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INTERNATIONAL CONFERENCE on OPTIMIZATION and DECISION SCIENCE

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AIRO
ASSOCIAZIONE ITALIANA DI RICERCA OPERATIVA
OPTIMIZATION AND DECISION SCIENCE

Efficient Last-Mile Delivery using UAVs: A Variational Inequality Approach for Optimizing Supply Chain Management

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University of Catania - Italy

G. Colajanni, P. Daniele, A. Nagurney, *Centralized Supply Chain Network Optimization with UAV-based Last Mile Deliveries*, Transportation Research Part C, to appear.


E-COMMERCE

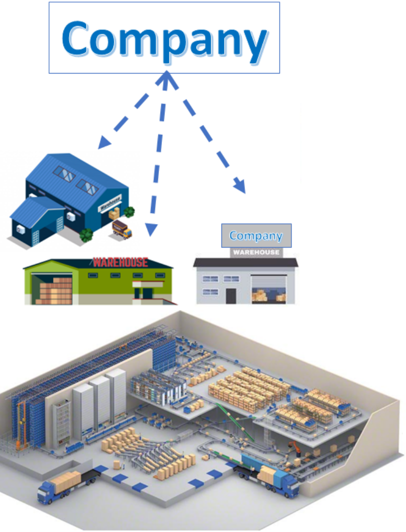
Companies

Companies

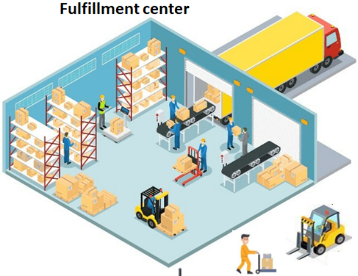
Customers

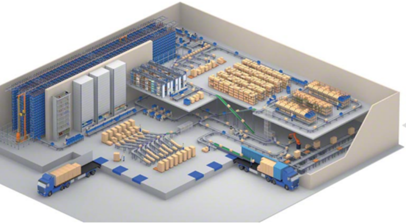

Support
24h
BUY NOW
ORDER
SALE
%

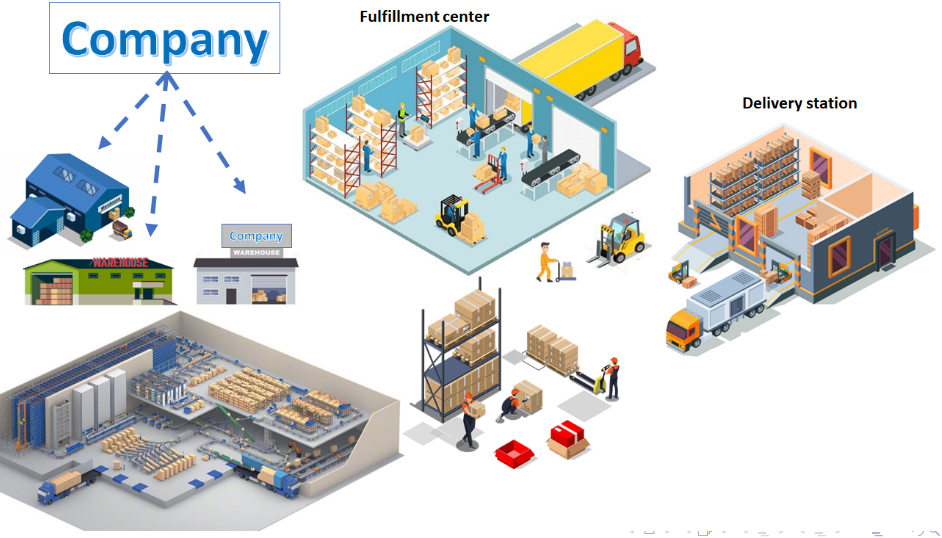

Company
ORDER
Support
24h
SALE
BUY NOW

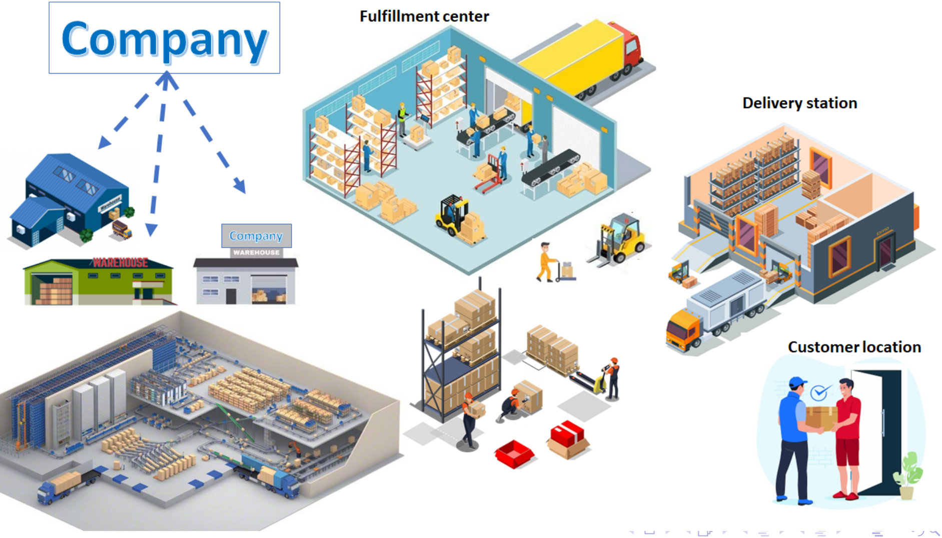

Company
WAREHOUSE
Company
WAREHOUSE


Company
Company
WAREHOUSE
WAREHOUSE


Fulfillment center



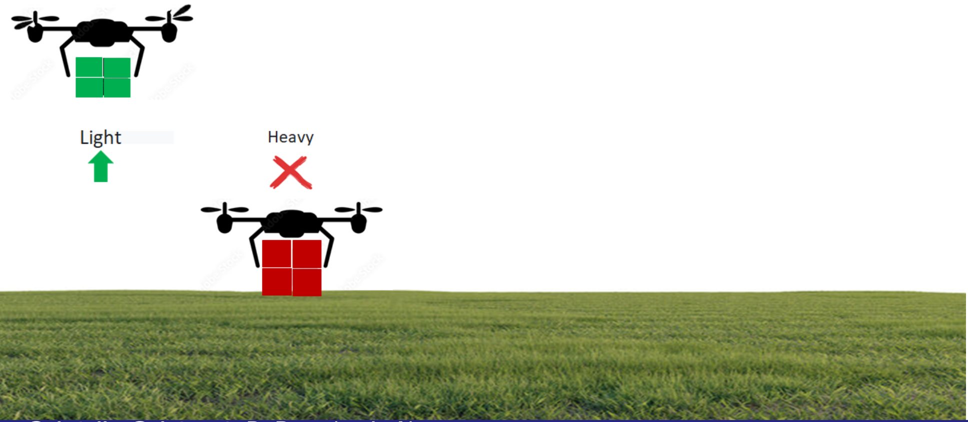

Company
Fulfillment center
Delivery station
Company
WAREHOUSE
WAREHOUSE


Company
Fulfillment center
Delivery station
WAREHOUSE
Company
WAREHOUSE
Customer location


Warehouse
Fulfillment Center
Delivery Station
Last mile
Customer Location

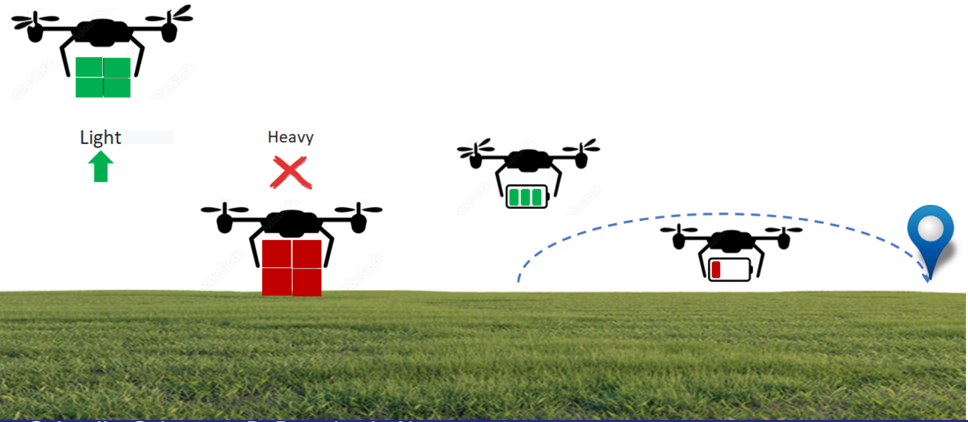
Limitations:

Weight

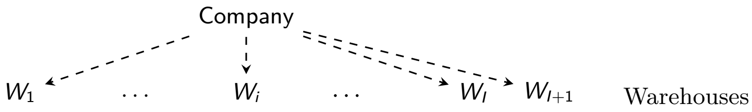


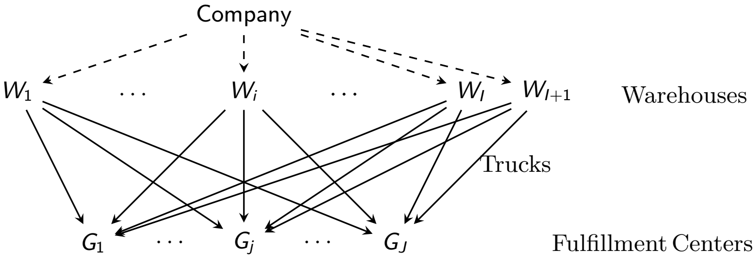
Limitations:

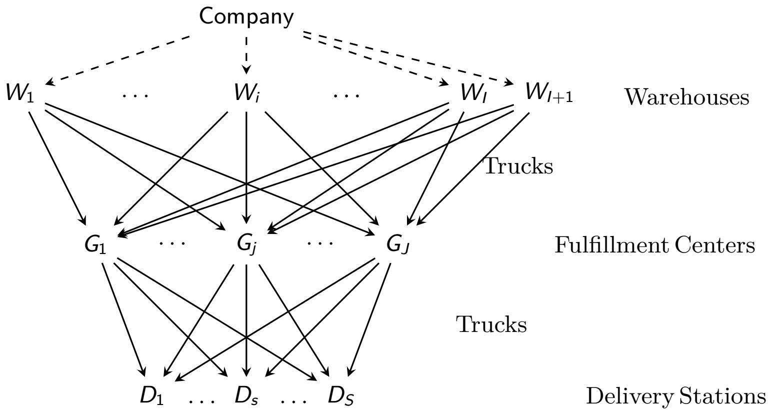
Weight **Battery $\Rightarrow$ Distance**

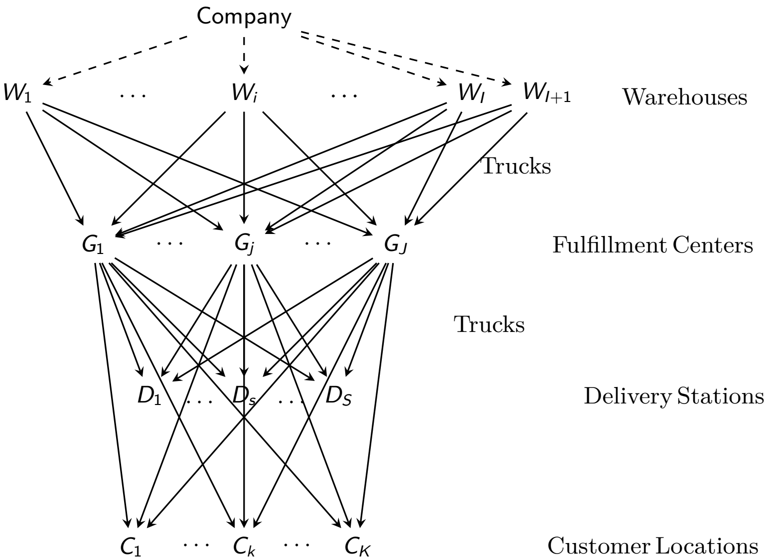


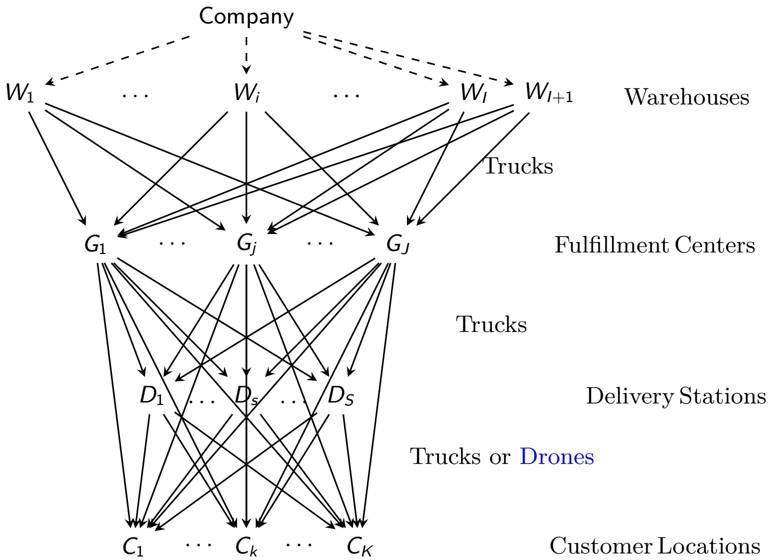
Company

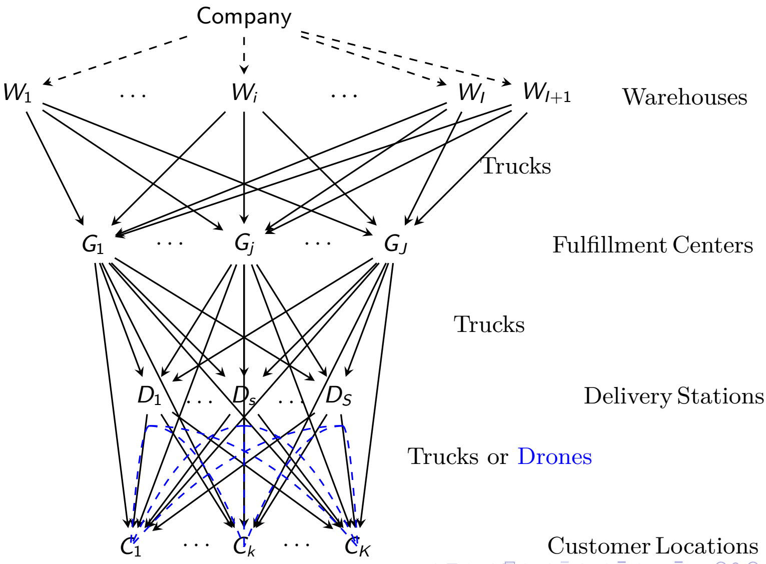

Company
W_1
. . .
W_i
. . .
W_I
W_{I+1}
Warehouses

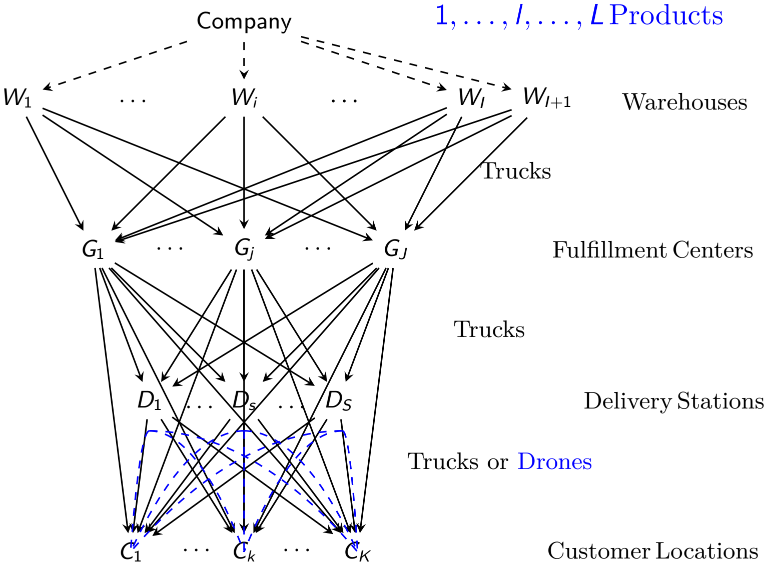

Company
W_1
$\cdots$
W_i
$\cdots$
W_I
W_{I+1}
Warehouses
Trucks
G_1
$\cdots$
G_j
$\cdots$
G_J
Fulfillment Centers


Company
W_1 . . . W_i . . . W_I W_{I+1}
Warehouses
Trucks
G_1 . . . G_j . . . G_J
Fulfillment Centers
Trucks
D_1 . . . D_s . . . D_S
Delivery Stations

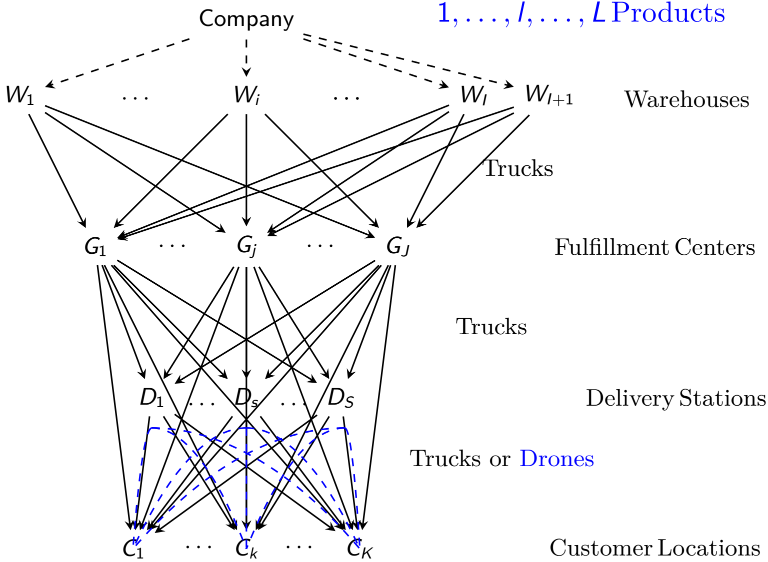

Company
W_1 . . . W_i . . . W_I W_{I+1}
Warehouses
Trucks
G_1 . . . G_j . . . G_J
Fulfillment Centers
Trucks
D_1 . . . D_s . . . D_S
Delivery Stations
C_1 . . . C_k . . . C_K
Customer Locations

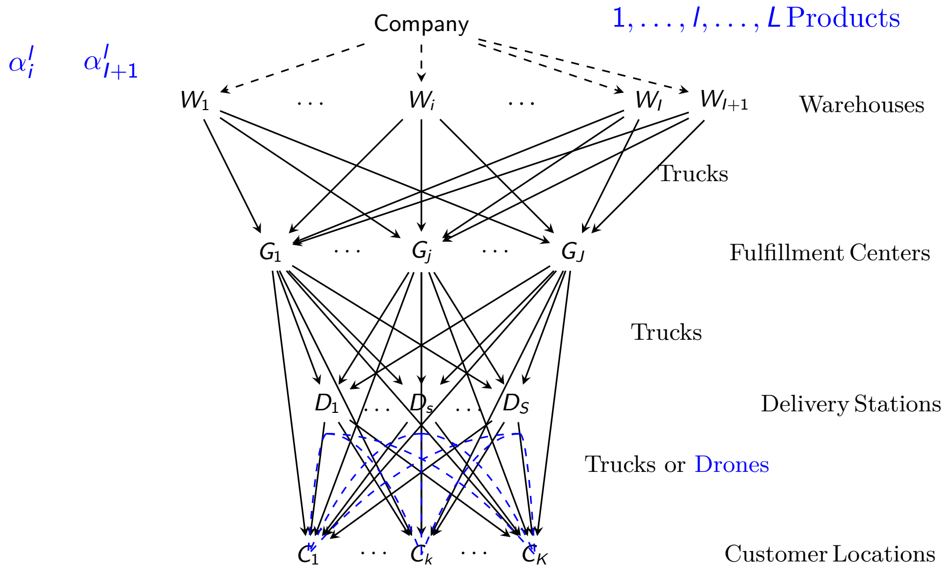

Company
W_1 ... W_i ... W_I W_{I+1}
Warehouses
Trucks
G_1 ... G_j ... G_J
Fulfillment Centers
Trucks
D_1 ... D_s ... D_S
Delivery Stations
Trucks or Drones
C_1 ... C_k ... C_K
Customer Locations


Company
W_1 . . . W_i . . . W_I W_{I+1}
Warehouses
Trucks
G_1 . . . G_j . . . G_J
Fulfillment Centers
Trucks
D_1 . . . D_s . . . D_S
Delivery Stations
Trucks or Drones
C_1 . . . C_k . . . C_K
Customer Locations


Company
$1, \dots, l, \dots, L$ Products
W_1 ... W_i ... W_I W_{I+1}
Warehouses
Trucks
G_1 ... G_j ... G_J
Fulfillment Centers
Trucks
D_1 ... D_s ... D_S
Delivery Stations
Trucks or Drones
C_1 ... C_k ... C_K
Customer Locations

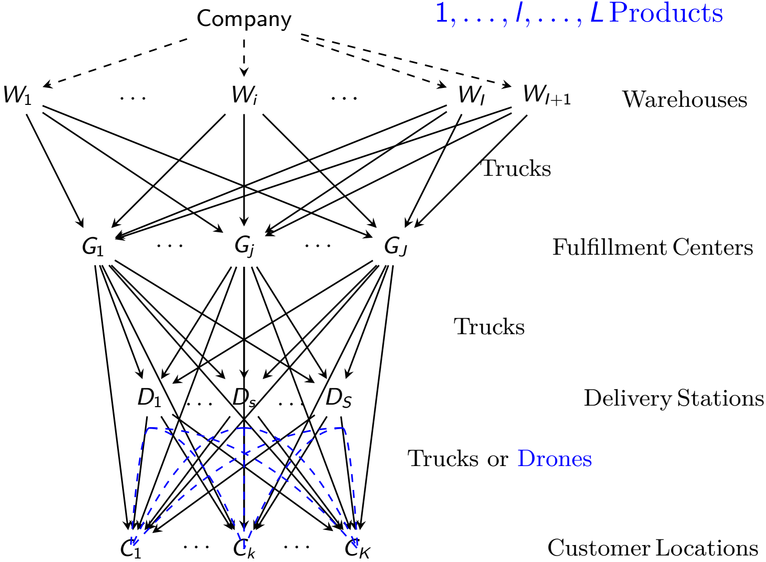
α_i^l

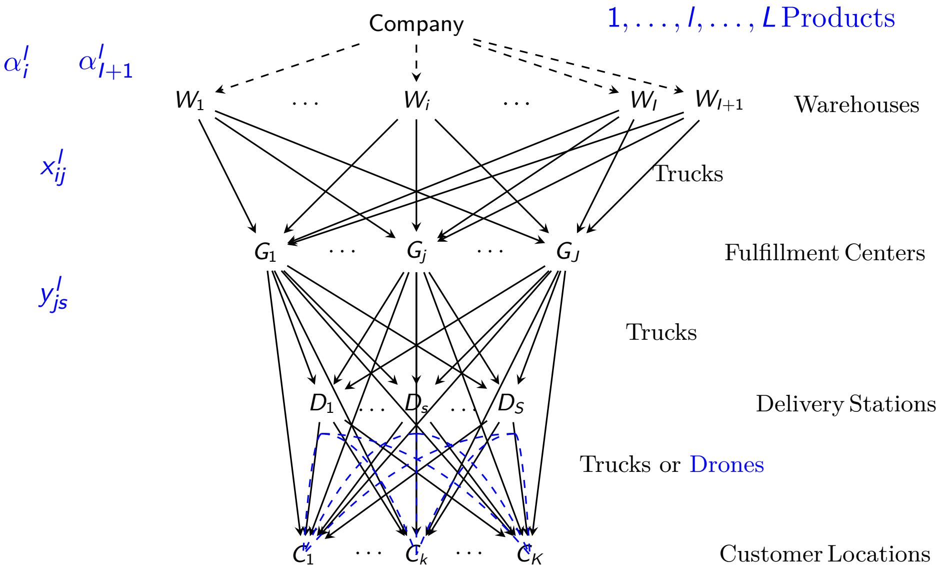

Company
$1, \ldots, l, \ldots, L$ Products
W_1 $\cdots$ W_i $\cdots$ W_I W_{I+1}
Warehouses
Trucks
G_1 $\cdots$ G_j $\cdots$ G_J
Fulfillment Centers
Trucks
D_1 $\cdots$ D_s $\cdots$ D_S
Delivery Stations
Trucks or Drones
C_1 $\cdots$ C_k $\cdots$ C_K
Customer Locations


$1, \ldots, l, \ldots, L$ Products
α_i^l α_{I+1}^l
Company
W_1 $\cdots$ W_i $\cdots$ W_I W_{I+1}
Warehouses
Trucks
G_1 $\cdots$ G_j $\cdots$ G_J
Fulfillment Centers
Trucks
D_1 $\cdots$ D_s $\cdots$ D_S
Delivery Stations
Trucks or Drones
C_1 $\cdots$ C_k $\cdots$ C_K
Customer Locations

α_i^l α_{I+1}^l

x_{ij}^l



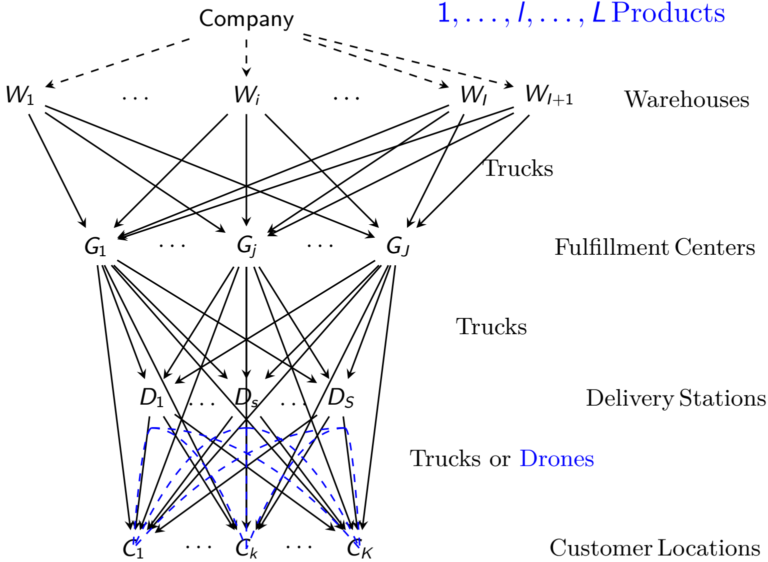

$1, \ldots, l, \ldots, L$ Products
α_i^l
α_{I+1}^l
x_{ij}^l
y_{js}^l
Company
W_1 $\cdots$ W_i $\cdots$ W_I W_{I+1}
Warehouses
Trucks
G_1 $\cdots$ G_j $\cdots$ G_J
Fulfillment Centers
Trucks
D_1 $\cdots$ D_s $\cdots$ D_S
Delivery Stations
Trucks or Drones
C_1 $\cdots$ C_k $\cdots$ C_K
Customer Locations

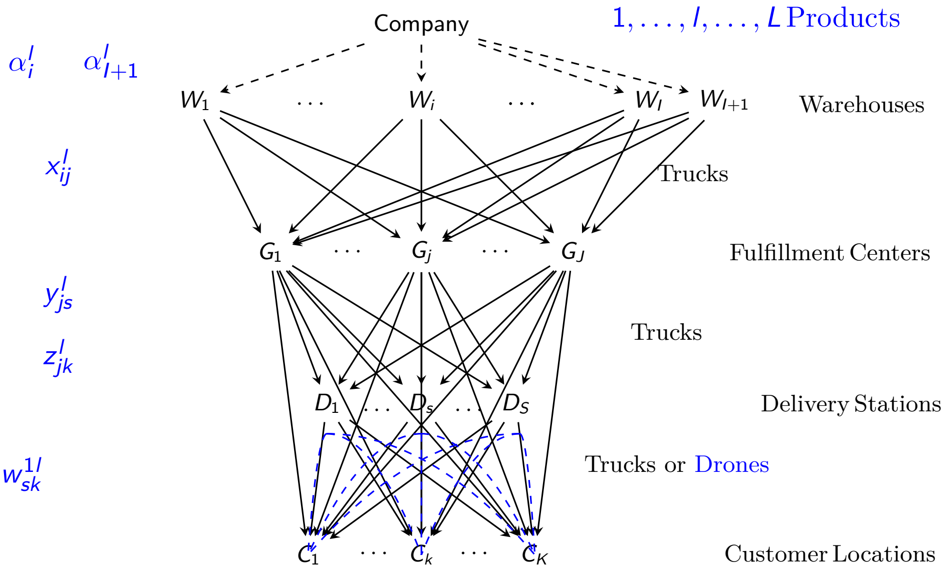
α_i^l α_{I+1}^l

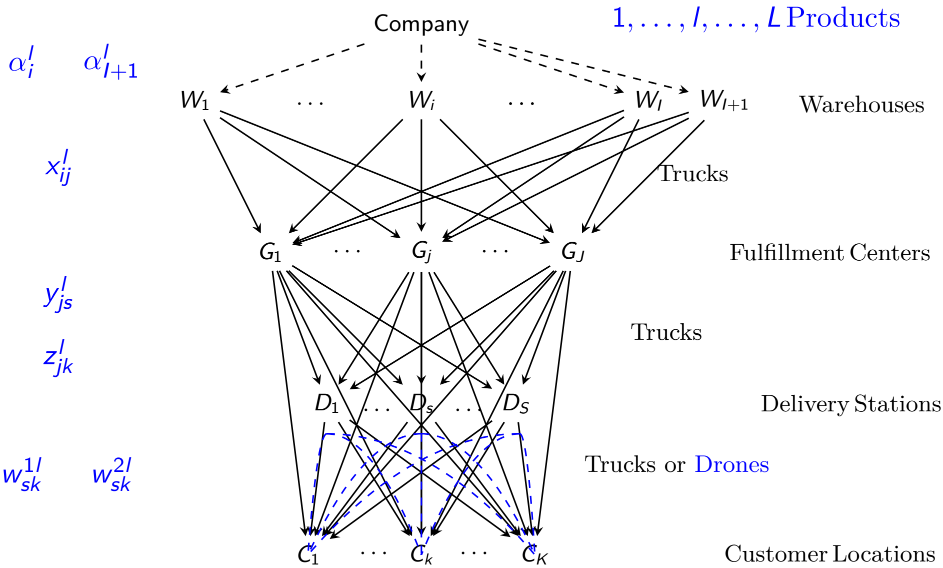
x_{ij}^l

y_{js}^l

z_{jk}^l




Company
$1, \ldots, l, \ldots, L$ Products
α_i^l α_{I+1}^l
W_1 $\cdots$ W_i $\cdots$ W_I W_{I+1}
Warehouses
x_{ij}^l
Trucks
G_1 $\cdots$ G_j $\cdots$ G_J
Fulfillment Centers
y_{js}^l
z_{jk}^l
Trucks
D_1 $\cdots$ D_s $\cdots$ D_S
Delivery Stations
w_{sk}^{1l}
Trucks or Drones
C_1 $\cdots$ C_k $\cdots$ C_K
Customer Locations


$1, \dots, l, \dots, L$ Products
α_i^l α_{I+1}^l
x_{ij}^l
y_{js}^l
z_{jk}^l
w_{sk}^{1l} w_{sk}^{2l}
Company
W_1 ... W_i ... W_I W_{I+1}
Warehouses
Trucks
G_1 ... G_j ... G_J
Fulfillment Centers
Trucks
D_1 ... D_s ... D_S
Delivery Stations
Trucks or Drones
C_1 ... C_k ... C_K
Customer Locations

$$\max_{\alpha, x, y, z, w^1, w^2} \left\{ \right. \tag{1}$$

$$\max_{\alpha,x,y,z,w^1,w^2} \Bigg\{ - c(\alpha, x, y, z, w^1, w^2) \qquad (1)$$

$$\max_{\alpha,x,y,z,w^1,w^2} \left\{ -c(\alpha,x,y,z,w^1,w^2) - \sum_{l=1}^{L}\sum_{i=1}^{l+1} \gamma_i^l \alpha_i^l \right. \tag{1}$$

$$
\max_{\alpha,x,y,z,w^1,w^2} \Bigg\{ -c(\alpha,x,y,z,w^1,w^2) - \sum_{l=1}^{L}\sum_{i=1}^{I+1}\gamma_i^l\alpha_i^l - \sum_{i=1}^{I+1}\sum_{j=1}^{J} c_{ij}(x_{ij})
- \sum_{j=1}^{J}\sum_{s=1}^{S}\hat{c}_{js}(y_{js},z_j) - \sum_{j=1}^{J}\sum_{k=1}^{K}\bar{c}_{jk}(y_j,z_{jk}) - \sum_{s=1}^{S}\sum_{k=1}^{K}\tilde{c}^1_{sk}(w^1_{sk})
- \sum_{s=1}^{S}\sum_{k=1}^{K}\tilde{c}^2_{sk}(w^2_{sk}) \qquad (1)
$$

$$\max_{\alpha,x,y,z,w^1,w^2} \Bigg\{ -c(\alpha,x,y,z,w^1,w^2) - \sum_{l=1}^{L}\sum_{i=1}^{I+1}\gamma_i^l\alpha_i^l - \sum_{i=1}^{I+1}\sum_{j=1}^{J} c_{ij}(x_{ij})$$
$$-\sum_{j=1}^{J}\sum_{s=1}^{S}\hat{c}_{js}(y_{js},z_j) - \sum_{j=1}^{J}\sum_{k=1}^{K}\bar{c}_{jk}(y_j,z_{jk}) - \sum_{s=1}^{S}\sum_{k=1}^{K}\tilde{c}_{sk}^1(w_{sk}^1)$$
$$-\sum_{s=1}^{S}\sum_{k=1}^{K}\tilde{c}_{sk}^2(w_{sk}^2) + \sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K} l_{sk}w_{sk}^{2l} \qquad (1)$$

$$\max_{\alpha,x,y,z,w^1,w^2} \left\{ -c(\alpha,x,y,z,w^1,w^2) - \sum_{l=1}^{L}\sum_{i=1}^{I+1} \gamma_i^l \alpha_i^l - \sum_{i=1}^{I+1}\sum_{j=1}^{J} c_{ij}(x_{ij}) \right.$$

$$- \sum_{j=1}^{J}\sum_{s=1}^{S} \hat{c}_{js}(y_{js}, z_j) - \sum_{j=1}^{J}\sum_{k=1}^{K} \bar{c}_{jk}(y_j, z_{jk}) - \sum_{s=1}^{S}\sum_{k=1}^{K} \tilde{c}^1_{sk}(w^1_{sk})$$

$$- \sum_{s=1}^{S}\sum_{k=1}^{K} \tilde{c}^2_{sk}(w^2_{sk}) + \sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K} l_{sk} w^{2l}_{sk}$$

$$- \sum_{i=1}^{I+1} c_i^{tr}(\alpha, w^2) - \sum_{s=1}^{S} c_s^{dr}(w^2) \qquad (1)$$

$$\max_{\alpha,x,y,z,w^1,w^2} \Bigg\{ -c(\alpha,x,y,z,w^1,w^2) - \sum_{l=1}^{L}\sum_{i=1}^{l+1}\gamma_i^l\alpha_i^l - \sum_{i=1}^{l+1}\sum_{j=1}^{J} c_{ij}(x_{ij})$$

$$-\sum_{j=1}^{J}\sum_{s=1}^{S}\hat{c}_{js}(y_{js},z_j) - \sum_{j=1}^{J}\sum_{k=1}^{K}\bar{c}_{jk}(y_j,z_{jk}) - \sum_{s=1}^{S}\sum_{k=1}^{K}\tilde{c}_{sk}^1(w_{sk}^1)$$

$$-\sum_{s=1}^{S}\sum_{k=1}^{K}\tilde{c}_{sk}^2(w_{sk}^2) + \sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K} I_{sk}w_{sk}^{2l}$$

$$-\sum_{i=1}^{l+1} c_i^{tr}(\alpha,w^2) - \sum_{s=1}^{S} c_s^{dr}(w^2)$$

$$+\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{k=1}^{K}\gamma_{jk}^l z_{jk}^l + \sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K}\gamma_{sk}^l w_{sk}^{1l} + \sum_{l=1}^{L}\sum_{s=1}^{S}\sum_{k=1}^{K}\gamma_{sk}^l w_{sk}^{2l}\Bigg\} \quad (1)$$

$$\alpha_i^l \leq Q_i^l, \quad \forall l=1,\ldots,L,\ \forall i=1,\ldots,I+1 \tag{2}$$

$$\alpha_i^l \leq Q_i^l, \quad \forall l=1,\ldots,L,\ \forall i=1,\ldots,I+1 \tag{2}$$

$$\sum_{j=1}^{J} x_{ij}^l \leq \alpha_i^l, \quad \forall l=1,\ldots,L,\ \forall i=1,\ldots,I+1 \tag{3}$$

$$\alpha_i^l \leq Q_i^l, \quad \forall l=1,...,L, \forall i=1,...,I+1 \tag{2}$$

$$\sum_{j=1}^{J} x_{ij}^l \leq \alpha_i^l, \quad \forall l=1,...,L, \forall i=1,...,I+1 \tag{3}$$

$$\sum_{i=1}^{I+1} x_{ij}^l \geq \sum_{s=1}^{S} y_{js}^l + \sum_{k=1}^{K} z_{jk}^l, \quad \forall j=1,...,J, \forall l=1,...,l \tag{4}$$

$$\alpha_i^l \leq Q_i^l, \quad \forall l{=}1,\ldots,L,\ \forall i{=}1,\ldots,I{+}1 \tag{2}$$

$$\sum_{j=1}^{J} x_{ij}^l \leq \alpha_i^l, \quad \forall l{=}1,\ldots,L,\ \forall i{=}1,\ldots,I{+}1 \tag{3}$$

$$\sum_{i=1}^{I+1} x_{ij}^l \geq \sum_{s=1}^{S} y_{js}^l + \sum_{k=1}^{K} z_{jk}^l, \quad \forall j{=}1,\ldots,J,\ \forall l{=}1,\ldots,l \tag{4}$$

$$\sum_{j=1}^{J} y_{js}^l \geq \sum_{k=1}^{K} w_{sk}^{1l} + \sum_{k=1}^{K} w_{sk}^{2l}, \quad \forall s{=}1,\ldots,S,\ \forall l{=}1,\ldots,L \tag{5}$$

$$\alpha_i^l \leq Q_i^l, \quad \forall l{=}1,\dots,L,\ \forall i{=}1,\dots,I{+}1 \tag{2}$$

$$\sum_{j=1}^{J} x_{ij}^l \leq \alpha_i^l, \quad \forall l{=}1,\dots,L,\ \forall i{=}1,\dots,I{+}1 \tag{3}$$

$$\sum_{i=1}^{I+1} x_{ij}^l \geq \sum_{s=1}^{S} y_{js}^l + \sum_{k=1}^{K} z_{jk}^l, \quad \forall j{=}1,\dots,J,\ \forall l{=}1,\dots,l \tag{4}$$

$$\sum_{j=1}^{J} y_{js}^l \geq \sum_{k=1}^{K} w_{sk}^{1l} + \sum_{k=1}^{K} w_{sk}^{2l}, \quad \forall s{=}1,\dots,S,\ \forall l{=}1,\dots,L \tag{5}$$

$$\sum_{j=1}^{J} z_{jk}^l + \sum_{s=1}^{S} w_{sk}^{1l} + \sum_{s=1}^{S} w_{sk}^{2l} = r_k^l, \quad \forall k{=}1,\dots,K,\ \forall l{=}1,\dots,L \tag{6}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J} x_{ij}^{l} \leq W \cdot n_i^{tr}, \quad \forall i{=}1,\ldots,I{+}1 \tag{7}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J} x_{ij}^{l} \leq W \cdot n_i^{tr}, \quad \forall i=1,\dots,I+1 \tag{7}$$

$$\sum_{l=1}^{L}\sum_{k=1}^{K} w_{sk}^{2l} \leq p_d n_s^{dr}, \quad \forall s=1,\dots,S \tag{8}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J} x_{ij}^{l} \leq W \cdot n_i^{tr}, \quad \forall i=1,\ldots,I+1 \tag{7}$$

$$\sum_{l=1}^{L}\sum_{k=1}^{K} w_{sk}^{2l} \leq p_d n_s^{dr}, \quad \forall s=1,\ldots,S \tag{8}$$

$$w_{sk}^{2l} \leq P_l^{dr}, \quad \forall l=1,\ldots,L, \forall s=1,\ldots,S, \forall k=1,\ldots,K \tag{9}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J} x_{ij}^{l} \leq W \cdot n_i^{tr}, \quad \forall i{=}1,\dots,I{+}1 \tag{7}$$

$$\sum_{l=1}^{L}\sum_{k=1}^{K} w_{sk}^{2l} \leq p_d n_s^{dr}, \quad \forall s{=}1,\dots,S \tag{8}$$

$$w_{sk}^{2l} \leq P_l^{dr}, \quad \forall l{=}1,\dots,L,\ \forall s{=}1,\dots,S,\ \forall k{=}1,\dots,K \tag{9}$$

$$w_{sk}^{2l} \leq B_{sk}^{dr}, \quad \forall s{=}1,\dots,L,\ \forall k{=}1,\dots,K,\ \forall l{=}1,\dots,L \tag{10}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J} x_{ij}^{l} \leq W \cdot n_i^{tr}, \quad \forall i{=}1,\dots,I{+}1 \tag{7}$$

$$\sum_{l=1}^{L}\sum_{k=1}^{K} w_{sk}^{2l} \leq p_d n_s^{dr}, \quad \forall s{=}1,\dots,S \tag{8}$$

$$w_{sk}^{2l} \leq P_l^{dr}, \quad \forall l{=}1,\dots,L,\ \forall s{=}1,\dots,S,\ \forall k{=}1,\dots,K \tag{9}$$

$$w_{sk}^{2l} \leq B_{sk}^{dr}, \quad \forall s{=}1,\dots,L,\ \forall k{=}1,\dots,K,\ \forall l{=}1,\dots,L \tag{10}$$

$$\alpha_i^l,\ x_{ij}^l,\ y_{js}^l,\ z_{jk}^l,\ w_{sk}^{1l},\ w_{sk}^{2l} \geq 0$$

$$\forall l{=}1,\dots,L, \forall i{=}1,\dots,I{+}1, \forall j{=}1,\dots,J, \forall s{=}1,\dots,S, \forall k{=}1,\dots,K. \tag{11}$$

Theorem

A vector $(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) \in \mathbb{K}$ *is an optimal solution to the problem* (1)-(11) *if and only if such a vector is a solution to the variational inequality:*
Find $(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) \in \mathbb{K}$ *such that:*

$$\sum_{i=1}^{I+1}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{\partial \alpha_i^l} + \gamma_i^l + \frac{\partial c_i^{tr}(\alpha^*, w^{2*})}{\partial \alpha_i^l}\right] \times (\alpha_i^l - \alpha_i^{l*})$$

$$+ \sum_{i=1}^{I+1}\sum_{j=1}^{J}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{\partial x_{ij}^l} + \frac{\partial c_{ij}(x_{ij}^*)}{\partial x_{ij}^l}\right] \times (x_{ij}^l - x_{ij}^{l*})$$

$$+ \sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{\partial y_{js}^l} + \frac{\partial \hat{c}_{js}(y_{js}^*, z_j^*)}{\partial y_{js}^l} + \frac{\partial \bar{c}_{jk}(y_j^*, z_{jk}^*)}{\partial y_{js}^l}\right] \times (y_{js}^l - y_{js}^{l*})$$

$$+ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})}{\partial z_{jk}^l} + \frac{\partial \hat{c}_{js}(y_{js}^*, z_j^*)}{\partial z_{jk}^l} + \frac{\partial \bar{c}_{jk}(y_j^*, z_{jk}^*)}{\partial z_{jk}^l} - \gamma_{jk}^l\right] \times (z_{jk}^l - z_{jk}^{l*})$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*,x^*,y^*,z^*,w^{1*},w^{2*})}{\partial w^{1l}_{sk}}+\frac{\partial \tilde{c}^{1}_{sk}(w^{1l*}_{sk})}{\partial w^{1l}_{sk}}-\gamma^{l}_{sk}\right]\times(w^{1l}_{sk}-w^{1l*}_{sk})$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\frac{\partial c(\alpha^*,x^*,y^*,z^*,w^{1*},w^{2*})}{\partial w^{2l}_{sk}}+\frac{\partial \tilde{c}^{2}_{sk}(w^{2l*}_{sk})}{\partial w^{2l}_{sk}}-l_{sk}\right.$$

$$\left.+\frac{\partial c^{tr}_{i}(\alpha^*,w^{2*})}{\partial w^{2l}_{sk}}+\frac{\partial \tilde{c}^{dr}_{s}(w^{2*})}{\partial w^{2l}_{sk}}-\gamma^{l}_{sk}\right]\times(w^{2l}_{sk}-w^{2l*}_{sk})\geq 0$$

$$\forall(\alpha,x,y,z,w^1,w^2)\in\mathbb{K}, \tag{12}$$

where

$$\mathbb{K}:=\left\{(\alpha,x,y,z,w^1,w^2)\in\mathbb{R}_+^{L[(I+1)+J(I+1)+JS+JK+2SK]}:\ (2)\text{-}(10)\ \text{hold}\right\}. \tag{13}$$

Theorem

Variational inequality problem (12) *admits at least one solution.*

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$VI(F,\mathcal{K})$: Find $X^* \in \mathcal{K}$ such that: $\quad \langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}.$

Theorem (Existence)

If $\mathcal{K}$ is a compact and convex set and F is a continuous function on $\mathcal{K}$, then variational inequality problem $VI(F,\mathcal{K})$ admits at least a solution X^*.

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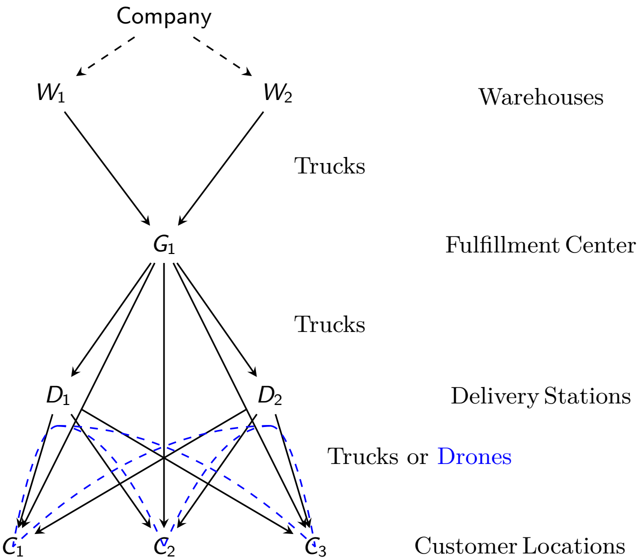
Theorem (Uniqueness)

Under the assumptions of the Existence Theorem, if the function $F(X)$ in VI(F,K) is strictly monotone on $\mathcal{K}$, that is:

$$\langle (F(X_1) - F(X_2))^T, X_1 - X_2 \rangle > 0, \quad \forall X_1,\ X_2 \in \mathcal{K}, X_1 \neq X_2,$$

then variational inequality (12), admits a unique solution.


Company
W_1
W_2
Warehouses
Trucks
G_1
Fulfillment Center
Trucks
D_1
D_2
Delivery Stations
Trucks or Drones
C_1
C_2
C_3
Customer Locations

$$c(x) = \beta_1 \cdot x^2 + \beta_2 \cdot x, \tag{14}$$

where $\beta_1 > 0$ and $\beta_2 \geq 0$.

Cost functions	Description	β_1	β_2
$c(\alpha, x, y, z, w^1, w^2)$	Handling	0.5	0.5
$c_{ij}(x_{ij})$	Transp. from W_1 to G_1	0.2	0.2
	Transp. from W_2 to G_1	0.1	0.1
$\hat{c}_{js}(y_{js}, z_j)$	Transp. from G_1 to D_1	0.1	0.1
	Transp. from G_1 to D_2	0.1	0.1
$\overline{c}_{jk}(y_j, z_{jk})$	Transp. from G_1 to C_1	50	50
	Transp. from G_1 to C_2	1	1
	Transp. from G_1 to C_3	50	50
$c_i^{tr}(\alpha, w^2)$	Trucks handling ($\forall i = 1, 2$)	0.3	0.3
$c_s^{dr}(w^2)$	UAVs handling ($\forall s = 1, 2$)	0.1	0.1

Table: Parameters of the cost functions numerical simulations

Cost functions	Description	β_1	β_2
$\tilde{c}^1_{sk}(w^1_{sk})$	Transp. from D_1 to C_1 (trucks)	2	2
	Transp. from D_1 to C_2 (trucks)	8	8
	Transp. from D_1 to C_3 (trucks)	8	8
	Transp. from D_2 to C_1 (trucks)	8	8
	Transp. from D_2 to C_2 (trucks)	9	9
	Transp. from D_2 to C_3 (trucks)	2	2
$\tilde{c}^2_{sk}(w^2_{sk})$	Transp. from D_1 to C_1 (UAVs)	1	1
	Transp. from D_1 to C_2 (UAVs)	4	4
	Transp. from D_1 to C_3 (UAVs)	4	4
	Transp. from D_2 to C_1 (UAVs)	4	4
	Transp. from D_2 to C_2 (UAVs)	4.5	4.5
	Transp. from D_2 to C_3 (UAVs)	1	1

Table: Parameters of the cost functions numerical simulations

Parameters	Values
Purchase price	$\gamma_1 = 3$
Production cost	$\gamma_2 = 4$
Incentive for UAVs	$I_{sk} = 2, \ \forall s = 1,2, \ \forall k = 1,2,3$
Selling price from G_j	$\gamma_{jk} = 149, \ \forall j = 1, \ \forall k = 1,2,3$
Selling price from D_s	$\gamma_{sk} = 150, \ \forall s = 1,2, \ \forall k = 1,2,3$
Maximum quantity at W_1	$Q_1 = 20$
Maximum quantity at W_2	$Q_2 = 10$
Demand from C_k	$r_k = 10, \ \forall k = 1,2,3$
Number of trucks at W_1	$n_1^{tr} = 4$
Number of trucks at W_2	$n_2^{tr} = 5$
Maximum capacity of trucks	$W = 6$
Number of UAVs at D_1	$n_1^{dr} = 5$
Number of UAVs at D_2	$n_2^{dr} = 5$
Maximum weight (UAV)	$p_d = 4$
Parameter on the weights	$P^{dr} = 30$
Parameter on distances	$B_{sk}^{dr} = 30, \ \forall s = 1,2, \ \forall k = 1,2,3$

Table: Parameter values for numerical simulations

Two main numerical simulations, which focus on:

S1: a UAV-based last mile network;

S2: a supply chain without using UAVs.

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Variables		Optimal solutions S1	Optimal solutions S2
α_i^{l*}	$i=1$	20	20
	$i=2$	10	10
x_{ij}^{l*}	$i=1, j=1$	20	20
	$i=2, j=1$	10	10
y_{js}^{l*}	$j=1, s=1$	11.38	10
	$j=1, s=2$	11.21	10
z_{jk}^{l*}	$j=1, k=1$	0	0
	$j=1, k=2$	7.41	10
	$j=1, k=3$	0	0

Variables		Optimal solutions S1	Optimal solutions S2
w_{sk}^{1l*}	$s=1, k=1$	0	0
	$s=1, k=2$	0	0
	$s=1, k=3$	0	10
	$s=2, k=1$	0	10
	$s=2, k=2$	0	0
	$s=2, k=3$	0	0
w_{sk}^{2l*}	$s=1, k=1$	7.98	–
	$s=1, k=2$	1.39	–
	$s=1, k=3$	1.92	–
	$s=2, k=1$	1.93	–
	$s=2, k=2$	1.20	–
	$s=2, k=3$	7.99	–

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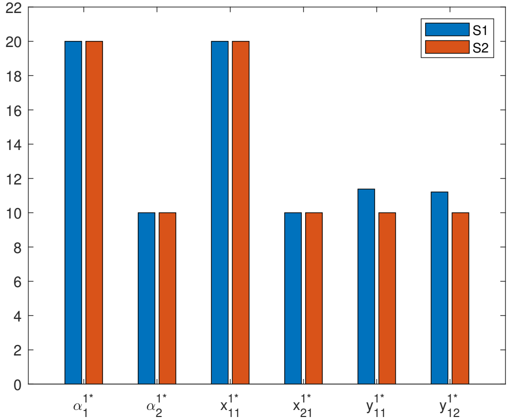

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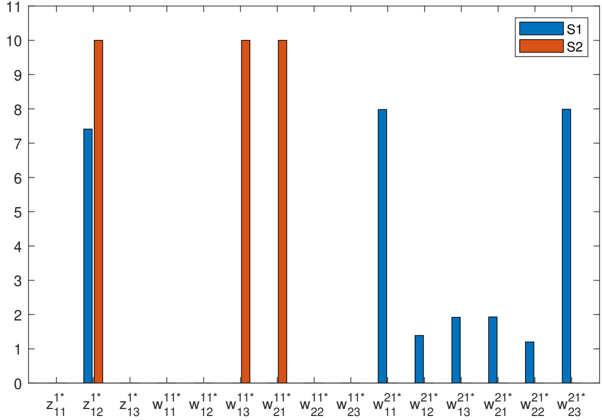

$$f^{(S1)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) = 2850.6;$$

$$f^{(S2)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*}) = 1137.$$

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$$PC = \left| \frac{\Delta f}{f^{(S1)}(\alpha^*, x^*, y^*, z^*, w^{1*}, w^{2*})} \right| \cdot 100\% = \left| \frac{1713.6}{2850.6} \right| \cdot 100\% = 60.11\%.$$

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The total GHG emission CO_2 equivalent emissions (expressed as grams, using the

EcoTransit World Model online environmental assessment tool):

$$E^{(S1)} = 3.2g, \quad E^{(S2)} = 27.3g.$$

Simulations	n_1^{dr}	n_2^{dr}	P^{dr}	B_{11}^{dr}	B_{12}^{dr}	B_{13}^{dr}	B_{21}^{dr}	B_{22}^{dr}	B_{23k}^{dr}
S1.1	5	5	30	30	30	30	30	30	30
S1.2	5	5	30	0	30	0	30	0	0
S1.3	5	2	30	30	30	30	30	30	30
S1.4	4	2	30	0	30	30	30	0	30
S1.5	5	5	0	30	30	30	30	30	30
S1.6	5	5	0	0	0	0	0	0	0

Table: Main parameters of the additional configurations

	S1.1	S1.2	S1.3	S1.4	S1.5	S1.6
α_1^*	20.00	20.00	20.00	20.00	20.00	20.00
α_2^*	10.00	10.00	10.00	10.00	10.00	10.00
x_{11}^*	20.00	20.00	20.00	20.00	20.00	20.00
x_{21}^*	10.00	10.00	10.00	10.00	10.00	10.00
y_{11}^*	15.00	14.38	20.05	18.26	15.00	15.00
y_{12}^*	15.00	15.62	9.95	11.74	15.00	15.00
z_{11}^*	0.00	0.00	0.00	0.00	0.00	0.00
z_{12}^*	0.00	0.00	0.00	0.00	0.00	0.00
z_{13}^*	0.00	0.00	0.00	0.00	0.00	0.00

Table: Optimal solutions of the additional configurations

	S1.1	S1.2	S1.3	S1.4	S1.5	S1.6
w_{11}^{1*}	0.00	1.82	0.33	3.20	6.83	6.83
w_{12}^{1*}	0.00	0.66	0.28	0.71	5.00	5.00
w_{13}^{1*}	0.00	3.19	0.08	0.24	3.17	3.17
w_{21}^{1*}	0.00	0.63	0.17	1.51	3.17	3.17
w_{22}^{1*}	0.00	0.62	0.62	0.92	5.00	5.00
w_{23}^{1*}	0.00	6.81	1.16	1.30	6.83	6.83
w_{11}^{2*}	6.62	0.00	8.15	0.00	0.00	0.00
w_{12}^{2*}	5.00	8.72	6.41	8.37	0.00	0.00
w_{13}^{2*}	3.38	0.00	4.80	5.75	0.00	0.00
w_{21}^{2*}	3.38	7.56	1.35	5.29	0.00	0.00
w_{22}^{2*}	5.00	0.00	2.69	0.00	0.00	0.00
w_{23}^{2*}	6.62	0.00	3.96	2.71	0.00	0.00

Table: Optimal solutions of the additional configurations

Sensitivity analysis on the incentive values

	SA1	SA2	SA3	SA4	SA5	SA6	SA7
I_{sk}	0	1	2	4	8	16	32

Table: Incentive value for each sensitivity analysis configuration

	SA1	SA2	SA3	SA4	SA5	SA6	SA7
I_{sk}	0	1	2	4	8	16	32

Table: Incentive value for each sensitivity analysis configuration

	SA1	SA2	SA3	SA4	SA5	SA6	SA7
I_{sk}	0	1	2	4	8	16	32
$W^{(tr)}$	11.95	11.70	11.16	8.66	7.09	0.00	0.00
$W^{(dr)}$	18.05	18.30	18.84	21.34	22.91	30.00	30.00
$I^{(SM)}$	0.00	18.30	37.68	85.34	183.25	480.00	960.00

Table: Sensitivity analysis: amount of product sent via trucks ($W^{(tr)}$) and via drones ($W^{(dr)}$) and incentive for sustainable mobility ($I^{(SM)}$)

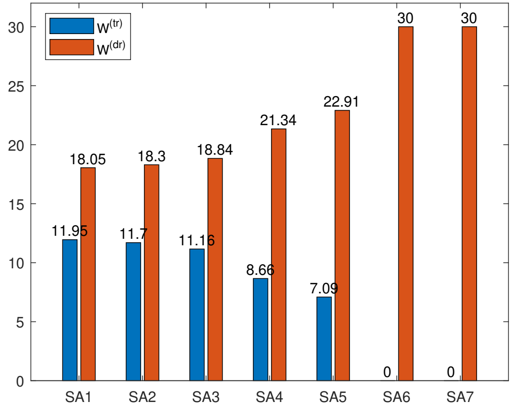

Figure: Total amount of product sent via trucks $(W^{(tr)})$ and via drones $(W^{(dr)})$ in the sensitivity analysis simulations

Conclusions

✓ Centralized supply chain network optimization model that maximizes the total profit obtained by a company that produces and/or outsources production, stores, ships and sells products to customers using a fleet made up of trucks and, in the last mile, also of drones.

Conclusions

- ✓ Centralized supply chain network optimization model that maximizes the total profit obtained by a company that produces and/or outsources production, stores, ships and sells products to customers using a fleet made up of trucks and, in the last mile, also of drones.
- ✓ Fundamental limitations such as weight limitations, low battery capacities and short delivery ranges.

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- ✓ Detailed numerical simulations that emphasize the advantages of the use of a hybrid fleet for last mile parcel deliveries.

Further research:

Include green charging stations and landing pads, the use of renewable energy and collection points in the supply chain.

Crowdsourcing delivery.

Main references

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Thank you!