.5f_{16}^2 + 3f_{16}$</td>
<td>$.5u_{16}^2 + u_{16}$</td>
<td>2.75</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>17</td>
<td>$.5f_{17}^2 + 2f_{17}$</td>
<td>$.5u_{17}^2 + u_{17}$</td>
<td>4.72</td>
<td>0.00</td>
<td>0.00</td>
<td>–</td>
</tr>
<tr>
<td>18</td>
<td>$10f_{18}$</td>
<td>–</td>
<td>5.00</td>
<td>–</td>
<td>–</td>
<td>34.13</td>
</tr>
<tr>
<td>19</td>
<td>$12f_{19}$</td>
<td>–</td>
<td>10.00</td>
<td>–</td>
<td>–</td>
<td>31.70</td>
</tr>
<tr>
<td>20</td>
<td>$15f_{20}$</td>
<td>–</td>
<td>5.00</td>
<td>–</td>
<td>–</td>
<td>29.66</td>
</tr>
</tbody>
</table>

The optimal supply chain network design for Example 3 has the initial topology since there are now positive flows on all the links. It is also interesting to note that there is a significant increase in production volumes by the organization at its manufacturing plants.
Figure 11: The Optimal Supply Chain Network Design for Example 3
Extensions to Perishable Products
without and with
Competition

Over 39,000 donations are needed everyday in the United States, and the blood supply is frequently reported to be just 2 days away from running out (American Red Cross (2010)).

Hospitals with as many days of surgical delays due to blood shortage as 120 a year have been observed (Whitaker et al. (2007)).

The national estimate for the number of units blood products outdated by blood centers and hospitals was 1,276,000 out of 15,688,000 units (Whitaker et al. (2007)).

The American Red Cross is the major supplier of blood products to hospitals and medical centers satisfying over 45% of the demand for blood components nationally (Walker (2010)).
Supply Chain Network Topology for a Regionalized Blood Bank

ARC Regional Division

Blood Collection Sites

Blood Centers

Component Labs

Storage Facilities

Distribution Centers

Demand Points

Blood Collection

Shipment of Collected Blood

Testing & Processing

Storage

Shipment

Distribution

Anna Nagurney

Grand Challenges in Supply Chain Networks
We developed a supply chain network optimization model for the management of the procurement, testing and processing, and distribution of a perishable product – that of human blood.

Novel features of the model include:

- It captures *perishability of this life-saving product* through the use of arc multipliers;
- It contains *discarding costs* associated with waste/disposal;
- It handles *uncertainty* associated with demand points;
- It assesses *costs associated with shortages/surpluses at the demand points*, and
- It quantifies the *supply-side risk* associated with procurement.
Relationship of the Model to Others in the Literature

If the demands are fixed, but there are additional processing tiers, as well as capacity investments as variables, the model is the medical nuclear supply chain design model in “Medical Nuclear Supply Chain Design: A Tractable Network Model and Computational Approach,” Anna Nagurney and Ladimer S. Nagurney, *International Journal of Production Economics* **140**(2) (2012), 865-874.
Figure 12: The Medical Nuclear Supply Chain Network Topology

Anna Nagurney
Grand Challenges in Supply Chain Networks
What About Competitive Behavior, Product Perishability, and Supply Chain Networks?
Recent results in this dimension:

In this paper, we develop a generalized network oligopoly model for pharmaceutical supply chain competition which takes into account brand differentiation of the product as well as waste management costs. It has the following original features:

1. It handles the perishability of the pharmaceutical product through the introduction of arc multipliers;
2. It allows each firm to the discarding cost of waste / perished medicine;
3. It captures product differentiation under oligopolistic competition through the branding of drugs, which can also include generics as distinct brands. This feature is important since the patents rights for many top-selling drugs are expiring.

The governing concept is that of Cournot-Nash equilibrium and we use variational inequality theory for the modeling, analysis, and computations.
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1. it handles the *perishability of the pharmaceutical product* through the introduction of arc multipliers;
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1. it handles the *perishability of the pharmaceutical product* through the introduction of arc multipliers;

2. it allows each firm to *the discarding cost of waste / perished medicine*;

3. it captures *product differentiation under oligopolistic competition through the branding of drugs*, which can also include generics as distinct brands. This feature is important since the patents rights for many top-selling drugs are expiring.
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1. it handles the *perishability of the pharmaceutical product* through the introduction of arc multipliers;

2. it allows each firm to *the discarding cost of waste / perished medicine*;

3. it captures *product differentiation under oligopolistic competition through the branding of drugs*, which can also include generics as distinct brands. This feature is important since the patents rights for many top-selling drugs are expiring.

*The governing concept is that of Cournot-Nash equilibrium and we use variational inequality theory for the modeling, analysis, and computations.*
Ironically, whereas some drugs may be unsold and unused and/or past their expiration dates, the number of drugs that were reported in short supply in the US in the first half of 2011 rose to 211 – close to an all-time record – with only 58 in short supply in 2004.
A pressure faced by pharmaceutical firms is the *impact of their medical waste*, which includes the perished excess medicine, and inappropriate disposal on the retailer / consumer end.
Figure 13: The Pharmaceutical Supply Chain Network Topology
Supply Chain Generalized Network Cournot-Nash Equilibrium

In the Cournot-Nash oligopolistic market framework, each firm selects its product path flows in a noncooperative manner, seeking to maximize its own profit, until an equilibrium is achieved, according to the definition below.

**Definition: Supply Chain Generalized Network Cournot-Nash Equilibrium**

A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ constitutes a supply chain generalized network Cournot-Nash equilibrium if for each firm $i$; $i = 1, \ldots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i,$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ and $K_i \equiv \{X_i | X_i \in R^{n_{p_i}}_+\}$. 
An equilibrium is established if no firm can unilaterally improve its profit by changing its production path flows, given the production path flow decisions of the other firms.

Next, we present the variational inequality formulations of the Cournot-Nash equilibrium for the pharmaceutical supply chain network under oligopolistic competition satisfying the Definition, in terms of both path flows (see Cournot (1838), Nash (1950, 1951), Gabay and Moulin (1980), and Nagurney (2006)).
The Variational Inequality Formulation

**Theorem**

Assume that, for each pharmaceutical firm $i; i = 1, \ldots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in $X_i$, and is continuously differentiable. Then $X^* \in K$ is a supply chain generalized network Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*)^T, X_i - X_i^* \rangle \geq 0, \quad \forall X \in K,$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to $X_i$. 
The previous variational inequality, for our model, is equivalent to the variational inequality: determine \( x^* \in K^1 \) such that:

\[
\sum_{i=1}^{l} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[ \frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \frac{\partial \hat{Z}_p(x^*)}{\partial x_p} - \rho_{ik}(x^*)\mu_p \right] \\
\sum_{l=1}^{n_R} \frac{\partial \rho_{il}(x^*)}{\partial d_{ik}} \mu_p \sum_{p \in P_i^l} \mu_p x_p^* \times [x_p - x_p^*] \geq 0, \quad \forall x \in K^1,
\]

where \( K^1 \equiv \{x| x \in R_+^{np}\} \), and, for notational convenience, we denote:

\[
\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{b \in L_i} \sum_{a \in L_i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \alpha_{ap} \quad \text{and} \quad \frac{\partial \hat{Z}_p(x)}{\partial x_p} \equiv \sum_{a \in L_i} \frac{\partial \hat{z}_a(f_a)}{\partial f_a} \alpha_{ap}.
\]
The variational inequality can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $n$-dimensional Euclidean space if we let $X \equiv x$ and

$$F(X) \equiv \left[ \frac{\partial \hat{C}_p(x)}{\partial x_p} + \frac{\partial \hat{Z}_p(x)}{\partial x_p} - \rho_{ik}(x) \mu_p - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(x)}{\partial d_{ik}} \mu_p \sum_{p \in P^i_k} \mu_p x_p; \ p \in P^i_k; \right]$$

$$i = 1, \ldots, I; \ k = 1, \ldots, n_R.$$
Explicit Formulae for the Euler Method Applied to the Supply Chain Generalized Network Oligopoly Variational Inequality

The elegance of this procedure for the computation of solutions to our supply chain generalized network oligopoly model with product differentiation can be seen in the following explicit formulae. In particular, we have the following closed form expressions for all the path flows $p \in P^i_k$, $\forall i, k$:

$$x^{\tau+1}_p = \max \left\{ 0, x^\tau_p + a_\tau (\rho_{ik}(x^\tau) \mu_p + \sum_{l=1}^{n^R} \frac{\partial \rho_{il}(x^\tau)}{\partial d_{ik}} \mu_p \sum_{p \in P^i_l} \mu_p x^\tau_p - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \frac{\partial \hat{Z}_p(x^\tau)}{\partial x_p} \right\}.$$
Numerical Case Study
This case is assumed to occur in the **third quarter of 2011** prior to the expiration of the patent for Lipitor.

Firm 1 represents a multinational pharmaceutical giant, hypothetically, **Pfizer, Inc.**, which still possesses the patent for **Lipitor**, the most popular brand of cholesterol-lowering drug.

Firm 2, on the other hand, which might represent, for example, **Merck & Co., Inc.**, been producing **Zocor**, another cholesterol regulating brand, whose patent expired in 2006.
The Pharmaceutical Supply Chain Network Topology for Case I

Pharmaceutical Firm 1

Pharmaceutical Firm 2

Figure 14: Case I Supply Chain Network
Case I (cont’d)

The demand price functions were as follows:

\[
\rho_{11}(d) = -1.1d_{11} - 0.9d_{21} + 275; \quad \rho_{21}(d) = -1.2d_{21} - 0.7d_{11} + 210;
\]

\[
\rho_{12}(d) = -0.9d_{12} - 0.8d_{22} + 255; \quad \rho_{22}(d) = -1.0d_{22} - 0.5d_{12} + 200;
\]

\[
\rho_{13}(d) = -1.4d_{13} - 1.0d_{23} + 265; \quad \rho_{23}(d) = -1.5d_{23} - 0.4d_{13} + 186.
\]

The Euler method for the solution of variational inequality was implemented in Matlab. The results can be seen in the following tables.
Link Multipliers, Total Cost Functions and Link Flow Solution for **Case I**

<table>
<thead>
<tr>
<th>Link $a$</th>
<th>$\alpha_a$</th>
<th>$\hat{c}_a(f_a)$</th>
<th>$\hat{z}_a(f_a)$</th>
<th>$f^*_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.95</td>
<td>$5f^2_1 + 8f_1$</td>
<td>.5$f^2_1$</td>
<td>13.73</td>
</tr>
<tr>
<td>2</td>
<td>.97</td>
<td>$7f^2_2 + 3f_2$</td>
<td>.4$f^2_2$</td>
<td>10.77</td>
</tr>
<tr>
<td>3</td>
<td>.96</td>
<td>$6.5f^2_3 + 4f_3$</td>
<td>.3$f^2_3$</td>
<td>8.42</td>
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<tr>
<td>4</td>
<td>.98</td>
<td>$5f^2_4 + 7f_4$</td>
<td>.35$f^2_4$</td>
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<tr>
<td>5</td>
<td>1.00</td>
<td>$.7f^2_5 + f_5$</td>
<td>.5$f^2_5$</td>
<td>5.21</td>
</tr>
<tr>
<td>6</td>
<td>.99</td>
<td>$.9f^2_6 + 2f_6$</td>
<td>.5$f^2_6$</td>
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</tr>
<tr>
<td>7</td>
<td>1.00</td>
<td>$.5f^2_7 + f_7$</td>
<td>.5$f^2_7$</td>
<td>4.47</td>
</tr>
<tr>
<td>8</td>
<td>.99</td>
<td>$f^2_8 + 2f_8$</td>
<td>.6$f^2_8$</td>
<td>3.02</td>
</tr>
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<td>1.00</td>
<td>$.7f^2_9 + 3f_9$</td>
<td>.6$f^2_9$</td>
<td>3.92</td>
</tr>
<tr>
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<td>1.00</td>
<td>$.6f^2_{10} + 1.5f_{10}$</td>
<td>.6$f^2_{10}$</td>
<td>3.50</td>
</tr>
<tr>
<td>11</td>
<td>.99</td>
<td>$.8f^2_{11} + 2f_{11}$</td>
<td>.4$f^2_{11}$</td>
<td>3.10</td>
</tr>
<tr>
<td>12</td>
<td>.99</td>
<td>$.8f^2_{12} + 5f_{12}$</td>
<td>.4$f^2_{12}$</td>
<td>2.36</td>
</tr>
<tr>
<td>13</td>
<td>.98</td>
<td>$.9f^2_{13} + 4f_{13}$</td>
<td>.4$f^2_{13}$</td>
<td>2.63</td>
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<td>$.8f^2_{14} + 2f_{14}$</td>
<td>.5$f^2_{14}$</td>
<td>3.79</td>
</tr>
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<td>15</td>
<td>.99</td>
<td>$.9f^2_{15} + 3f_{15}$</td>
<td>.5$f^2_{15}$</td>
<td>3.12</td>
</tr>
<tr>
<td>16</td>
<td>1.00</td>
<td>$1.1f^2_{16} + 3f_{16}$</td>
<td>.6$f^2_{16}$</td>
<td>3.43</td>
</tr>
<tr>
<td>17</td>
<td>.98</td>
<td>$2f^2_{17} + 3f_{17}$</td>
<td>.45$f^2_{17}$</td>
<td>8.20</td>
</tr>
<tr>
<td>18</td>
<td>.99</td>
<td>$2.5f^2_{18} + f_{18}$</td>
<td>.55$f^2_{18}$</td>
<td>7.25</td>
</tr>
<tr>
<td>19</td>
<td>.98</td>
<td>$2.4f^2_{19} + 1.5f_{19}$</td>
<td>.5$f^2_{19}$</td>
<td>7.97</td>
</tr>
<tr>
<td>20</td>
<td>.98</td>
<td>$1.8f^2_{20} + 3f_{20}$</td>
<td>.3$f^2_{20}$</td>
<td>6.85</td>
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</table>
## Link Multipliers, Total Cost Functions and Solution for **Case I** (cont’d)

<table>
<thead>
<tr>
<th>Link ( a )</th>
<th>( \alpha_a )</th>
<th>( \hat{c}_a(f_a) )</th>
<th>( \hat{z}_a(f_a) )</th>
<th>( f_a^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>.98</td>
<td>( 2.1f_{21}^2 + 3f_{21} )</td>
<td>.35( f_{21}^2 )</td>
<td>5.42</td>
</tr>
<tr>
<td>22</td>
<td>.99</td>
<td>( 1.9f_{22}^2 + 2.5f_{22} )</td>
<td>.5( f_{22}^2 )</td>
<td>6.00</td>
</tr>
<tr>
<td>23</td>
<td>1.00</td>
<td>( .5f_{23}^2 + 2f_{23} )</td>
<td>.6( f_{23}^2 )</td>
<td>3.56</td>
</tr>
<tr>
<td>24</td>
<td>1.00</td>
<td>( .7f_{24}^2 + f_{24} )</td>
<td>.6( f_{24}^2 )</td>
<td>1.66</td>
</tr>
<tr>
<td>25</td>
<td>.99</td>
<td>( .5f_{25}^2 + .8f_{25} )</td>
<td>.6( f_{25}^2 )</td>
<td>2.82</td>
</tr>
<tr>
<td>26</td>
<td>.99</td>
<td>( .6f_{26}^2 + f_{26} )</td>
<td>.45( f_{26}^2 )</td>
<td>3.34</td>
</tr>
<tr>
<td>27</td>
<td>.99</td>
<td>( .7f_{27}^2 + .8f_{27} )</td>
<td>.4( f_{27}^2 )</td>
<td>1.24</td>
</tr>
<tr>
<td>28</td>
<td>.98</td>
<td>( .4f_{28}^2 + .8f_{28} )</td>
<td>.45( f_{28}^2 )</td>
<td>2.59</td>
</tr>
<tr>
<td>29</td>
<td>1.00</td>
<td>( .3f_{29}^2 + 3f_{29} )</td>
<td>.55( f_{29}^2 )</td>
<td>3.45</td>
</tr>
<tr>
<td>30</td>
<td>1.00</td>
<td>( .75f_{30}^2 + f_{30} )</td>
<td>.55( f_{30}^2 )</td>
<td>1.28</td>
</tr>
<tr>
<td>31</td>
<td>1.00</td>
<td>( .65f_{31}^2 + f_{31} )</td>
<td>.55( f_{31}^2 )</td>
<td>3.09</td>
</tr>
<tr>
<td>32</td>
<td>.99</td>
<td>( .5f_{32}^2 + 2f_{32} )</td>
<td>.3( f_{32}^2 )</td>
<td>2.54</td>
</tr>
<tr>
<td>33</td>
<td>.99</td>
<td>( .4f_{33}^2 + 3f_{33} )</td>
<td>.3( f_{33}^2 )</td>
<td>3.43</td>
</tr>
<tr>
<td>34</td>
<td>1.00</td>
<td>( .5f_{34}^2 + 3.5f_{34} )</td>
<td>.4( f_{34}^2 )</td>
<td>0.75</td>
</tr>
<tr>
<td>35</td>
<td>.98</td>
<td>( .4f_{35}^2 + 2f_{35} )</td>
<td>.55( f_{35}^2 )</td>
<td>1.72</td>
</tr>
<tr>
<td>36</td>
<td>.98</td>
<td>( .3f_{36}^2 + 2.5f_{36} )</td>
<td>.55( f_{36}^2 )</td>
<td>2.64</td>
</tr>
<tr>
<td>37</td>
<td>.99</td>
<td>( .55f_{37}^2 + 2f_{37} )</td>
<td>.55( f_{37}^2 )</td>
<td>0.95</td>
</tr>
<tr>
<td>38</td>
<td>1.00</td>
<td>( .35f_{38}^2 + 2f_{38} )</td>
<td>.4( f_{38}^2 )</td>
<td>3.47</td>
</tr>
<tr>
<td>39</td>
<td>1.00</td>
<td>( .4f_{39}^2 + 5f_{39} )</td>
<td>.4( f_{39}^2 )</td>
<td>2.47</td>
</tr>
<tr>
<td>40</td>
<td>.98</td>
<td>( .55f_{40}^2 + 2f_{40} )</td>
<td>.6( f_{40}^2 )</td>
<td>0.00</td>
</tr>
</tbody>
</table>
The computed equilibrium demands for each of the two brands were:

\[ d_{11}^* = 10.32, \quad d_{21}^* = 7.66, \]
\[ d_{12}^* = 4.17, \quad d_{22}^* = 8.46, \]
\[ d_{13}^* = 8.41, \quad d_{23}^* = 1.69. \]

The incurred equilibrium prices associated with the branded drugs at each demand market were as follows:

\[ \rho_{11}(d^*) = 256.75, \quad \rho_{21}(d^*) = 193.58, \]
\[ \rho_{12}(d^*) = 244.48, \quad \rho_{22}(d^*) = 189.46, \]
\[ \rho_{13}(d^*) = 251.52, \quad \rho_{23}(d^*) = 180.09. \]
Case I: Result Analysis

Firm 1, which produces the top-selling product, captures the majority of the market share at demand markets 1 and 3, despite the higher price. In fact, it has almost entirely seized demand market 3 forcing several links connecting Firm 2 to demand market 3 to have insignificant flows including link 40 with a flow equal to zero.

Firm 2 dominates demand market 2, due to the consumers’ willingness to lean towards this product there, perhaps as a consequence of the lower price, or the perception of quality, etc.

The profits of the two firms are:

\[ U_1(X^*) = 2,936.52 \] and \[ U_2(X^*) = 1,675.89. \]
A Case Study – Case II

In this case, we consider the scenario in which Firm 1 has just lost the exclusive patent right of its highly popular cholesterol regulator. A manufacturer of generic drugs, say, Sanofi, here denoted by Firm 3, has recently introduced a generic substitute for Lipitor by reproducing its active ingredient Atorvastatin (Smith (2011)). Firm 3 is assumed to have two manufacturing plants, two distribution centers as well as two storage facilities in order to supply the same three demand markets as in Case I (See Figure).
The Pharmaceutical Supply Chain Network Topology for Cases II and III

Pharmaceutical Firm 1  Pharmaceutical Firm 3  Pharmaceutical Firm 2

Anna Nagurney  Grand Challenges in Supply Chain Networks
Firm 1 has just lost the exclusive patent right of its highly popular cholesterol regulator. A manufacturer of generic drugs, say, Ranbaxy Laboratories, here denoted by Firm 3, has recently introduced a generic substitute for Lipitor by reproducing its active ingredients.

The demand price functions for the products of Firm 1 and 2 will stay the same as in Case I. The demand price functions corresponding to the product of Firm 3 are as follows:

\[
\begin{align*}
\rho_{31}(d) &= -0.9d_{31} - 0.6d_{11} - 0.8d_{21} + 150; \\
\rho_{32}(d) &= -0.8d_{32} - 0.5d_{12} - 0.6d_{22} + 130; \\
\rho_{33}(d) &= -0.9d_{33} - 0.7d_{13} - 0.5d_{23} + 133.
\end{align*}
\]
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**Link Multipliers, Total Cost Functions and Link Flow Solution for Case II**
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Link Multipliers, Total Cost Functions and Solution for Case II (cont’d)
Case II: Result Analysis

The equilibrium product flows of Firms 1 and 2 on links 1 through 40 are identical to the corresponding values in Case I.

When the new product produced by Firm 3 is just introduced, the manufacturers of the two existing products will not experience an immediate impact on their respective demands of branded drugs.

The equilibrium computed demands for the products of Firms 1 and 2 at the demand markets will remain as in Case I, and the equilibrium amounts of demand for the new product of Firm 3 at each demand market is equal to:

\[ d_{31}^* = 5.17, \quad d_{32}^* = 3.18, \quad \text{and} \quad d_{33}^* = 3.01. \]
The equilibrium prices associated with the branded drugs 1 and 2 at the demand markets will not change, whereas the incurred equilibrium prices of generic drug 3 are as follows:

\[ \rho_{31}(d^*) = 133.02, \quad \rho_{32}(d^*) = 120.30, \quad \text{and} \quad \rho_{33}(d^*) = 123.55, \]

which is significantly lower than the respective prices of its competitors in all the demand markets.

Thus, the profit that Firm 3 derived from manufacturing and delivering the new generic substitute to these 3 markets is:

\[ U_3(X^*) = 637.38, \]

while the profits of Firms 1 and 2 remain unchanged.
Case III

The generic product of Firm 3 has now been well-established, and has affected the behavior of the consumers through the demand price functions of the relatively more recognized products of Firms 1 and 2. The demand price functions associated are now given by:

**Firm 1:**
\[
\rho_{11}(d) = -1.1d_{11} - 0.9d_{21} - 1.0d_{31} + 192;
\]
\[
\rho_{21}(d) = -1.2d_{21} - 0.7d_{11} - 0.8d_{31} + 176;
\]
\[
\rho_{31} = -0.9d_{31} - 0.6d_{11} - 0.8d_{21} + 170;
\]

**Firm 2:**
\[
\rho_{12}(d) = -0.9d_{12} - 0.8d_{22} - 0.7d_{32} + 166;
\]
\[
\rho_{22}(d) = -1.0d_{22} - 0.5d_{12} - 0.8d_{32} + 146;
\]
\[
\rho_{32}(d) = -0.8d_{32} - 0.5d_{12} - 0.6d_{22} + 153;
\]

**Firm 3:**
\[
\rho_{13}(d) = -1.4d_{13} - 1.0d_{23} - 0.5d_{33} + 173;
\]
\[
\rho_{23}(d) = -1.5d_{23} - 0.4d_{13} - 0.7d_{33} + 164;
\]
\[
\rho_{33}(d) = -0.9d_{33} - 0.7d_{13} - 0.5d_{23} + 157.
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Link Multipliers, Total Cost Functions and Link Flow Solution for Case III
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<td>$.4f_{28}^2 + .8f_{28}$</td>
<td>$.45f_{28}^2$</td>
<td>0.66</td>
</tr>
<tr>
<td>29</td>
<td>1.00</td>
<td>$.3f_{29}^2 + 3f_{29}$</td>
<td>$.55f_{29}^2$</td>
<td>2.29</td>
</tr>
<tr>
<td>30</td>
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<td>$.55f_{30}^2$</td>
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<tr>
<td>31</td>
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<td>$.55f_{31}^2$</td>
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<tr>
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<tr>
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<tr>
<td>34</td>
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<td>$.4f_{34}^2$</td>
<td>2.39</td>
</tr>
<tr>
<td>35</td>
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<tr>
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<tr>
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<td>$.4f_{38}^2$</td>
<td>3.46</td>
</tr>
<tr>
<td>39</td>
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<td>$.4f_{39}^2 + 5f_{39}$</td>
<td>$.4f_{39}^2$</td>
<td>0.00</td>
</tr>
<tr>
<td>40</td>
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<td>$.55f_{40}^2 + 2f_{40}$</td>
<td>$.6f_{40}^2$</td>
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<tr>
<td>41</td>
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<tr>
<td>42</td>
<td>.96</td>
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<td>$.4f_{42}^2$</td>
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</tr>
<tr>
<td>43</td>
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<td>$.45f_{43}^2$</td>
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</tr>
<tr>
<td>44</td>
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<td>$.45f_{44}^2$</td>
<td>3.63</td>
</tr>
<tr>
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<td>$.5f_{45}^2$</td>
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<td>$.55f_{46}^2$</td>
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</tr>
<tr>
<td>47</td>
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<td>$1.5f_{47}^2 + 4f_{47}$</td>
<td>$.4f_{47}^2$</td>
<td>8.60</td>
</tr>
<tr>
<td>48</td>
<td>.98</td>
<td>$2.1f_{48}^2 + 6f_{48}$</td>
<td>$.45f_{48}^2$</td>
<td>6.72</td>
</tr>
<tr>
<td>49</td>
<td>.99</td>
<td>$6f_{49}^2 + 3f_{49}$</td>
<td>$.55f_{49}^2$</td>
<td>3.63</td>
</tr>
<tr>
<td>50</td>
<td>1.00</td>
<td>$.7f_{50}^2 + 2f_{50}$</td>
<td>$.7f_{50}^2$</td>
<td>3.39</td>
</tr>
<tr>
<td>51</td>
<td>.98</td>
<td>$6f_{51}^2 + 7f_{51}$</td>
<td>$.45f_{51}^2$</td>
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<tr>
<td>52</td>
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<td>$.9f_{52}^2 + 9f_{52}$</td>
<td>$.5f_{52}^2$</td>
<td>1.12</td>
</tr>
<tr>
<td>53</td>
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<td>$.55f_{53}^2 + 6f_{53}$</td>
<td>$.55f_{53}^2$</td>
<td>2.86</td>
</tr>
<tr>
<td>54</td>
<td>.98</td>
<td>$.8f_{54}^2 + 4f_{54}$</td>
<td>$.5f_{54}^2$</td>
<td>2.60</td>
</tr>
</tbody>
</table>

**Link Multipliers, Total Cost Functions and Solution for Case III (cont’d)**
Case III: Results

The computed equilibrium demands and sales prices for the products of Firms 1, 2, and 3 are as follows:

\[
\begin{align*}
    d_{11}^* &= 7.18, & d_{21}^* &= 7.96, & d_{31}^* &= 4.70, \\
    d_{12}^* &= 4.06, & d_{22}^* &= 0.00, & d_{32}^* &= 6.25, \\
    d_{13}^* &= 2.93, & d_{23}^* &= 5.60, & \text{and } d_{33}^* &= 3.93.
\end{align*}
\]

\[
\begin{align*}
    \rho_{11}(d^*) &= 172.24, & \rho_{21}(d^*) &= 157.66, & \rho_{31}(d^*) &= 155.09, \\
    \rho_{12}(d^*) &= 157.97, & \rho_{22}(d^*) &= 138.97, & \rho_{32}(d^*) &= 145.97, \\
    \rho_{13}(d^*) &= 161.33, & \rho_{23}(d^*) &= 151.67, & \text{and } \rho_{33}(d^*) &= 148.61.
\end{align*}
\]

The computed amounts of firms’ profits:

\[
\begin{align*}
    U_1(X^*) &= 1,199.87, & U_2(X^*) &= 1,062.73, & \text{and } U_3(X^*) &= 980.83.
\end{align*}
\]
As a result of the consumers’ growing inclination towards the generic substitute of the previously popular Lipitor, Firm 2 has lost its entire share of market 2 to its competitors, resulting in zero flows on several links. Similarly, Firm 1 now has declining sales of its brand in demand markets 1 and 3.

As expected, the introduction of the generic substitute has also caused remarkable drops in the prices of the existing brands. Interestingly, the decrease in the price of Lipitor in demand markets 2 and 3 exceeds 35%.

Note that simultaneous declines in the amounts of demand and sales price has caused a severe reduction in the profits of Firms 1 and 2. This decline for Firm 1 is observed to be as high as 60%.
As noted by Johnson (2011), the market share of a branded drug may decrease by as much as 40%-80% after the introduction of its generic rival. Thus, the model captures the observed decrease in the US market share.
As noted by Johnson (2011), the market share of a branded drug may decrease by as much as 40%-80% after the introduction of its generic rival. Thus, the model captures the observed decrease in the US market share.

The reduction in demand and price due to the patent expiration has been observed in the market sales. The US sales of Lipitor have dropped over 75% (Forbes (2012) and Firecepharma (2012)).
## Paths Definition and Optimal Path Flow Pattern - Firm 1

<table>
<thead>
<tr>
<th>O/D Pair (1, R₁)</th>
<th>Path Definition</th>
<th>Path Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1 = (1, 5, 17, 23)$</td>
<td>$x_1^* = 1.87$</td>
<td></td>
</tr>
<tr>
<td>$p_2 = (1, 6, 18, 26)$</td>
<td>$x_2^* = 1.46$</td>
<td></td>
</tr>
<tr>
<td>$p_3 = (1, 7, 19, 29)$</td>
<td>$x_3^* = 1.57$</td>
<td></td>
</tr>
<tr>
<td>$p_4 = (2, 8, 17, 23)$</td>
<td>$x_4^* = 0.73$</td>
<td></td>
</tr>
<tr>
<td>$p_5 = (2, 9, 18, 26)$</td>
<td>$x_5^* = 1.17$</td>
<td></td>
</tr>
<tr>
<td>$p_6 = (2, 10, 19, 29)$</td>
<td>$x_6^* = 0.87$</td>
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</table>

<table>
<thead>
<tr>
<th>O/D Pair (1, R₂)</th>
<th>Path Definition</th>
<th>Path Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_7 = (1, 5, 17, 24)$</td>
<td>$x_7^* = 0.89$</td>
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<tr>
<td>$p_8 = (1, 6, 18, 27)$</td>
<td>$x_8^* = 0.57$</td>
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</tr>
<tr>
<td>$p_9 = (1, 7, 19, 30)$</td>
<td>$x_9^* = 0.66$</td>
<td></td>
</tr>
<tr>
<td>$p_{10} = (2, 8, 17, 24)$</td>
<td>$x_{10}^* = 0.68$</td>
<td></td>
</tr>
<tr>
<td>$p_{11} = (2, 9, 18, 27)$</td>
<td>$x_{11}^* = 0.82$</td>
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</tr>
<tr>
<td>$p_{12} = (2, 10, 19, 30)$</td>
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<table>
<thead>
<tr>
<th>O/D Pair (1, R₃)</th>
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</thead>
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<td>$p_{13} = (1, 5, 17, 25)$</td>
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<tr>
<td>$p_{14} = (1, 6, 18, 28)$</td>
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<tr>
<td>$p_{15} = (1, 7, 19, 31)$</td>
<td>$x_{15}^* = 0.64$</td>
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<tr>
<td>$p_{16} = (2, 8, 17, 25)$</td>
<td>$x_{16}^* = 0.49$</td>
<td></td>
</tr>
<tr>
<td>$p_{17} = (2, 9, 18, 28)$</td>
<td>$x_{17}^* = 0.53$</td>
<td></td>
</tr>
<tr>
<td>$p_{18} = (2, 10, 19, 31)$</td>
<td>$x_{18}^* = 0.72$</td>
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</tr>
<tr>
<td>O/D Pair</td>
<td>Path Definition</td>
<td>Path Flow</td>
</tr>
<tr>
<td>------------</td>
<td>-------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>(2, R₁)</td>
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<td>( x_{p_{19}}^{*} = 1.26 )</td>
</tr>
<tr>
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<td>( p_{20} = (3, 12, 21, 35) )</td>
<td>( x_{p_{20}}^{*} = 0.77 )</td>
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<tr>
<td></td>
<td>( p_{21} = (3, 13, 22, 38) )</td>
<td>( x_{p_{21}}^{*} = 1.51 )</td>
</tr>
<tr>
<td></td>
<td>( p_{22} = (4, 14, 20, 32) )</td>
<td>( x_{p_{22}}^{*} = 1.63 )</td>
</tr>
<tr>
<td></td>
<td>( p_{23} = (4, 15, 21, 35) )</td>
<td>( x_{p_{23}}^{*} = 1.16 )</td>
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<tr>
<td></td>
<td>( p_{24} = (4, 16, 22, 38) )</td>
<td>( x_{p_{24}}^{*} = 2.12 )</td>
</tr>
<tr>
<td></td>
<td>( p_{25} = (3, 11, 20, 33) )</td>
<td>( x_{p_{25}}^{*} = 0.00 )</td>
</tr>
<tr>
<td></td>
<td>( p_{26} = (3, 12, 21, 36) )</td>
<td>( x_{p_{26}}^{*} = 0.00 )</td>
</tr>
<tr>
<td></td>
<td>( p_{27} = (3, 13, 22, 39) )</td>
<td>( x_{p_{27}}^{*} = 0.00 )</td>
</tr>
<tr>
<td></td>
<td>( p_{28} = (4, 14, 20, 33) )</td>
<td>( x_{p_{28}}^{*} = 0.00 )</td>
</tr>
<tr>
<td></td>
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<td>( x_{p_{29}}^{*} = 0.00 )</td>
</tr>
<tr>
<td></td>
<td>( p_{30} = (4, 16, 22, 39) )</td>
<td>( x_{p_{30}}^{*} = 0.00 )</td>
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<tr>
<td>(2, R₂)</td>
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<td>( x_{p_{31}}^{*} = 1.26 )</td>
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<tr>
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<td>( p_{32} = (3, 12, 21, 37) )</td>
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<tr>
<td></td>
<td>( p_{33} = (3, 13, 22, 40) )</td>
<td>( x_{p_{33}}^{*} = 0.57 )</td>
</tr>
<tr>
<td></td>
<td>( p_{34} = (4, 14, 20, 34) )</td>
<td>( x_{p_{34}}^{*} = 1.26 )</td>
</tr>
<tr>
<td></td>
<td>( p_{35} = (4, 15, 21, 37) )</td>
<td>( x_{p_{35}}^{*} = 1.29 )</td>
</tr>
<tr>
<td></td>
<td>( p_{36} = (4, 16, 22, 40) )</td>
<td>( x_{p_{36}}^{*} = 0.54 )</td>
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<tr>
<td>(2, R₃)</td>
<td>( p_{33} = (3, 11, 20, 34) )</td>
<td>( x_{p_{33}}^{*} = 1.26 )</td>
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<td>( p_{34} = (3, 12, 21, 37) )</td>
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<tr>
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<td>( p_{35} = (3, 13, 22, 40) )</td>
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<tr>
<td></td>
<td>( p_{36} = (4, 14, 20, 34) )</td>
<td>( x_{p_{36}}^{*} = 1.26 )</td>
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<tr>
<td></td>
<td>( p_{37} = (4, 15, 21, 37) )</td>
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<tr>
<td></td>
<td>( p_{38} = (4, 16, 22, 40) )</td>
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</table>
## Paths Definition and Optimal Path Flow Pattern - Firm 3

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<th>Path Flow</th>
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<tr>
<td>$p_{37}$ = (41, 43, 47, 49)</td>
<td>$x^*<em>{p</em>{37}} = 1.87$</td>
<td></td>
</tr>
<tr>
<td>$p_{38}$ = (41, 44, 48, 52)</td>
<td>$x^*<em>{p</em>{38}} = 1.78$</td>
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</tr>
<tr>
<td>$p_{39}$ = (42, 45, 47, 49)</td>
<td>$x^*<em>{p</em>{39}} = 0.70$</td>
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</tr>
<tr>
<td>$p_{40}$ = (42, 46, 48, 52)</td>
<td>$x^*<em>{p</em>{40}} = 0.68$</td>
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</tr>
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</table>

<table>
<thead>
<tr>
<th>O/D Pair (3, $R_2$)</th>
<th>Path Definition</th>
<th>Path Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{41}$ = (41, 43, 47, 50)</td>
<td>$x^*<em>{p</em>{41}} = 1.61$</td>
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</tr>
<tr>
<td>$p_{42}$ = (41, 44, 48, 53)</td>
<td>$x^*<em>{p</em>{42}} = 1.46$</td>
<td></td>
</tr>
<tr>
<td>$p_{43}$ = (42, 45, 47, 50)</td>
<td>$x^*<em>{p</em>{43}} = 2.07$</td>
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</tr>
<tr>
<td>$p_{44}$ = (42, 46, 48, 53)</td>
<td>$x^*<em>{p</em>{44}} = 1.90$</td>
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</table>

<table>
<thead>
<tr>
<th>O/D Pair (3, $R_3$)</th>
<th>Path Definition</th>
<th>Path Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{45}$ = (41, 43, 47, 51)</td>
<td>$x^*<em>{p</em>{45}} = 0.84$</td>
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</tr>
<tr>
<td>$p_{46}$ = (41, 44, 48, 54)</td>
<td>$x^*<em>{p</em>{46}} = 0.53$</td>
<td></td>
</tr>
<tr>
<td>$p_{47}$ = (42, 45, 47, 51)</td>
<td>$x^*<em>{p</em>{47}} = 1.46$</td>
<td></td>
</tr>
<tr>
<td>$p_{48}$ = (42, 46, 48, 54)</td>
<td>$x^*<em>{p</em>{48}} = 1.33$</td>
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</table>
Path Flow Trajectories

The Trajectories of Product Flows on Paths $p_1 - p_6$
Path Flow Trajectories

The Trajectories of Product Flows on Paths $p_7 - p_{12}$
Path Flow Trajectories

The Trajectories of Product Flows on Paths $p_{13} - p_{18}$
Path Flow Trajectories

The Trajectories of Product Flows on Paths $p_{19} - p_{24}$
Path Flow Trajectories

The Trajectories of Product Flows on Paths $p_{25} - p_{30}$
Path Flow Trajectories

The Trajectories of Product Flows on Paths $p_{31} - p_{36}$
Path Flow Trajectories

The Trajectories of Product Flows on Paths $p_{37} - p_{42}$
Path Flow Trajectories

The Trajectories of Product Flows on Paths $p_{43} - p_{48}$
A variety of perishable product supply chain models, computational procedures, and applications can be found in our new book:

**Networks Against Time**

Supply Chain Analytics for Perishable Products

Anna Nagurney
Min Yu
Amir H. Masoumi
Ladimer S. Nagurney

**Springer Briefs in Optimization**

Springer
Summary, Conclusions, and Suggestions for Future Research
We emphasized the **importance of capturing behavior** in supply chain modeling, analysis, and design.

We discussed a **variety of network design approaches**: the addition of links; the integration of networks as in mergers and acquisitions; and the design from scratch (and redesign).

We developed an **integrated framework for the design of supply chain networks for critical products** with outsourcing.

The model utilizes cost minimization within a system-optimization perspective as the primary objective and captures rigorously the uncertainty associated with the demand for critical products at the various demand points.
The supply chain network design model allows for the investment of enhanced link capacities and the investigation of whether the product should be outsourced or not.

The framework *can be applied in numerous situations* in which the goal is to produce and deliver a critical product at minimal cost so as to satisfy the demand at various demand points, as closely as possible, given associated penalties for under- and over-supply.
In additional research, we have been heavily involved in constructing mathematical models that capture the impacts of foreign exchange risk and competition intensity on supply chain companies who are involved in offshore outsourcing activities.

Our research in supply chains has also led us to other time-sensitive products, such as fast fashion.

We have also been focusing on design of sustainable supply chains.

Finally, we have also developed models for the dynamics and equilibrium states in ecological predator-prey networks, that is, supply chains in nature.
We expect that future research will include design for robustness and resiliency.

Thank You!

For more information, see: http://supernet.isenberg.umass.edu