Tariffs and Quotas in Global Trade: What Networks, Game Theory, and Variational Inequalities Reveal

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Muchas gracias por todo, fue un gran honor hablar con ustedes.



The conference location is beautiful and the scientific talks excellent!

Outline

- Some Background
- ► The Global Supply Chain Network Model with TRQs
- ▶ A Case Study on Avocados
- Supply Chain Network Models with Quality Under Strict Quotas or Tariffs
- ► A Case Study on Soybeans
- ► Some Final Thoughts

Some Background

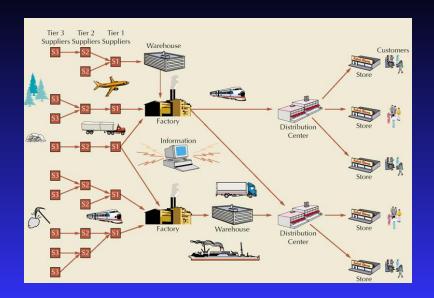
I Work on the Modeling of Network Systems



Much of My Recent Research Has Been on Supply Chains



A General Supply Chain



Characteristics of Supply Chains and Networks Today

- ► *large-scale nature* and complexity of network topology;
- congestion, which leads to nonlinearities;
- alternative behavior of users of the networks, which may lead to paradoxical phenomena;
- possibly conflicting criteria associated with optimization;
- ➤ interactions among the underlying networks themselves, such as the Internet with electric power networks, financial networks, and transportation and logistical networks;
- ► recognition of *their fragility and vulnerability*;
- policies surrounding networks today may have major impacts not only economically, but also socially, politically, and security-wise.

Supply Chains Are Essential to Global Trade

- ➤ Global supply chain networks have made possible the wide distribution of goods, from agricultural products to textiles and apparel as well as aluminum and steel.
- Nations engage in trade to increase their productivity levels, employment rates, and general economic welfare.
- ➤ The increased level of world trade has also garnered the attention of government policy makers.
- Governments may attempt to protect their domestic firms from the possible effects of the highly competitive global arena.



Some of the Biggest Agricultural Exports of Colombia



Global Trade Policies

Examples of policy instruments that have been applied by governments to modify trade patterns included: tariffs, quotas, and a combination thereof - tariff rate quotas.



Tariffs Are in the News Every Day!

The imposition of tariffs by certain countries is leading to retaliation by other countries with ramifications across multiple supply chains, and a **trade war**.

With Higher Tariffs, China Retaliates Against the U.S.



The Yangshan Deep Water Port in Shanghai, China. The Chinese government said on Monday that it would raise tariffs on goods from the United States as of June 1, giving nesotiators from the two countries time to strike a deal. Alv Song-Reuters

The New York Times, May 13, 2019

Global Trade Policies

Our research community needs to construct **computable operational mathematical models** that enable the assessment of the impacts of trade policy instruments such as tariff rate quotas on consumer prices, trade flows, as well as on the profits of producers/firms.

However, this is very challenging research!

How Our Journey Began

In the early 1990s, we were contacted by Charles E. Nicholson and Phillip M. Bishop of the Department of Agricultural, Resource and Managerial Economics at Cornell University, who were interested in modeling ad valorem tariffs associated with the dairy industry and Mexico and were faced with challenges.

We had been doing a lot of research on spatial price equilibrium modeling:

- ► A. Nagurney, T. Takayama, and D. Zhang (1995), Massively parallel computation of spatial price equilibria as dynamical systems, Journal of Economic Dynamics and Control 19(1-2), 3-37.
- ▶ A. Nagurney, S. Thore, and J. Pan (1996), Spatial market policy modeling with goal targets, *Operations Research* **44(2)**, 393-406.
- ► A. Nagurney and L. Zhao (1991), A network equilibrium formulation of market disequilibrium and variational inequalities, *Networks* 21, 109-132.

How Our Journey Began

We ended up publishing a series of papers, including:

A. Nagurney, C.F. Nicholson, and P.M. Bishop (1996), Massively parallel computation of large-scale spatial price equilibrium models with discriminatory ad valorem tariffs, *Annals of Operations Research* **68(2)**, 281-300.





This part of the presentation is based on the paper:

A. Nagurney, D. Besik, and L.S. Nagurney (2019), Global supply chain networks and tariff tate quotas: Equilibrium analysis with application to agricultural products, which is in press in the *Journal of Global Optimization*.



Motivation

- A tariff rate quota (TRQ) is a two-tiered tariff, in which a lower in-quota tariff is applied to imports until a quota is attained and then a higher over-quota tariff is applied to all subsequent imports.
- ▶ The Uruguay Round in 1996 induced the creation of more than 1,300 new TRQs. Currently, 43 World Trade Organization members have a total of 1,425 tariff quotas in their commitments. TRQs are widely utilized especially in agricultural trade for products such as: milk and dairy products, bananas, chocolate, sugar, beef, peanuts, eggs, poultry, soybeans, potatoes, among others.
- ► The world's four most important food crops: rice, wheat, corn, and bananas have all been subject to tariff rate quotas.



Literature Review

Perfect Competition

- ➤ Tariff rate quotas (TRQs) have been deemed challenging to formulate; models have focused almost exclusively on spatial price equilibrium.
- ➤ Spatial price equilibrium models are perfectly competitive models with numerous producers (Samuelson (1964), Takayama and Judge (1964, 1971)).
- ► For more recent applications of spatial price equilibrium models, utilizing variational inequality theory, see Nagurney (1999, 2006), Daniele (2004), Li, Nagurney, and Yu (2018)).
- ► For the inclusion of tariff rate quotas into spatial price equilibrium models using variational inequality theory, see the EJOR paper by Nagurney, Besik, and Dong (2019).

Literature Review

A. Nagurney, D. Besik, and J. Dong (2019), Tariffs and quotas in world trade: A unified variational inequality framework, *European Journal of Operational Research* **275(1)**, 347-360.



Imperfect Competition

- ▶ In many industrial sectors, the more appropriate framework is that of imperfect competition, as in the case of oligopolistic competition.
- Shono (2001) relaxed the assumption of perfect competition, and incorporated TRQs, under oligopolistic competition and that the computable framework consisted of linear functions.
- ➤ Maeda, Suzuki, and Kaiser (2001, 2005) considered oligopolistic competition and TRQs but assumed that there is a single producer in each country.

Specifically, we:

- ▶ Introduce the global supply chain network model consisting of firms that seek to maximize their profits by determining how much of the product to manufacture/produce at the production sites, which can be located in multiple countries;
- Incorporate tariff rate quotas into the supply chain network equilibrium model, and
- ► Provide a case study on the agricultural product of avocados, a very popular fruit in the United States, with growing consumer demand even in China.



Notation Related to Tariff Rate Quotas

- ▶ The groups G_g ; $g = 1, ..., n_G$, consist of the middle tier nodes $\{h\}$ corresponding to the production sites in the countries from which imports are to be restricted under the tariff quota regime and the demand markets $\{I\}$ in the country that is imposing the tariff rate quota.
- lacktriangle Associated with each group G_g is an under-quota tariff $au_{G_g}^u$.
- ▶ Associated with each group G_g is an over-quota tariff $\tau_{G_g}^o$, where $\tau_{G_g}^u < \tau_{G_g}^o$.

The Variables

 Q_{ihl} : denotes the volume of the product manufactured/produced by firm i at production site $h \in \mathcal{J}_i$ and then shipped to demand market l for consumption.

 Q_i : is the vector of nonnegative product flows, where $Q_i = \{Q_{ihl}; h \in \mathcal{J}_i, l \in \mathcal{K}\}.$

Q is then the vector of all the Q_i s.

 λ_{G_g} : denotes the quota rent equivalent for G_g .

The Production Cost Functions

Each firm i; i = 1, ..., I, is faced with a production cost function f_{ih} associated with manufacturing the product at h such that:

$$f_{ih} = f_{ih}(Q), \quad \forall h \in \mathcal{J}_i.$$
 (1)

The Transportation Cost Functions

Each firm i; i = 1, ..., I, encumbers a transportation cost c_{ihl} associated with transporting the product from production site node h to demand market node l:

$$c_{ihl} = c_{ihl}(Q), \quad \forall h \in \mathcal{J}_i, \forall l \in \mathcal{K}.$$
 (2)

The Conservation of Flow Equations

The demand at each demand node I; $\forall I \in \mathcal{K}$, is denoted by d_I , and satisfies:

$$\sum_{i=1}^{l} \sum_{h \in \mathcal{I}_i} Q_{ihl} = d_l. \tag{3}$$

The Demand Price Functions

The consumers, located at the demand markets, reflect their willingness to pay for the product through the demand price functions ρ_I , $\forall I \in \mathcal{K}$, with these functions being expressed as:

$$\rho_I = \rho_I(d), \tag{4a}$$

where d is the vector of all the demands.

In view of (3), we can redefine the demand price functions (4a) as follows:

$$\hat{\rho}_I = \hat{\rho}_I(Q) \equiv \rho_I(d), \quad \forall I \in \mathcal{K}.$$
 (4b)

The Utility Function for a Firm Under a TRQ

For a firm i affected by a TRQ, we define the utility function U_i^G as

$$U_{l}^{G} \equiv \sum_{h \in \mathcal{J}_{l}} \sum_{l \in \mathcal{K}} \hat{\rho}_{l}(Q) Q_{lhl} - \sum_{h \in \mathcal{J}_{l}} f_{lh}(Q) - \sum_{h \in \mathcal{J}_{l}} \sum_{l \in \mathcal{K}} c_{lhl}(Q)$$
$$- \sum_{G_{g} \in \mathcal{I}^{l}} (\tau_{G_{g}}^{u} + \lambda_{G_{g}}^{*}) \sum_{(h,l) \in G_{g}} Q_{lhl}$$
(5a)

where $\lambda_{G_g}^*$ is the equilibrium economic rent equivalent for group G_g , assuming values as in Definition 1 below. We group the $\lambda_{G_g}^*$ s into the vector λ^* .

The Utility Function for a Firm Not Under A TRQ

For any other firm i, we define its utility function U_i , as

$$U_{i} \equiv \sum_{h \in \mathcal{J}_{i}} \sum_{l \in \mathcal{K}} \hat{\rho}_{l}(Q) Q_{ihl} - \sum_{h \in \mathcal{J}_{i}} f_{ih}(Q) - \sum_{h \in \mathcal{J}_{i}} \sum_{l \in \mathcal{K}} c_{ihl}(Q).$$
 (5b)

We then define $\hat{U}_i \equiv U_i^G$ for all firms i with plants associated with groups and $\hat{U}_i \equiv U_i$ for all firms without plants in countries subject to tariff rate quotas.

Also, we define the feasible sets: $K_i \equiv \{Q_i | Q_i \in R_{j=1}^{\sum_{j=1}^J K n_j^i}\}$, $\forall i$.

We assume that the utility functions are concave and continuously differentiable.

Definition 1: Global Supply Chain Network Equilibrium Under TRQs

A product flow pattern Q^* and quota rent equivalent λ^* is a global supply chain network equilibrium under tariff rate quotas if, for each firm i; i = 1, ..., I, the following conditions hold:

$$\hat{U}_i(Q_i^*, Q_{-i}^*, \lambda^*) \ge \hat{U}_i(Q_i, Q_{-i}^*, \lambda^*), \quad \forall Q_i \in K_i,$$
 (6)

where $Q_{-i}^* \equiv (Q_1^*, \dots, Q_{i-1}^*, Q_{i+1}^*, \dots, Q_i^*)$, and for all groups G_g :

$$\lambda_{G_{g}}^{*} \begin{cases} = \tau_{G_{g}}^{\circ} - \tau_{G_{g}}^{u}, & \text{if} \quad \sum_{i=1}^{I} \sum_{(h,l) \in G_{g}} Q_{ihl}^{*} > \bar{Q}_{G_{g}}, \\ \leq \tau_{G_{g}}^{\circ} - \tau_{G_{g}}^{u}, & \text{if} \quad \sum_{i=1}^{I} \sum_{(h,l) \in G_{g}} Q_{ihl}^{*} = \bar{Q}_{G_{g}}, \\ = 0, & \text{if} \quad \sum_{i=1}^{I} \sum_{(h,l) \in G_{g}} Q_{ihl}^{*} < \bar{Q}_{G_{g}}. \end{cases}$$
(7)

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation of the Global Supply Chain Network Equilibrium Under TRQs

Under the assumption that the utility functions are concave and continuously differentiable, the product flow and quota rent equivalent pattern $(Q^*, \lambda^*) \in \mathcal{H}$ is a global supply chain network equilibrium under tariff rate quotas according to Definition 1 if and only if it satisfies the variational inequality (VI):

$$-\sum_{i=1}^{l}\sum_{h\in\mathcal{J}_{i}}\sum_{l\in\mathcal{K}}rac{\partial\hat{U}_{i}(Q^{*},\lambda^{*})}{\partial Q_{ihl}} imes(Q_{ihl}-Q_{ihl}^{*})$$

$$+\sum_{\mathcal{G}}\left[ar{Q}_{G_{\mathcal{G}}}-\sum_{i=1}^{l}\sum_{(h,l)\in G_{\mathcal{G}}}Q_{ihl}^{*}
ight] imes\left[\lambda_{G_{\mathcal{G}}}-\lambda_{G_{\mathcal{G}}}^{*}
ight]\geq0,\quadorall(Q,\lambda)\in\mathcal{H},$$

where $\mathcal{H} \equiv \{(Q, \lambda) | Q \in \bar{K}, \lambda \in R^{n_G}_+ | 0 \le \lambda_{G_g} \le \tau^o_{G_g} - \tau^u_{G_g}, \forall g \}.$

Variational Inequality Formulation

Corollary 1: Variational Inequality Formulation for the Global Supply Chain Network Without TRQs

In the absence of tariff rate quotas, the equilibrium of the resulting global supply chain network model collapses to the solution of the VI: determine $Q^* \in \bar{K}$, satisfying:

$$-\sum_{i=1}^{I}\sum_{h\in\mathcal{I}:}\sum_{l\in\mathcal{K}}\frac{\partial U_{i}(Q^{*})}{\partial Q_{ihl}}\times(Q_{ihl}-Q_{ihl}^{*})\geq0,\quad\forall Q\in\bar{K},\qquad(9)$$

where
$$\bar{K} \equiv \prod_{i=1}^{I} K_i$$
.

Variants of the Model

Unit Tariffs

The framework can be adapted to handle the simpler trade policy of unit tariffs with an appended term: $-\sum_{h\in\mathcal{J}_l}\sum_{l\in\mathcal{K}}\tau_{hl}Q_{ml}$, where τ_{hl} denotes the unit tariff assessed on a product flow from h to l, with $\tau_{hl}=0$, if h,l corresponds to a production site and demand market pair in countries not under a tariff.

Variants of the Model

Strict Quotas

If there is a strict quota regime, for those firms i that are affected, the utility function U_i^G in (5a) is modified to U_i^Q as:

$$U_{i}^{Q} = \sum_{h \in \mathcal{J}_{i}} \sum_{l \in \mathcal{K}} \hat{\rho}_{l}(Q) Q_{ihl} - \sum_{h \in \mathcal{J}_{i}} f_{ih}(Q) - \sum_{h \in \mathcal{J}_{i}} \sum_{l \in \mathcal{K}} c_{ihl}(Q)$$
$$- \sum_{G_{g} \in \mathcal{I}^{i}} \lambda_{G_{g}}^{*} \sum_{(h,l) \in G_{g}} Q_{ihl}, \tag{10}$$

where the groups G_g , $\forall g$, now correspond to those node pairs under strict quotas.

The Nash Equilibrium conditions (6) are still relevant but the system (7) is replaced with the system below: for all groups G_g :

$$\bar{Q}_{G_g} - \sum_{i=1}^{I} \sum_{(h,l) \in G_g} Q_{ihl}^* \begin{cases} = 0, & \text{if } \lambda_{G_g}^* > 0, \\ \ge 0, & \text{if } \lambda_{G_g} = 0. \end{cases}$$
(11)

Qualitative Properties

The Standard Variational Inequality Form

We now put variational inequality (8) into standard form (cf. Nagurney (1999)): determine $X^* \in \mathcal{L} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \ge 0, \quad \forall X \in \mathcal{L},$$
 (12)

where X and F(X) are \mathcal{N} -dimensional vectors, \mathcal{L} is a closed, convex set, and F is a given continuous function from \mathcal{L} to $R^{\mathcal{N}}$.

Indeed, we can define $X \equiv (Q,\lambda)$ and $F(X) \equiv (F_1(X),F_2(X))$, where $F_1(X)$ consists of $\sum_{i=1}^{I} \sum_{j=1}^{J} K n_j^i$ elements: $-\frac{\partial \hat{U}_i(Q,\lambda)}{\partial Q_{ihl}}$ for all i,h,l, and $F_2(X)$ consists of n_G elements, with the g-th element given by: $\left[\bar{Q}_{G_g} - \sum_{i=1}^{I} \sum_{(h,l) \in G_g} Q_{ihl}\right]$. Also, here $\mathcal{N} = \sum_{i=1}^{I} \sum_{j=1}^{J} K n_j^i + n_G$ and $\mathcal{L} \equiv \mathcal{H}$.

Qualitative Properties

Theorem 2: Existence of a Solution X^* to (12)

Existence of a solution X^* to the variational inequality governing the global supply chain network model with tariff rate quotas given by (12); equivalently, (8), is guaranteed.

Proposition 1: Monotonicity of F(X) in (12)

F(X) in (12) is monotone, that is,

$$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \ge 0, \quad \forall X^1, X^2 \in \mathcal{L},$$
 (13)

if and only if $\hat{F}(X)$ is monotone, where the (i, h, l)-component of $\hat{F}(X)$, $\forall i, h, l$, consists of

$$\left[\sum_{j\in\mathcal{J}_{i}}\frac{\partial f_{ij}(Q)}{\partial Q_{ihl}}+\sum_{j\in\mathcal{J}_{i}}\sum_{k\in\mathcal{K}}\frac{\partial c_{ijk}(Q)}{\partial Q_{ihl}}-\hat{\rho}_{l}(Q)-\sum_{j\in\mathcal{J}_{i}}\sum_{k\in\mathcal{K}}\frac{\partial \hat{\rho}_{k}(Q)}{\partial Q_{ihl}}Q_{ijk}\right].$$
(14)

The Algorithm: The Modified Projection Method

Step 0: Initialization

Initialize with $X^0 \in \mathcal{L}$. Set t := 1 and select β , such that $0 < \beta \le \frac{1}{L}$, where L is the Lipschitz constant for function F in the variational inequality problem.

Step 1: Construction and Computation

Compute \bar{X}^t by solving the VI subproblem:

$$\langle \bar{X}^t + \beta F(X^{t-1}) - X^{t-1}, X - \bar{X}^t \rangle \ge 0, \quad \forall X \in \mathcal{L}.$$
 (15)

Step 2: Adaptation

Compute X^t by solving the VI subproblem:

$$\langle X^t + \beta F(\bar{X}^t) - X^{t-1}, X - X^t \rangle \ge 0, \quad \forall X \in \mathcal{L}.$$
 (16)

Step 3: Convergence Verification

If $|X^t - X^{t-1}| < \epsilon$, for $\epsilon > 0$, a specified tolerance, then, stop; otherwise, set t := t + 1 and go to Step 1.

The Algorithm: The Modified Projection Method

Explicit formulae for our model for Step 1.

Path Flows

For each Q_{ihl} with (h, l) associated with a group G_g , $\forall g$:

$$\bar{Q}_{ihl}^{t} = \max\{0, \beta\left(-\sum_{j \in \mathcal{J}_{i}} \frac{\partial f_{ij}(Q^{t-1})}{\partial Q_{ihl}} - \sum_{j \in \mathcal{J}_{i}} \sum_{k \in \mathcal{K}} \frac{\partial c_{ijk}(Q^{t-1})}{\partial Q_{ihl}} + \hat{\rho}_{l}(Q^{t-1})\right) + \sum_{j \in \mathcal{J}_{i}} \sum_{k \in \mathcal{K}} \frac{\partial \hat{\rho}_{k}(Q^{t-1})}{\partial Q_{ihl}} Q_{ijk}^{t-1} - \tau_{G_{g}}^{u} - \lambda_{G_{g}}^{t-1}\right) + Q_{ihl}^{t-1}\}, \tag{17}$$

and for each Q_{ihl} with (h, l) not associated with a TRQ:

$$\bar{Q}_{ihl}^{t} = \max\{0, \beta(-\sum_{j \in \mathcal{J}_{i}} \frac{\partial f_{ij}(Q^{t-1})}{\partial Q_{ihl}} - \sum_{j \in \mathcal{J}_{i}} \sum_{k \in \mathcal{K}} \frac{\partial c_{ijk}(Q^{t-1})}{\partial Q_{ihl}} + \hat{\rho}_{l}(Q^{t-1})$$

$$+\sum\sum\frac{\partial\hat{\rho}_k(Q^{t-1})}{\partial Q_{ibl}}Q_{ijk}^{t-1})+Q_{ibl}^{t-1}\}.$$
 (18)

The Algorithm: The Modified Projection Method

The Quota Rent Equivalents

The closed form expression for the quota rent equivalent for group G_{σ} ; $g = 1, ..., n_{G}$, is:

$$\bar{\lambda}_{G_g}^t = \max\{0, \min\{\beta(\sum_{i=1}^{l} \sum_{(h,l) \in G_g} Q_{ihl}^{t-1} - \bar{Q}_{G_g}) + \lambda_{G_g}^{t-1}, \tau_{G_g}^o - \tau_{G_g}^u\}\}.$$
(19)

- Mexico produces more avocados than any other country in the world, about a third of the global total.
- ► In 2017, Mexico exported more than 1.7 billion pounds of Haas avocados to the US.
- ► With about 90% of the avocados imported from Mexico to the United States coming from Michoacan.
- ► The Mexican state of Jalisco, the second-largest avocado-producing state in Mexico, accounts for about 6 percent of total Mexican production.



• The volume of avocado imports into the United States has surpassed even the volume by weight of bananas imported into the US.



- US domestic avocado consumption has risen to approximately 6.5 pounds per person annually, as compared to only 1.4 in 1990.
- The US is among the world's top ten avocado producers, producing between 160,000 and 270,000 tons of avocados a year.
- In terms of other major demand markets, Mexico was the largest supplier of avocados to China until 2017.

The United States' recent imposition of a variety of tariffs, in turn, has resulted in retaliatory tariffs by multiple countries, notably, by Mexico and China and on agricultural products produced in the US.



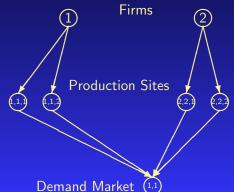
Figure 1: Front page of The New York Times, June 2, 2019



Example 1: Baseline Example Without Tariff Rate Quotas

Firm 1 has two US production sites: one in San Diego county, and the other in San Luis Obispo in California. Firm 2 also has two production sites available, but located in Mexico, in Michoacan and Jalisco. There is a single demand market in this example, located in the US.

We are interested in determining the equilibrium product flows: Q_{111}^* , Q_{121}^* , Q_{231}^* , and Q_{241}^* .



Example 1: Baseline Example Without Tariff Rate Quotas

The production cost functions faced by Firm 1 at its two production sites are:

$$f_{11}(Q) = .005Q_{111}^2 + .8Q_{111}, \quad f_{12}(Q) = .01Q_{121}^2 + 1.1Q_{121}.$$

The transportation cost functions associated with Firm 1 transporting the avocados to the demand market are:

$$c_{111}(Q) = .1Q_{111}^2 + .5Q_{111}, \quad c_{121}(Q) = .1Q_{121}^2 + .4Q_{121}.$$

The production cost functions faced by Firm 2, with the production sites at the two locations in Mexico, are:

$$f_{23}(Q) = .0005Q_{231}^2 + .15Q_{231}, \quad f_{24}(Q) = .0005Q_{241}^2 + .5Q_{241},$$

and its transportation costs to the demand market are:

$$c_{231}(Q) = .04Q_{231}^2 + .5Q_{231}, \quad c_{241}(Q) = .045Q_{241}^2 + .5Q_{241}.$$

The demand price function is: $\rho_1(d) = -.01d_1 + 3$.

Equilibrium Product Flows, Demand Prices, and Profits

We consider the time horizon of a week and the quantities of avocados are reported in millions of pounds. The currency is US dollars.

► The modified projection method yielded the equilibrium avocado product flow pattern:

$$Q_{111}^* = 5.63$$
, $Q_{121}^* = 4.52$, $Q_{231}^* = 20.75$, $Q_{241}^* = 15.24$.

- ▶ Demand price per pound of avocados is $\rho_1 = 2.54$.
- ► Firm 1 achieves a utility (profit) of 6.09 in millions of dollars whereas Firm 2 enjoys a utility (profit) of 34.63 in millions of dollars.

Since consumers in the United States consume about 80% of their avocados from Mexico and about 20% from the US, the above results are very reasonable and also correspond well to the weekly consumption of avocados by US consumers.

Example 2: Tariff Rate Quotas on Avocados from Mexico

The United States assigns the tariff rate quota on group G_1 , which consists of the Mexican production sites that ship to the United States, and the demand market.

In this example, $\bar{Q}_1 = 30$, $\tau^u_{G_1} = .25$, and $\tau^o_{G_1} = .50$.

► The equilibrium avocado product trade flow and economic rent equivalent patterns are:

$$Q_{111}^* = 5.88, \quad Q_{121}^* = 4.76, \quad Q_{231}^* = 17.60, \quad Q_{241}^* = 12.40,$$
 $\lambda_{G_1}^* = .09.$

- ▶ The demand market price per pound of avocados is: $\rho_1 = 2.59$.
- ► The utility (profit) of Firm 1 is: 6.69 in millions of dollars and that of Firm 2: 24.18 in millions of dollars.
- ► The US government acquires tariff payments of 10.24 in millions of dollars.

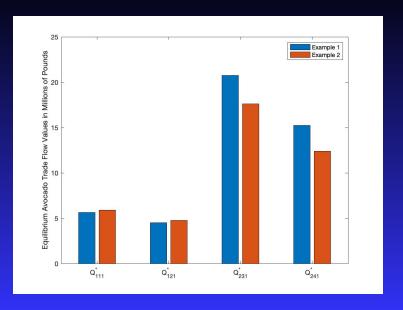
Example 2: Tariff Rate Quotas on Avocados from Mexico

Since the demand market price per pound of avocados ρ_1 is now 2.59, the consumers are faced with a higher price.

Observe that the imports from Mexico to the United States are precisely equal to the imposed quota.

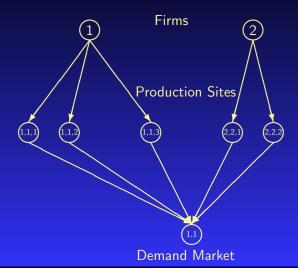
The imposition of the TRQs increases the profit of the US firm by about 10% and decreases the profit of the Mexican firm by about 33%.

Equilibrium Avocado Trade Flows in Examples 1 and 2



Example 3: Addition of a New Production Site in the US

Example 3 has the same data as Example 2 except that now Firm 1 has added a production site in Florida.



Example 3: Addition of a New Production Site in the US

► The new computed equilibrium avocado product flow pattern is:

$$Q_{111}^* = 5.57$$
, $Q_{121}^* = 4.45$, $Q_{151}^* = 7.56$, $Q_{231}^* = 17.60$, $Q_{241}^* = 12.40$.

- ▶ The volume of imports from Mexico remain at the quota $\bar{G}_1 = 30$ million pounds and the equilibrium $\lambda_{G_1}^* = .02$.
- ► The utility (profit) of Firm 1 is now 12.36 in millions of dollars and 24.18 for Firm 2 in millions of dollars.
- ► The US government now acquires tariff payments of 8.10 in millions of dollars.

Example 3: Addition of a New Production Site in the US

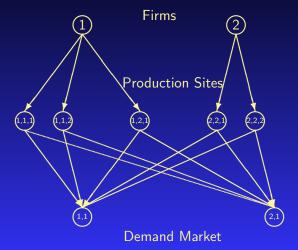
The almost doubling of profits for Firm 1 in this example signals that it should expand the number of its production sites.



Consumers also benefit since the demand market price decreases to 2.52.

Example 4: Addition of a New Demand Market in China

There has been growing interest among consumers in China for avocados.



Example 4: Addition of a New Demand Market in China

► The new computed equilibrium avocado product flow pattern is:

$$Q_{111}^* = 5.03$$
, $Q_{112}^* = 13.33$, $Q_{121}^* = 3.48$, $Q_{122}^* = 11.96$, $Q_{151}^* = 7.60$, $Q_{152}^* = 1.07$, $Q_{231}^* = 17.51$, $Q_{232}^* = 40.09$, $Q_{241}^* = 12.49$, $Q_{242}^* = 21.82$.

- ▶ The incurred demand market prices at the equilibrium in the United States and China are $\rho_1 = 2.54$ and $\rho_2 = 6.12$, respectively.
- ► The utility (profit) of Firm 1 is now 68.35 and that of Firm 2: 174.97.
- ▶ The imports from Mexico to the United States are at the quota with $\lambda_{G_1}^* = .01$.
- ► The US government income from tariff payments is now: 7.8 in millions of dollars.

Example 4: Addition of a New Demand Market in China

With the opening of a major new market for avocados, the utilities (profits) of both firms increase significantly, with those of Firm 1 more than five-fold, and those of Firm 2 about seven-fold.

The price of avocados in the US, however, increases, albeit only slightly.

The price per pound of avocados in China is very reasonable and reflects reality.

Example 5: Tariff Rate Quota Imposed by China on Imports from the US

- $ightharpoonup G_2$ consists of the production sites corresponding to the United States and the demand market in China.
- ▶ The added data are: $ar{Q}_{G_2}=15$, $au_{G_2}^u=1$ and $au_{G_2}^o=2$.
- The modified projection method yielded the following equilibrium avocado product flow pattern:

$$Q_{111}^* = 5.25, \quad Q_{112}^* = 7.80, \quad Q_{121}^* = 3.92, \quad Q_{122}^* = 6.58,$$

$$Q_{151}^* = 7.58, \quad Q_{152}^* = .63,$$

$$Q_{231}^* = 17.50, \quad Q_{232}^* = 40.99, \quad Q_{241}^* = 12.48, \quad Q_{242}^* = 22.30.$$

- ▶ The demand prices are: $\rho_1 = 2.53$ for a pound of avocados in the United States and $\rho_2 = 6.22$ for a pound of avocados in China.
- ► The equilibrium economic rents are: $\lambda_{G_1}^* = 0.00$ and $\lambda_{G_2}^* = .87$.

The US government gathers 7.49 million in tariff payments, whereas the Chinese government gains 28.05 million dollars in tariff payments.

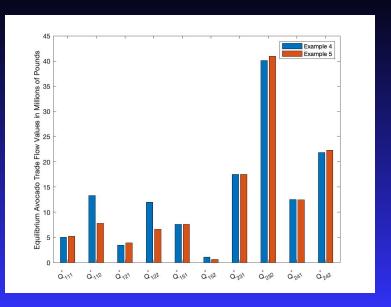
Example 5: Tariff Rate Quota Imposed by China on Imports from the US

Firm 1 now has a reduced utility (profit) of 30.60 in millions of dollars, whereas Firm 2 has a utility (profit) of 181.67 in millions of dollars.

Under the tariff quota regime imposed by China on the United States, Firm 1 experiences a drop in profits of over 50% as compared to Example 4, whereas Firm 2 enjoys a small increase in profits.

The Chinese government clearly benefits from the imposition of the tariff rate quota against the United States; however, consumers in China must pay a higher price.

Equilibrium Avocado Trade Flows in Examples 4 and 5

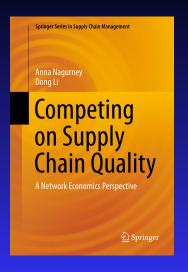


Findings

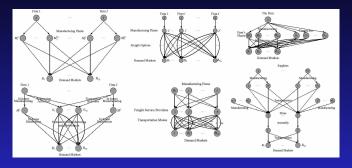
- ► To-date, there has been limited modeling work of imperfectly competitive firms in supply chain network, in the presence of trade policies such as TRQs, which have been challenging to model.
- ► The theory of variational inequalities is very useful for the formulation, analysis, and solution of oligopolistic supply chain network equilibrium problems under TRQs.
- ➤ The numerical examples that comprise the case study quantify impacts of tariff rate quotas on consumer prices, on product flows, as well as on the firms' profits.
- The results demonstrate that TRQs can be effective in reducing product flows from countries on which they are imposed but at the expense of the consumers in terms of prices.

Supply Chain Network Models with Quality Under Strict Quotas or Tariffs

Our Research on Quality



In the book, we present supply chain network models and tools to investigate information asymmetry, impacts of outsourcing on quality, minimum quality standards, applications to industries such as pharma and high tech, freight services and quality.



Also, relevant is the paper, D. Li and A. Nagurney (2017), Supply chain performance assessment and supplier and component importance identification in a general competitive multitiered supply chain network model, *Journal of Global Optimization* **67(1)**, 223-250.

Food and Product Quality



Consumers are increasingly demanding quality in the food)fresh produce) that they eat as well as many other products.

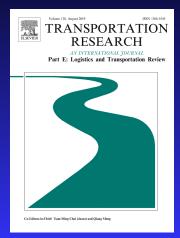
Supply Chain Network Models with Quality Under Strict Quotas or Tariffs

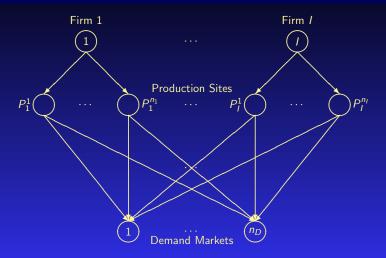
Now we turn to research on the incorporation of quality as a strategic variable and the exploration of the relationship between strict quotas and tariffs.

Specifically, we are interested in the impact of tariffs or strict quotas on consumer welfare.

This part of the presentation is based on the paper:

A. Nagurney, D. Besik, and D. Li (2019), Strict quotas or tariffs? Implications for product quality and consumer welfare in differentiated product supply chains, *Transportation Research E* **29**, pp 136-161.





Notation for the Supply Chain Network Models with Quality

Notation	Definition
Q _{ijk}	the nonnegative amount of firm i 's product produced at production site P_i^j and shipped to demand market k . The $\{Q_{ijk}\}$ elements for all j and k are grouped into the vector $Q_i \in R_+^{n_i n_D}$. We then further group the Q_i ; $i=1,\ldots,I$, into the vector $Q \in R_+^{\sum_{j=1}^{j} n_j n_D}$.
q _{ij}	the quality level, or, simply, the quality, of product i , which is produced by firm i at its site P_i^j . The quality levels of each firm i ; $i=1,\ldots,I$, the $\{q_{ij}\}$, are grouped into the vector $q_i \in \mathcal{R}_+^{n_i}$. Then the quality levels of all firms are grouped into the vector $q \in \mathcal{R}_+^{\sum_{i=1}^{l} n_i}$.
Ĝik	the average quality of firm i 's product at demand market k ; $i=1,\ldots,l$; $k=1,\ldots,n_D$, where $\hat{q}_{ik}=\frac{\sum_{j=1}^{n_i}q_{ij}Q_{jk}}{d_{ik}}$. We group the average quality levels of all firms at all the demand markets into the vector $\hat{q}\in R_+^{ln_D}$.
Q	the strict quota defined for production sites in a particular country over which the quota is imposed for the product by another country to its demand markets.
λ	the Lagrange multiplier associated with the quota constraint.
$f_{ij}(s,q)$	the production cost at firm i 's site P_i^j .
$\hat{c}_{ijk}(Q,q)$	the total transportation cost associated with distributing firm i 's product, produced at site P_i^j , to demand market k .
$\rho_{ik}(d,\hat{q})$	the demand price function for firm i 's product at demand market k .

The Conservation of Flow Equations

The production output at firm i's production site P_i^I and the demand for the product at each demand market k must satisfy:

$$s_{ij} = \sum_{k=1}^{n_D} Q_{ijk}, \quad i = 1, \dots, I; j = 1, \dots, n_i,$$
 (20)

$$d_{ik} = \sum_{i=1}^{n_i} Q_{ijk}, \quad k = 1, \dots, n_D.$$
 (21)

The Nonnegativity Constraints

In addition, the product shipments must be nonnegative, that is:

$$Q_{ijk} \ge 0, \quad i = 1, \dots, I; j = 1, \dots, n_i; k = 1, \dots, n_D.$$
 (22)

Bounds on Quality

The quality levels, in turn, must meet or exceed the nonnegative minimum quality standards (MQSs) at the production sites, but they cannot exceed their respective upper bounds of quality:

$$\bar{q}_{ij} \geq q_{ij} \geq \underline{q}_{ij}, \quad i = 1, \dots, I; j = 1, \dots, n_i.$$
 (23)

Let K^i denote the feasible set corresponding to firm i, where $K^i \equiv \{(Q_i, q_i) | (22) \text{ and } (23) \text{ hold} \}$ and define $K \equiv \prod_{i=1}^{I} K^i$.

The Utility of Firm i

The strategic variables of firm i are its product shipments Q_i and its quality levels q_i , with the profit/utility U_i of firm i; i = 1, ..., I, given by the difference between its total revenue and its total costs:

$$U_{i} = \sum_{k=1}^{n_{D}} \hat{\rho}_{ik}(Q, q) \sum_{j=1}^{n_{i}} Q_{ijk} - \sum_{j=1}^{n_{i}} \hat{f}_{ij}(Q, q) - \sum_{k=1}^{n_{D}} \sum_{j=1}^{n_{i}} \hat{c}_{ijk}(Q, q).$$
 (24)

Definition 2: A Differentiated Product Supply Chain Network Equilibrium with Quality

A product shipment and quality level pattern $(Q^*, q^*) \in K$ is said to constitute a differentiated product supply chain network equilibrium with quality if for each firm i; i = 1, ..., I,

$$U_i(Q_i^*, Q_{-i}^*, q_i^*, q_{-i}^*) \ge U_i(Q_i, Q_{-i}^*, q_i, q_{-i}^*), \quad \forall (Q_i, q_i) \in K^i,$$
 (25)

where

$$Q_{-i}^* \equiv (Q_1^*, \dots, Q_{i-1}^*, Q_{i+1}^*, \dots, Q_I^*) \quad \text{and} \quad q_{-i}^* \equiv (q_1^*, \dots, q_{i-1}^*, q_{i+1}^*, \dots, q_I^*).$$

A differentiated product supply chain equilibrium is established if no firm can unilaterally improve upon its profits by choosing an alternative vector of product shipments and quality levels of its product.

A Differentiated Product Supply Chain Network Equilibrium Model with Quality

Theorem 3: VI Formulation of the Differentiated Product Supply Chain Network Equilibrium Model with Quality

Assume that for each firm i; i = 1, ..., I, the profit function $U_i(Q, q)$ is concave with respect to the variables in Q_i and q_i , and is continuously differentiable. Then the product shipment and quality pattern $(Q^*, q^*) \in K$ is a differentiated product supply chain network equilibrium with quality according to Definition 2 if and only if it satisfies the VI:

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_{i}}\sum_{k=1}^{n_{D}}\frac{\partial U_{i}(Q^{*},q^{*})}{\partial Q_{ijk}}\times(Q_{ijk}-Q_{ijk}^{*})-\sum_{i=1}^{I}\sum_{j=1}^{n_{i}}\frac{\partial U_{i}(Q^{*},q^{*})}{\partial q_{ij}}\times(q_{ij}-q_{ij}^{*})\geq0,$$

$$\forall (Q,q) \in K. \tag{26}$$

- ▶ We consider a model with a strict quota.
- Now, a country imposes a strict quota on the product flows from another country.
- ▶ The group \mathcal{G} consists of all the relevant production site and demand market pairs (j, k).

Additional Shared Constraint

The Quota Constraint

Under a strict quota regime, the following additional constraint must be satisfied:

$$\sum_{i=1}^{I} \sum_{(j,k)\in\mathcal{G}} Q_{ijk} \le \bar{Q}. \tag{27}$$

This is a common or *shared* constraint associated with firms having production sites in the country on which the quota is imposed.

We define:

$$S \equiv \{Q|(27) \text{ holds}\}. \tag{28}$$

Additional Shared Constraint

The utility function of a firm i depends on not only its own strategies, but also on those of the other firms, and now their feasible sets do as well since the new feasible set will correspond to $K \cap S$.

Generalized Nash Equilibrium (GNE) (see Facchinei and Kanzow (2010) and Kulkarni and Shanbhag (2012) is needed to formulate the equilibrium conditions with shared constraints.

See also: Nagurney, Yu, and Besik (2017), *Journal of Global Optimization* **69(1)**, 231-254.

Definition 4: A Differentiated Product Supply Chain Generalized Nash Network Equilibrium with Quality and a Strict Quota

A product shipment and quality level pattern $(Q^*, q^*) \in K \cap S$ is a differentiated product supply chain Generalized Nash Network Equilibrium with quality if for each firm i; i = 1, ..., I,

$$U_{i}(Q_{i}^{*}, Q_{-i}^{*}, q_{i}^{*}, q_{-i}^{*}) \geq U_{i}(Q_{i}, Q_{-i}^{*}, q_{i}, q_{-i}^{*}), \quad \forall (Q_{i}, q_{i}) \in K^{i} \cap \mathcal{S}.$$
(29)

Definition 5: Variational Equilibrium

A vector $(Q^*, q^*) \in K \cap S$ is said to be a variational equilibrium of the above Generalized Nash Network Equilibrium if it is a solution of the variational inequality

$$-\sum_{i=1}^{l}\sum_{j=1}^{n_{i}}\sum_{k=1}^{n_{D}}\frac{\partial U_{i}(Q^{*},q^{*})}{\partial Q_{ijk}}\times(Q_{ijk}-Q_{ijk}^{*})$$

$$-\sum_{i=1}^{l}\sum_{j=1}^{n_{i}}\frac{\partial U_{i}(Q^{*},q^{*})}{\partial q_{ij}}\times(q_{ij}-q_{ij}^{*})\geq0,$$

$$\forall(Q,q)\in K\cap\mathcal{S}.$$
(30)

In a GNE defined by a variational equilibrium, the Lagrange multipliers associated with the shared/coupling constraints are all the same.

Corollary 2: Alternative VI Formulation of the Differentiated Product Supply Chain Network Equilibrium Model with Quality and a Strict Quota

An equivalent variational inequality to (30) is: determine $(Q^*, q^*, \lambda^*) \in \mathcal{K}$ such that

$$-\sum_{i=1}^{I} \sum_{(j,k) \notin \mathcal{G}} \frac{\partial U_{i}(Q^{*}, q^{*})}{\partial Q_{ijk}} \times (Q_{ijk} - Q_{ijk}^{*}) + \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} (-\frac{\partial U_{i}(Q^{*}, q^{*})}{\partial Q_{ijk}} + \lambda^{*}) \times (Q_{ijk} - Q_{ijk}^{*})$$

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_{i}} \frac{\partial U_{i}(Q^{*}, q^{*})}{\partial q_{ij}} \times (q_{ij} - q_{ij}^{*})$$

$$+(\bar{Q} - \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} Q_{ijk}^{*}) \times (\lambda - \lambda^{*}) \geq 0, \quad \forall (Q, q, \lambda) \in \mathcal{K},$$

$$(31)$$

where K which consists of $(Q, q) \in K$ and $\lambda \in R^1_+$.

The Lagrange multiplier λ is associated with the strict quota constraint. The proof utilizes the results in Nagurney (2018) and Gossler et al. (2019).

The Differentiated Product Supply Chain Network Equilibrium Model with a Tariff

- ightharpoonup A tariff au^* on the production sites in the country subject to the trade policy instrument is imposed.
- ightharpoonup A group $\mathcal G$ as in the strict quota model is considered.
- lacktriangle The utility functions \hat{U}_i ; $i=1,\ldots,I$, now take the form

$$\hat{U}_i = U_i - \sum_{i=1}^l \sum_{(i,k) \in G} \tau^* Q_{ijk},$$

with U_i ; i = 1, ..., I, as in (24).

The Differentiated Product Supply Chain Network Equilibrium Model with a Tariff

Theorem 4: VI Formulation of the Differentiated Product Model Supply Chain Network Equilibrium Model with Quality and a Tariff

Under the same assumptions as in Theorem 3, a product shipment and quality pattern $(Q^*, q^*) \in K$ is a differentiated product supply chain network equilibrium according to Definition 4 with \hat{U}_i replacing U_i for $i=1,\ldots,I$, if and only if it satisfies the VI:

$$-\sum_{i=1}^{I} \sum_{(j,k) \notin \mathcal{G}} \frac{\partial U_{i}(Q^{*}, q^{*})}{\partial Q_{ijk}} \times (Q_{ijk} - Q_{ijk}^{*}) + \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} (-\frac{\partial U_{i}(Q^{*}, q^{*})}{\partial Q_{ijk}} + \tau^{*}) \times (Q_{ijk} - Q_{ijk}^{*})$$

$$-\sum_{i=1}^{I} \sum_{i=1}^{n_{i}} \frac{\partial U_{i}(Q^{*}, q^{*})}{\partial q_{ij}} \times (q_{ij} - q_{ij}^{*}) \ge 0, \quad \forall (Q, q) \in K.$$
(32)

Equivalence Between the Model with a Strict Quota and the Model with a Tariff

We have established theoretically (and also illustrate numerically in our paper) that if the strict quota is tight and the tariff for the same group is set to the associated Lagrange multiplier then the equilibrium product flows and quality levels of both the strict quota and the tariff models coincide.

The provides government decision makers and policy makers the the flexibility of imposing either trade policy instrument, under certain conditions.

Of course, under a tariff regime the government gets added income.

What About the Impact on Consumers?

Consumer Welfare with or without Tariffs or Quotas

The consumer welfare associated with product i at demand market k at equilibrium with or without a strict quota, CW_{ik} , is

$$CW_{ik} = \int_0^{d_{ik}^*} \rho_{ik}(d_{-ik}^*, d_{ik}, \hat{q}^*) d(d_{ik}) - \rho_{ik}(d^*, \hat{q}^*) d_{ik}^*, \, \forall i, k, \quad (33)$$

where $d_{-ij}^* \equiv (d_{11}^*, \dots, d_{i,j-1}^*, d_{i,j+1}^*, \dots, d_{mn}^*)$ (cf. Spence (1975) and Wildman (1984)).

The first term in (33) calculates the maximum total price that consumers at demand market k are willing to pay for the amount of product i that satisfies their demand at equilibrium. The second term expresses the actual total price paid by consumers for their demand. The difference then measures the benefit that consumers obtain when purchasing the product, which is the consumer welfare associated with product i at demand market k at equilibrium.

A Case Study on Soybeans

A Case Study on Soybeans

- Soybeans were discovered and domesticated in China over 3000 years ago.
- ▶ In the United States, soybean production and export have become essential parts of the agricultural economy, with soybeans ranked second among crops in farm value in 2005.
- ▶ In 2018, soybean production in the United States reached 5.11 billion bushels with an export of 2.13 billion bushels.
- China is the largest importer of soybeans due to its rapidly increasing population size.

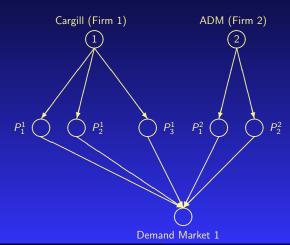


A Case Study on Soybeans

- ➤ The consumption of soybeans in China, in 2017, was reported to be 112.18 million tons, but the domestic production volume was only 13 million tons.
- ▶ In 2018, the trade war between China and the United States escalated, with the Chinese government imposing quotas and tariffs on the soybeans exported from the United States in retaliation.
- ► This created an opportunity for other large soybean exporters, such as Brazil and Argentina.
- ▶ In 2017, Brazil exported 53.8 million tons of soybeans to China, corresponding to 75% of its production volume.



- ► Cargill has 3 production sites: The United States, Brazil, and Argentina.
- Archer Daniels Midland Company (ADM) has 2 production sites: The United States, and Brazil. The demand market is in China.



- ► Example 6: No Trade Interventions
- Example 7: With a Strict Quota on the US by China (Example 6 with a Strict Quota of $\bar{Q}=1200$)
- ightharpoonup Example 8: With a Tariff on the US by China (Example 6 with a Tariff of $au^*=10$)

Detailed functional data can be found in our paper.

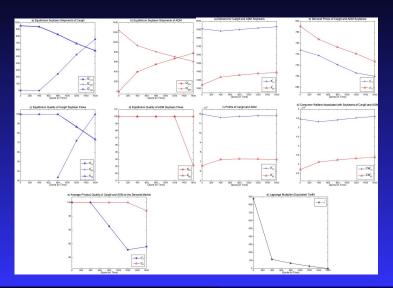
Location	Flows	Example 6	Example 7 (strict quota)	Example 8 (tariff)
US	Q_{111}^*	756.70	528.96	685.38
Brazil	Q_{121}^{*}	591.26	697.99	624.46
Argentina	Q_{131}^{*}	585.90	692.63	619.09
US	Q_{211}^{*}	779.32	671.04	735.79
Brazil	Q_{221}^{*}	612.31	708.75	653.62
US	q_{11}^*	100.00	72.13	93.46
Brazil	q_{12}^{*}	73.90	87.25	78.06
Argentina	q_{13}^{*}	73.24	86.58	77.39
US	q_{21}^{*}	100.00	100.00	100.00
Brazil	q ₂₂ *	93.18	100.00	99.46

Firm	Notation	Example 6	Example 7	Example 8	
Cargill	d_{11}^{*}	1,933.86	1,919.58	1,928.93	
ADM	d_{21}^*	1,391.64	1,379.79	1,389.42	
Cargill	\hat{q}_{11}	83.91	82.84	83.32	
ADM	\hat{q}_{21}	97.00	100.00	99.75	
Cargill	ρ_{11}	700.25	706.16	701.76	
ADM	$ ho_{21}$	726.76	741.20	734.52	
Cargill	<i>CW</i> ₁₁	560,966.35	552,718.33	558,116.66	
ADM	CW_{21}	338,916.81	333,167.37	337,834.91	
Cargill	U_1	1,180,812.05	1,181,876.72	1,174,437.42	
ADM	U_2	724,196.08	728,637.06	718,677.19	

► The equilibrium Lagrange multiplier in Example 7, which is the equivalent tariff, is:

$$\lambda^* = 29.90.$$

Sensitivity Analysis for Example 7: Equilibrium Values as Strict Quota Decreases



Managerial Insights

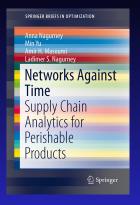
- ► From the consumer's perspective, the results consistently and unanimously show that consumer welfare declines for consumers in the country imposing a strict quota or tariff on an imported product. Hence, a government may wish to loosen a quota (equivalently, reduce a tariff) so as not to adversely affect its own consumers.
- ➤ Producing firms, as also critical stakeholders in competitive supply chain networks, should expand their demand markets within their own countries. This allows for a basic, but, effective, redesign of the supply chain network under a tariff or quota and results in higher profits for the firms.

Managerial Insights

- ➤ Also, firms should expand the number of production sites to countries not under a tariff or quota to maintain or improve upon their profits if some of their production sites are in countries subject to such trade policy instruments.
- ➤ Governments have the flexibility of imposing either a tariff or a quota to obtain equivalent trade flows and product quality levels. The imposition of a tariff may be more advisable/favored by a government, since it requires less "policing" and also yields financial rewards.

Some Final Thoughts

Research on Product Quality and Multidisciplinarity



Our recent research is using explicit formulae to model product quality in the case of fresh produce using chemistry, physics, etc.; see: D. Besik and A. Nagurney (2017), Quality in competitive fresh produce supply chains with application to farmers' markets, *Socio-Economic Planning Sciences* **60**, 62-76.

Muchas gracias!



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