Dynamics of Quality as a Strategic Variable in Complex Food Supply Chain Network Competition

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Outline

- Background and Motivation
- Preliminaries on Quality
- The Food Supply Chain Network Game Theory Model
- Relationship of the Model to Others in the Literature
- The Algorithm
- A Case Study on Peaches, Including Disruptions
- Summary and Conclusions

Background and Motivation

Food Supply Chains

Food is something anyone can relate to.

Fascinating Facts About Food Perishability

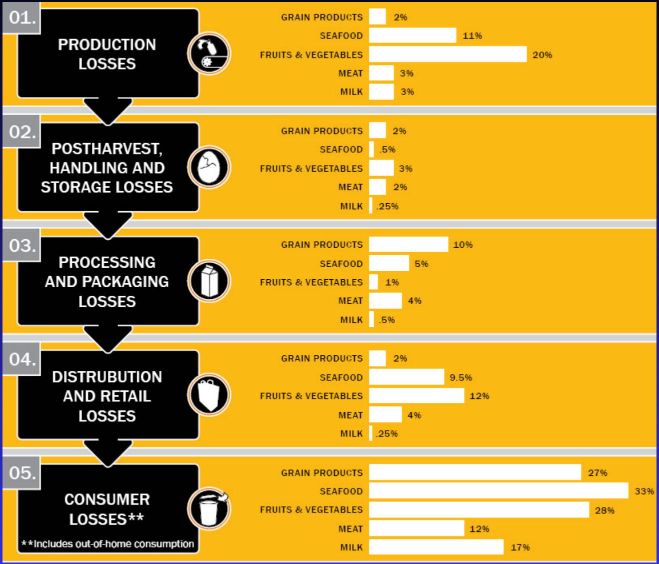
Source: Food and Agriculture Organization 2011

Fascinating Facts About Food Perishability

THE SHELF LIFE OF FOOD

Foods unopened, uncut or uncooked unless stated otherwise	COUNTER/PANTRY (1 DAY ◂——▸ 1 MONTH)	REFRIGERATOR (1 DAY ◂——▸ 3 MONTHS)	FREEZER (1 MONTH ◂——▸ 1 YEAR)
APPLES	2-4 weeks	1-2 months	8-12 months
BANANAS	2-7 days	5-9 days	2-3 months
CANTALOUPE	Until ripe	1 week	8-12 months
CARROTS	Up to 4 days	4-5 weeks	8-12 months
CUCUMBERS	1-3 days	1 week	8-12 months
EGGS	Few hours	3-4 weeks	Do not freeze
MILK	Few hours	5-7 days	1 month
YOGURT	Few hours	2-3 weeks	1-2 months

Fascinating Facts About Food Perishability



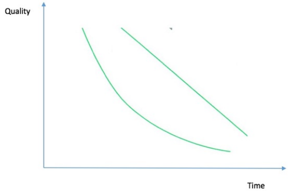
Source: Food and Agriculture Organization 2011

Background and Motivation

- **Food supply chains**, as noted in Yu and Nagurney (2013), are distinct from other product supply chains.
- Fresh produce is exposed to continuous and significant **change in the quality** of food products throughout the entire supply chain from the points of production/harvesting to points of demand/consumption.
- The quality of food products is **decreasing with time**, even with the use of advanced facilities and under the best **processing, handling, storage, and shipment** conditions (Sloof, Tijskens, and Wilkinson (1996) and Zhang, Habenicht, and Spieß (2003)).

Background and Motivation

- It has been discovered that the quality of fresh produce can be determined scientifically using **chemical formulae**, which include both **time** and **temperature**.
- The **initial quality** is also very important and food producers, such as farmers, have significant control over this important **strategic variable** at their production/harvesting sites.
- There are great opportunities for enhanced decision-making in this realm that can be supported by **appropriate models** and **methodological tools**.



Literature Review

- We note that early contributions focused on perishability and, in particular, on **inventory management** (see Ghare and Schrader (1963), Nahmias (1982, 2011) and Silver, Pyke, and Peterson (1998) for reviews).
- More recently, some studies have proposed integrating **more than a single supply chain network activity** (see, e.g., Zhang, Habenicht, and Spieß (2003), Widodo et al. (2006), Ahumada and Villalobos (2011), and Kopanos, Puigjaner, and Georgiadis (2012)).
- Yu and Nagurney (2013) have emphasized the need to bring greater realism to the underlying **economics** and **competition** on food supply chains.
- Additional modeling and methodological contributions in the **food supply chain** and **quality** domain have been made by Blackburn and Scudder (2009) and by Rong, Akkerman, and Grunow (2011).
- Besik and Nagurney (2017) formulated **short fresh produce supply chains** with the inclusion of the **dynamics of quality**, in the context of **farmers' markets**, while also capturing competition.

Contributions

- We construct a competitive supply chain network model for fresh produce under **oligopolistic competition** among the food firms, who are profit-maximizers.

- The firms have, as their **strategic variables**, not only the **product flows** on the pathways of their supply chain networks from the production/harvesting locations to the ultimate points of demand, but also the **initial quality** of the produce that they grow at their production locations.

- The consumers at the retail outlets (demand points), **differentiate the fresh produce** from the distinct firms and reflect their preferences through the **prices** that they are willing to pay which depend on quantities of the produce as well as **the average quality** of the produce associated with the firm and retail outlet pair(s).

- **Quality of the produce** reaching a destination node depends on its **initial quality** and **on the path** that it took with each particular path consisting of specific links, with particular characteristics of physical features of **time, temperature, etc.**.

Preliminaries on Quality

Preliminaries on Quality

Fresh foods **deteriorate** since they are biological products, and, therefore, **lose quality over time**. The rate of **quality deterioration** can be represented as a function of the microenvironment, the gas composition, the relative humidity, and the temperature (Taoukis and Labuza (1989)).

Labuza (1984) demonstrated that the quality of a food attribute, Q, over time t, which can correspond, depending on the fruit or vegetable, to **the color change**, **the moisture content**, **the amount of nutrition such as vitamin C**, or **the softening of the texture**, can be formulated via the differential equation:

$$\frac{\partial Q}{\partial t} = -kQ^n = -Ae^{(-E/RT)}Q^n, \tag{1}$$

where k is the reaction rate and is defined by the Arrhenius formula, $Ae^{(-E/RT)}$, A is a pre-exponential constant, T is the temperature, E is the activation energy, and R is the universal gas constant (cf. Arrhenius (1889)).

Preliminaries on Quality

If the reaction order n is zero, that is, $\frac{\partial Q}{\partial t} = -k$, and the **initial quality** is denoted by Q_0, we can quantify the **remaining quality** Q_t at time t (Tijskens and Polderdijk (1996)) according to:

$$Q_t = Q_0 - kt. \tag{2}$$

Examples of fresh produce that follow a reaction order of zero include **watermelons** and **spinach**.

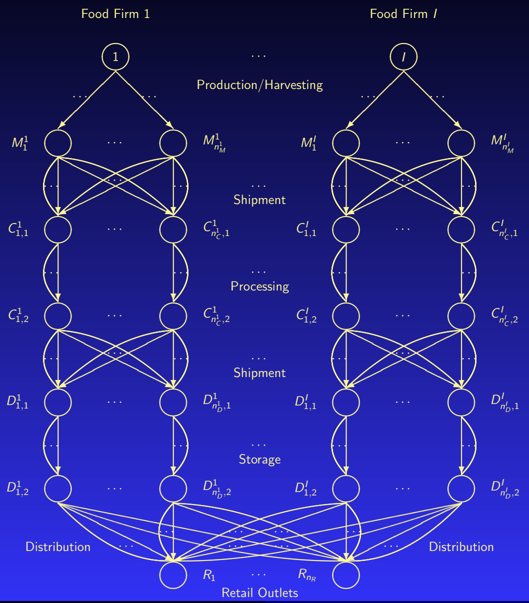
If the reaction order is 1, known as a *first order reaction*, the quality decay function is then given by the expression:

$$Q_t = Q_0 e^{-kt}. \tag{3}$$

Popular fruits that follow first order kinetics include **peaches**, and **strawberries**, as well as vegetables such as: **peas, beans**, **carrots**, **avocados**, and **tomatoes**.

The Food Supply Chain Network Game Theory Model

The Food Supply Chain Network Topology



Quality Over a Path

Let L^i denote the set of directed links in the supply chain network of food firm i; where $i = 1, \ldots, I$, which consists of **a set of production links**, L_1^i, and **a set of post-harvest links**, L_2^i, that is, $L^i \equiv L_1^i \cup L_2^i$.

Let β_b denote the **quality decay incurred on link** b, for $b \in L_2^i$, which is a factor that depends on the reaction order n, the reaction rate k_b, and the time t_b on link b, according to:

$$\beta_b \equiv \begin{cases} -k_b t_b, , & \text{if } n = 0, \forall b \in L_2^i, \forall i, \\ e^{-k_b t_b}, & \text{if } n = 1, \forall b \in L_2^i, \forall i, \end{cases} \tag{4}$$

where the reaction rate is

$$k_b = A e^{(-E/RT_b)}, \quad \forall b \in L_2^i, \forall i. \tag{5}$$

Quality Over a Path

We can have **multiple paths** from an origin node i to a destination node k, P^i_k denotes the set of all paths that have origin i and destination k.

The quality q_p, over a path p, joining the origin node i, with a destination node k, with the incorporation of the quality deterioration of the fresh produce, is:

$$q_p \equiv \begin{cases} \mathbf{q^i_{0a}} + \sum_{b \in p \cap L^i_2} \beta_b, & \text{if} \quad n = 0,\ p \in P^i_k, \forall i, k, \\ \\ q^i_{0a} \prod_{\mathbf{b \in p \cap L^i_2}} \beta_{\mathbf{b}}, & \text{if} \quad n = 1,\ p \in P^i_k, \forall i, k. \end{cases} \tag{6}$$

Here q^i_{0a} **is the initial quality** of the fresh produce on a top-most link a from an origin node i and in the path p under consideration.

The Food Supply Chain Network Model with Quality

Nonnegativity of the Path Flows

For each path p, joining an origin node i with a destination node k, the following nonnegativity condition must hold:

$$x_p \geq 0, \quad \forall p \in P^i_k;\ i = 1, \ldots, I;\ k = R_1, \ldots, R_{n_R}. \tag{7}$$

Nonnegativity of the Initial Quality Levels

The initial quality of the fresh produce on the top-most links a of an origin node i, must be nonnegative, that is:

$$q^i_{0a} \geq 0, \quad \forall a \in L^i_1;\ i = 1, \ldots, I. \tag{8}$$

Maximum Initial Quality Levels

We assume that the quality is bounded from above by a maximum value; hence, we have that:

$$q^i_{0a} \leq \bar{q}^i_{0a}, \quad \forall a \in L^i_1;\ i = 1, \ldots, I. \tag{9}$$

Link Flows

The conservation of flow equations that relate the link flows of each food firm i; $i = 1, \ldots, I$, to the path flows are given by:

$$f_l = \sum_{p \in P} x_p \delta_{lp}, \quad \forall l \in L^i; \; i = 1, \ldots, I, \tag{10}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise.

Link Capacities

Link flows must satisfy capacity constraints:

$$f_l \leq u_l, \quad \forall l \in L. \tag{11}$$

Capacities in Terms of Path Flows

In view of (10), we can rewrite (11) as:

$$\sum_{p \in P} x_p \delta_{lp} \leq u_l, \quad \forall l \in L. \tag{12}$$

The Food Supply Chain Network Model with Quality

Average Quality Levels

The average quality product of firm i, at retail outlet k, is given by:

$$\hat{q}_{ik} = \frac{\sum_{p \in P_k^i} q_p x_p}{\sum_{p \in P_k^i} x_p}, \quad i = 1, \ldots, I; \; k = R_1, \ldots, R_{n_R}. \tag{13}$$

Demands

The demand for food firm i's fresh food product at retail outlet k, d_{ik}, is equal to the sum of all the fresh produce flows on paths joining (i, k):

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad i = 1, \ldots, I; \; k = R_1, \ldots, R_{n_R}. \tag{14}$$

Demand Price Functions

The demand price of food firm i's product at retail outlet k is:

$$\rho_{ik} = \rho_{ik}(d, \hat{q}), \quad i = 1, \ldots, I; \; k = R_1, \ldots, R_{n_R}. \tag{15}$$

The Food Supply Chain Network Model with Quality

Costs of Production/Harvesting

The cost of production/harvesting at firm i's site a:

$$\hat{z}_a = \hat{z}_a(f_a, q^i_{0a}), \quad \forall a \in L^i_1;\ i = 1, \dots, I. \tag{16}$$

Operational Cost Functions

The operational cost functions associated with the remaining links in the supply chain network are:

$$\hat{c}_b = \hat{c}_b(f), \quad \forall b \in L^i_2;\ i = 1, \dots, I. \tag{17}$$

Vector of Path Flow Strategies

The vector of path flows of firm i; $i = 1, \dots, I$ is:

$$X_i \equiv \{\{x_p\} | p \in P^i\}\} \in R_+^{n_{P^i}}, P^i \equiv \cup_{k=R_1 \dots, R_{n_R}} P^i_k. \tag{18}$$

Vector of Initial Quality Strategies

The vector of initial quality levels of firm i; $i = 1, \dots, I$ is:

$$q^i_0 \equiv \{\{q^i_{0a}\} | a \in L^i_1\}\} \in R_+^{n_{L^i_1}}. \tag{19}$$

Utility Functions

The utility of firm i; $i = 1, \ldots, I$, is expressed as:

$$U_i = \sum_{k=R_1}^{R_{n_R}} \rho_{ik}(d, \hat{q}) d_{ik} - \Big(\sum_{a \in L_1^i} \hat{z}_a(f_a, q_{0a}^i) + \sum_{b \in L_2^i} \hat{c}_b(f)\Big). \tag{20}$$

Rewritten Demand Price Functions

In view of (6), (13), and (14), we can rewrite (15) as:

$$\hat{\rho}_{ik}(x, q_0) \equiv \rho_{ik}(d, \hat{q}), \quad i = 1, \ldots, I; \; k = R_1, \ldots, R_{n_R}. \tag{21}$$

Vector of the Profits

The I-dimensional vector $\hat{U}$ of profits of all firms i; $i = 1, \ldots, I$, is:

$$\hat{U} \equiv \hat{U}(X, q_0). \tag{22}$$

Definition: Food Supply Chain Network Nash Equilibrium

A fresh produce path flow pattern and initial quality level $(X^*, q_0^*) \in K = \prod_{i=1}^{I} K_i$ *constitutes a food supply chain network Nash Equilibrium if for each food firm* i*;* $i = 1, \ldots, I$*:*

$$\hat{U}_i(X_i^*, X_{-i}^*, q_0^{i*}, q_0^{-i*}) \geq \hat{U}_i(X_i, X_{-i}^*, q_0^i, q_0^{-i*}), \quad \forall (X_i, q_0^i) \in K_i, \quad (23)$$

where $X_{-i}^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$,
$q_0^{-i*} \equiv (q_0^{1*}, \ldots, q_0^{i-1*}, q_0^{i+1*}, \ldots, q_0^{I*})$, *and*
$K_i \equiv \{(X_i, q_0^i) | X_i \in R_+^{n_{P^i}}, q_0^i \in R_+^{n_{L_1^i}},$ (9) *and* (12) *hold for* $l \in L^i\}$.

Variational Inequality Formulation

An equilibrium is established if no food firm can unilaterally improve upon its profit by altering its product flows and initial quality at production sites in its supply chain network, given the product flows and initial quality decisions of the other firms.

Theorem: Variational Inequality Formulation
Assume that, for each food firm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X, q_0)$ is concave with respect to the variables X_i and q_0^i, and is continuously differentiable. Then $(X^*, q_0^*) \in K$ is a supply chain network Nash Equilibrium according to the Definition if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*, q_0^*), X_i - X_i^* \rangle - \sum_{i=1}^{I} \langle \nabla_{q_0^i} \hat{U}_i(X^*, q_0^*), q_0^i - q_0^{i*} \rangle \geq 0,$$

$$\forall (X, q_0) \in K, \qquad (24)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and ∇ denotes the gradient.

An Equivalent Variational Inequality Formulation

An equivalent VI is:

Determine $(x^*, q_0^*, \lambda^*, \gamma^*) \in K^1$ *satisfying:*

$$\sum_{i=1}^{I} \sum_{k=R_1}^{R_{n_R}} \sum_{p \in P_k^i} \left[\frac{\partial \hat{Z}^i(x^*, q_0^{i*})}{\partial x_p} + \frac{\partial \hat{C}^i(x^*)}{\partial x_p} + \sum_{l \in L^i} \gamma_l^* \delta_{lp} - \hat{\rho}_{ik}(x^*, q_0^*) - \sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^*, q_0^*)}{\partial x_p} \sum_{r \in P_j^i} x_r^* \right]$$

$$\times [x_p - x_p^*] + \sum_{i=1}^{I} \sum_{a \in L_1^i} \left[\frac{\partial \hat{Z}^i(x^*, q_0^{i*})}{\partial q_{0a}^i} + \lambda_a^* - \sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^*, q_0^*)}{\partial q_{0a}^i} \sum_{r \in P_j^i} x_r^* \right] \times [q_{0a}^i - q_{0a}^{i*}]$$

$$+ \sum_{i=1}^{I} \sum_{a \in L_1^i} [\bar{q}_{0a}^i - q_{0a}^{i*}] \times [\lambda_a - \lambda_a^*] + \sum_{i=1}^{I} \sum_{l \in L^i} \left[u_l - \sum_{r \in P} x_r^* \delta_{lr} \right] \times [\gamma_l - \gamma_l^*] \geq 0,$$

$$\forall (x, q_0, \lambda, \gamma) \in K^1, \qquad (25)$$

where $K^1 \equiv \{(x, q_0, \lambda, \gamma) | x \in R_+^{n_P},\ q_0 \in R_+^{n_{L_1}},\ \lambda \in R_+^{n_{L_1}},\ \gamma \in R_+^{n_L}\}$.

An Equivalent Variational Inequality Formulation

For each path p; $p \in P_k^i$; $i = 1, \ldots, I$; $k = R_1, \ldots, R_{n_R}$:

$$\frac{\partial \hat{Z}^i(x, q_0^i)}{\partial x_p} \equiv \sum_{a \in L_1^i} \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial f_a} \delta_{ap}, \tag{26a}$$

$$\frac{\partial \hat{C}^i(x)}{\partial x_p} \equiv \sum_{b \in L_2^i} \sum_{l \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_l} \delta_{lp}, \tag{26b}$$

$$\frac{\partial \hat{\rho}_{ij}(x, q_0)}{\partial x_p} \equiv \frac{\partial \rho_{ij}(d, \hat{q})}{\partial d_{ik}} + \frac{\partial \rho_{ij}(d, \hat{q})}{\partial \hat{q}_{ik}} \left(\frac{q_p}{\sum_{r \in P_k^i} x_r} - \frac{\sum_{r \in P_k^i} q_r x_r}{(\sum_{r \in P_k^i} x_r)^2} \right). \tag{26c}$$

For each a; $a \in L_1^i$; $i = 1, \ldots, I$,

$$\frac{\partial \hat{Z}^i(x, q_0^i)}{\partial q_{0a}^i} \equiv \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial q_{0a}^i}, \tag{26d}$$

$$\frac{\partial \hat{\rho}_{ij}(x, q_0)}{\partial q_{0a}^i} \equiv \sum_{h=R_1}^{R_{n_R}} \sum_{s \in P_h^i} \frac{x_s}{\sum_{r \in P_h^i} x_r} \frac{\partial \rho_{ij}(d, \hat{q})}{\partial \hat{q}_{ih}} \frac{\partial q_s}{\partial q_{0a}^i}. \tag{26e}$$

In particular, if link a is not included in path s, $\frac{\partial q_s}{\partial q_{0a}^i} = 0$; *if link a is included in path s, following (6), we have:*

$$\frac{\partial q_s}{\partial q_{0a}^i} = \begin{cases} 1, & \text{if } n = 0, \\ \prod_{b \in s \cap L_2^i} \beta_b, & \text{if } n = 1. \end{cases} \tag{26f}$$

Relationship of the Model to Others in the Literature

Relationship of the Model to Others in the Literature

The above model is now related to several models in the literature.

If quality is not a strategic variable and the product is not perishable, then the model is related to the sustainable fashion supply chain network model of Nagurney and Yu in the *International Journal of Production Economics* **135** (2012), pp 532-540. In that model, however, the other criterion, in addition to the profit maximization one, was emission minimization, rather than waste cost minimization, as in the model in this paper.

Relationship of the Model to Others in the Literature

If the demands are fixed, and there is a single organization, but there are additional processing tiers, as well as capacity investments as variables, along with arc multipliers for perishability, then the model is the medical nuclear supply chain design model of Nagurney and Nagurney, *International Journal of Production Economics* (2012).

Relationship of the Model to Others in the Literature

If the demands are fixed, and there is a single organization, but there are additional processing tiers, as well as capacity investments as variables, along with arc multipliers for perishability, then the model is the medical nuclear supply chain design model of Nagurney and Nagurney, *International Journal of Production Economics* (2012).

If there is only a single organization / firm, and the demands are subject to uncertainty, with the inclusion of expected costs due to shortages or excess supplies, the total operational cost functions are separable, and a criterion of risk is added, then the model above is related to the blood supply chain network operations management model of Nagurney, Masoumi, and Yu, *Computational Management Science* (2012).

Relationship of the Model to Others in the Literature

If the product is homogeneous, and there is no quality and associated deterioration, and the total costs are assumed to be separable, then the above model collapses to the supply chain network oligopoly model of Nagurney (2010) in which synergies associated with mergers and acquisitions were assessed.

The Original Supply Chain Network Oligopoly Model

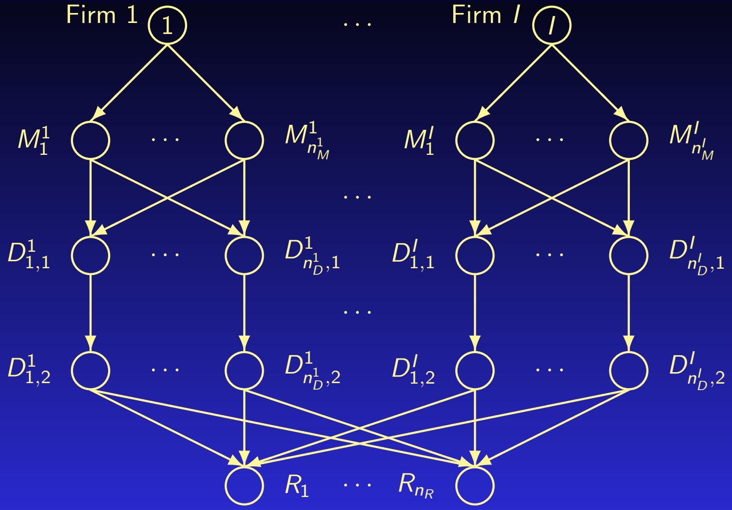


Figure: Supply Chain Network Structure of the Oligopoly Without Perishability; Nagurney, *Computational Management Science* **7**(2010), pp 377-401.

Mergers Through Coalition Formation

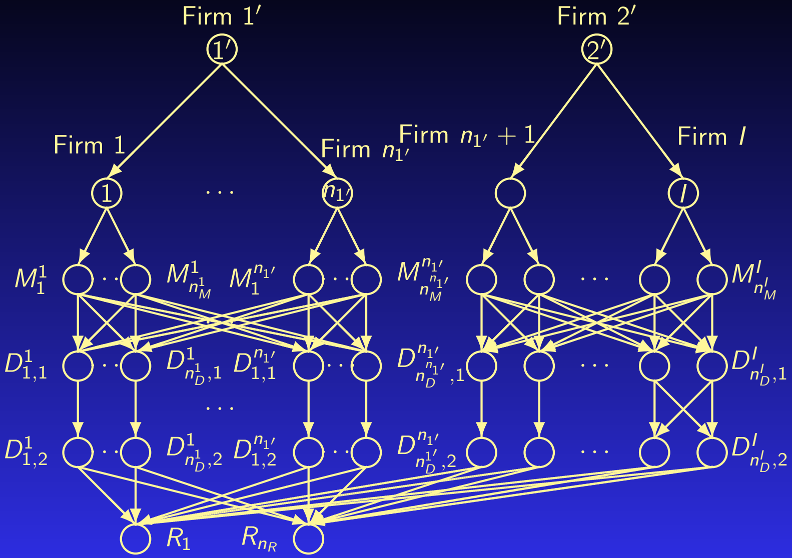


Figure: Mergers of the First $n_{1'}$ Firms and the Next $n_{2'}$ Firms

The Algorithm

The Algorithm - Euler Method

The Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993), is applied to this model.

Specifically, iteration τ of the Euler method applied to solve the VI problem in standard form is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})),$$

The Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

The Algorithm

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \rightarrow 0$, as $\tau \rightarrow \infty$.

Conditions for convergence of this scheme as well as various applications to the solutions of network oligopolies can be found in Nagurney and Zhang (1996), Nagurney, Dupuis, and Zhang (1994), Nagurney (2010a), and Nagurney and Yu (2011).

The Euler Method Explicit Formulae at Iteration $\tau + 1$

For each path $p \in P^i_j$, $\forall i, j$, compute:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + \alpha_\tau(\hat{\rho}_{ik}(x^\tau, q_0^\tau) + \sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^\tau, q_0^\tau)}{\partial x_p} \sum_{r \in P^i_j} x_r^\tau$$

$$- \frac{\partial \hat{Z}^i(x^\tau, q_0^{i\tau})}{\partial x_p} - \frac{\partial \hat{C}^i(x^\tau)}{\partial x_p} - \sum_{l \in L^i} \gamma_l^\tau \delta_{lp})\}.$$

For each initial quality level $a \in L^i_1$, $\forall i$, in turn, compute:

$$q_{0a}^{i\tau+1} = \max\{0, q_{0a}^{i\tau} + \alpha_\tau(\sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^\tau, q_0^\tau)}{\partial q_{0a}^i} \sum_{r \in P^i_j} x_r^\tau - \frac{\partial \hat{Z}^i(x^\tau, q_0^{i\tau})}{\partial q_{0a}^i} - \lambda_a^\tau)\}.$$

The Euler Method Explicit Formulae at Iteration $\tau + 1$

The Lagrange multiplier for each top-most link $a \in L_1^i$; $i = 1, \ldots, I$, associated with the initial quality bounds is computed as:

$$\lambda_a^{\tau+1} = \max\{0, \lambda_a^{\tau} + \alpha_\tau(q_{0a}^{i\tau} - \bar{q}_{0a}^{i})\}.$$

The Lagrange multiplier for each link $l \in L^i$; $i = 1, \ldots, I$, associated with the link capacities is computed according to:

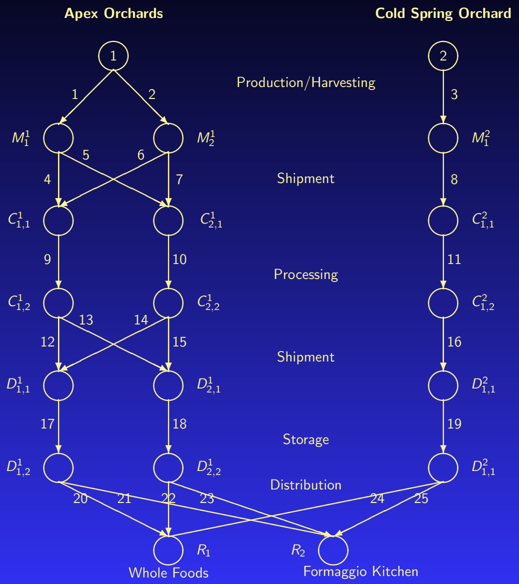
$$\gamma_l^{\tau+1} = \max\{0, \gamma_l^{\tau} + \alpha_\tau(\sum_{r \in P} x_r^{\tau} \delta_{lr} - u_l)\}.$$

A Case Study on Peaches, Including Disruptions

A Case Study on Peaches

- We focus on the **peach market** in the United States, specifically in **Western Massachusetts**.
- It is noted that, in 2015, the United States peach production was **825,415 tons** in volume, and 606 million dollars in worth (USDA NASS (2016), Zhao et al. (2017)).
- We selected two orchards from Western Massachusetts: **Apex Orchards** and **Cold Spring Orchard**, located, respectively, in Shelburne, MA and Belchertown, MA.
- The orchards sell their peaches to two retailers, **Whole Foods**, located in Hadley, MA, and **Formaggio Kitchen**, located in Cambridge, MA.
- The mode of **transportation** for both of the orchards is **trucks**.
- The **color change attribute** of peaches, in the form of **browning**, follows **a first-order**, that is, an **exponential decay function**.

Supply Chain Network Topology for the Case Study



Parameters for the Calculation of Quality Decay

Link b	Hours	Temperature (Celsius)	β_b $(n = 1)$
4	1	23	0.9961
5	2	23	0.9922
6	2	23	0.9922
7	1	23	0.9961
8	2	27	0.9913
9	3	18	0.9906
10	3	18	0.9906
11	4	25	0.9836
12	1	23	0.9961
13	2	23	0.9922
14	2	23	0.9922
15	1	23	0.9961
16	3	27	0.9870
17	48	1	1.0000
18	72	1	1.0000
19	96	18	0.7397
20	2	27	0.9913
21	4	27	0.9827
22	1	27	0.9956
23	4	27	0.9827
24	0.5	27	0.9978
25	4	27	0.9827

Cost Functions, Capacities and Upper Bounds for the Numerical Examples

Table: Total Production / Harvesting Cost Functions, Link Capacities, and Upper Bounds on Initial Quality

Link a	$\hat{z}_a(f_a, q^i_{0a})$	u_a	$\bar{q}^i_{0a}$
1	$.002f_1^2 + f_1 + 0.7q^1_{01} + .01(q^1_{01})^2$	200	98
2	$.002f_2^2 + f_2 + 0.7q^1_{02} + .01(q^1_{02})^2$	200	95
3	$.002f_3^2 + f_3 + 0.5q^2_{03} + .001(q^1_{03})^2$	150	90

Table: Total Operational Link Cost Functions and Link Capacities

Link b	$\hat{c}_b(f)$	u_b
4	$.001f_4^2 + .7f_4$	150
5	$.002f_5^2 + .7f_5$	150
6	$.001_6^2 + .5f_6$	120
7	$.002f_7^2 + .5f_7$	120
8	$.002f_8^2 + .9f_8$	100
9	$.0025f_9^2 + 1.2f_9$	200
10	$.0025f_{10}^2 + 1.2_{10}$	200
11	$.0026f_{11}^2 + 1.5f_{11}$	150
12	$.001f_{12}^2 + .6f_{12}$	150
13	$.002f_{13}^2 + .6f_{13}$	150
14	$.001f_{14}^2 + .6f_{14}$	150
15	$.002f_{15}^2 + .6f_{15}$	150
16	$.002f_{16}^2 + .6f_{16}$	120
17	$.003f_{17}^2 + .5f_{17}$	150
18	$.0037f_{18}^2 + .9f_{18}$	150
19	$.002f_{19}^2 + .7f_{19}$	120
20	$.002f_{20}^2 + .6f_{20}$	150
21	$.003f_{21}^2 + .7f_{21}$	120
22	$.002f_{22}^2 + .6f_{22}$	150
23	$.003f_{23}^2 + .7f_{23}$	100
24	$.002f_{24}^2 + .6f_{24}$	100
25	$.003f_{25}^2 + .7f_{25}$	100

- We report the total **production / harvesting cost functions**,**the upper bounds on the initial quality**, the total operational cost functions, and the link flow capacities.
- The Euler method is implemented in FORTRAN and a Linux system at the University of Massachusetts used for the computations.
- The data is gathered from Sumner and Murdock (2017) and Dris and Jain (2007), in which the authors made a sample cost analysis.
- The **time horizon**, under consideration, is that of **a week**.
- The Euler method is implemented in FORTRAN and a Linux system at the University of Massachusetts used for the computations.
- The sequence is, $a_\tau = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \ldots\}$, with the convergence tolerance being 10^{-7}.

Example 1 - Baseline

- It is known that both retailers sell **high quality food** products, with **Formaggio Kitchen** selling peaches at a higher price due to its emphasis on **quality**.
- Through conversations at the retailers, we concluded that **Apex Orchards** sell their **peaches** at a higher price.

  Demand Price Functions of Apex Orchards:

$$\rho_{11} = -.02d_{11} - .01d_{21} + 0.008\hat{q}_{11} + 20,$$

$$\rho_{12} = -.02d_{12} - .01d_{22} + 0.01\hat{q}_{12} + 22.$$

  Demand Price Functions of Cold Spring Orchard:

$$\rho_{21} = -.02d_{21} - .015d_{11} + 0.008\hat{q}_{21} + 18,$$

$$\rho_{22} = -.02d_{22} - .015d_{12} + 0.01\hat{q}_{22} + 19.$$

Equilibrium Flows, Equilibrium Initial Quality, and the Equilibrium Lagrange Multipliers

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	133.43	97.54	0.00	0.00
2	**166.57**	**95.00**	**0.00**	**0.05**
3	100.00	65.61	0.00	0.00

Table: Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

Table: Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

Link b	f_b^*	γ_b^*
4	69.31	0.00
5	64.12	0.00
6	90.33	0.00
7	76.24	0.00
8	**100.00**	**6.53**
9	159.64	0.00
10	140.36	0.00
11	100.00	0.00
12	81.96	0.00
13	77.68	0.00
14	68.04	0.00
15	72.32	0.00
16	100.00	0.00
17	**150.00**	**6.95**
18	**150.00**	**6.19**
19	100.00	0.00
20	64.84	0.00
21	85.16	0.00
22	65.10	0.00
23	84.90	0.00
24	47.80	0.00
25	52.20	0.00

Equilibrium Prices, Demands, Average Quality, and Profits

Equilibrium Prices at the Demand Markets:

$$\rho_{11} = 17.67, \ \boldsymbol{\rho_{12} = 19.00}, \ \rho_{21} = 15.47, \ \rho_{22} = 15.86.$$

Equilibrium Demands:

$$d^*_{11} = 129.95, \ \mathbf{d^*_{12} = 170.05}, \ d^*_{21} = 47.80, \ d^*_{22} = 52.20.$$

Average Quality:

$$\hat{\mathbf{q}}_{\mathbf{11}} = \mathbf{93.40}, \ \hat{q}_{12} = 92.56, \ \hat{q}_{21} = 46.60, \ \hat{q}_{22} = 45.90.$$

Profits:

$$\mathbf{U_1 = 3,302.01}, \quad U_2 = 787.65.$$

Sensitivity Analysis and Insights

The orchards may wish to invest in enhancing their capacity with Apex Orchards focusing on the storage facilities and Cold Spring Orchard on its freight shipment capacity.

- **When we raised u_8 to 150, while keeping all the other data as above, the profit of Cold Spring Orchard increased to 921.74** whereas that of Apex Orchards (because of the competition) decreased to 3,272.11.

- **When we raised both u_{17} and u_{18} to 200 and kept all the other data as in Example 1 above, then the profit enjoyed by Apex Orchards increased to 3,884.80** and that of Cold Spring Orchard decreased to 696.87.

Sensitivity Analysis and Insights

- Finally, **we had both orchards make investments so that $u_8 = 150$ and u_{17} and u_{18} were equal to 200. The profit of Apex Orchards was now 3,844.89 and that of Cold Spring: 815.37**. Both firms gain as compared to the profit values in Example 1. The demand prices are now lower but the average quality higher with $\rho_{11} = 16.54$, $\rho_{12} = 17.94$, $\rho_{21} = 14.64$, and $\rho_{22} = 15.10$, and $\hat{q}_{11} = 93.79$, $\hat{q}_{12} = 92.92$, $\hat{q}_{21} = 63.93$, and $\hat{q}_{22} = 67.96$.

By investing in supply chain infrastructure both producers and consumers gain.

Example 2 - Disruption Scenario 1

- We now consider a **disruption scenario** in which a **natural disaster** has significantly affected the **capacity** of the orchard production sites of both orchards.
- Such an incident occurred in **2016 in the Northeast of the United States** when extreme weather in terms of cold temperatures "decimated" the **peach crop**.
- We now have the following capacities on the production/harvesting links:

$$u_1 = 100, \; u_2 = 150, \; u_3 = 80.$$

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	**100.00**	75.54	**8.17**	0.00
2	**150.00**	75.54	**8.18**	0.00
3	**80.00**	11.02	**7.78**	0.00

Table: Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

Link b	f_b^*	γ_b^*
4	50.28	0.00
5	49.72	0.00
6	82.76	0.00
7	67.24	0.00
8	80.00	0.00
9	133.04	0.00
10	116.96	0.00
11	80.00	0.00
12	80.78	0.00
13	52.25	0.00
14	69.22	0.00
15	47.75	0.00
16	80.00	0.00
17	**150.00**	**0.17**
18	100.00	0.00
19	80.00	0.00
20	67.2	0.00
21	82.79	0.00
22	37.46	0.00
23	62.54	0.00
24	37.67	0.00
25	42.33	0.00

Equilibrium Prices, Demands, Average Quality, and Profits

Equilibrium Prices at the Demand Markets:

$$\rho_{11} = 18.12, \ \boldsymbol{\rho_{12} = 19.40}, \ \rho_{21} = 15.74, \ \rho_{22} = 16.05.$$

Equilibrium Demands:

$$d^*_{11} = 104.67, \ \mathbf{d^*_{12} = 145.33}, \ d^*_{21} = 37.67, \ d^*_{22} = 42.33.$$

Average Quality:

$$\hat{\mathbf{q}}_{\mathbf{11}} = \mathbf{73.42}, \ \hat{q}_{12} = 72.68, \ \hat{q}_{21} = 7.83, \ \hat{q}_{22} = 7.71.$$

Profits:

$$\mathbf{U_1 = 2,984.07}, \quad U_2 = 675.72.$$

Insights

Observe from the above equilibrium solution that all the production sites are now at their capacities and, hence, the corresponding link Lagrange multipliers are all positive.

Also, observe that the average quality of each orchard's peaches has decreased at each retailer, as compared to the results for Example 1. The demand prices have increased but more for the peaches of Apex Orchards than those from Cold Spring Orchard.

The profit is reduced for both orchards because of the limitations on how many pecks of peaches they can produce and harvest due to the disruption caused by the natural disaster.

Example 3 - Disruption Scenario 2

- We consider a **disruption** that affects **transportation** in that the links 5 and 6 associated with the supply chain network of **Apex Orchards** are no longer available.
- This can occur and has occurred in western Massachusetts as a result of flooding.
- We now have the following capacities on those links:

$$u_5 = 0, \;\; u_6 = 0.$$

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	150.00	84.50	0.00	0.00
2	120.00	84.50	0.00	0.00
3	100.00	65.59	0.00	0.00

Table: Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

Link b	f_b^*	γ_b^*
4	150.00	6.94
5	**0.00**	**78.33**
6	**0.00**	**79.92**
7	**120.00**	**7.27**
8	**100.00**	**6.75**
9	150.00	0.00
10	120.00	0.00
11	100.00	0.00
12	85.36	0.00
13	64.64	0.00
14	64.64	0.00
15	55.36	0.00
16	100.00	0.00
17	**150.00**	**0.40**
18	**120.00**	**6.19**
19	100.00	0.00
20	66.22	0.00
21	83.78	0.00
22	48.52	0.00
23	71.48	0.00
24	47.86	0.00
25	52.14	0.00

Equilibrium Prices, Demands, Average Quality, and Profits

Equilibrium Prices at the Demand Markets:

$$\rho_{11} = 17.89, \; \boldsymbol{\rho_{12} = 19.19}, \; \rho_{21} = 15.69, \; \rho_{22} = 16.09.$$

Equilibrium Demands:

$$d^*_{11} = 114.74, \; \mathbf{d^*_{12} = 155.26}, \; d^*_{21} = 47.86, \; d^*_{22} = 52.14.$$

Average Quality:

$$\hat{\mathbf{q}}_{\mathbf{11}} = \mathbf{82.32}, \; \hat{q}_{12} = 81.46, \; \hat{q}_{21} = 46.59, \; \hat{q}_{22} = 45.88.$$

Profits:

$$\mathbf{U_1 = 3,074.72}, \quad U_2 = 811.35.$$

Sensitivity Analysis and Insights

Apex Orchards farm experiences a loss in profits, whereas its competitor, Cold Spring Orchards, garners a higher profit, as compared to the baseline Example 1.

Both orchards raise their prices and the average quality of their produce drops although much more significantly for Apex Orchards, which has suffered a supply chain disruption in terms of transportation/shipment possibilities.

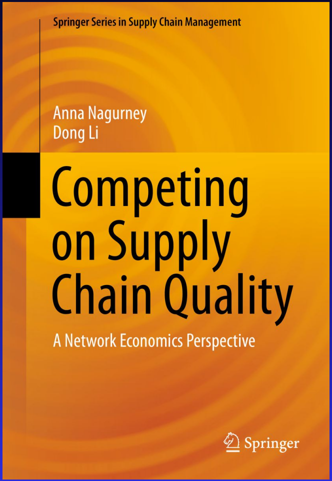
Sensitivity Analysis and Insights

We then addressed the following questions: What would be the impact on profits if only link 5 was restored to its original capacity of 150 (and link 6 remained unavailable)? What would be the impact on profits if only link 6 was restored to its original capacity of 120 (and link 5 remained unavailable)?

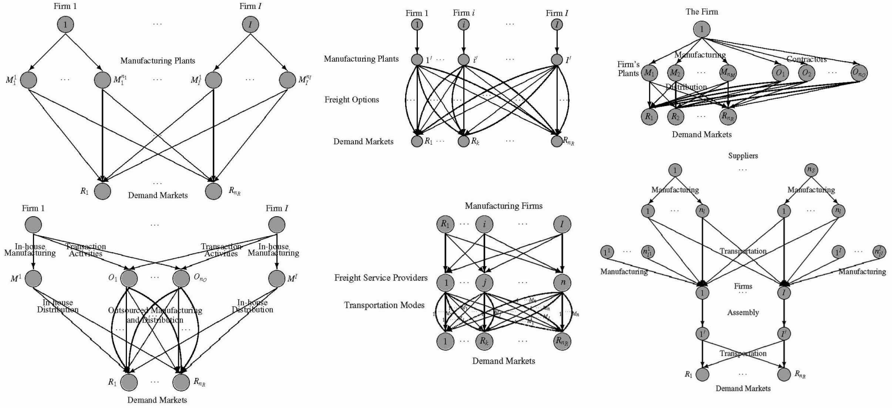
The profit of Apex Orchards was 3,272.78 with link 5 restored only and that of Cold Spring Orchard was: 787.64. On the other hand, if link 6 was restored only, then Apex Orchards garnered 3,283.32 in profit and Cold Spring Orchard 787.67 in profit.

Given the choice, Apex Orchards should advocate for restoration of link 6 versus link 5 if only one link restoration is feasible.

Our Recent Book



In the book, we present supply chain network models and tools to investigate, amongst other topics, information asymmetry, impacts of outsourcing on quality, minimum quality standards, applications to industries such as pharma and high tech, freight services and quality, and the identification of which suppliers matter the most to both individual firms' supply chains and to that of the supply chain network economy.



Summary and Conclusions

- We emphasized the *importance of product perishability*.

- We developed a *competitive supply chain network model* focused on food with initial quality and path flows as strategic variables and with explicit formulae for quality deterioration.

- The model was formulated and solved as a variational inequality problem.

- We also related the model to several others in the literatures with applications ranging from medical nuclear supply chains to blood supply chains.

- The framework *can be applied in numerous situations*, with some minor modifications, to capture oligopolistic competition for perishable and time-sensitive products.

THANK YOU!

For more information, see: http://supernet.isenberg.umass.edu