An Integrated Multitiered Supply Chain Network Model of Competing Agricultural Firms and Processing Firms: The Case of Fresh Produce and Quality

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Outline

Background

- Agricultural supply chains (ASCs) are very intricate local, regional, and global networks, creating pathways from farms to consumers.
- ASCs encompass sets of activities such as: farming/production, processing, storage, transportation, and distribution.
- The agricultural product industry includes small and large scale farms as well as global commercial food firms such as Tyson Foods, Dole, Cargill, etc.
- The latter are also involved in the processing of food, as in the form of cutting, canning, freezing, pasteurization, modified atmosphere packaging, etc.
- The processing firms procure their raw materials in the form of fresh produce from agricultural firms.

Background

- Consumers today have the option to buy fresh produce or minimally processed produce at grocery stores and supermarkets.
- For example, we can buy fresh whole broccoli or chopped packaged ones, fresh whole fruits or fresh-cut fruits.
- The processed fruits and vegetables market is projected to cross USD 465 billion by 2027 (Globe Newswire (2021)).
- Minimally processed ready to eat fresh food as well as frozen produce have gained popularity in recent times due to the convenience of use and health benefits.



Background

- Since fresh and minimally processed produce are substitutable, we see the agricultural and processing firms competing with each other at demand markets.
- However, there can be major quality concerns when it comes to processed produce.
- Quality of agricultural products change continuously from the point of production to the point of consumption

Research Questions

- How do agricultural firms and processing firms compete with each other based on their differentiated products at the demand markets?
- What is the trade off for agricultural firms between selling their produce to processing firms, vs at the demand markets?
- How does competition between agricultural and processing firms affect price at the demand markets?
- How does quality degradation of the product affect price at the demand markets?

Literature Review

- **Perishability & Quality**: Jonkman, Barbosa-Povoa, and Bloemhof (2019), Lejarza and Baldea (2021), Van Der Vorst et al. (2009).
- **Competition**: Yu and Nagurney (2013), Nagurney, Besik, and Yu (2018)
- **Multitiered Supply Chain Structures** : Yamada et al. (2011), Vorst, Beulens, and van Beek (2000), Li and Nagurney (2015)

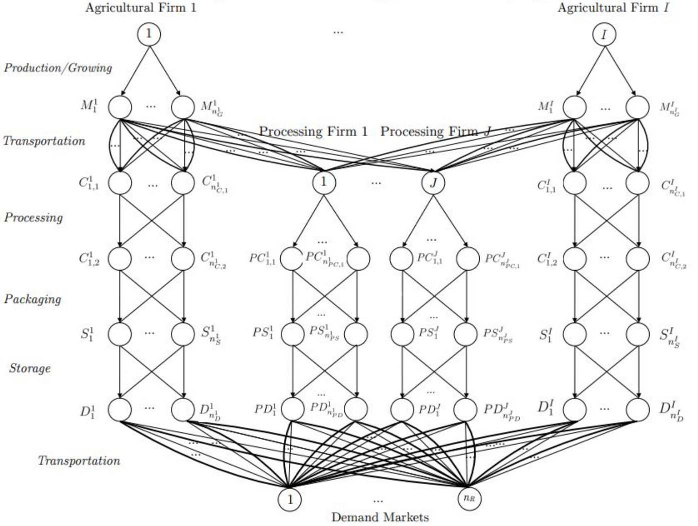
Contributions

We developed an integrated multitiered competitive agricultural supply chain network model in which agricultural firms and processing firms compete to sell their differentiated products.

- We capture competition at **two tiers**. We include in our model explicit functions to capture the **quality decay** of the harvested fresh produce along the entire, multitiered supply chain.
- We capture the **cost of all supply chain activities**.
- In our numerical study, we include supply chains of **both fresh and minimally processed produce**; specifically, that of carrots, that follow different kinetic functions of quality decay.
- The literature on the use of **game theory** for the modeling, analysis, and solution of supply chain network problems associated with agricultural products, including that of fresh produce, is much more limited.

Integrated Multitiered Supply Chain Network Model

Figure 1: Multitiered Food Supply Chain Network Topology



Integrated Multitiered Supply Chain Network Model

Table 1: Preliminary Notation for the Supply Chain Network Models

Notation	Definition
$x_{p_{AF}}$	The nonnegative flow of the agricultural product on a path p_{AF} sent from an agricultural firm i to a demand market k. Let P_k^i denote the set of all paths joining agricultural firm i with demand market k directly by agricultural firm i's set of links L^i, P denote the set of all paths from all agricultural firms to demand markets, and n_P is the number of such paths.
Q_{ij}	The nonnegative amount of agricultural firm i's product shipment from its production site to processing firm j through the link set $\hat{L}_1^j$.
ρ_{1ij}^*	The price that the processing firm j is willing to pay for the agricultural product shipment of agricultural firm i.
$x_{p_{PF}}$	The nonnegative flow of agricultural product on a path p_{PF} sent from a processing firm j to a demand market k. Let P_k^j denote the set of all paths joining processing firm j with demand market k directly by processing firm j's set of links $\hat{L}_2^j$, $\hat{P}$ denote the set of all paths from all processing firms to demand markets, and $n_{\hat{P}}$ is the number of such paths.
d_{ik}^{AF}	The demand for agricultural firm i's product at demand market k. We group all d_{ik}^{AF} elements into the vector $d^{AF} \in R_+^{In_R}$.
d_{jk}^{PF}	The demand for processing firm j's product at demand market k. We group all d_{jk} elements into the vector $d^{PF} \in R_+^{Jn_R}$.

Integrated Multitiered Supply Chain Network Model

Decision variables Flows:

- from agricultural firms to demand markets,
- from agricultural firms to processing firms,
- from processing firms to demand markets.

Demand

The demand at the demand market k for the agricultural product of agricultural firm i; $i = 1, \dots, I$, is given by:

$$d_{ik}^{AF} = \sum_{p_{AF} \in P_k^i} x_{p_{AF}}, \quad k = 1, \dots, n_R.$$

The demand at the demand market k for the agricultural product of processing firm j must satisy:

$$d_{jk}^{PF} = \sum_{p_{PF} \in P_k^j} x_{p_{PF}}, \quad j = 1, \dots, J; \; k = 1, \dots, n_R.$$

Quality Deterioration

Let $\beta_{\bar{a}}$ denote the quality decay incurred on link $\bar{a}$, for $\bar{a} \in L^i \cup \hat{L}_j$, which is a factor that depends on the reaction order n, the reaction rate $k_{\bar{a}}$, and the time $t_{\bar{a}}$ on link $\bar{a}$, that is

$$\beta_{\bar{a}} \equiv \begin{cases} -k_{\bar{a}} t_{\bar{a}}, & \text{if } n = 0, \forall \bar{a} \in L, \\ e^{-k_{\bar{a}} t_{\bar{a}}}, & \text{if } n \neq 0, \forall \bar{a} \in L. \end{cases}$$

Reaction rate given by the Arrhenius formula,

$$k_{\bar{a}} = Ae^{(-E_A/RT_{\bar{a}})}$$

.

Quality Deterioration

Quality for agricultural firms:

$$q_{p_{AF}} \equiv \begin{cases} q_{0i}^{AF} + \sum_{\bar{a} \in p_{AF}} \beta_{\bar{a}}, & \text{if} \;\; n = 0, \forall \bar{a} \in L^i, \;\; p_{AF} \in P_k^i, \forall i, k, \\ q_{0i}^{AF} \prod_{\bar{a} \in p_{AF}} \beta_{\bar{a}}, & \text{if} \;\; n = 1, \forall \bar{a} \in L^i, p_{AF} \in P_k^i, \forall i, k. \end{cases}$$

Quality for processing firms:

$$q_{p_{PF}} \equiv \begin{cases} q_{0j}^{PF} + \sum_{\bar{a} \in p_{PF}} \beta_{\bar{a}}, & \text{if} \;\; n = 0, \forall \bar{a} \in \hat{L}_2^j, \;\; p_{AF} \in P_k^j, \forall j, k, \\ q_{0j}^{PF} \prod_{\bar{a} \in p_{PF}} \beta_{\bar{a}}, & \text{if} \;\; n = 1, \forall \bar{a} \in \hat{L}_2^j, \;\; p_{AF} \in P_k^j, \forall j, k. \end{cases}$$

Quality Deterioration

Initial quality for processing firms,

$$q_{0j}^{PF} \equiv \begin{cases} \dfrac{\sum_{i=1}^{I}\left(q_{0i}^{AF} + \displaystyle\sum_{\bar{a}\in L_1^i \cup \hat{L}_1^j} \beta_{\bar{a}}\right)}{I}, & \text{if} \quad n = 0, \forall \bar{a} \in L_1^i \cup \hat{L}_1^j, \\ \dfrac{\sum_{i=1}^{I}\left(q_{0i}^{AF} \displaystyle\prod_{\bar{a}\in L_1^i \cup \hat{L}_1^j} \beta_{\bar{a}}\right)}{I}, & \text{if} \quad n = 1, \forall \bar{a} \in L_1^i \cup \hat{L}_1^j. \end{cases}$$

Average Quality

$$\hat{q}_{ik} = \frac{\sum_{p_{AF}\in P_k^i} q_{p_{AF}} x_{p_{AF}}}{\sum_{p_{AF}\in P_k^i} x_{p_{AF}}}.$$

$$\bar{\hat{q}}_{jk} = \frac{\sum_{p_{PF}\in P_k^j} q_{p_{PF}} x_{p_{PF}}}{\sum_{p_{PF}\in P_k^j} x_{p_{PF}}}.$$

Behavior of Agricultural Firms

Constraints

- Conservation of flow equations must hold for each agricultural firm i; $i = 1, \ldots, I$:

$$f_a = \sum_{k=1}^{n_R} \sum_{p_{AF} \in P_k^i} x_{p_{AF}} \delta_{ap_{AF}} + \sum_{j=1}^{J} Q_{ij}, \quad \forall a \in L_1^i;\ i = 1, \ldots, I.$$

$$f_b = \sum_{k=1}^{n_R} \sum_{p_{AF} \in P_k^i} x_{p_{AF}} \delta_{bp_{AF}}, \quad \forall b \in L_2^i;\ i = 1, \ldots, I,$$

Behavior of Agricultural Firms

Constraints

- Production capacity constraint:

$$CAP_i \geq \sum_{k=1}^{n_R} \sum_{p_{AF} \in P_k^i} x_{p_{AF}} + \sum_{j=1}^{J} Q_{ij}.$$

- Non-negativity constraints:

$$x_{p_{AF}} \geq 0, \quad \forall p_{AF} \in P_k^i;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R.$$

$$Q_{ij} \geq 0, \quad i = 1, \ldots, I;\ j = 1, \ldots, J.$$

Behavior of Agricultural Firms

Profit function:

$$U_i^{AF} = \sum_{k=1}^{n_R} \rho_{ik}^{AF}(d^{AF}, d^{PF}, \hat{q}, \bar{\hat{q}}) d_{ik}^{AF} + \sum_{j=1}^{J} \rho_{1ij}^{*} Q_{ij} - \sum_{a \in L_1^i} \hat{h}_a(f^1) - \sum_{b \in L_2^i} \hat{c}_b(f^2).$$

Revenue from demand markets

Revenue from processing firms

Total operational cost

Definition 1: A Cournot-Nash Equilibrium

Agricultural product path flows from agricultural firms to the demand markets, and agricultural product shipments from agricultural firms to processing firms $(X^{AF*}, Q^*) \in R_+^{n_P + IJ}$ are said to constitute a Cournot-Nash equilibrium if for each agricultural firm i; $i = 1, \ldots, I$,

$$\hat{U}_i^{AF}(X_i^{AF*}, \hat{X}_i^{AF*}, Q_i^*, \hat{Q}_i^*, X^{PF*}) \geq \hat{U}_i^{AF}(X_i^{AF}, \hat{X}_i^{AF*}, Q_i, \hat{Q}_i^*, X^{PF*}), \quad \forall (X^{AF}, Q) \in R_+^{n_P + IJ},$$

where

$$\hat{X}_i^{AF*} \equiv (X_1^{AF*}, \ldots, X_{i-1}^{AF*}, X_{i+1}^{AF*}, \ldots, X_I^{AF*}) \quad \text{and} \quad \hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*).$$

According to the above condition, a Cournot-Nash equilibrium is established if no agricultural firm can unilaterally improve upon its profit by selecting an alternative vector of agricultural product path flows to demand markets, and product shipments from agricultural firms to processing firms.

Theorem 1: Variational Inequality Formulation of Cournot-Nash Equilibrium Conditions of Agricultural Firms

An agricultural product path flow from agricultural firms to the demand markets, and agricultural product shipment from agricultural firms to processing firm pattern, $(X^{AF*}, Q^*) \in R_+^{n_P+IJ}$, is a Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF*},Q^*)}{\partial x_{p_{AF}}}+\frac{\partial\hat{C}_{p_{AF}}(X^{AF*})}{\partial x_{p_{AF}}}-\hat{\rho}_{ik}^{AF}(X^{AF*},X^{PF*},\hat{q},\bar{\hat{q}})\right.$$

$$\left.-\sum_{l=1}^{n_R}\frac{\partial\hat{\rho}_{il}^{AF}(X^{AF*},X^{PF*},\hat{q},\bar{\hat{q}})}{\partial x_{p_{AF}}}\sum_{r_{AF}\in P_k^i}x^*_{r_{AF}}+\lambda_i^*\right]\times[x_{p_{AF}}-x^*_{p_{AF}}]$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\frac{\sum_{a\in L_1^i}\partial\bar{\hat{h}}_a(X^{AF*},Q^*)}{\partial Q_{ij}}-\rho^*_{1ij}+\lambda_i^*\right]\times[Q_{ij}-Q^*_{ij}]$$

$$\sum_{i=1}^{I}\left[CAP_i-\sum_{k=1}^{n_R}\sum_{p_{AF}\in P_k^i}x^*_{p_{AF}}-\sum_{j=1}^{J}Q^*_{ij}\right]\times[\lambda_i-\lambda_i^*]\geq 0,\quad\forall(X^{AF},Q,\lambda)\in R_+^{n_P+IJ+I}.$$

Behavior of Processing Firms

Constraints

- The total amount of agricultural product shipments, by processing firm $j; j = 1, \ldots, J$, to the demand markets cannot exceed the total amount that it receives from its contracted agricultural firms. Hence,

$$\sum_{i=1}^{I} Q_{ij} \geq \sum_{k=1}^{n_R} \sum_{p_{PF} \in P_k^j} x_{p_{PF}}.$$

- Conservation of flow constraint:

$$f_e = \sum_{k=1}^{n_R} \sum_{p_{PF} \in P_k^j} x_{p_{PF}} \delta_{e_{p_{PF}}}, \quad \forall e \in \hat{L}_2^j; \, j = 1, \ldots, J.$$

- Non-negativity constraint:

$$x_{p_{PF}} \geq 0, \quad \forall p_{PF} \in P_k^j; \, j = 1, \ldots, J; \, k = 1, \ldots, n_R.$$

Behavior of Processing Firms

Profit function:

$$U_j^{PF} = \sum_{k=1}^{n_R} \rho_{jk}^{PF}(d^{PF}, d^{AF}, \hat{q}, \bar{\hat{q}})d_{jk} - \sum_{i=1}^{I} \rho_{1ij}^{*} Q_{ij} - \sum_{b \in \hat{L}_1^j} \hat{z}_b(Q) - \sum_{e \in \hat{L}_2^j} \hat{c}_e(f^3).$$

Revenue from demand markets

Cost of procurement

Cost of shipping

Total cost of supply chain activities

Definition 2: A Cournot-Nash Equilibrium of Processing Firms

Agricultural product path flows from processing firms to demand markets, and product shipments from agricultural firms to processing firms, $(X^{PF*}, Q^*) \in R_+^{n_{\hat{P}}+IJ}$, are said to constitute a Cournot-Nash equilibrium if for each processing firm j; $j = 1, \ldots, J$,

$$\hat{U}_j^{PF}(X_j^{PF*}, \hat{X}_j^{PF*}, Q_j^*, \hat{Q}_j^*, X^{AF*}) \geq \hat{U}_j^{PF}(X_j^{PF}, \hat{X}_j^{PF*}, Q_j, \hat{Q}_j^*, X^{AF*}), \quad \forall (X^{PF}, Q) \in R_+^{n_{\hat{P}}+IJ},$$

where

$$\hat{X}_j^{PF*} \equiv (X_1^{PF*}, \ldots, X_{j-1}^{PF*}, X_{j+1}^{PF*}, \ldots, X_J^{PF*}) \quad \textbf{and} \quad \hat{Q}_j^* \equiv (Q_1^*, \ldots, Q_{j-1}^*, Q_{j+1}^*, \ldots, Q_J^*).$$

According to the above a Cournot-Nash equilibrium is established if no processing firm can unilaterally improve upon its profit by selecting an alternative vector of agricultural product path flows to demand markets, and agricultural product shipments from agricultural firms to processing firms.

Behavior of Processing Firms

Theorem 2: Variational Inequality Formulation of Cournot-Nash Equilibrium Conditions for Processing Firms

An agricultural product path flow from processing firms to the demand markets, and agricultural product shipment from agricultural firms to processing firm pattern, $(X^{PF*}, Q^*) \in R_+^{n_{\hat{P}}+IJ}$ is a Cournot-Nash equilibrium according to Definition 2 if and only if it satisfies the variational inequality:

$$\sum_{j=1}^{J} \sum_{k=1}^{n_R} \sum_{p_{PF} \in P_k^j} \left[\frac{\partial \hat{C}_{p_{PF}}(X^{PF*})}{\partial x_{p_{PF}}} - \hat{\bar{\rho}}_{jk}^{PF}(X^{PF*}, X^{AF*}, \hat{q}, \bar{\hat{q}}) - \sum_{h=1}^{n_R} \frac{\partial \hat{\bar{\rho}}_{jh}^{PF}(X^{PF*}, X^{AF*}, \hat{q}, \bar{\hat{q}})}{\partial x_{p_{PF}}} \sum_{s_{PF} \in P_k^j} x_{s_{PF}}^* \right.$$

$$\left. + \eta_j^* \right] \times [x_{p_{PF}} - x_{p_{PF}}^*] + \sum_{j=1}^{J} \sum_{i=1}^{I} \left[\frac{\sum_{b \in \hat{L}_1^j} \partial \hat{z}_b(Q^*)}{\partial Q_{ij}} + \rho_{1ij}^* - \eta_j^* \right] \times [Q_{ij} - Q_{ij}^*]$$

$$+ \sum_{j=1}^{J} \left[\sum_{i=1}^{I} Q_{ij}^* - \sum_{k=1}^{n_R} \sum_{p_{PF} \in P_k^j} x_{p_{PF}}^* \right] \times [\eta_j - \eta_j^*] \geq 0, \quad \forall (X^{PF}, Q, \eta) \in R_+^{n_{\hat{P}}+IJ+J}.$$

Integrated Multitiered Agricultural Supply Chain Network Equilibrium

The equilibrium state of the integrated multitiered agricultural supply chain network with agricultural firms and processing firms is one where both variational inequalities mentioned earlier hold simultaneously.

Carrot Supply Chain: Fresh Vs. Frozen

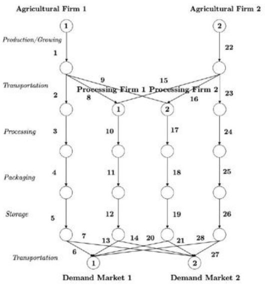
Quality attribute: Color Change (Frozen zero order, Fresh first order)

- Scenario 1 Baseline Example
- Scenario 2 - Supply Chain Disruption at Agricultural Firm 1
- Scenario 3 - Removal of Paths from Agricultural Firm 1 to Demand Markets
- Scenario 4 - Quality issues for Agricultural Firm 1 and Processing Firm 2
- Scenario 5 - Removal of Paths from Agricultural Firm 1 and Agricultural Firm 2 to Demand Markets

Numerical Study

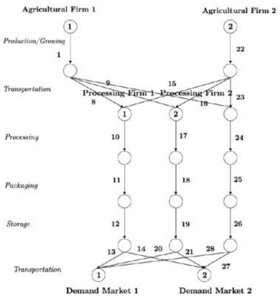
The algorithm that we use for the computation of the solution to the integrated multitiered supply chain network model with competing agricultural firms and processing firms is **the Euler method**.

Figure 2: Multitiered Supply Chain Network Topology for Scenario 1



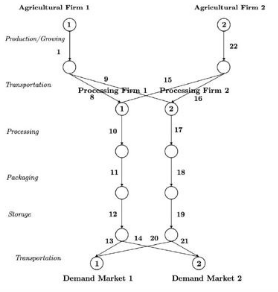
Scenarios 1, 2

Figure 3: Multitiered Supply Chain Network Topology for Scenario 2



Scenarios 3, 4

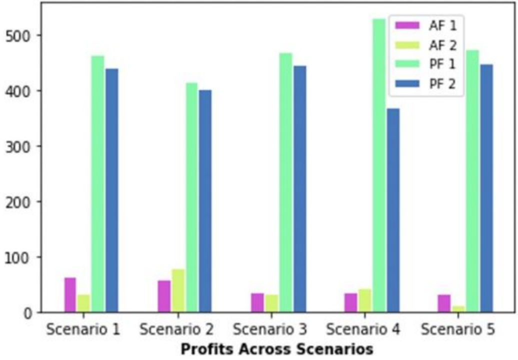
Figure 4: Multitiered Supply Chain Network Topology for Scenario 4



Scenario 5

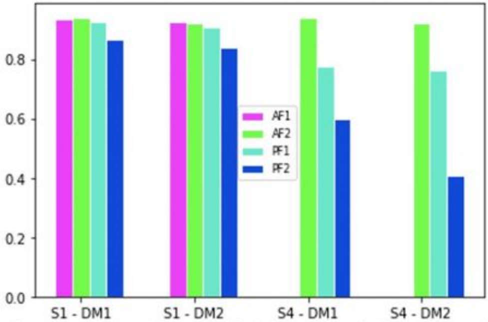
Managerial Insights and Conlcusions

- Scenario 1: Both processing firms enjoy higher profits than the agricultural firms.
- Scenario 2: With the production capacity reduction, Agricultural Firm 1's profit decreases as expected.
- Profit of Agricultural Firm 2 increases as it takes advantage of the production capacity reduction of Agricultural Firm 1.



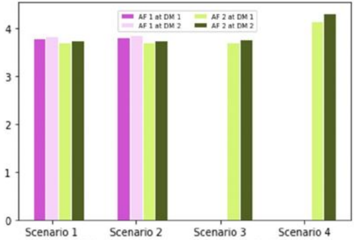
Managerial Insights and Conlcusions

- From Scenario 3 to Scenario 4, there is a decrease in the profits of Agricultural Firm 1 and Processing Firm 2 due to the quality problems that these two firms face. This result shows the importance of quality preservation in the fresh food industry.
- Scenario 5: Profit of agricultural firms decrease when they sell their produce only to processing firms.

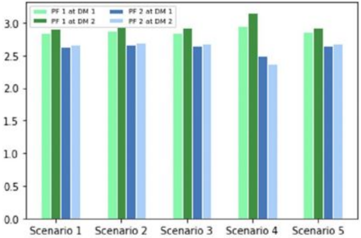


Average Quality at Demand Markets in Scenario 1 and Scenario 4

Managerial Insights and Conlcusions



Demand Prices Across Scenarios for Agricultural Firms



Demand Prices Across Scenarios for Processing Firms

- The demand market prices charged by the agricultural firms are higher than that of processing firms which aligns with what we observe in reality.
- In the absence of competition from Agricultural Firm 1, the demand market price of Agricultural Firm 2 increases.
- The demand market prices charged by Processing Firm 2 in Scenario 4 decrease significantly due to its reduced quality.

Acknowledgement

The authors dedicate this paper to freedom-loving people around the globe with special acknowledgment of those fighting for the freedom of Ukrainians and the world.

Thank you!