Supply Chain Network Competition Among Blood Service Organizations: A Generalized Nash Equilibrium Network Framework

Anna Nagurney
John F. Smith Memorial Professor
Director of the Virtual Center for Supernetworks

Pritha Dutta
Ph.D. Student

Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, MA 01003

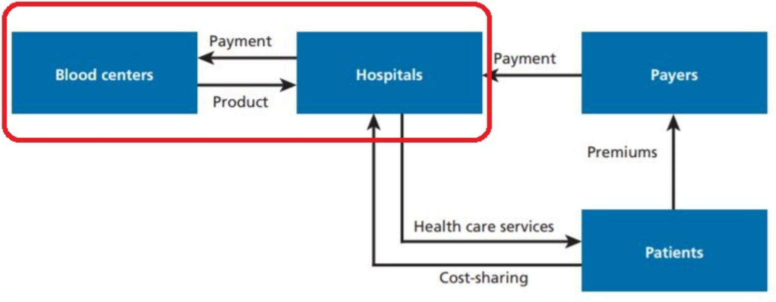
60th Annual CORS Conference
Halifax, Nova Scotia
June 4-6, 2018

Background

- The face of the blood banking industry is changing rapidly.
- Innovation and technology play significant roles in the advancement of this industry.
- It is highly capital intensive but at the same time predominantly non-profit.
- Economic sustainability of the blood suppliers is of utmost importance for the safe and steady supply of blood in the country.

Background

- In this paper we focus on the **operational challenges faced by the blood service organizations** and their economics.



RAND *RR1575-5.1*

Background

Inherent challenges

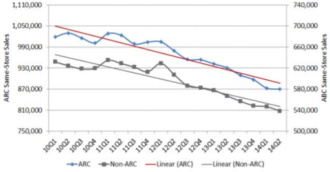
- **Shortages: Less than 10%** of the eligible population in US actually donates blood each year. American Red Cross reported shortage of 28,000 blood donations in November and December last year due to effects of seasonality.

- **Perishability:** Red blood cells have a shelf life of 42 days while platelets expire after 5 days.
- **Wastage:** Due to short shelf life and uncertain demand pattern, collected blood units can easily get wasted.

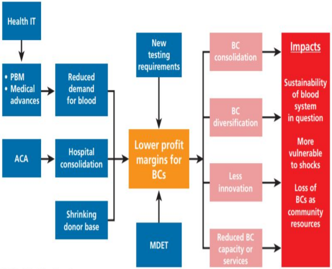
Background

New challenges

- **Declining demand:** According to the American Red Cross, the leading supplier of blood in the US, with about 40% of the market, there was **a 33% decrease in blood transfusions in the period 2010-2014.**
- **Rise of competition** among the blood service organizations.
- On one hand, they compete for the **limited pool of eligible blood donors** and on the other hand, for **supply contracts with hospitals**.



America's Blood Centers; January 2010-June 2014 DW Sales/Pricing Universe



NOTE: BC = blood center.

RAND RR1575-4.2

Background

Examples of competition among blood service organizations:

- At the end of 2016, **American Red Cross** lost its business in **Central Arkansas to Arkansas Blood Institute**, an aliate of the Oklahoma Blood Institute (Brantley (2017)).
- In 2013 Eastern Maine Medical Center ended its contract with **American Red Cross** to do business with **Puget Sound Blood Center**, a Seattle-based community blood bank (Barber (2013)).
- Since 2011, a small **Sarasota-based blood bank, SunCoast Communities Blood Bank**, had been competing for blood donations with a much larger organization, **Florida Blood Services**, that served hospitals in Tampa and neighboring areas (Smith (2011)).

Background

Our contributions:

- Our model is, to the best of our knowledge, the first one to capture **competition among blood service organizations**.
- We use**common/shared capacities** on the blood donations to incorporate **supply side** competition.
- We also use **common/shared constraints** on the **demand side** to address issues of shortage and wastage.
- We use arc-path multipliers to capture **perishability** of blood products.
- We formulate the problem as a **Generalized Nash Equilibrium** one due to presence of shared constraints and provide alternative variational inequality formulation along with **Lagrange analysis**.

Background

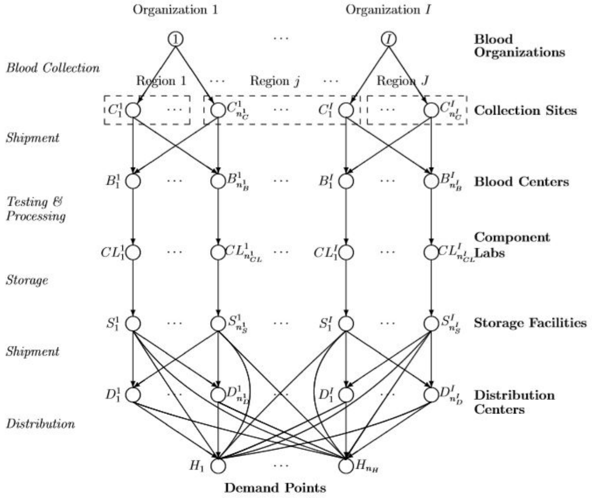
Generalized Nash Equilibrium (GNE)

- In **Nash equilibrium** problems, the **feasible set** of each decision maker in the noncooperative game **depends only on his/her own strategies**.
- In **Generalized Nash Equilibrium** games, the strategies of the decision makers depend not only on their **own feasible sets**, but **also on that of their competitors**.
- In this paper, we make use of the concept of **variational equilibrium** (cf. Facchinei and Kanzow (2010), Kulkarni and Shanbhag (2012)), which is a specific kind of GNE.

Relevant Literature

- **Blood supply chain optimization:** Nagurney, Masoumi, and Yu (2012), Duan and Liao (2014), Fahimnia et al. (2017), Masoumi, Yu and Nagurney (2017), Ramezanian and Behboodi (2017).
- **Supply chain competition among nonprofits and game theory:** Castaneda, Garen, and Thornton (2008), Zhuang, Saxton, and Wu (2011), Nagurney, Alvarez Flores, and Soylu (2016).
- **Supply capacity constraints and Generalized Nash Equilibrium:** Goh, Lim, and Meng (2007), Nagurney, Alvarez Flores, and Soylu (2016), Nagurney, Yu, and Besik (2017).

The Multiple Blood Service Organizations Supply Chain Network Competition Model



The Multiple Blood Service Organizations Supply Chain Network Competition Model

Perishability

Notation	Definition
α_a	The arc multiplier associated with link a, which represents the percentage of throughput on link a. $\alpha_a \in (0,1]$; $a \in L$.
α_{ap}	The arc-path multiplier, which is the product of the multipliers of the links on path p that precede link a; $a \in L$ and $p \in P$; that is, $$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod\limits_{b \in \{a' < a\}_p} \alpha_b, & \text{if } \{a' < a\}_p \neq \emptyset, \\ \delta_{ap}, & \text{if } \{a' < a\}_p = \emptyset, \end{cases}$$ where $\{a' < a\}_p$ denotes the set of the links preceding link a in path p and $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.
μ_p	The multiplier corresponding to the percentage of throughput on path p; that is, $\mu_p \equiv \prod\limits_{a \in p} \alpha_a$; $p \in P$.

The Multiple Blood Service Organizations Supply Chain Network Competition Model

Demands

The conservation of flow equation that has to hold for each blood service organization i; $i = 1, \ldots, I$, at each demand point k; $k = H_1, \ldots, H_{n_H}$, is

$$\sum_{p \in P^i_k} \mu_p x_p = d_{ik}, \qquad (1)$$

where P^i_k denotes the set of all paths joining blood service organization node i with destination node H_k.

Nonnegativity Constraints on Path Flows

$$x_p \geq 0, \quad \forall p \in P, \qquad (2)$$

where P denotes the set of all paths in the network.

The Multiple Blood Service Organizations Supply Chain Network Competition Model

Relationship between Link Flows and Path Flows

$$f_a = \sum_{p \in P} x_p \alpha_{ap}, \quad \forall a \in L. \tag{3}$$

Supply Constraints: Shared Constraints

$$\sum_{a \in L_1^j} f_a \leq S^j, \tag{4}$$

S^j represents the total population eligible to donate blood in a given week in region j; $j = 1, \dots, J$.

The Multiple Blood Service Organizations Supply Chain Network Competition Model

Capacity Constraints

$$f_a \leq u_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \tag{5}$$

Bounds on Demands: Shared Constraints

$$\sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p \geq \underline{d}_k, \quad k = H_1, \ldots, H_{n_H}, \tag{6}$$

$$\sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p \leq \bar{d}_k, \quad k = H_1, \ldots, H_{n_H}. \tag{7}$$

The Multiple Blood Service Organizations Supply Chain Network Competition Model

Utility Function

Each blood service organization i seeks to maximize its transaction utility, U_i given as

$$U_i = \sum_{k=H_1}^{H_{n_H}} \rho_{ik}(d) d_{ik} + \omega_i \sum_{k=H_1}^{H_{n_H}} \gamma_{ik} d_{ik} - \sum_{a \in L^i} \hat{c}_a(f). \tag{8}$$

- **Revenue**
- **Monetized altruism**
- **Cost**

The Multiple Blood Service Organizations Supply Chain Network Competition Model

Vector of Strategies

$$X_i \equiv \{\{x_p\}|p \in P^i\} \in R_+^{n_{P^i}}. \tag{9}$$

Capacity Constraints on Path Flows

$$\sum_{a \in L_1^j} \sum_{p \in P} x_p \delta_{ap} \leq S^j, \quad j = 1, \ldots, J. \tag{10}$$

$$\sum_{p \in P} x_p \alpha_{ap} \leq u_a, \quad \forall a \in L^i, \quad i = 1, \ldots, I. \tag{11}$$

The Multiple Blood Service Organizations Supply Chain Network Competition Model

Feasible Sets

We define the i-th blood bank's individual feasible set, K_i, as

$$K_i \equiv \{X_i | (2) \text{ and } (11) \text{ hold for } i\}. \tag{12}$$

Feasible set consisting of the shared constraints

$$\mathcal{S} \equiv \{X | \, (10), \, (6), \text{ and } (7) \text{ hold}\}. \tag{13}$$

The Multiple Blood Service Organizations Supply Chain Network Competition Model

Definition 1: Blood Supply Chain Network Generalized Nash Equilibrium

A blood product path flow pattern $X^* \in K \equiv \prod_{i=1}^{I} K^i, X^* \in \mathcal{S}$, constitutes a blood supply chain network Generalized Nash Equilibrium if for each blood service organization i; $i = 1, ..., I$:

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K^i, \forall X \in \mathcal{S}, \tag{14}$$

where

$$\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*).$$

- An equilibrium is established if no blood service organization can unilaterally improve upon its utility by selecting an alternative vector of blood product flows, given the blood product flows of the other blood service organizations, and subject to the capacity constraints, both individual and shared ones, the shared demand constraints, and the nonnegativity constraints.

The Multiple Blood Service Organizations Supply Chain Network Competition Model

Definition 2: Variational Equilibirum

A strategy vector X^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $X^* \in K, X^* \in \mathcal{S}$ is a solution of the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \forall X \in \mathcal{S}. \tag{15}$$

Variational Inequality Formulation

Determine $x^* \in K, x^* \in \mathcal{S}$ such that:

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \omega_i \gamma_{ik} \mu_p - \hat{\rho}_{ik}(x^*) \mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \right]$$

$$\times [x_p - x_p^*] \geq 0, \quad \forall x \in K, x \in \mathcal{S}. \tag{16}$$

The Multiple Blood Service Organizations Supply Chain Network Competition Model

We can put the variational inequality formulations into standard variational inequality form (see Nagurney (1999)), that is: determine $Y^* \in \mathcal{K} \subset R^N$, such that

$$\langle F(Y^*), Y - Y^* \rangle \geq 0, \quad \forall Y \in \mathcal{K}, \tag{17}$$

where F is a given continuous function from $\mathcal{K}$ to R^N and $\mathcal{K}$ is a closed and convex set.

The Multiple Blood Service Organizations Supply Chain Network Competition Model

Alternative Variational Inequality Formulation with Lagrange Multipliers

Determine the vector of equilibrium path flows and Lagrange multipliers, $(x^*, \eta^*, \theta^*, \sigma^*, \epsilon^*) \in \mathcal{K}^3$, such that:

$$\sum_{i=1}^{I} \sum_{k=H_1}^{H_{n_H}} \sum_{p \in P_k^i} \Big[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^* \delta_{ap} + \sum_{a \in L^i} \theta_a^* \alpha_{ap} - \omega_i \gamma_{ik} \mu_p - \sigma_k^* \mu_p$$

$$+ \epsilon_k^* \mu_p - \hat{\rho}_{ik}(x^*) \mu_p - \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^* \Big] \times [x_p - x_p^*] + \sum_{j=1}^{J} \Big[S^j - \sum_{a \in L_1^j} \sum_{p \in P} x_p^* \delta_{ap} \Big] \times [\eta_j - \eta_j^*]$$

$$+ \sum_{i=1}^{I} \sum_{a \in L^i} \Big[u_a - \sum_{p \in P} x_p^* \alpha_{ap} \Big] \times [\theta_a - \theta_a^*] + \sum_{k=H_1}^{H_{n_H}} \Big(\sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^* - \underline{d}_k \Big) \times (\sigma_k - \sigma_k^*)$$

$$+ \sum^{H_{n_H}} (\bar{d}_k - \sum^{I} \sum \mu_p x_p^*) \times (\epsilon_k - \epsilon_k^*) \geq 0, \quad \forall (x, \eta, \theta, \sigma, \epsilon) \in \mathcal{K}^3. \qquad (18)$$

The Multiple Blood Service Organizations Supply Chain Network Competition Model

Feasible Set

The feasible set $\mathcal{K}^3$ in the previous alternative variational inequality formulation is given as:

$$\mathcal{K}^3 \equiv \{(x, \eta, \theta, \sigma, \epsilon) | x \in R_+^{n_P}, \eta \in R_+^J, \theta \in R_+^{n_L}, \sigma \in R_+^{n_H}, \epsilon \in R_+^{n_H}\}. \quad (19)$$

Economic Interpretation of Lagrange Analysis

Case I: None of the associated constraints are active

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} = \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \quad (20)$$

Case II: The associated donor supply constraints are active but other capacity and demand constraints associated with the path are not

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} < \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \quad (21)$$

Case III: One or more links on the path are at their capacities but no other associated capacity or demand constraints are active

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} < \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \quad (22)$$

Economic Interpretation of Lagrange Analysis

Case IV: The demand point that the path is destined to has its demand at the lower bound whereas no other associated constraints are active

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} > \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \quad (23)$$

Case V: The demand point that the path is destined to has its demand at the upper bound whereas no other associated constraints are active

$$\frac{\partial \hat{C}_p(x^*)}{\partial x_p} < \omega_i \gamma_{ik} \mu_p + \hat{\rho}_{ik}(x^*)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^*. \quad (24)$$

The Algorithm

An iteration $\tau + 1$ of the Euler method induced by the iterative scheme of Dupuis and Nagurney (1993) is given by:

$$Y^{\tau+1} = P_{\mathcal{K}}(Y^{\tau} - a_{\tau} F(Y^{\tau})), \tag{25}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the utility function. For convergence of the general iterative scheme, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} = \infty, a_{\tau} > 0, a_{\tau} \to 0$, as $\tau \to \infty$. (Nagurney and Zhang (1996)).

The Algorithm

Explicit Formula for Euler method

For this problem, we have the following closed form expressions for the path flows at iteration $\tau + 1$. For each path $p \in P_k^i, \forall i, k$, we have:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\hat{\rho}_{ik}(x^\tau)\mu_p + \sum_{l=H_1}^{H_{n_H}} \frac{\partial \hat{\rho}_{il}(x^\tau)}{\partial x_p} \sum_{q \in P_l^i} x_q^\tau \mu_q + \omega_i \gamma_{ik} \mu_p - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p}$$

$$- \sum_{j=1}^{J} \sum_{a \in L_1^j} \eta_j^\tau \delta_{ap} - \sum_{a \in L^i} \theta_a^\tau \alpha_{ap} + \sigma_k^\tau \mu_p - \epsilon_k^\tau \mu_p)\}. \quad (26)$$

The Lagrange multipliers associated with blood collection links $a \in L_1^j$; $j = 1, \ldots, J$, are computed according to:

$$\eta_j^{\tau+1} = \max\{0, \eta_j^\tau + a_\tau(\sum_{a \in L_1^j} \sum_{p \in P} x_p^\tau \delta_{ap} - S^j)\}. \quad (27)$$

Explicit Formula for Euler method

The closed form expression for the Lagrange multipliers for the capacity constraint on link $a \in L^i$; $i = 1, \ldots, I$ is:

$$\theta_a^{\tau+1} = \max\{0, \theta_a^{\tau} + a_\tau(\sum_{p \in P} x_p^{\tau} \alpha_{ap} - u_a)\}. \quad (28)$$

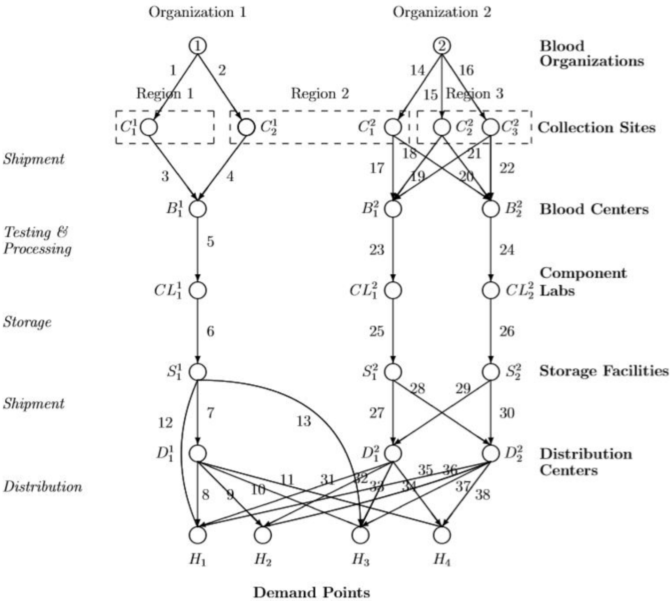
The explicit formulae for the Lagrange multipliers associated with the lower bounds on the demands at demand points: $k = H_1, \ldots, H_{n_H}$, are:

$$\sigma_k^{\tau+1} = \max\{0, \sigma_k^{\tau} + a_\tau(\underline{d}_k - \sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^{\tau})\}. \quad (29)$$

The Lagrange multipliers associated with the upper bounds on the demands at the demand points: $k = H_1, \ldots, H_{n_H}$, in turn, are computed according to:

$$\epsilon_k^{\tau+1} = \max\{0, \epsilon_k^{\tau} + a_\tau(\sum_{i=1}^{I} \sum_{p \in P_k^i} \mu_p x_p^{\tau} - \overline{d}_k)\}. \quad (30)$$

Numerical Examples



Numerical Examples: Example 1

The number of people eligible to donate blood in each of the regions in a week are:

$$S^1 = 6000, S^2 = 2400, S^3 = 3000.$$

The upper and lower bounds on the demand at each hospital are given below:

$$\underline{d}_1 = 200, \quad \overline{d}_1 = 350,$$
$$\underline{d}_2 = 60, \quad \overline{d}_2 = 150,$$
$$\underline{d}_3 = 200, \quad \overline{d}_3 = 300,$$
$$\underline{d}_4 = 100, \quad \overline{d}_4 = 120.$$

Numerical Examples: Example 1

The demand price functions are given as follows:
Organization 1:

$$\rho_{11}(d) = -0.07d_{11} - 0.02d_{21} + 300, \quad \rho_{12}(d) = -0.08d_{12} - 0.03d_{22} + 310,$$

$$\rho_{13}(d) = -0.05d_{13} - 0.01d_{23} + 300, \quad \rho_{14}(d) = -0.04d_{14} - 0.02d_{24} + 280.$$

Organization 2:

$$\rho_{21}(d) = -0.05d_{21} - 0.01d_{11} + 280, \quad \rho_{22}(d) = -0.07d_{22} - 0.04d_{12} + 290,$$

$$\rho_{23}(d) = -0.03d_{23} - 0.01d_{13} + 280, \quad \rho_{24}(d) = -0.05d_{24} - 0.02d_{14} + 270.$$

The values for the altruism component are as follows:

$$\omega_1 = \omega_2 = 1,$$

$$\gamma_{11} = 2, \gamma_{12} = 2, \gamma_{13} = 2, \gamma_{14} = 1,$$

$$\gamma_{21} = 2, \gamma_{22} = 2, \gamma_{23} = 2, \gamma_{24} = 1.$$

Numerical Examples: Example 1

Table: Definition of Links, Associated Weekly Capacities, Total Operational Costs, and Solution for Example 1

Link a	From Node	To Node	u_a	α_a	$\hat{c}_a(f)$	f_a^*	θ_a^*
1	1	C_1^1	250	1.00	$0.24f_1^2 + 0.6f_1$	139.33	0.00
2	1	C_2^1	200	1.00	$0.4f_2^2 + 0.9f_2$	87.59	0.00
3	C_1^1	B_1^1	300	1.00	$0.06f_3^2 + 0.1f_3$	139.33	0.00
4	C_2^1	B_1^1	250	1.00	$0.07f_4^2 + 0.16f_4$	87.59	0.00
5	B_1^1	CL_1^1	500	0.97	$0.36f_5^2 + 0.45f_5$	226.92	0.00
6	CL_1^1	S_1^1	500	1.00	$0.02f_6^2 + 0.04f_6$	220.11	0.00
7	S_1^1	D_1^1	500	1.00	$0.03f_7^2 + 0.09f_7$	166.40	0.00
8	D_1^1	H_1	50	1.00	$0.4f_8^2 + 0.7f_8$	21.37	0.00
9	D_1^1	H_2	50	1.00	$0.5f_9^2 + 0.9f_9$	28.38	0.00
10	D_1^1	H_3	100	1.00	$0.15f_{10}^2 + 0.8f_{10}$	76.64	0.00
11	D_1^1	H_4	60	1.00	$0.35f_{11}^2 + 0.6f_{11}$	40.00	0.00
12	S_1^1	H_1	50	1.00	$0.4f_{12}^2 + 0.9f_{12}$	33.71	0.00
13	S_1^1	H_3	**20**	1.00	$0.7f_{13}^2 + 1f_{13}$	**20.00**	**5.02**
14	2	C_1^2	250	1.00	$0.25f_{14}^2 + 0.7f_{14}$	130.81	0.00
15	2	C_2^2	300	1.00	$0.2f_{15}^2 + 0.8f_{15}$	148.27	0.00
16	2	C_3^2	200	1.00	$0.3f_{16}^2 + 0.5f_{16}$	112.99	0.00
17	C_1^2	B_1^2	100	1.00	$0.12f_{17}^2 + 0.3f_{17}$	70.11	0.00
18	C_1^2	B_2^2	150	1.00	$0.08f_{18}^2 + 0.27f_{18}$	60.71	0.00
19	C_2^2	B_1^2	100	1.00	$0.16f_{19}^2 + 0.45f_{19}$	70.86	0.00
20	C_2^2	B_2^2	200	1.00	$0.1f_{20}^2 + 0.5f_{20}$	77.41	0.00

Numerical Examples: Example 1

Table: Definition of Links, Associated Weekly Capacities, Total Operational Costs, and Solution for Example 1

Link a	From Node	To Node	u_a	α_a	$\hat{c}_a(f)$	f_a^*	θ_a^*
21	C_3^2	B_1^2	100	1.00	$0.2f_{21}^2 + 0.6f_{21}$	35.85	0.00
22	C_3^2	B_2^2	100	1.00	$0.05f_{22}^2 + 0.08f_{22}$	77.14	0.00
23	B_1^2	CL_1^2	600	0.98	$0.36f_{23}^2 + 0.8f_{23}$	176.81	0.00
24	B_2^2	CL_2^2	500	0.96	$0.3f_{24}^2 + 0.7f_{24}$	215.25	0.00
25	CL_1^2	S_1^2	500	1.00	$0.02f_{25}^2 + 0.05f_{25}$	173.28	0.00
26	CL_2^2	S_2^2	500	1.00	$0.03f_{26}^2 + 0.04f_{26}$	206.64	0.00
27	S_1^2	D_1^2	150	1.00	$0.15f_{27}^2 + 0.4f_{27}$	88.02	0.00
28	S_1^2	D_2^2	150	1.00	$0.18f_{28}^2 + 0.65f_{28}$	85.25	0.00
29	S_2^2	D_1^2	200	1.00	$0.09f_{29}^2 + 0.12f_{29}$	116.35	0.00
30	S_2^2	D_2^2	150	1.00	$0.14f_{30}^2 + 0.5f_{30}$	90.30	0.00
31	D_1^2	H_1	100	1.00	$0.24f_{31}^2 + 0.8f_{31}$	48.90	0.00
32	D_1^2	H_2	80	1.00	$0.32f_{32}^2 + 0.9f_{32}$	51.65	0.00
33	D_1^2	H_3	100	1.00	$0.25f_{33}^2 + f_{33}$	63.82	0.00
34	D_1^2	H_4	**40**	1.00	$0.5f_{34}^2 + 0.8f_{34}$	**40.00**	**3.02**
35	D_2^2	H_1	150	1.00	$0.1f_{35}^2 + 0.35f_{35}$	96.01	0.00
36	D_2^2	H_2	**20**	1.00	$0.5f_{36}^2 + 0.8f_{36}$	**20.00**	**8.80**
37	D_2^2	H_3	80	1.00	$0.35f_{37}^2 + 0.7f_{37}$	39.53	0.00
38	D_2^2	H_4	**20**	1.00	$0.4f_{38}^2 + 0.9f_{38}$	**20.00**	**22.84**

Numerical Examples: Example 1

Equilibrium demands

Organization 1: $\mathbf{d}_{11}^{*} = \mathbf{55.09}$, $\mathbf{d}_{12} = \mathbf{28.39}$, $\mathbf{d}_{13} = \mathbf{96.64}$, $\mathbf{d}_{14} = \mathbf{40.00}$.

Organization 2: $\mathbf{d}_{21}^{*} = \mathbf{144.91}$, $\mathbf{d}_{22} = \mathbf{71.65}$, $\mathbf{d}_{23} = \mathbf{103.36}$, $\mathbf{d}_{24} = \mathbf{60.00}$.

Equilibrium prices

Organization 1:
$\rho_{\mathbf{11}}(\mathbf{d}^{*}) = \mathbf{293.25}$, $\rho_{\mathbf{12}}(\mathbf{d}^{*}) = \mathbf{305.58}$, $\rho_{\mathbf{13}}(\mathbf{d}^{*}) = \mathbf{294.13}$, $\rho_{\mathbf{14}}(\mathbf{d}^{*}) = \mathbf{277.20}$.
Organization 2:
$\rho_{\mathbf{21}}(\mathbf{d}^{*}) = \mathbf{272.20}$, $\rho_{\mathbf{22}}(\mathbf{d}^{*}) = \mathbf{283.85}$, $\rho_{\mathbf{23}}(\mathbf{d}^{*}) = \mathbf{275.93}$, $\rho_{\mathbf{24}}(\mathbf{d}^{*}) = \mathbf{266.20}$.

Numerical Examples: Example 1

Lagrange multipliers at equilibrium

- None of the supply /donor upper bound constraints are binding, hence all η_j^*s are equal to 0.
- None of the demands are at the imposed upper bound, hence all ϵ_k^*s are equal to 0.
- Three of the demands at the lower bounds.

$$\sigma_1^* = 0.55, \quad \sigma_2^* = 0.00, \quad \sigma_3^* = 5.80, \quad \sigma_4^* = 27.63.$$

Costs at equilibrium

Organization 1 : **$33,099.85**. Organization 2: **$86,525.06**.

Revenues at equilibrium

Organization 1 : **$64,341.70**. Organization 2: **$104,275.07**.

Net revenue at equilibrium

Organization 1 : **$31,241.85**. Organization 2: **$17,750.01**.

Numerical Examples: Example 2

The network and the data remain the same as Example 1 except that the demand upper and lower bound constraints are removed.

Equilibrium demands

Organization 1: $\mathbf{d}^*_{11} = \mathbf{67.08}$, $\mathbf{d}_{12} = \mathbf{34.48}$, $\mathbf{d}_{13} = \mathbf{99.99}$, $\mathbf{d}_{14} = \mathbf{13.32}$.

Organization 2: $\mathbf{d}^*_{21} = \mathbf{158.41}$, $\mathbf{d}_{22} = \mathbf{77.42}$, $\mathbf{d}_{23} = \mathbf{97.00}$, $\mathbf{d}_{24} = \mathbf{41.32}$.

Total demand at hospitals:

$d^*_1 = 225.49$, $d^*_2 = 111.90$, $d^*_3 = 197.03$, $d^*_4 = 54.64$.

Equilibrium prices

Organization 1:

$\rho_{11}(\mathbf{d}^*) = \mathbf{292.14}$, $\rho_{12}(\mathbf{d}^*) = \mathbf{304.92}$, $\rho_{13}(\mathbf{d}^*) = \mathbf{294.03}$, $\rho_{14}(\mathbf{d}^*) = \mathbf{278.64}$.

Organization 2:

$\rho_{21}(\mathbf{d}^*) = \mathbf{271.41}$, $\rho_{22}(\mathbf{d}^*) = \mathbf{283.20}$, $\rho_{23}(\mathbf{d}^*) = \mathbf{276.09}$, $\rho_{24}(\mathbf{d}^*) = \mathbf{267.67}$.

Numerical Examples: Example 2

Table: Link, Equilibrium Link solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 2

Link a	f_a^*	θ_a^*
1	136.03	0.00
2	85.49	0.00
3	136.03	0.00
4	85.49	0.00
5	221.51	0.00
6	214.87	0.00
7	155.56	0.00
8	27.78	0.00
9	34.48	0.00
10	79.99	0.00
11	13.32	0.00
12	39.30	0.00
13	**20.00**	**13.10**
14	128.84	0.00
15	146.03	0.00
16	111.29	0.00
17	69.08	0.00
18	59.76	0.00
19	69.08	0.00
20	76.22	0.00

Table: Link, Equilibrium Link solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 2

Link a	f_a^*	θ_a^*
21	35.31	0.00
22	75.98	0.00
23	174.20	0.00
24	211.96	0.00
25	170.71	0.00
26	203.48	0.00
27	84.32	0.00
28	86.39	0.00
29	111.31	0.00
30	92.17	0.00
31	54.97	0.00
32	57.42	0.00
33	61.40	0.00
34	21.84	0.00
35	103.44	0.00
36	20.00	0.00
37	35.64	0.00
38	**19.84**	**13.60**

Numerical Examples: Example 2

Lagrange multipliers at equilibrium

None of the supply /donor upper bound constraints are binding, hence all η_j^*s are equal to 0.

Costs at equilibrium

Organization 1: **$31,685.55**. Organization 2: **$83,461.70**.

Revenues at equilibrium

Organization 1 : **$63,221.34**. Organization 2: **$102,772.48**.

Net revenue at equilibrium

Organization 1 : **$31,535.79**. Organization 2: **$19,310.78**.

Numerical Examples: Example 3

The network and data remain identical to the one in Example 1 except that we now consider a major disruption in the form of a disease causing number of eligible donors to decrease considerably.

The number of people eligible to donate blood in each of the regions in a week are:

$$S^1 = 500, S^2 = 220, S^3 = 120.$$

Equilibrium demands

Organization 1: $\mathbf{d}^*_{11} = \mathbf{77.39}$, $\mathbf{d}_{12} = \mathbf{23.21}$, $\mathbf{d}_{13} = \mathbf{116.17}$, $\mathbf{d}_{14} = \mathbf{45.68}$.

Organization 2: $\mathbf{d}^*_{21} = \mathbf{122.61}$, $\mathbf{d}_{22} = \mathbf{36.79}$, $\mathbf{d}_{23} = \mathbf{83.33}$, $\mathbf{d}_{24} = \mathbf{54.33}$.

Equilibrium prices

Organization 1:
$\rho_{11}(\mathbf{d}^*) = \mathbf{292.13}$, $\rho_{12}(\mathbf{d}^*) = \mathbf{307.04}$, $\rho_{13}(\mathbf{d}^*) = \mathbf{293.33}$, $\rho_{14}(\mathbf{d}^*) = \mathbf{277.09}$.
Organization 2:
$\rho_{21}(\mathbf{d}^*) = \mathbf{273.10}$, $\rho_{22}(\mathbf{d}^*) = \mathbf{286.50}$, $\rho_{23}(\mathbf{d}^*) = \mathbf{276.33}$, $\rho_{24}(\mathbf{d}^*) = \mathbf{266.37}$.

Numerical Examples: Example 3

Table: Link, Equilibrium Link solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 3

Link a	f_a^*	θ_a^*
1	237.60	0.00
2	33.49	0.00
3	237.60	0.00
4	33.49	0.00
5	271.09	0.00
6	262.96	0.00
7	196.94	0.00
8	3.38	0.00
9	23.21	0.00
10	96.67	0.00
11	45.68	0.00
12	46.01	0.00
13	**20.00**	**13.10**
14	186.51	0.00
15	68.06	0.00
16	51.94	0.00
17	86.42	0.00
18	100.09	0.00
19	35.42	0.00
20	32.64	0.00

Numerical Examples: Example 3

Table: Link, Equilibrium Link solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 3

Link a	f_a^*	θ_a^*
21	18.86	0.00
22	33.07	0.00
23	140.70	0.00
24	165.80	0.00
25	137.89	0.00
26	159.17	0.00
27	68.17	0.00
28	9.72	0.00
29	86.81	0.00
30	72.36	0.00
31	42.78	0.00
32	25.43	0.00
33	52.44	0.00
34	34.33	0.00
35	79.83	0.00
36	11.36	0.00
37	30.89	0.00
38	**20.00**	**13.60**

Numerical Examples: Example 3

Lagrange multipliers at equilibrium

- Due to decreased supply constraints for both Regions 2 and 3 are now tight. $\eta_1^* = 0, \quad \eta_2^* = 109.82, \quad \eta_3^* = 85.00.$
- Lagrange multipliers associated with the lower and upper bounds at the four demand points are:

$$\sigma_1^* = 107.14, \quad \sigma_2^* = 90.43, \quad \sigma_3^* = 110.02, \quad \sigma_4^* = 129.07,$$

$$\epsilon_1^* = 0.00, \quad \epsilon_2^* = 0.00, \quad \epsilon_3^* = 0.00, \quad \epsilon_4^* = 0.00.$$

Costs at equilibrium

Organization 1: **$50,978.78**. Organization 2: **$87,042.11**.

Revenues at equilibrium

Organization 1 : **$76,616.49**. Organization 2: **$81,522.08**.

Net revenue at equilibrium

Organization 1 : **$25,637.71**. Organization 2: -**$5,520.03**.

Numerical Examples: Example 4

This example is also based on Example 1 but in Example 4 we decrease capacities associated with BSO 2's testing and processing and storage links 24 and 26 due to a natural disaster. All other data remaining same, here we have $u_{24} = 200$ and $u_{26} = 200$.

Equilibrium demands

Organization 1: $\mathbf{d}^*_{11} = \mathbf{57.31}$, $\mathbf{d}_{12} = \mathbf{26.22}$, $\mathbf{d}_{13} = \mathbf{98.68}$, $\mathbf{d}_{14} = \mathbf{40.00}$.

Organization 2: $\mathbf{d}^*_{21} = \mathbf{142.69}$, $\mathbf{d}_{22} = \mathbf{66.50}$, $\mathbf{d}_{23} = \mathbf{101.32}$, $\mathbf{d}_{24} = \mathbf{60.00}$.

Equilibrium prices

Organization 1:
$\rho_{11}(\mathbf{d}^*) = \mathbf{293.13}$, $\rho_{12}(\mathbf{d}^*) = \mathbf{305.91}$, $\rho_{13}(\mathbf{d}^*) = \mathbf{294.05}$, $\rho_{14}(\mathbf{d}^*) = \mathbf{277.20}$.
Organization 2:
$\rho_{21}(\mathbf{d}^*) = \mathbf{272.29}$, $\rho_{22}(\mathbf{d}^*) = \mathbf{284.30}$, $\rho_{23}(\mathbf{d}^*) = \mathbf{275.97}$, $\rho_{24}(\mathbf{d}^*) = \mathbf{266.20}$.

Numerical Examples: Example 4

Table: Link, Equilibrium Link solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 4

Link a	f_a^*	θ_a^*
1	140.65	0.00
2	88.43	0.00
3	140.65	0.00
4	88.43	0.00
5	229.08	0.00
6	222.21	0.00
7	167.35	0.00
8	22.45	0.00
9	26.22	0.00
10	78.68	0.00
11	40.00	0.00
12	34.86	0.00
13	**20.00**	**5.71**
14	127.67	0.00
15	144.65	0.00
16	109.84	0.00
17	72.21	0.00
18	55.46	0.00
19	72.05	0.00
20	72.60	0.00

Numerical Examples: Example 4

Table: Link, Equilibrium Link solution, and Link Capacity Equilibrium Lagrange Multipliers for Example 4

Link a	f_a^*	θ_a^*
21	37.80	0.00
22	71.94	0.00
23	182.15	0.00
24	200.00	0.00
25	178.51	0.00
26	192.00	0.00
27	90.63	0.00
28	87.88	0.00
29	107.11	0.00
30	84.89	0.00
31	48.47	0.00
32	46.50	0.00
33	62.77	0.00
34	40.00	0.00
35	94.22	0.00
36	20.00	0.00
37	38.55	0.00
38	**20.00**	**21.38**

Numerical Examples: Example 4

Lagrange multipliers at equilibrium

- $\eta_1^* = 0, \quad \eta_2^* = 0, \quad \eta_3^* = 0.$
- Lagrange multipliers associated with the lower and upper bounds at the four demand points are:

$$\sigma_1^* = 4.11, \quad \sigma_2^* = 0, \quad \sigma_3^* = 9.08, \quad \sigma_4^* = 30.20,$$

$$\epsilon_1^* = 0.00, \quad \epsilon_2^* = 0.00, \quad \epsilon_3^* = 0.00, \quad \epsilon_4^* = 0.00.$$

Costs at equilibrium

Organization 1: **$33,706.32**. Organization 2: **$84,635.16**.

Revenues at equilibrium

Organization 1 : **$64,925.04**. Organization 2: **$101,693.17**.

Net revenue at equilibrium

Organization 1 : **$31,218.71**. Organization 2: **$17,058.01**.

Summary and Scope for Future Research

- We have **capacities on the links** representing the economic activities associated with blood supply chain networks.
- We have incorporated **upper bounds on donations in different regions**.
- We have added **lower bounds and upper bounds associated with the demand** for RBCs at the various demand points to ensure that each hospital or medical center has the minimum amount needed for a given week while also guaranteeing that waste will be reduced because of the upper bounds.
- The novel features of the competitive supply chain network game theory model result in a **Generalized Nash Equilibrium**.

Summary and Scope for Future Research

- **Lagrange multipliers** associated with **shared constraints are equal** among the competitors, thereby providing a nice **economic fairness interpretation**.
- Illustrative examples are used to demonstrate **impacts of plausible disruptions** on demand, price and net revenues generated by blood service organizations.
- Future work of interest includes modeling **cooperation** among blood banks in terms of their various supply chain network activities.

Thank you !

https://supernet.isenberg.umass.edu/visuals.html