Design of Sustainable Supply Chain Networks for Sustainable Cities

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- Supply Chains
- Methodology
- Applied Supply Chain Network Game Theory Models of Relevance to Sustainable Cities
- The Sustainable Supply Chain Network Model with Frequency of Activities
- Numerical Examples
- An Extension to Capture Competition
- Summary and Conclusions

Background and Motivation

What is Sustainability?

The general definition of sustainability is development that meets the needs of the present without compromising the ability of future generations to meet their own needs (World Commission Environment and Development (WCED) (1987)).

The Debates Continue

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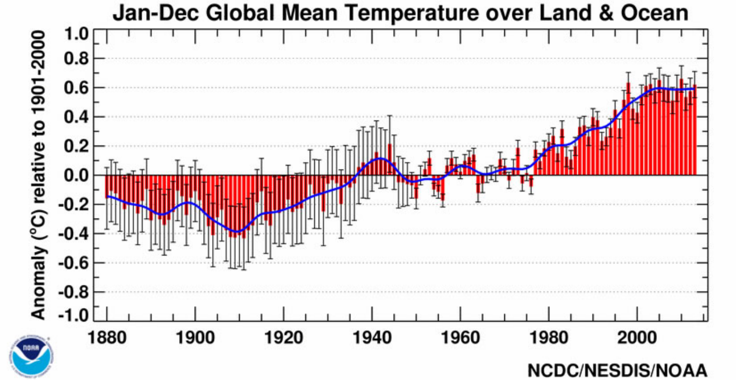
- What *resources* will future generations require?
- What *level of emissions* can be released without negatively affecting future generations?
- What *policies* are required to achieve sustainability?
- What are the effects of *market forces*, etc.?

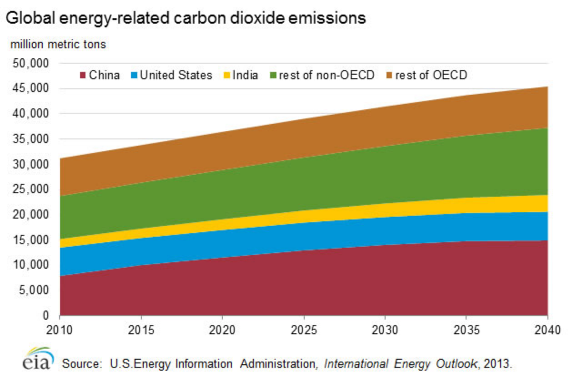
Climate Change



In the Northern Hemisphere, where most of Earth's land mass is located, the three decades from 1983 to 2012 were likely the warmest 30-year period of the last 1400 years, according to the IPCC.

Pollution and Environmental Impacts

In the US alone, greenhouse gas emissions are projected to rise 35% between 2005 and 2030 due to fewer forests and agricultural land to absorb the carbon, an increasing population, expansion of the economy, and an increased use of fossil fuel powered power plants to generate energy (Creyts et al. (2007)).



Cities and Sustainability

Cities, *as dynamic complex networks, are the systems in which more people now live than don't* and which represent the economic engines for commerce, research and development, education, health care, and even culture.

Cities have evolved over space and time on built infrastructure from transportation networks to telecommunication and electric power networks.

At the same time, cities are *the centers of resource usage* from electricity and other forms of energy and fuel, to food, water, and a plethora of other products.

Cities and Population

Today, 54% of the world's population lives in urban areas, a proportion that is expected to increase to 66% by 2050, the Population Division of the UN Department of Economic and Social Affairs reports in its 2014 revision of the World Urbanization Prospects report.

Some Examples of Cities

Cities and Sustainability

Cities are also *the repositories and generators of waste output and other environmental pollutants, such as carbon and other emissions, sewage, noise, etc.*

Some Negative Externalities Associated with Cities

Cities and Supply Chains

Cities are supplied by a complex array of supply chains servicing an immense spectrum of economic activities from food stores and restaurants, office supplies and high tech equipment, apparel, construction materials, as well as raw materials, to name just a few.

Supply Chains

Supply Chains

Supply chains are the *critical infrastructure and backbones* for the production, distribution, and consumption of goods as well as services in our globalized *Network Economy*.

Supply chains, in their most fundamental realization, *consist of manufacturers and suppliers, distributors, retailers, and consumers at the demand markets*.

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Today, supply chains may span thousands of miles across the globe, involve numerous suppliers, retailers, and consumers, and be underpinned by multimodal transportation and telecommunication networks.

Characteristics of Supply Chains and Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*, which leads to nonlinearities;
- *alternative behavior of users of the networks*, which may lead to paradoxical phenomena (Braess paradox);
- *possibly conflicting criteria associated with optimization*;
- *interactions among the underlying networks themselves*, such as the Internet with electric power networks, financial networks, and transportation and logistical networks;
- recognition of *their fragility and vulnerability*;
- policies surrounding networks today may have major impacts not only economically, but also *environmentally, socially, politically, and security-wise*.

Supply Chains Are Network Systems

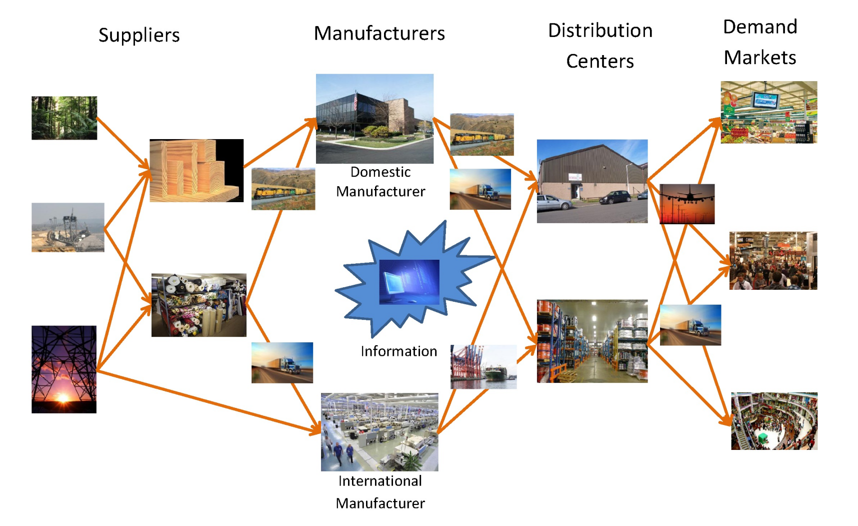
Supply chains are, in fact, *Complex Network Systems*.

Hence, *any formalism that seeks to model supply chains and to provide quantifiable insights and measures must be a system-wide one and network-based.*

Such crucial issues as the stability and resiliency of supply chains, as well as their adaptability and responsiveness to events in *a global environment of increasing risk and uncertainty* can only be rigorously examined from the view of supply chains as network systems.

Supply chains may be characterized by *decentralized decision-making* associated with the different economic agents or by *centralized* decision-making.

A General Supply Chain



Examples of Supply Chains

- food and food products
- high tech products
- automotive
- energy (oil, electric power, etc.)
- clothing and toys
- humanitarian relief
- healthcare supply chains.

Examples of Supply Chains

Sustainability of supply chains is, hence, a precursor to the sustainability of our cities. According to a Business for Social Responsibility (2009) paper, it is now widely acknowledged that making significant progress on mitigating the impact of climate change depends on reducing the negative environmental impacts of supply chains through their redesign and enhanced management (see also McKinsey Quarterly (2008)).

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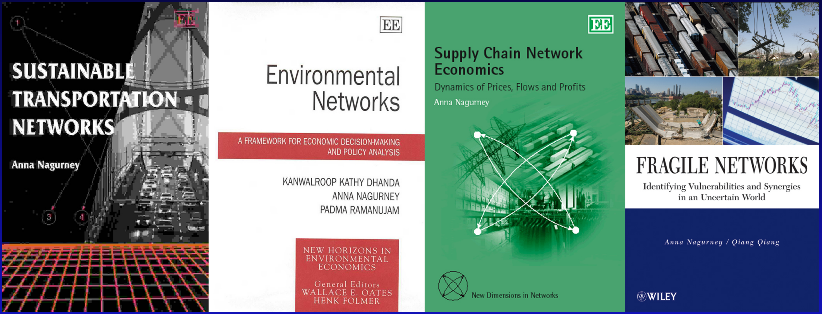
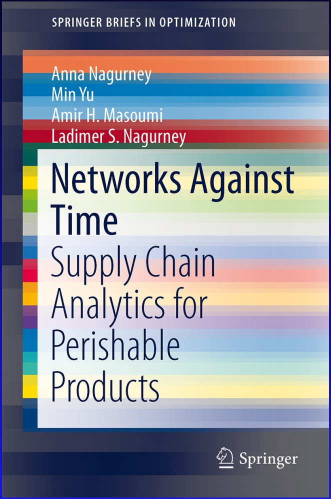
As noted by Capgemini in its 2008 report: *2016: Future Supply Chain*, *"Preserving energy and raw materials and other resources like water will become a crucial aspect in future supply chains, as costs will likely remain volatile and supplies will continue to dwindle."* These conditions may well create substantial pressure on current supply chain models.

Our Approach to Sustainability



Our Most Recent Book



Why More Research is Needed

Although the importance of sustainable supply chains to the sustainability of cities is being increasingly recognized (cf. Grant Thornton (2011)), in terms of not only the enhancement of business processes in terms of efficiency and cost reduction but also the reduction of negative environmental externalities as well as waste *there have been only limited modeling efforts that capture supply chains within a cities framework*.

Why More Research is Needed

Although the importance of sustainable supply chains to the sustainability of cities is being increasingly recognized (cf. Grant Thornton (2011)), in terms of not only the enhancement of business processes in terms of efficiency and cost reduction but also the reduction of negative environmental externalities as well as waste *there have been only limited modeling efforts that capture supply chains within a cities framework*.

Models of sustainable supply chains are important since *they enable the evaluation (before expensive investments are actually made) as to alternative network designs, technologies, as well as sensitivities to cost and demand structures*.

Why More Research is Needed

The edited volume of Taniguchi and Thompson (2004), which focuses on logistics systems for sustainable cities, *emphasized the unique features of urban logistical systems, which may include more frequent freight shipments and deliveries, with the concomitant negative externalities*.

Why More Research is Needed

Geroliminis and Daganzo (2005) further emphasize that the environmental impacts of logistical activities are most severe where population densities are highest, that is, in cities.

They have identified *innovative practices of cities around the globe in terms of their logistics systems and sustainability*, including the use of alternative modes of transportation, such as, for example, even bicycles for deliveries in Amsterdam and electric trams in Gothenburg.

Methodology

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized supply chain network problems.

Definition: The Variational Inequality Problem
The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$*, is to determine a vector* $X^* \in \mathcal{K}$*, such that:*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F *is a given continuous function from* $\mathcal{K}$ *to* R^N*,* $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* R^N*.*

Methodology - The Variational Inequality Problem

The vector X consists of the decision variables – typically, the flows (products, prices, etc.).

$\mathcal{K}$ is the feasible set representing how the decision variables are constrained – for example, the flows may have to be nonnegative; budget constraints may have to be satisfied; similarly, quality and/or time constraints may have to be satisfied.

The function F that enters the variational inequality represents functions that capture the behavior in the form of the functions such as costs, profits, risk, etc.

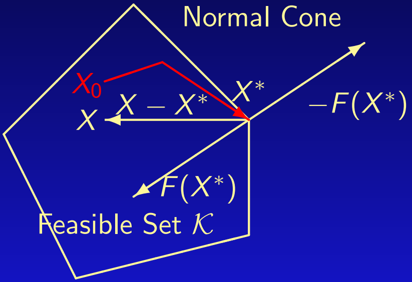
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- game theory problems, operating under Nash equilibrium,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of supply chain network problems from centralized to decentralized ones as well as to design problems.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System (Dupuis and Nagurney (1993), Nagurney and Zhang (1996))

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides natural underlying dynamics associated with travel (and other) behavior to the equilibrium.

To model the *dynamic behavior of complex networks*, including supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in

evolutionary games (Sandholm (2005, 2011)),

ecological predator-prey networks (Nagurney and Nagurney (2011a, b)), and

even *neuroscience* (Girard et al. (2008)).

Applied Supply Chain Network Game Theory Models of Relevance to Sustainable Cities

Electric Power Supply Chains

We developed *an empirical, large-scale electric supply chain network equilibrium model*, formulated it as a VI problem, and were able to solve it by *exploiting the connection between electric power supply chain networks and transportation networks* using our proof of a hypothesis posed in the classic book, *Studies in the Economics of Transportation*, by Beckmann, McGuire, and Winsten (1956).

The paper, "An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England," by Z. Liu and A. Nagurney was published in *Naval Research Logistics* **56** (2009) pp 600-624;

http://supernet.isenberg.umass.edu/articles/LiuNagurneyNRL.pdf

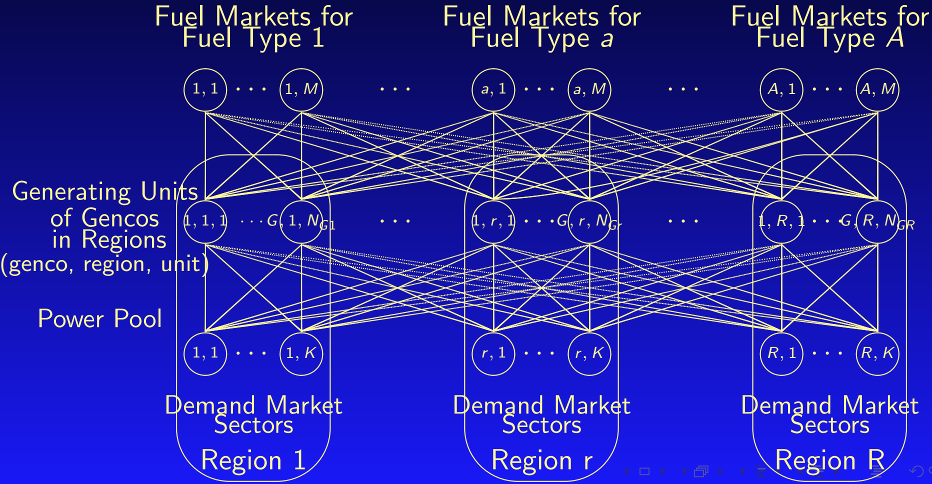
An Empirical Example of an Electric Power Supply Chain for New England

There are 82 generating companies who own and operate 573 generating units. We considered 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal. The whole area was divided into 10 regions:

1. Maine,
2. New Hampshire,
3. Vermont,
4. Connecticut (excluding Southwest Connecticut),
5. Southwestern Connecticut (excluding the Norwalk-Stamford area),
6. Norwalk-Stamford area,
7. Rhode Island,
8. Southeastern Massachusetts,
9. Western and Central Massachusetts,
10. Boston/Northeast Massachusetts.

Graphic of New England

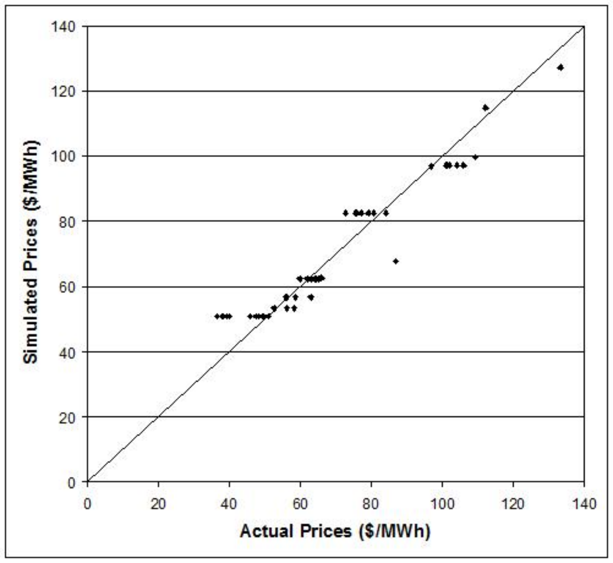
The Electric Power Supply Chain Network with Fuel Supply Markets



We tested the model on the data of July 2006 which included $24 \times 31 = 744$ hourly demand/price scenarios. We sorted the scenarios based on the total hourly demand, and constructed the load duration curve. We divided the duration curve into 6 blocks ($L_1 = 94$ hours, and $L_w = 130$ hours; $w = 2, ..., 6$) and calculated the average regional demands and the average weighted regional prices for each block.

The empirical model had on the order of $20,000$ *variables.*

Actual Prices Vs. Simulated Prices ($/Mwh)



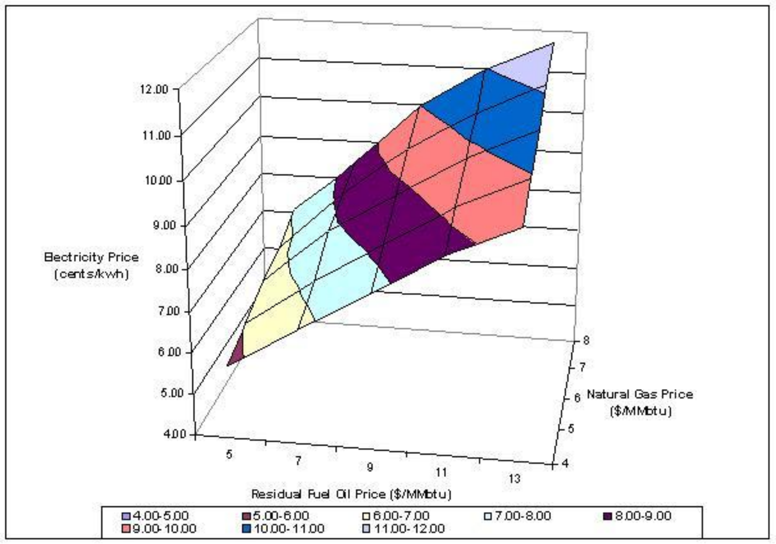
Sensitivity Analysis

We used the same demand data, and then varied the prices of natural gas and residual fuel oil. We assumed that the percentage change of distillate fuel oil and jet fuel prices were the same as that of the residual fuel oil price.

The next figure presents the average electricity price for the two peak blocks under oil/gas price variations.

The surface in the figure represents the average peak electricity prices under different natural gas and oil price combinations.

Sensitivity Analysis



Food Supply Chains

Food is something anyone can relate to.

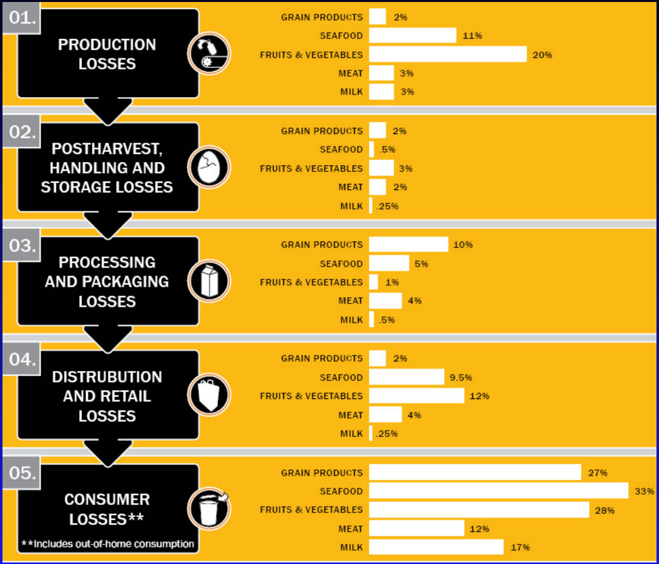
Fascinating Facts About Food Perishability

THE

SHELF LIFE OF FOOD

Foods unopened, uncut or uncooked unless stated otherwise	COUNTER/PANTRY (1 DAY ◂—▸ 1 MONTH)	REFRIGERATOR (1 DAY ◂—▸ 3 MONTHS)	FREEZER (1 MONTH ◂—▸ 1 YEAR)
APPLES	2-4 weeks	1-2 months	8-12 months
BANANAS	2-7 days	5-9 days	2-3 months
CANTALOUPE	Until ripe	1 week	8-12 months
CARROTS	Up to 4 days	4-5 weeks	8-12 months
CUCUMBERS	1-3 days	1 week	8-12 months
EGGS	Few hours	3-4 weeks	Do not freeze
MILK	Few hours	5-7 days	1 month
YOGURT	Few hours	2-3 weeks	1-2 months

Fascinating Facts About Food Perishability



Source: Food and Agriculture Organization 2011

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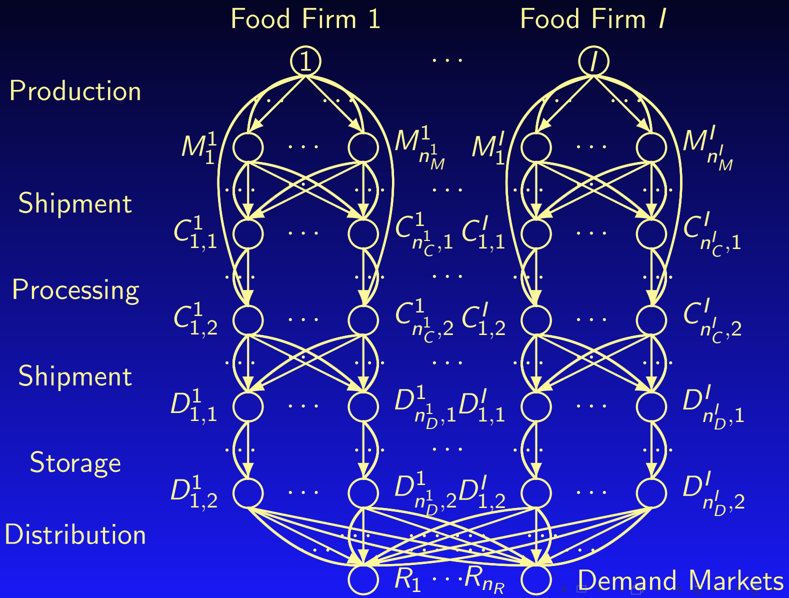
Fresh Produce Food Supply Chains

We developed a fresh produce supply chain network oligopoly model that

1. captures the deterioration of fresh food along the entire supply chain from a network perspective;
2. handles the exponential time decay through the introduction of arc multipliers;
3. formulates oligopolistic competition with product differentiation;
4. includes the disposal of the spoiled food products, along with the associated costs;
5. allows for the assessment of alternative technologies involved in each supply chain activity.

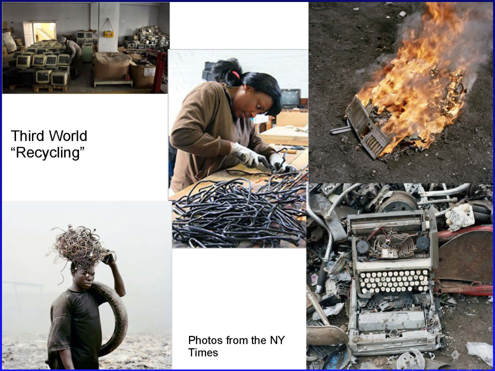
Reference: "Competitive Food Supply Chain Networks with Application to Fresh Produce," M. Yu and A. Nagurney, *European Journal of Operational Research* **224(2)** (2013) pp 273-282.

Fresh Produce Food Supply Chains

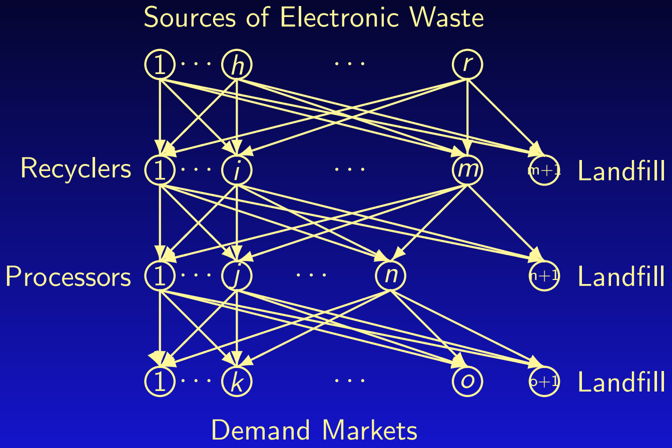


Electronic Waste

An estimated 50 million tons of e-waste are produced each year. The US discards 30 million computers each year and 100 million phones are disposed of in Europe each year. Some of the electronic waste is recycled, some is dumped illegally in landfills, and some is exported to other countries. There are valuable components that should be recycled and the impact of e-waste minimized.



Closed Loop Supply Chains and Electronic Recycling



Reference: "Reverse Supply Chain Management and Electronic Waste Recycling: A Multitiered Network Equilibrium Framework for E-Cycling," A. Nagurney and F. Toyasaki *Transportation*

The Sustainable Supply Chain Network Model
with
Frequency of Activities

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- Logistics in cities are often characterized by more frequent shipments, especially using primarily freight vehicles such as trucks.

- However, the scope of our model is broader and we also capture the optimal frequencies of the other activities, that is, those of manufacturing, storage, etc.

The Sustainable Supply Chain Network Model with Frequency of Activities

This part of the presentation is based on the paper, "Design of Sustainable Supply Chains for Sustainable Cities," A. Nagurney, *Environment & Planning B* **42(1)** (2015) pp 40-57.

It is inspired, in part, by the paper of M.J. Beckmann: "Vehicle and Passenger Flows in Mass Transportation: Optimal Routing of Buses and Planes," at the Symposium on Transportation Network Design and Economics, Northwestern University, January 29, 2010.

The Sustainable Supply Chain Network Model with Frequencies

We consider the supply chain network topology depicted in the Figure but note that this network is simply representative and more disaggregation can be included, depending on the application.

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The top level (origin) node 0 corresponds to the firm and the bottom level (destination) nodes correspond to the demand sites, which can denote, for example, retailers or consumers, that the firm wishes to supply.

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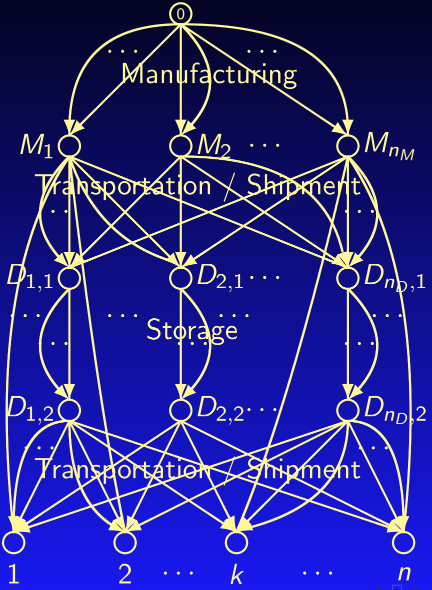
The top level (origin) node 0 corresponds to the firm and the bottom level (destination) nodes correspond to the demand sites, which can denote, for example, retailers or consumers, that the firm wishes to supply.

The paths joining the origin node to the destination nodes depict sequences of supply chain network activities that guarantee that the product is produced and is delivered to the demand sites.

The Supply Chain Network Topology



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The supply chain network consisting of the graph $G = [N, L]$, with N denoting the set of nodes and L the set of directed links.

The firm seeks to minimize the total costs associated with its production, storage, and transportation/distribution activities, along with the total cost of link operation frequencies, plus the total cost of environmental impact and waste, which we elaborate upon below, subject to the demand being satisfied at the demand sites.

The Sustainable Supply Chain Network Model with Frequencies

We denote the links by a, b, etc., and the total cost on a link a by $\hat{c}_a$. For the sake of generality, we note that the total costs are generalized costs and may include, for example, risk, time, etc. (see also Nagurney (2010)).

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A path p in the network joining node 0, which is the origin node, to a demand node, which is a destination node, represents the activities and their sequence associated with producing the product and having it, ultimately, delivered. Let w_k denote the pair of origin/destination (O/D) nodes $(0, k)$ and let P_{w_k} denote the set of paths, which represent alternative associated possible supply chain network processes, joining $(0, k)$. P is the set of all paths joining node 0 to the demand nodes. n_P denotes the number of paths from the organization to the demand markets.

The Sustainable Supply Chain Network Model with Frequencies

Let x_p represent the nonnegative flow of the product on path p joining (origin) node 0 with a (destination) demand node. Let d_k denote the demand, which is assumed to be known and fixed, for the product at demand location k.

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The following conservation of flow equation must hold:

$$d_k \equiv \sum_{p \in P_{w_k}} x_p, \quad k = 1, \ldots, n, \tag{1}$$

that is, the demand must be satisfied at each demand site.

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Let f_a denote the flow of the product on link a. The following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{2}$$

where $\delta_{ap} = 1$, if link a is contained in path p, and $\delta_{ap} = 0$, otherwise.

The Sustainable Supply Chain Network Model with Frequencies

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P, \tag{3}$$

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The total operational cost on a link, be it a manufacturing / production link, a transportation / shipment link, or a storage link is assumed to be a function of the flow of the product on the link; see, for example, Nagurney and Nagurney (2010) and Nagurney (2006) and the references therein. We have, thus, that

$$\hat{c}_a = \hat{c}_a(f_a), \quad \forall a \in L. \tag{4}$$

The Sustainable Supply Chain Network Model with Frequencies

We assume that the total cost on each link is convex and is continuously differentiable.

We denote the total cost of operating link a at a frequency γ_a by $\hat{\pi}_a$, $\forall a \in L$, and assume that

$$\hat{\pi}_a = \hat{\pi}_a(\gamma_a), \quad \forall a \in L. \tag{5}$$

These frequency operational cost functions are assumed to be convex and continuously differentiable and to have bounded second order partial derivatives.

The Sustainable Supply Chain Network Model with Frequencies

The firm seeks to determine the optimal levels of product processed on each supply chain network link coupled with the optimal levels of frequency link operation subject to the minimization of the total cost.

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The firm is faced with the following objective function:

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \hat{\pi}_a(\gamma_a). \tag{6}$$

The Sustainable Supply Chain Network Model with Frequencies

The firm is concerned with the environmental impact of its activities, which can include not only the emissions generated but also noise pollution, as well as other types of pollution and infrastructure deterioration.

Let $\hat{e}_a(f_a, \gamma_a)$, $\forall a \in L$, denote the environmental impact function associated with link a. Let $\hat{z}_a(f_a)$, $a \in L$, denote the waste management cost associated with link a, $a \in L$. They are assumed to be convex and continuously differentiable and to have bounded second order partial derivatives, as are the ones above.

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The second objective of the firm is then given by:

$$\text{Minimize} \quad \sum_{a \in L} \hat{e}_a(f_a, \gamma_a) + \hat{z}_a(f_a). \tag{7}$$

The Multicriteria Optimization Problem for Sustainable Supply Chain Network Design with Frequency of Activities

A nonnegative constant ω is now assigned to the environmental criterion (7). The constant ω is a weight that the firm assigns. Of course, ω can also be interpreted as a "tax" imposed by the governmental/environmental authority (see, e.g., Wu et al. (2006)).

The Sustainable Supply Chain Network Model with Frequencies

We assume, as given, a parameter $\bar{u}_a$, $\forall a \in L$. These parameters denote the existing capacities of the links.

For example, for a manufacturing link $\bar{u}_a$ would denote the capacity of production, that is, the volume of the product that could be produced on the link; for a storage link a, the capacity would denote how much of the product could be stored there, and, similarly, for a transportation/shipment link, $\bar{u}_a$ would represent the amount that could be shipped (could denote a truckload, for example).

The Sustainable Supply Chain Network Model with Frequencies

Using results from multicriteria optimization (see, e.g., Nagurney and Dong (2002)), one can then construct the following:

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \hat{\pi}_a(\gamma_a) + \omega(\sum_{a \in L} \hat{e}_a(f_a, \gamma_a) + \hat{z}_a(f_a)). \quad (8)$$

The firm seeks to solve (8), subject to the constraints: (1), (2), (3), and

$$f_a \leq \bar{u}_a \gamma_a, \quad \forall a \in L. \quad (9)$$

$$0 \leq \gamma_a, \quad \forall a \in L. \quad (10)$$

Constraint (9) guarantees that the product flow on a link does not exceed that link's capacity times the frequency of replenishment.

The Sustainable Supply Chain Network Model with Frequencies

We associate the Lagrange multiplier μ_a with constraint (9) for each link $a \in L$ and denote the associated optimal Lagrange multiplier by μ_a^*. These terms may also be interpreted as the price or value of an additional unit of "capacity" on link a. We group these Lagrange multipliers into the respective vectors μ and μ^*.

The Variational Inequality Formulation

Theorem 1

The optimization problem (8), subject to the constraints (1) – (3), (9), and (10), is equivalent to the variational inequality problem: determine the vectors of link flows, link operation frequencies, and Lagrange multipliers $(f^*, \gamma^*, \mu^*) \in K$, *such that:*

$$\sum_{a \in L} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \omega \frac{\partial \hat{e}_a(f_a^*, \gamma_a^*)}{\partial f_a} + \omega \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} + \mu_a^* \right] \times [f_a - f_a^*]$$

$$+ \sum_{a \in L} \left[\frac{\partial \hat{\pi}_a(\gamma_a^*)}{\partial \gamma_a} + \omega \frac{\partial \hat{e}_a(f_a^*, \gamma_a^*)}{\partial \gamma_a} - \bar{u}_a \mu_a^* \right] \times [\gamma_a - \gamma_a^*]$$

$$+ \sum_{a \in L} [\bar{u}_a \gamma_a^* - f_a^*] \times [\mu_a - \mu_a^*] \geq 0, \quad \forall (f, \gamma, \mu) \in K, \qquad (11)$$

$K \equiv \{(f, \gamma, \mu) | \exists x \geq 0,$ *and* $(1), (2),$ *and* (10) *hold, and* $\mu \geq 0\}$, *where* f *is the vector of link flows,* γ *is the vector of link operation frequencies, and* μ *is the vector of Lagrange multipliers.*

The Variational Inequality Formulation

Variational inequality (11) can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{12}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space. If we define the column vectors: $X \equiv (f, \gamma, \mu)$ and $F(X) \equiv (F_1(X), F_2(X), F_3(X))$, such that

$$F_1(X) \equiv [\frac{\partial \hat{c}_a(f_a)}{\partial f_a} + \omega \frac{\partial \hat{e}_a(f_a, \gamma_a)}{\partial f_a} + \omega \frac{\partial \hat{z}_a(f_a)}{\partial f_a} + \mu_a; \quad a \in L], \tag{13}$$

$$F_2(X) \equiv \left[\frac{\partial \hat{\pi}_a(\gamma_a)}{\partial \gamma_a} + \omega \frac{\partial \hat{e}_a(f_a, \gamma_a)}{\partial \gamma_a} - \bar{u}_a \mu_a; a \in L \right], \tag{14}$$

$$F_3(X) \equiv [\bar{u}_a \gamma_a - f_a; \quad a \in L], \tag{15}$$

and define $\mathcal{K} \equiv K$, then (11) can be re-expressed as (12).

Special Case of the Model

Let $\bar{u}_a = 1$ and let $\hat{\pi}_a$ now denote the total cost associated with investment to a level of operation γ_a on link a, for $a \in L$.

Special Case of the Model

Corollary 1

Under the preceding assumptions, the optimality conditions for the sustainable supply chain network model take on the following variational inequality form: determine the vectors of link flows, link capacity investments, and Lagrange multipliers $(f^*, \gamma^*, \mu^*) \in K^1$*:*

$$\sum_{a \in L} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \omega \frac{\partial \hat{e}_a(f_a^*, \gamma_a^*)}{\partial f_a} + \omega \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} + \mu_a^* \right] \times [f_a - f_a^*]$$

$$+ \sum_{a \in L} \left[\frac{\partial \hat{\pi}_a(\gamma_a^*)}{\partial \gamma_a} + \omega \frac{\partial \hat{e}_a(f_a^*, \gamma_a^*)}{\partial \gamma_a} - \mu_a^* \right] \times [\gamma_a - \gamma_a^*]$$

$$+ \sum_{a \in L} [\gamma_a^* - f_a^*] \times [\mu_a - \mu_a^*] \geq 0, \quad \forall (f, \gamma, \mu) \in K^1, \tag{16}$$

$K^1 \equiv \{(f, \gamma, \mu) | \exists x \geq 0, (1), (2),$ *and* (10) *hold with* $\bar{u}_a = 1, \forall a, \mu \geq 0\}$.

Theorem 2

The function $F(X)$ as defined following (12) (see (13) – (15)), under the assumptions above, is monotone, that is,

$$\langle (F(X^1) - F(X^2)), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \qquad (17)$$

Qualitative Properties

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Theorem 3
The function $F(X)$ as defined following (12) is Lipschitz continuous, that is,

$$\|F(X^1) - F(X^2)\| \leq \|X^1 - X^2\|, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{18}$$

It is important to realize that linear functions are convex and continuously differentiable. Hence, our model can be applied (and solved) under many different not unreasonable cost settings.

Convergence of the Algorithmic Scheme

Theorem 4: Convergence

Assume that the function that enters the variational inequality (11) (or (12)) has at least one solution and satisfies the conditions in Theorem 2 and in Theorem 3. Then the modified projection method converges to the solution of variational inequality (11) (or (12)) and, similarly, due to Corollary 1, to the solution of (16).

Numerical Examples

Numerical Examples

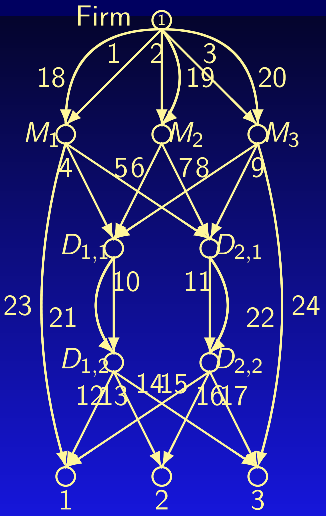
The modified projected method was implemented and a Unix system at the University of Massachusetts Amherst was used for all the computations.

We initialized the algorithm by equally distributing the demand at each demand site among all the paths joining the firm node 0 to the demand node.

All other variables (the link frequencies and the Lagrange multipliers) were initialized to zero. We used the equilibration algorithm (cf. Dafermos and Sparrow (1969) and Nagurney (1999)) for the solution of the embedded quadratic programming network optimization problems.

The numerical examples were solved to a high degree of accuracy since the imposed convergence criterion guaranteed that the absolute value of successive iterates differed by no more than 10^{-5}.

The Supply Chain Network Topology $G = [N, L]$ for the Examples



Numerical Examples

The numerical examples consisted of a firm faced with 3 possible manufacturing plants, each of which had 2 possible technologies, 2 distribution centers, each of which also had 2 distinct technologies, and the firm had to supply the 3 demand points.

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The first two examples had link capacities as reported in Table 1. The third numerical example (since it was a supply chain network design example) had $\bar{u}_a = 1$ for all links a, with the interpretation that the optimal values for the γ_a^*, for all links $a \in L$, would reflect the effective optimal capacities of the corresponding links (see, e.g., Nagurney (2010)).

The Input Data

Table 1: Total Operating and Frequency Cost Functions, Environmental Impact Cost and Waste Cost Functions, and Link Capacities for Numerical Examples 1 and 2

Link	$\hat{c}_a(f_a)$	$\hat{\pi}_a(\gamma_a)$	$\hat{e}_a(f_a, \gamma_a)$	$\hat{z}_a(f_a)$	$\bar{u}_a$
1	$.5f_1^2 + 2f_1$	$.5\gamma_1^2 + \gamma_1$	$.05f_1^2 + f_1 + 1.5\gamma_1^2 + 2\gamma_1$	$.05f_1^2 + f_1$	100.
2	$.5f_2^2 + f_2$	$2.5\gamma_2^2 + \gamma_2$	$.1f_2^2 + f_2 + 2\gamma_2^2 + 2\gamma_2$	$.1f_2^2 + 2f_2$	100.
3	$.5f_3^2 + f_3$	$\gamma_3^2 + 2\gamma_3$	$.15f_3^2 + 2f_3 + 2.5\gamma_3^2 + \gamma_3$	$.25f_3^2 + 5f_3$	200.
4	$1.5f_4^2 + 2f_4$	$\gamma_4^2 + \gamma_4$	$.05f_4^2 + .1f_4 + .1\gamma_4^2 + .2\gamma_4$	$.05f_4^2 + 2f_4$	20.
5	$f_5^2 + 3f_5$	$2.5\gamma_5^2 + 2\gamma_5$	$.05f_5^2 + .1f_5 + .05\gamma_5^2 + .1\gamma_5$	$.1f_5^2 + 3f_5$	20.
6	$f_6^2 + 2f_6$	$.5\gamma_6^2 + \gamma_6$	$.1f_6^2 + .1f_6 + .05\gamma_6^2 + .1\gamma_6$	$.05f_6^2 + f_6$	20.
7	$.5f_7^2 + 2f_7$	$.5\gamma_7^2 + \gamma_7$	$.05f_7^2 + .2f_7 + .1\gamma_7^2 + .2\gamma_7$	$.25f_7^2 + f_7$	20.
8	$.5f_8^2 + 2f_8$	$1.5\gamma_8^2 + \gamma_8$	$.05f_8^2 + .1f_8 + .1\gamma_8^2 + .3\gamma_8$	$.2f_8^2 + 2f_8$	10.
9	$f_9^2 + 5f_9$	$2\gamma_9^2 + 3\gamma_9$	$.05f_9^2 + .1f_9 + .1\gamma_9^2 + .2\gamma_9$	$.1f_9^2 + 5f_9$	10.
10	$.5f_{10}^2 + 2f_{10}$	$\gamma_{10}^2 + 5\gamma_{10}$	$.2f_{10}^2 + f_{10} + 1.5\gamma_{10}^2 + 3\gamma_{10}$	$.05f_{10}^2 + 5f_{10}$	50.
11	$f_{11}^2 + f_{11}$	$.5\gamma_{11}^2 + 3\gamma_{11}$	$.25f_{11}^2 + 3f_{11} + 2\gamma_{11}^2 + 3\gamma_{11}$	$.1f_{11}^2 + 2f_{11}$	50.
12	$.5f_{12}^2 + 2f_{12}$	$.5\gamma_{12}^2 + \gamma_{12}$	$.05f_{12}^2 + .1f_{12} + .1_{12}^2 + .2\gamma_{12}$	$.05f_{12}^2 + 3f_{12}$	15.
13	$.5f_{13}^2 + 5f_{13}$	$.5\gamma_{13}^2 + \gamma_{13}$	$.1f_{13}^2 + .1f_{13} + .05\gamma_{13}^2 + .1\gamma_{13}$	$.05f_{13}^2 + 5f_{13}$	15.
14	$f_{14}^2 + 7f_{14}$	$2\gamma_{14}^2 + 5\gamma_{14}$	$.15f_{14}^2 + .2f_{14} + .1\gamma_{14}^2 + .1\gamma_{14}$	$.05f_{14}^2 + 3f_{14}$	15.
15	$f_{15}^2 + 2f_{15}$	$.5\gamma_{15}^2 + \gamma_{15}$	$.05f_{15}^2 + .3f_{15} + .1\gamma_{15}^2 + .2\gamma_{15}$	$.1f_{15}^2 + 5f_{15}$	20.

The Input Data

Table 2: Total Operating and Frequency Cost Functions, Environmental Impact Cost and Waste Cost Functions, and Link Capacities for Numerical Examples 1 and 2

16	$.5f_{16}^2 + 3f_{16}$	$\gamma_{16}^2 + \gamma_{16}$	$.05f_{16}^2 + .1f_{16} + .1\gamma_{16}^2 + .1\gamma_{16}$	$.15f_{16}^2 + 3f_{16}$	20.
17	$.5f_{17}^2 + 2f_{17}$	$.5\gamma_{17}^2 + \gamma_{17}$	$.15f_{17}^2 + .3f_{17} + .05\gamma_{17}^2 + .1\gamma_{17}$	$.1f_{17}^2 + 5f_{17}$	20.
18	$.5f_{18}^2 + f_{18}$	$\gamma_{18}^2 + 2\gamma_{18}$	$.2f_{18}^2 + 2f_{18} + 2\gamma_{18}^2 + 3\gamma_{18}$	$.05f_{18}^2 + f_{18}$	100.
19	$.5f_{19}^2 + 2f_{19}$	$\gamma_{19}^2 + \gamma_{19}$	$.25f_{19}^2 + 3f_{19} + 3\gamma_{19}^2 + 4\gamma_{19}$	$.1f_{19}^2 + 2f_{19}$	200.
20	$1.5f_{20}^2 + f_{20}$	$\gamma_{20}^2 + \gamma_{20}$	$.3f_{20}^2 + 3f_{20} + 2.5\gamma_{20}^2 + 5\gamma_{20}$	$.15f_{20}^2 + f_{20}$	100.
21	$.5f_{21}^2 + 2f_{21}$	$\gamma_{21}^2 + 3\gamma_{21}$	$.1f_{21}^2 + 3f_{21} + 1.5\gamma_{21}^2 + 4\gamma_{21}$	$.15f_{21}^2 + 3f_{21}$	100.
22	$f_{22}^2 + 3f_{22}$	$.5\gamma_{22}^2 + 2\gamma_{22}$	$.05f_{22}^2 + 4f_{22} + 2.5\gamma_{22}^2 + 4\gamma_{22}$	$.25f_{22}^2 + 5f_{22}$	100.
23	$.5f_{23}^2 + f_{23}$	$.25\gamma_{23}^2 + \gamma_{23}$	$.2f_{23}^2 + f_{23} + \gamma_{23}^2 + 2\gamma_{23}$	$.2f_{23}^2 + 4f_{23}$	150.
24	$f_{24}^2 + f_{24}$	$.25\gamma_{24}^2 + \gamma_{24}$	$.1f_{24}^2 + 3f_{24} + .05\gamma_{24}^2 + 2\gamma_{24}$	$.1f_{24}^2 + 2f_{24}$	150.

Example 1

In Example 1 the demands were:

$$d_1 = 100, \quad d_2 = 200, \quad d_3 = 100.$$

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$$d_1 = 100, \quad d_2 = 200, \quad d_3 = 100.$$

In Example 1 we assumed that the firm did not care about the environmental impact and the waste generated generated in its supply chain and, hence, $\omega = 0$. The computed solution is reported in Table 2.

Example 1 Solution

Table 3: Example 1 Optimal Solution

Link	f_a^*	γ_a^*	μ_a^*
1	74.61	.7461	.0175
2	58.08	.5808	.0390
3	100.71	.5035	.0150
4	25.30	1.2651	.1765
5	24.89	1.2443	.4111
6	46.75	2.3373	.1669
7	68.45	3.4228	.2211
8	49.52	4.9520	1.5856
9	11.20	1.1202	.7481
10	60.73	1.2146	.1486
11	52.76	1.0551	.0811
12	0.00	.0000	.0000
13	108.47	7.2307	.5481
14	13.10	.83733	.5662
15	0.00	.0000	.0000

Example 1 Solution

Table 4: Example 1 Optimal Solution

Link a	f_a^*	γ_a^*	μ_a^*
16	91.53	4.5766	.5076
17	13.01	.6506	.0826
18	75.58	.7558	.0351
19	57.12	.2856	.0079
20	33.90	.3390	.0168
21	60.84	.6084	.0422
22	51.79	.5179	.0252
23	100.00	.6667	.0089
24	73.89	.4926	.0083

Example 1 Solution

The total cost (see objective function (6)) was: 55,920.97. The total environmental impact cost (see objective function (7)) was: 11,966.57, and the total waste costs were: 15,551.25. The value of the objective function (8) was, hence, 55.920.97.

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All the demand for demand market 1 is fulfilled through link 23 since links 12 and 15 have zero product flow on them. Of course, the corresponding frequencies of operating these links is also zero.

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All the demand for demand market 1 is fulfilled through link 23 since links 12 and 15 have zero product flow on them. Of course, the corresponding frequencies of operating these links is also zero.

Also, since, in this example, the firm is not at all concerned about its environmental impact and wastes generated, the value of the objective function corresponds to the total operational and frequency costs.

Example 2

Example 2 had the identical data as in Example 1 except that the firm was now concerned about the environment with $\omega = 1$. The new computed solution is given in Table 3.

Example 2 Solution

Table 5: Example 2 Optimal Solution

Link a	f_a^*	γ_a^*	μ_a^*
1	90.32	.9032	.0661
2	62.87	.6287	.0866
3	84.87	.4223	.0298
4	31.83	1.5913	.2351
5	30.93	1.5471	.4994
6	53.64	2.6821	.2026
7	59.35	2.9677	.2381
8	35.89	3.5892	1.2784
9	8.70	.8703	.6857
10	60.29	1.2057	.2806
11	52.30	1.0461	.2246
12	0.00	.0000	.0000
13	109.41	7.2947	.6089
14	11.95	.7966	.5630
15	0.00	.0000	.0000

Example 2 Solution

Table 6: Example 2 Optimal Solution

Link a	f_a^*	γ_a^*	μ_a^*
16	90.59	4.5294	.5533
17	8.41	.4204	.0781
18	72.45	.7245	.0935
19	50.13	.2506	.0350
20	39.77	.3977	.0878
21	61.07	.6107	.1005
22	46.69	.4669	.0880
23	100.00	.6667	.0311
24	79.64	.5310	.0085

Example 2 Solution

The total cost (see objective function (6)) was now: 56,632.07. The environmental impact cost (see objective function (7)) was now: 11,468.64. The waste cost was: 14,326.37. The value of the objective function (8) was, hence, 82,427.09. Due to the higher weight on the environmental and waste costs, the impact on the environment was reduced. However, as a consequence, the total cost is now higher than in Example 1 although not substantially so.

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Links 12 and 15, which are transportation/shipment links, are not used/operated, as was also the case in Example 1.

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Links 12 and 15, which are transportation/shipment links, are not used/operated, as was also the case in Example 1.

As expected, there is a transfer of production to the more environmentally-friendly manufacturing plants, with the associated technologies of production.

Example 3

Example 3 had the same data as Example 2 except that the $\bar{u}_1 = 1$ for all links $a = 1, \ldots, 24$. hence, the firm, in Example 1, was interested in designing a sustainable supply chain network for the product, with concern for the environment.

Example 3

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We kept the cost data for Example 3 as in Example 2 for comparison purposes. For actual design purposes one would need to increase the values of the $\hat{\pi}_a$ functions for all links $a \in L$, since these would then reflect actual construction/investment costs in the links (cf. Nagurney (2010)).

Here our goal was to demonstrate the flexibility of the modeling and computational framework.

The computed solution is reported in Table 4.

Example 3 Solution

Table 7: Example 3 Optimal Solution

Link a	f_a^*	γ_a^*	μ_a^*
1	97.25	97.25	391.9421
2	52.05	52.05	471.3385
3	70.95	70.95	499.5281
4	39.78	39.78	88.7147
5	24.65	24.65	127.7666
6	53.33	53.33	59.7681
7	54.01	4.01	66.0133
8	19.80	19.80	64.6667
9	8.43	8.43	38.5899
10	56.30	56.30	289.4470
11	47.05	47.05	241.1960
12	0.00	.0000	.0000
13	112.92	112.92	125.3066
14	0.00	.0000	5.1009
15	0.00	.0000	.3203

Example 3 Solution

Table 8: Example 3 Optimal Solution

Link a	f_a^*	γ_a^*	μ_a^*
16	87.08	87.08	192.6962
17	0.00	.0000	.0000
18	67.18	67.18	407.9846
19	55.29	55.29	447.2140
20	57.28	57.28	406.8561
21	56.61	56.61	289.9844
22	40.04	40.04	246.2007
23	100.00	100.00	252.9854
24	100.00	100.00	51.0052

Example 3 Solution

The total cost was: 122,625.56. The environmental impact was now: 102,133.26. The waste cost was: 13,464.07. The value of the objective function (8) was 238,222.89.

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Since links 12, 14, 15, and 17, have zero flows and zero effective capacities on those links, the optimal sustainable supply chain network design topology is given by the topology in the next Figure.

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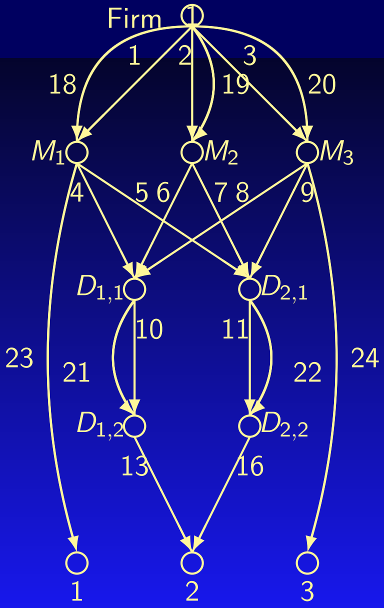
Demand points 1 and 3 are now served exclusively through direct shipments following the manufacture of the product.

The Optimal Supply Chain Network Topology for Example 3



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When we doubled the demands at each of the three demand points in Examples 1 through 3, the same links had zero flows as under the original demands. This kind of information is useful for a firm. One can also explore increases in the weight ω and improvements in environmental technologies.

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The above examples, although stylized, illustrate the practicality and flexibility of the sustainable supply chain network modeling approach and algorithm.

An Extension to Capture Competition

An Extension to Capture Competition

In the paper, "Supply Chain Network Sustainability Under Competition and Frequencies of Activities from Production to Distribution," A. Nagurney, M. Yu, and J. Floden, *Computational Management Science* **10(4)** (2013) pp 397-422, we extended the model to include competition and product differentiation.

Now the firms are profit-maximizers and control both their product flows and the frequencies associated with the supply chain network activities.

An Extension to Capture Competition

We utilize game theory and the Nash equilibrium concept and formulate the governing equilibrium conditions as a variational inequality problem.

We construct a weighted utility function associated with the two criteria faced by each firm. The term ω_i is assumed to be the price that firm i would be willing to pay for each unit of emission on each of its links and it is nonnegative.

The multicriteria decision-making problem faced by firm i; $i = 1, \ldots, I$, is:

$$U_i = \sum_{k=1}^{n_R} \hat{\rho}_{ik}(x) \sum_{p \in P_k^i} x_p - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{g}_a(\gamma_a) - \omega_i \sum_{a \in L^i} \hat{e}_a(f_a, \gamma_a). \quad (19)$$

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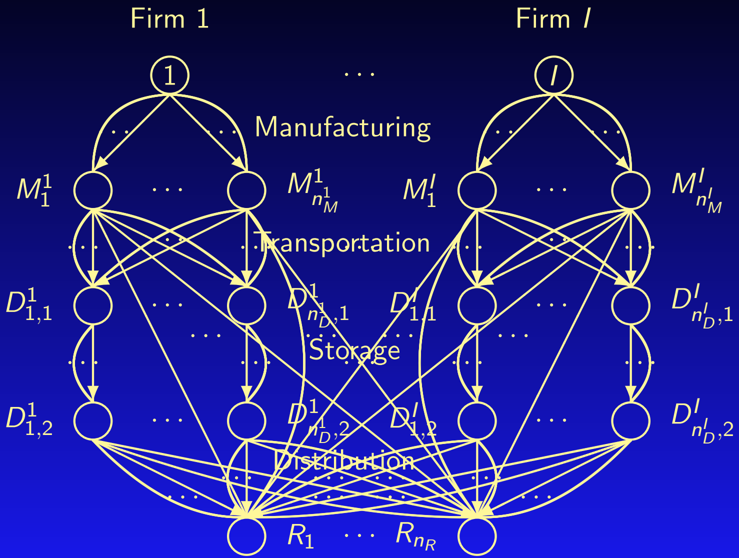

Figure 2: The Sustainable Supply Chain Network Topology that Captures

Definition: Supply Chain Network Cournot-Nash Equilibrium
A path flow and link frequency pattern $Y^* \in K = \prod_{i=1}^{I} K_i$ *is said to constitute a supply chain network Cournot-Nash equilibrium if for each firm* i*;* $i = 1, \ldots, I$*:*

$$\hat{U}_i(Y_i^*, \hat{Y}_i^*) \geq \hat{U}_i(Y_i, \hat{Y}_i^*), \quad \forall Y_i \in K_i, \tag{20}$$

where $\hat{Y}_i^* \equiv (Y_1^*, \ldots, Y_{i-1}^*, Y_{i+1}^*, \ldots, Y_I^*)$ *and*
$K_i \equiv \{Y_i | Y_i \in R_+^{n_{P^i}+n_{L^i}}\}$.

According to (20), an equilibrium is established if no firm can individually improve its utility, by changing its production path flows and its activity frequencies, given the decisions of the other firms.

Theorem 5

Assume that for each firm i; $i = 1, \ldots, I$, the utility function $\hat{U}_i(Y)$ is concave with respect to the variables in Y_i, and is continuously differentiable. Then $Y^* \in K$ is a sustainable supply chain network Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{Y_i} \hat{U}_i(Y^*), Y_i - Y_i^* \rangle \geq 0, \quad \forall Y \in K, \tag{21}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{Y_i} \hat{U}_i(Y)$ denotes the gradient of $\hat{U}_i(Y)$ with respect to Y_i.

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- Numerical examples illustrated the relevance and scope of the theoretical and computational approach.
- We also noted recent work in game theory and sustainable supply chain network competition.

Additional References

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