Optimal Endogenous Carbon Taxes for Electric Power Supply Chains with Power Plants

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Paper Outline

- Develop a modeling and computational framework that allows for the determination of optimal carbon taxes applied to electric power plants in the context of electric power supply chain (generation/distribution/consumption) networks.

- The general framework that we develop allows for three distinct types of carbon taxation environmental policies
  - A completely decentralized scheme in which taxes can be applied to each individual power generator/ power plant in order to guarantee that each assigned emission bound is not exceeded
  - A centralized scheme which assumes a fixed bound over the entire electric power supply chain in terms of total carbon emissions
  - A centralized scheme which assumes the bound to be a function of the tax.

- Twelve numerical examples are presented in which the optimal carbon taxes, as well as the equilibrium electric power flows and demands, are computed.

Motivation

- The electrical industry is growing, with the total global consumption of electricity to reach 23.1 trillion kilowatt hours in 2025.
- Of the total U.S. emissions of carbon dioxide and nitrous oxide, more than a third arises from generating electricity.
- Accumulated evidence of global warming.
- Need for environmental-energy modeling which include carbon taxes to address market failures in energy.

Literature

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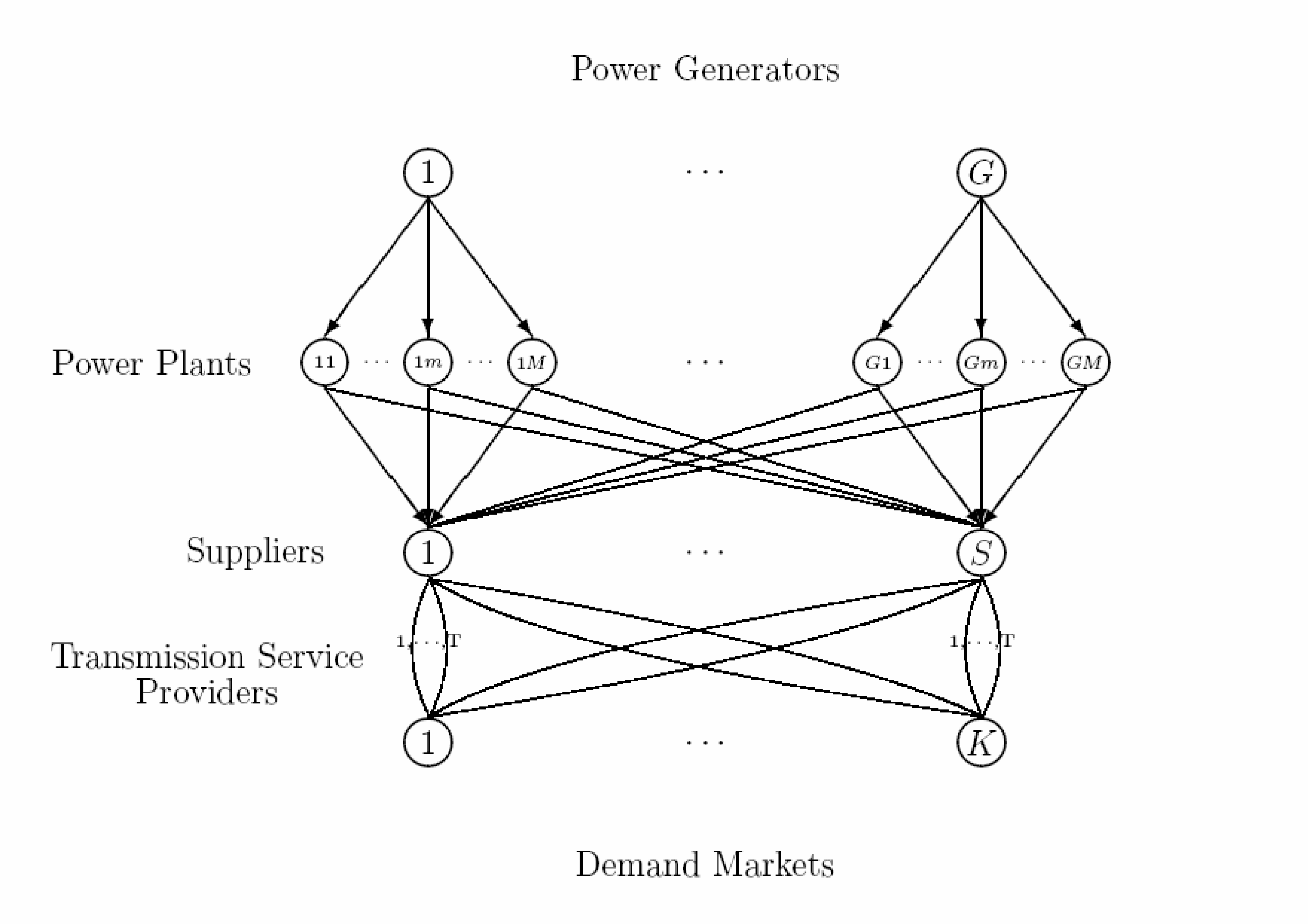
The Electric Power Supply Chain Network with Power Plants



Notation for the Electric Power Supply Chain Network Model

Notation	Definition
q_{gm}	quantity of electricity produced by generator g using power plant m, where $g = 1, \ldots, G; m = 1, \ldots, M$
q_m	G-dimensional vector of electric power generated by the gencos using power plant m with components: $q_{1m}, \ldots, q_{Gm}$
q	GM-dimensional vector of all the electric power outputs generated by the gencos at the power plants
Q^1	GMS-dimensional vector of electric power flows between the power plants of the power generators and the power suppliers with component gms denoted by q_{gms}
Q^2	STK-dimensional vector of power flows between suppliers and demand markets with component stk denoted by q^t_{sk} and denoting the flow between supplier s and demand market k via transmission provider t
d	K-dimensional vector of market demands with component k denoted by d_k
$f_{gm}(q_m)$	power generating cost function of power generator g using power plant m with marginal power generating cost with respect to q_{gm} denoted by $\frac{\partial f_{gm}}{\partial q_{gm}}$
$c_{gms}(q_{gms})$	transaction cost incurred by power generator g using power plant m in transacting with power supplier s with marginal transaction cost denoted by $\frac{\partial c_{gms}(q_{gms})}{\partial q_{gms}}$

Notation for the Electric Power Supply Chain Network Model

e_{gm}	amount of carbon emitted by genco g using power plant m per unit of electric power produced
h	S-dimensional vector of the power suppliers' supplies of the electric power with component s denoted by h_s, with $h_s \equiv \sum_{g=1}^{G} \sum_{m=1}^{M} q_{gms}$
$c_s(h) \equiv c_s(Q^1)$	operating cost of power supplier s with marginal operating cost with respect to h_s denoted by $\frac{\partial c_s}{\partial h_s}$ and the marginal operating cost with respect to q_{gms} denoted by $\frac{\partial c_s(Q^1)}{\partial q_{gms}}$
$c^t_{sk}(q^t_{sk})$	transaction cost incurred by power supplier s in transacting with demand market k via transmission provider t with marginal transaction cost with respect to q^t_{sk} denoted by $\frac{\partial c^t_{sk}(q^t_{sk})}{\partial q^t_{sk}}$
$\hat{c}_{gms}(q_{gms})$	transaction cost incurred by power supplier s in transacting with power generator g for power generated by plant m with marginal transaction cost denoted by $\frac{\partial \hat{c}_{gms}(q_{gms})}{\partial q_{gms}}$
$\hat{c}^t_{sk}(Q^2)$	unit transaction cost incurred by consumers at demand market k in transacting with power supplier s via transmission provider t
$\rho_{3k}(d)$	demand market price function at demand market k

A Decentralized Carbon Taxation Scheme

The Optimization problem of the power generator can be expressed as follows:

$$\text{Maximize} \quad \sum_{m=1}^{M}\sum_{s=1}^{S}\rho_{1gms}^{*}q_{gms} - \sum_{m=1}^{M} f_{gm}(q_m) - \sum_{m=1}^{M}\sum_{s=1}^{S} c_{gms}(q_{gms}) - \sum_{m=1}^{M}\tau_{gm}^{*}e_{gm}q_{gm}$$

subject to:

$$\sum_{s=1}^{S} q_{gms} = q_{gm}, \quad m = 1,\dots,M,$$

$$q_{gms} \geq 0, \quad m = 1,\dots,M; s = 1,\dots,S.$$

The Optimization problem faced by the supplier can be expressed as follows:

$$\text{Maximize} \quad \sum_{k=1}^{K}\sum_{t=1}^{T}\rho_{2sk}^{t*}q_{sk}^{t} - c_s(Q^1) - \sum_{g=1}^{G}\sum_{m=1}^{M}\rho_{1gms}^{*}q_{gms} - \sum_{g=1}^{G}\sum_{m=1}^{M}\hat{c}_{gms}(q_{gms}) - \sum_{k=1}^{K}\sum_{t=1}^{T}c_{sk}^{t}(q_{sk}^{t})$$

subject to:

$$\sum_{k=1}^{K}\sum_{t=1}^{T} q_{sk}^{t} = \sum_{g=1}^{G}\sum_{m=1}^{M} q_{gms}$$

$$q_{gms} \geq 0, \quad g = 1,\dots,G, \quad m = 1,\dots,M,$$

$$q_{sk}^{t} \geq 0, \quad k = 1,\dots,K; t = 1,\dots,T.$$

A Decentralized Carbon Taxation Scheme

Market Equilibrium Conditions at Demand Market k

$$\rho_{2sk}^{t*} + \hat{c}_{sk}^{t}(Q^{2*}) \begin{cases} = \rho_{3k}(d^*), & \text{if} \quad q_{sk}^{t*} > 0, \\ \geq \rho_{3k}(d^*), & \text{if} \quad q_{sk}^{t*} = 0. \end{cases}$$

Decentralized Carbon Tax Equilibrium conditions

$$\bar{B}_{gm} - e_{gm}q_{gm}^{*} \begin{cases} = 0, & \text{if} \quad \tau_{gm}^{*} > 0, \\ \geq 0, & \text{if} \quad \tau_{gm}^{*} = 0. \end{cases}$$

Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Decentralized Carbon Taxes

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*, \tau^*) \in \mathcal{K}^5$ *satisfying:*

$$\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm}^* e_{gm}\right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^{S}\frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}}\right] \times [q_{gms} - q_{gms}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*})\right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^{K}\rho_{3k}(d^*) \times [d_k - d_k^*]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\bar{B}_{gm} - e_{gm}q_{gm}^*\right] \times \left[\tau_{gm} - \tau_{gm}^*\right] \geq 0,$$

$$\forall (q, h, Q^1, Q^2, d, \tau) \in \mathcal{K}^5,$$

where

$$\mathcal{K}^5 \equiv \{(q, h, Q^1, Q^2, d, \tau) | (q, h, Q^1, Q^2, d, \tau) \in R_+^{2GM+S+GMS+TSK+K}$$

and the constraints hold.

A Centralized Carbon Taxation Scheme

Centralized Carbon Tax Equilibrium conditions with a Fixed Bound

$$\bar{B} - \sum_{g=1}^{G} \sum_{m=1}^{M} e_{gm} q_{gm}^{*} \begin{cases} = 0, & \text{if} \quad \mathcal{T}^{*} > 0, \\ \geq 0, & \text{if} \quad \mathcal{T}^{*} = 0. \end{cases} \tag{23}$$

Clearly equilibrium conditions (23) can be formulated as the inequality:

$$\left[\bar{B} - \sum_{g=1}^{G} \sum_{m=1}^{M} e_{gm} q_{gm}^{*} \right] \times [\mathcal{T} - \mathcal{T}^{*}] \geq 0, \quad \forall \mathcal{T} \geq 0.$$

Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Centralized Carbon Taxes and a Fixed Upper Bound

The equilibrium conditions governing the electric power supply chain network according to Definition 2 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*, \mathcal{T}^*) \in \mathcal{K}^6$ *satisfying:*

$$\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \mathcal{T}^* e_{gm}\right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^{S}\frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}}\right] \times [q_{gms} - q_{gms}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*})\right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^{K}\rho_{3k}(d^*) \times [d_k - d_k^*]$$

$$+\left[\bar{B} - \sum_{g=1}^{G}\sum_{m=1}^{M} e_{gm} q_{gm}^*\right] \times [\mathcal{T} - \mathcal{T}^*] \geq 0, \quad \forall (q, h, Q^1, Q^2, d, \mathcal{T}) \in \mathcal{K}^4,$$

where

$$\mathcal{K}^5 \equiv \{(q, h, Q^1, Q^2, d, \mathcal{T}) | (q, h, Q^1, Q^2, d, \mathcal{T}) \in R_+^{GM+S+GMS+TSK+K+1}$$

and the constraints hold.

A Centralized Carbon Taxation Scheme

Centralized Carbon Tax Equilibrium conditions with an Elastic Bound

In this case, the carbon tax equilibrium conditions (cf. (23)) would be as follows.

$$B(\mathcal{T}^*) - \sum_{g=1}^{G} \sum_{m=1}^{M} e_{gm} q_{gm}^* \begin{cases} = 0, & \text{if} \quad \mathcal{T}^* > 0, \\ \geq 0, & \text{if} \quad \mathcal{T}^* = 0. \end{cases}$$

Clearly equilibrium conditions (23) can be formulated as the inequality:

$$\left[B(\mathcal{T}^*) - \sum_{g=1}^{G} \sum_{m=1}^{M} e_{gm} q_{gm}^* \right] \times [\mathcal{T} - \mathcal{T}^*] \geq 0, \quad \forall \mathcal{T} \geq 0.$$

Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Centralized Carbon Taxes and an Elastic Carbon Emission Bound

The equilibrium conditions governing the electric power supply chain network according to Definition 3 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*, \mathcal{T}^*) \in \mathcal{K}^6$ *satisfying:*

$$\sum_{g=1}^{G}\sum_{m=1}^{M}\left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \mathcal{T}^* e_{gm}\right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^{S}\frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}}\right] \times [q_{gms} - q_{gms}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*})\right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^{K}\rho_{3k}(d^*) \times [d_k - d_k^*]$$

$$+\left[B(\mathcal{T}^*) - \sum_{g=1}^{G}\sum_{m=1}^{M} e_{gm} q_{gm}^*\right] \times [\mathcal{T} - \mathcal{T}^*] \geq 0,$$

$$\forall (q, h, Q^1, Q^2, d, \mathcal{T}) \in \mathcal{K}^6.$$

Euler Method

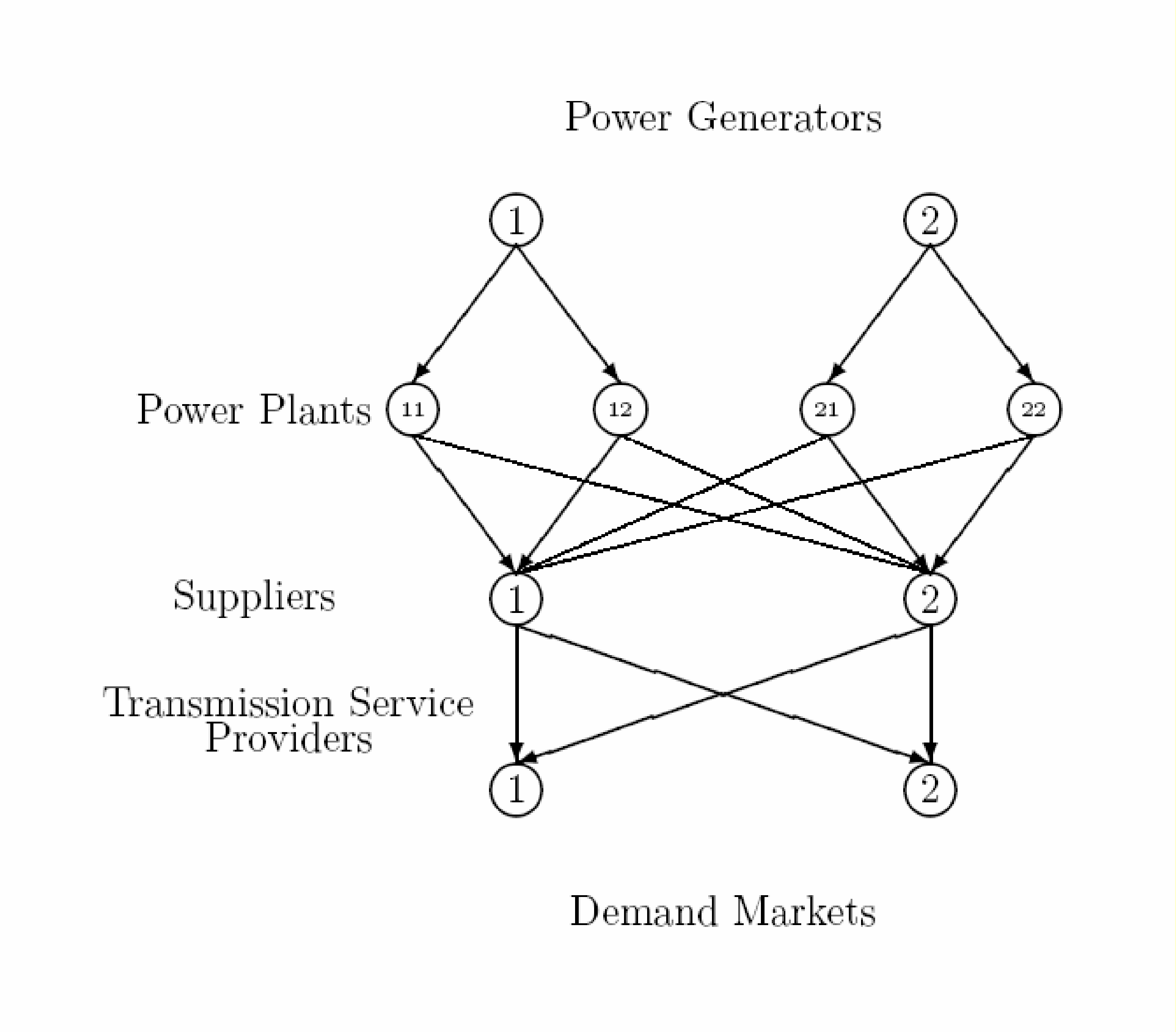
$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K, \tag{30}$$

For the solution of (30), the Euler method takes the form: at iteration l compute x^{l+1} by solving the variational inequality problem:

$$x^{l+1} = P_K(x^l - a_l F(x^l)),$$

where P_K is the projection operator, and the sequence $\{a_l\}$ must satisfy the conditions: $\sum_{l=0}^{\infty} a_l = \infty$, $a_l > 0$, for all l, and $a_l \to 0$, as $l \to \infty$.

12 Numerical Examples



Numerical Examples 1 ,2 ,3, and 4

Instances of an electric power supply chain network equilibrium model with a *Decentralized* Carbon taxation scheme.

Numerical Examples 5, 6, 7, and 8

Instances of an electric power supply chain network equilibrium model with a *Centralized* Carbon taxation scheme with a *Fixed* bound on the carbon emissions.

Numerical Examples 9, 10, 11, and 12

Instances of an electric power supply chain network equilibrium model with a *Centralized* Carbon taxation scheme with a *Variable* bound on the carbon emissions.

Numerical Example 1

The carbon emission terms: e_{gm}; $g = 1, 2$; $m = 1, 2$ were all equal to 1.

The power generating cost functions for the power generators were given by:

$$f_{11}(q_1) = 2.5q_{11}^2 + q_{11}q_{21} + 2q_{11}, \quad f_{12}(q_2) = 2.5q_{12}^2 + q_{11}q_{12} + 2q_{22}, \quad f_{21}(q_1) = .5q_{21}^2 + .5q_{11}q_{21} + 2q_{21}$$

$$f_{22}(q_2) = .5q_{22}^2 + q_{12}q_{22} + 2q_{22}.$$

The transaction cost functions faced by the power generators and associated with transacting with the power suppliers were given by:

$$c_{111}(q_{111}) = .5q_{111}^2 + 3.5q_{111}, \; c_{112}(q_{112}) = .5q_{112}^2 + 3.5q_{112}, \; c_{121}(q_{121}) = .5q_{121}^2 + 3.5q_{121},$$

$$c_{122}(q_{122}) = .5q_{122}^2 + 3.5q_{122},$$

$$c_{211}(q_{211}) = .5q_{211}^2 + 2q_{211}, \; c_{212}(q_{212}) = .5q_{212}^2 + 2q_{212}, \; c_{221}(q_{221}) = .5q_{221}^2 + 2q_{221},$$

$$c_{222}(q_{222}) = .5q_{222}^2 + 2q_{222}.$$

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Numerical Example 1

The operating costs of the power generators, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2} q_{i2})^2.$$

The demand market price functions at the demand markets were:

$$\rho_{31}(d) = -1.33d_1 + 366.6, \quad \rho_{32} = -1.33d_2 + 366.6,$$

and the transaction costs between the power suppliers and the consumers at the demand markets were given by:

$$\hat{c}^1_{sk}(q^1_{sk}) = q^1_{sk} + 5, \quad s = 1, 2; k = 1, 2.$$

All other transaction costs were assumed to be equal to zero.

In Example 1, the carbon emission bounds were: $\bar{B}_{11} = \bar{B}_{12} = \bar{B}_{21} = \bar{B}_{22} = 100$.

Numerical Example 2

Example 2 was constructed from Example 1 as follows. It had the identical data but since genco 2/power plant 2 emitted over 92 units of carbon, we set $\bar{B}_{22} = 23$

Numerical Example 3

Example 3 was constructed from Example 2. The data were identical to the data in Example 2, except that now we set all the bounds as follows: $\bar{B}_{11} = \bar{B}_{12} = \bar{B}_{21} = \bar{B}_{22} = 23$.

Numerical Example 4

Example 4 was constructed from Example 3 except that now the emission factor $e_{11} = 2$. Hence, its value was now doubled.

Solutions to Numerical Examples 1 ,2 ,3, and 4

Equilibrium Solution	Example 1	Example 2	Example 3	Example 4
Computed Equilibrium Power Flows				
q_{11}^*	22.56	29.86	23.00	11.51
q_{12}^*	9.93	31.17	23.00	23.02
q_{21}^*	22.90	30.20	23.00	23.02
q_{22}^*	92.38	23.00	23.00	23.05
q_{111}^*	11.28	14.93	11.50	5.76
q_{112}^*	11.28	14.93	11.50	5.76
q_{121}^*	4.97	15.59	11.50	11.51
q_{122}^*	4.97	15.59	11.50	11.51
q_{211}^*	11.45	15.10	11.50	11.51
q_{212}^*	11.45	15.10	11.50	11.51
q_{221}^*	46.19	11.50	11.50	11.52
q_{222}^*	46.19	11.50	11.50	11.52
h_1^*	73.89	57.12	46.00	40.30
h_2^*	73.89	57.12	46.00	40.30
q_{11}^{1*}	36.94	28.56	23.00	20.15
q_{12}^{1*}	36.94	28.56	23.00	20.15
q_{21}^{1*}	36.94	28.56	23.00	20.15
q_{22}^{1*}	36.94	28.56	23.00	20.15
Computed Equilibrium Demands				
d_1^*	73.89	57.12	46.00	40.30
d_2^*	73.89	57.12	46.00	40.30
Computed Optimal Taxes				
τ_{11}^*	0.00	0.00	76.43	77.86
τ_{12}^*	0.00	0.00	76.43	92.38
τ_{21}^*	0.00	0.00	77.93	105.41
τ_{22}^*	0.00	130.26	169.93	185.96

Centralized Carbon Taxation Scheme with a Fixed bound on Carbon Emission

Numerical Example 5

The e_{gm}; $g = 1, 2$; $m = 1, 2$ were set equal to 1.

Remaining data were as given in Example 1 except that in Example 5 we set $\bar{B} = 100$. Note that this bound represents the bound on the total amount of carbon emitted by all the power plants of all the gencos in the electric power supply chain network.

Numerical Example 6

Example 6 had data identical to that in Example 5 except that we now tightened the bound on the carbon emissions and had that $\bar{B} = 50$.

Centralized Carbon Taxation Scheme with a Fixed bound on Carbon Emission

Numerical Example 7

Example 7, in turn, had the same data as Example 6 (and Example 5) except that we further tightened the carbon emission bound so that $\bar{B} = 20$.

Numerical Example 8

Example 8 was constructed from Example 7 and had the same data except that we changed the second demand price function to

$$\rho_{32}(d) = -1.33d_2 + 733.3,$$

which means that the consumers at the second demand market are willing to pay a higher price for electric power than in Examples 5 through 7.

Solutions to Numerical Examples 5, 6, 7, and 8

Equilibrium Solution	Example 5	Example 6	Example 7	Example 8
Computed Equilibrium Power Flows				
q_{11}^*	15.20	7.48	2.85	2.87
q_{12}^*	6.63	3.17	1.10	1.10
q_{21}^*	15.53	7.82	3.19	3.20
q_{22}^*	62.65	31.53	12.86	12.91
q_{111}^*	7.60	3.74	1.43	1.43
q_{112}^*	7.60	3.74	1.43	1.43
q_{121}^*	3.31	1.59	0.55	0.55
q_{122}^*	3.31	1.59	0.55	0.55
q_{211}^*	7.76	3.91	1.59	1.60
q_{212}^*	7.76	3.91	1.59	1.60
q_{221}^*	31.32	15.77	6.43	6.46
q_{222}^*	31.32	15.77	6.43	6.46
h_1^*	50.00	25.00	10.00	10.00
h_2^*	50.00	25.00	10.00	10.00
q_{11}^{1*}	25.00	12.50	5.00	0.00
q_{12}^{1*}	25.00	12.50	5.00	10.00
q_{21}^{1*}	25.00	12.50	5.00	0.00
q_{22}^{1*}	25.00	12.50	5.00	10.00
Computed Equilibrium Demands				
d_1^*	50.00	25.00	10.00	0.00
d_2^*	50.00	25.00	10.00	20.00
Computed Optimal Tax				
$\mathcal{T}^*$	115.50	236.38	308.91	656.96

Centralized Carbon Taxation Scheme with a Variable bound on Carbon Emission

Numerical Example 9

Example 9 had the same input data as Example 5 but the bound on the carbon emissions was now elastic and given by:

$$B(\mathcal{T}) = \mathcal{T} + 100.$$

Numerical Example 10

Example 10 had the same data as Example 9 except that the carbon emission bound function was now given by:

$$B(\mathcal{T}) = \mathcal{T} + 50.$$

Centralized Carbon Taxation Scheme with a Variable bound on Carbon Emission

Numerical Example 11

Example 11 had the same data as the two preceding examples but differed in the elastic carbon emission bound function which was now even "tighter" and given by:

$$B(\mathcal{T}) = \mathcal{T} + 20.$$

Numerical Example 12

Example 12 was constructed from Example 10 and had the identical data except that we now modified the second demand market demand function to be as in Example 8.

Solutions to Numerical Examples 9, 10, 11, and 12

Equilibrium Solution	Example 9	Example 10	Example 11	Example 12
Computed Equilibrium Power Flows				
q_{11}^*	20.41	18.15	16.80	27.32
q_{12}^*	8.96	7.95	7.35	12.06
q_{21}^*	20.74	18.48	17.13	27.65
q_{22}^*	83.68	74.58	69.11	111.57
q_{111}^*	10.20	9.08	8.40	13.66
q_{112}^*	10.20	9.08	8.40	13.66
q_{121}^*	4.48	3.98	3.67	6.03
q_{122}^*	4.48	3.98	3.67	6.03
q_{211}^*	10.37	9.24	8.57	13.83
q_{212}^*	10.37	9.24	8.57	13.83
q_{221}^*	41.84	37.29	34.56	55.79
q_{222}^*	1.84	37.29	34.56	55.79
h_1^*	66.90	59.58	55.19	89.31
h_2^*	66.90	59.58	55.19	89.31
q_{11}^{1*}	33.45	29.79	27.60	0.00
q_{12}^{1*}	33.45	29.79	27.60	89.31
q_{21}^{1*}	33.45	29.79	27.60	0.00
q_{22}^{1*}	33.45	29.79	27.60	89.31
Computed Equilibrium Demands				
d_1^*	66.90	59.58	55.19	0.00
d_2^*	66.90	59.58	55.19	178.61
Computed Optimal Tax				
$\mathcal{T}^*$	33.79	69.16	90.38	128.61

Conclusions

- The model presented in this paper may help policy-makers to determine the optimal carbon taxes on the power plants in the electric power generation industry.

- The first model, a completely decentralized scheme, allows the policy-makers to determine the optimal tax for each individual electric power plant which guarantees that the emission bound or quota of each plant is not exceeded.

- The second and third models, on the other hand, both enforce a “global” emission bound on the entire industry by imposing a uniform tax rate on the generating plants.

Conclusions

- The numerical results demonstrate, as the theory predicts, that the carbon taxes achieve the desired goal, in that the imposed bounds on the carbon emissions are not exceeded.

- Moreover, they illustrate the spectrum of scenarios that can be explored in terms of changes in the bounds on the carbon emissions; changes in emission factors; changes in the demand functions, etc.

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Thank you!

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