User-optimized and System-optimized Travel Behavior

Anna Nagurney

John F. Smith Memorial Professor
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

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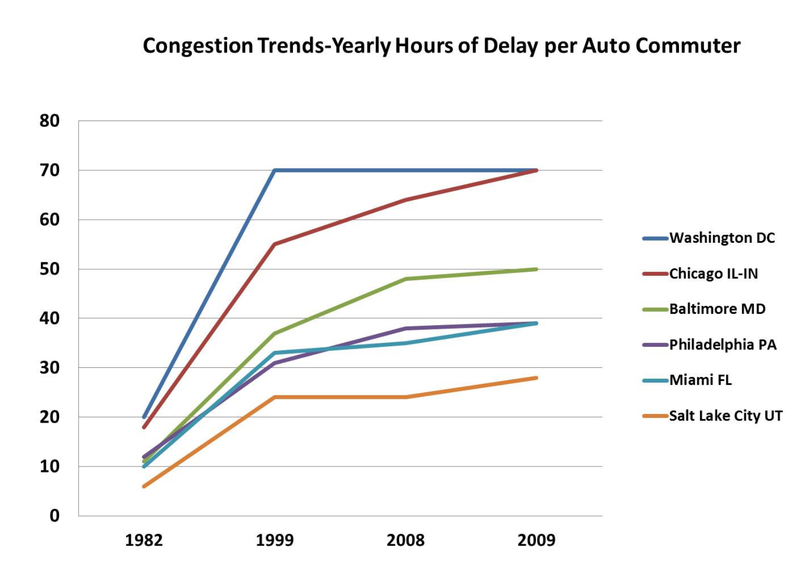
The study of the efficient operation of transportation networks dates to ancient Rome with a classical example being the publicly provided Roman road network and the time of day chariot policy, whereby chariots were banned from the ancient city of Rome at particular times of day.

Characteristics of Traffic Networks Today

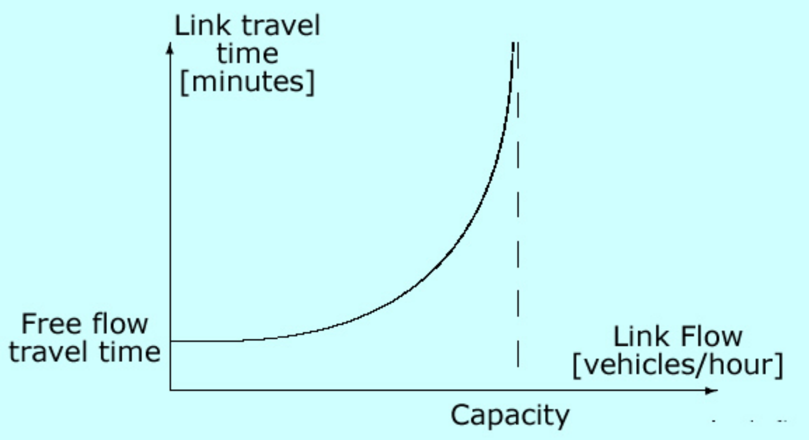
- *large-scale nature* and complexity of network topology;
- *congestion*, which leads to nonlinearities;
- *alternative behavior of users of the networks*, which may lead to paradoxical phenomena;
- *possibly conflicting criteria associated with optimization*;
- *interactions among the underlying networks themselves*, such as the Internet with electric power, financial, and transportation and logistical networks;
- recognition of *their fragility and vulnerability*;
- policies surrounding networks today may have major impacts not only economically, but also *socially, politically, and security-wise*.

Change in Annual Average Congestion Delay Hours for Commuters from 1982 - 2009



Congestion costs continue to rise: measured in constant 2009 dollars, *the cost of congestion has risen from $24 billion in 1982 to $115 billion in 2009 in the United States.* (Texas Transportation Institute's Urban Mobility Report (2010)).

Capturing Link Congestion



In a typical user link travel time (or cost) function, the free flow travel time referes to the travel time to traverse the link when there is zero flow or traffic on the link (zero vehicles).

Network Design Must Capture the Behavior of Users

Decision-makers select their cost-minimizing routes.

User-Optimized or Equilibrium

Decentralized	Selfish	U–O
vs. ⇔	vs. ⇔	vs.
Centralized	Unselfish	S–O

System-Optimized

Flows are routed so as to minimize the total cost to society.

Two fundamental principles of travel behavior, due to Wardrop (1952), with terms coined by Dafermos and Sparrow (1969).

User-optimized (U-O) (network equilibrium) Problem – each user determines his/her cost minimizing route of travel between an origin/destination, until an equilibrium is reached, in which no user can decrease his/her cost of travel by unilateral action (in the sense of Nash).

System-optimized (S-O) Problem – users are allocated among the routes so as to minimize the total cost in the system, where the total cost is equal to the sum over all the links of the link's user cost times its flow.

The U-O problems, under certain simplifying assumptions, possesses optimization reformulations. But now we can handle cost asymmetries, multiple modes of transport, and different classes of travelers, without such assumptions.

Multimodal Transportation

We Can State These Conditions Mathematically!

The U-O and S-O Conditions

Definition: U-O or Network Equilibrium – Fixed Demands
A path flow pattern x^*, with nonnegative path flows and O/D pair demand satisfaction, is said to be U-O or in equilibrium, if the following condition holds for each O/D pair $w \in W$ and each path $p \in P_w$:

$$C_p(x^*) \begin{cases} = \lambda_w, & \textit{if} \quad x_p^* > 0, \\ \geq \lambda_w, & \textit{if} \quad x_p^* = 0. \end{cases}$$

Definition: S-O Conditions
A path flow pattern x with nonnegative path flows and O/D pair demand satisfaction, is said to be S-O, if for each O/D pair $w \in W$ and each path $p \in P_w$:

$$\hat{C}'_p(x) \begin{cases} = \mu_w, & \text{if} \quad x_p > 0, \\ \geq \mu_w, & \text{if} \quad x_p = 0, \end{cases}$$

where $\hat{C}'_p(x) = \sum_{a \in \mathcal{L}} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}$, and μ_w is a Lagrange multiplier.

The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized network problems.

Definition: The Variational Inequality Problem

The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$, is to determine a vector $X^* \in \mathcal{K}$, such that:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F is a given continuous function from $\mathcal{K}$ to R^N, $\mathcal{K}$ is a given closed convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in R^N.

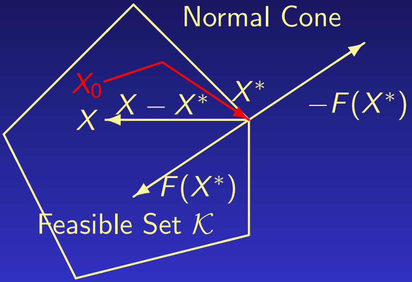
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of congested network problems from centralized to decentralized ones as well as to design problems.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System (Dupuis and Nagurney, Nagurney and Zhang)

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides a natural underlying dynamics associated with travel (and other) behavior to the equilibrium.

The Braess Paradox Illustrates

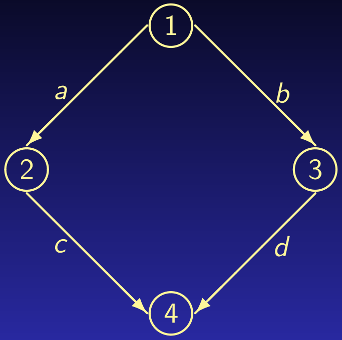
Why Behavior on Traffic Networks is Important

The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand of **6**, the equilibrium path flows are $x^*_{p_1} = x^*_{p_2} = 3$ and

The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83.$



$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path $p_3 = (a, e, d)$.

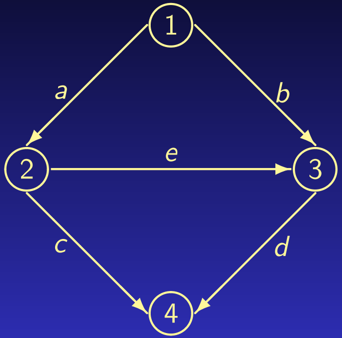
The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, $C_{p_3} = 70$.

The new equilibrium flow pattern network is

$x^*_{p_1} = x^*_{p_2} = x^*_{p_3} = 2$.

The equilibrium path travel cost:

$C_{p_1} = C_{p_2} = C_{p_3} = 92$.



$c_e(f_e) = f_e + 10$

Under S-O behavior, the total cost in the network is minimized, and the new route p_3, under the same demand of 6, would not be used.

The Braess paradox never occurs in S-O networks.

The 1968 Braess article has been translated from German to English and appears as:

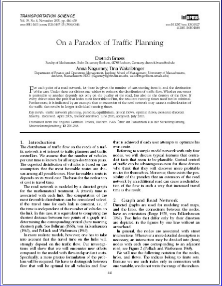
"On a Paradox of Traffic Planning,"

D. Braess, A. Nagurney, and T. Wakolbinger (2005) *Transportation Science* **39**, 446-450.


Über ein Paradoxon aus der Verkehrsplanung

Von D. Braess, Münster [1])

Eingegangen am 28. März 1968


TRANSPORTATION SCIENCE

On a Paradox of Traffic Planning

Dietrich Braess

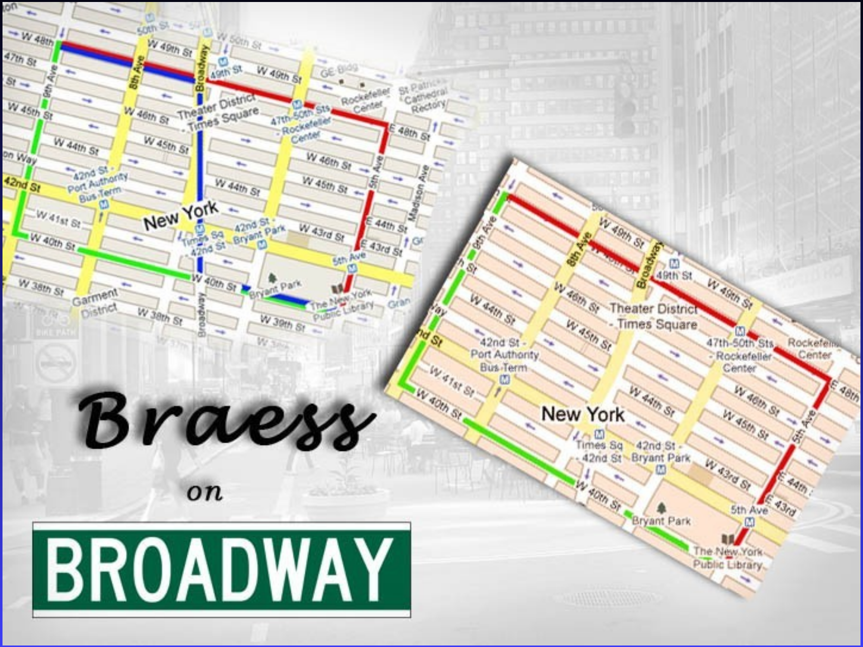
Anna Nagurney, Tina Wakolbinger

The Braess Paradox Around the World

1969 - Stuttgart, Germany - The traffic worsened until a newly built road was closed.

1990 - Earth Day - New York City - 42^{nd} Street was closed and traffic flow improved.

2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.


Braess
on
BROADWAY
New York
Theater District
- Times Square
47th-50th Sts
- Rockefeller
Center
42nd St -
Port Authority
Bus Term
Times Sq
- 42nd St
42nd St -
Bryant Park
Bryant Park
The New York
Public Library
W 49th St
W 46th St
W 45th St
W 44th St
W 43rd St
W 41st St
W 40th St
8th Ave
Broadway
5th Ave
E 48th St
E 44th St
E 43rd St

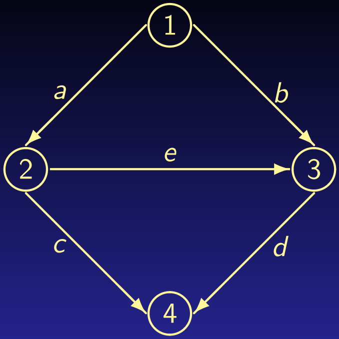
Question: When does the U-O solution coincide with the S-O solution?

Answer: In a general network, when the user link cost functions are given by:

$$c_a(f_a) = c_a^0 f_a^{\beta},$$

for all links, with $c_a^0 \geq 0$, and $\beta \geq 0$.

In particular, if $c_a(f_a) = c_a^0$, that is, in the case of *uncongested networks*, this result always holds.



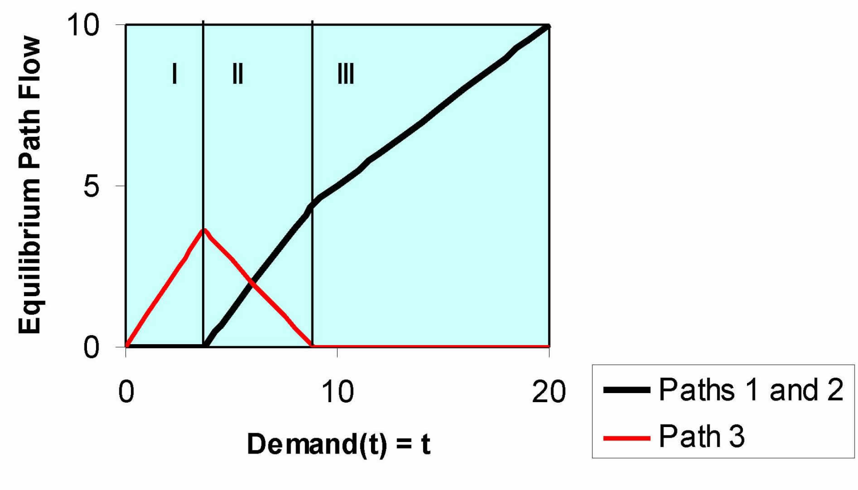
Recall the Braess network with the added link e.

What happens as the demand increases?

For Networks with Time-Dependent Demands

We Use Evolutionary Variational Inequalities

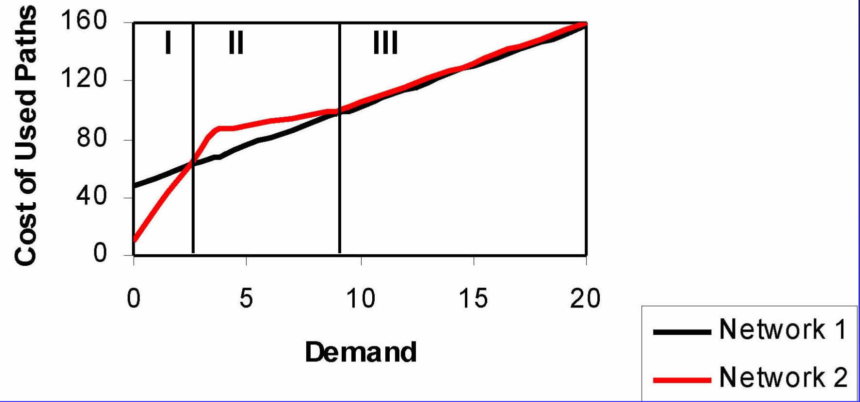
The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands Solved as an *Evolutionary Variational Inequality* (Nagurney, Daniele, and Parkes (2007)).



In Demand Regime I, Only the New Path is Used.

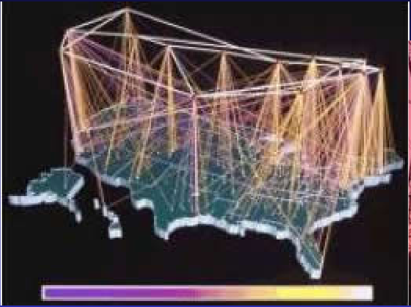
In Demand Regime II, the travel demand lies in the range [2.58, 8.89], and *the Addition of a New Link (Path) Makes Everyone Worse Off*!

In Demand Regime III, when the travel demand exceeds 8.89, *Only the Original Paths are Used!*



The new path is never used, under U-O behavior, when the demand exceeds 8.89, even out to infinity!

Other Networks that Behave like Traffic Networks




The Internet and electric power networks

We showed that, *for the specific Braess network*, the paradox no longer occurred as the demand for travel increased, once the demand reached a certain level.

This leads us to the *hypothesis:*

Under a higher demand, the Braess Paradox is negated in that the new route, which resulted in increased travel time at a particular demand, will no longer be used.

Theorem (Nagurney (2010))

Under the preceding assumptions, there exists a Δd_{w_1} *positive for which the Braess Paradox is negated in that the flow on the path* r *that resulted in the Braess Paradox occurring at a fixed level of demand, will no longer occur at the new level of demand since that path will not be used and, hence, it cannot result in an increase in travel cost.*

Interpretation with Fascinating Implications

The Theorem demonstrates that, as demand increases, the Braess Paradox *works itself out.*

One would expect that at a higher level of demand the network gets even more congested and that more of the paths/routes would then be used!

However, the Theorem establishes that the route that resulted in the Braess Paradox at a particular level of demand will no longer be used at a higher level of demand.

This suggests that there may be an underlying *Wisdom of Crowds Phenomenon* taking place.

It is worth noting that the qualitative results in the above Theorem also hold for nonlinear, strongly monotone cost functions.

Transportation networks are the *fundamental critical infrastructure* for the movement of people and goods in our globalized *Network Economy*.

Transportation networks also serve as the primary conduit for *rescue, recovery, and reconstruction in disasters.*

Recent disasters have vividly demonstrated the importance and vulnerability of our transportation and critical infrastructure systems

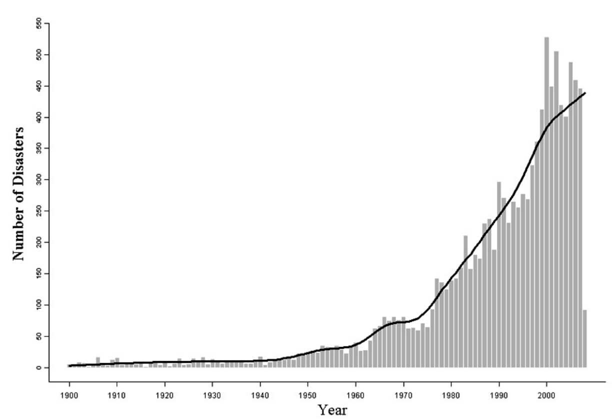
- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 – one in the UK and the other in Italy and Switzerland;
- The Indonesian tsunami (and earthquake), December 26, 2004;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge collapse, August 1, 2007;
- The Mediterranean cable destruction, January 30, 2008;
- The Sichuan earthquake on May 12, 2008;
- The Haiti earthquake that struck on January 12, 2010 and the Chilean one on February 27, 2010;
- The recent floods in northeastern Australia and Brazil.

Hurricane Katrina in 2005

Hurricane Katrina has been called an *"American tragedy,"* in which essential services failed completely.

The Haitian and Chilean Earthquakes

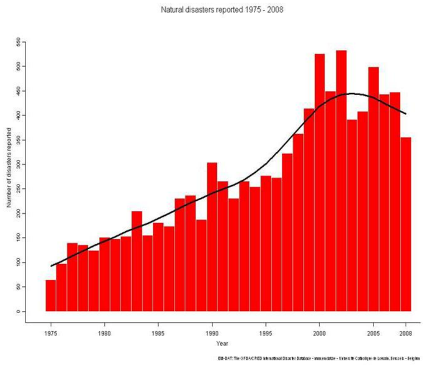
Disasters have brought an unprecedented impact on human lives in the 21st century and the number of disasters is growing. From January to October 2005, *an estimated 97,490 people were killed in disasters globally; 88,117 of them because of natural disasters*.

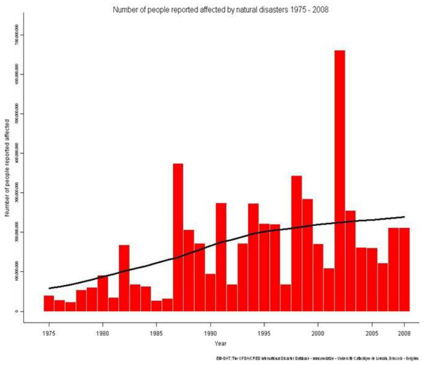


Frequency of disasters [Source: Emergency Events Database (2008)]

Disasters have a catastrophic effect on human lives and a region's or even a nation's resources.

Natural Disasters (1975–2008)





Which Nodes and Links Really Matter?

The Nagurney and Qiang (N-Q) Network Efficiency / Performance Measure

Definition: A Unified Network Performance Measure

The network performance/efficiency measure, $\mathcal{E}(\mathcal{G}, d)$, for a given network topology $\mathcal{G}$ and the equilibrium (or fixed) demand vector d, is:

$$\mathcal{E} = \mathcal{E}(\mathcal{G}, d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where recall that n_W is the number of O/D pairs in the network, and d_w and λ_w denote, for simplicity, the equilibrium (or fixed) demand and the equilibrium disutility for O/D pair w, respectively.

Definition: Importance of a Network Component

The importance of a network component $g \in \mathcal{G}$, $I(g)$, is measured by the relative network efficiency drop after g is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(\mathcal{G}, d) - \mathcal{E}(\mathcal{G} - g, d)}{\mathcal{E}(\mathcal{G}, d)}$$

where $\mathcal{G} - g$ is the resulting network after component g is removed from network $\mathcal{G}$.

The Approach to Identifying the Importance of Network Components

The elimination of a link is treated in the N-Q network efficiency measure by removing that link while the removal of a node is managed by removing the links entering and exiting that node.

In the case that the removal results in no path connecting an O/D pair, we simply assign the demand for that O/D pair to an abstract path with a cost of infinity.

The N-Q measure is well-defined even in the case of disconnected networks.

The Advantages of the N-Q Network Efficiency Measure

- The measure captures *demands, flows, costs, and behavior of users*, in addition to network topology.

- The resulting importance definition of network components is applicable and *well-defined even in the case of disconnected networks*.

- It can be used to identify the *importance (and ranking) of either nodes, or links, or both*.

- It can be applied to *assess the efficiency/performance of a wide range of network systems, including financial systems and supply chains under risk and uncertainty*.

- It is applicable also to *elastic demand networks*.

- It is *applicable to dynamic networks*, *including the Internet*.

Some Applications of the N-Q Measure

The Sioux Falls Network

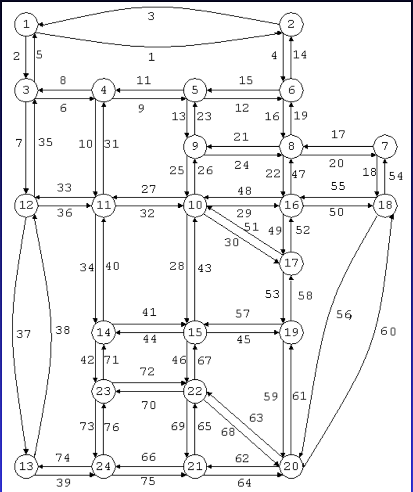


Figure 1: The Sioux Falls network with 24 nodes, 76 links, and 528 O/D pairs of nodes.

Importance of Links in the Sioux Falls Network

The computed network efficiency measure $\mathcal{E}$ for the Sioux Falls network is $\mathcal{E} = 47.6092$. Links 56, 60, 36, and 37 are the most important links, and hence special attention should be paid to protect these links accordingly, while the removal of links 10, 31, 4, and 14 would cause the least efficiency loss.

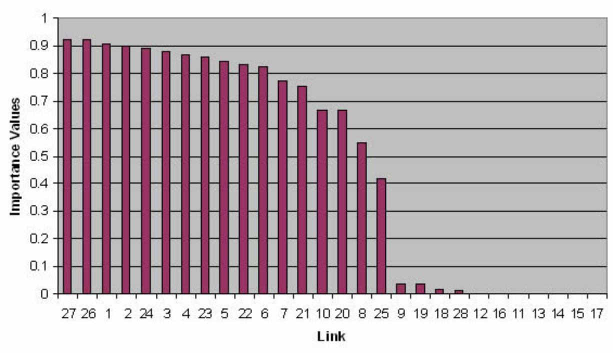


Figure 2: The Sioux Falls network link importance rankings

According to the European Environment Agency (2004), *since 1990, the annual number of extreme weather and climate related events has doubled, in comparison to the previous decade*. These events account for approximately 80% of all economic losses caused by catastrophic events. In the course of climate change, catastrophic events are projected to occur more frequently (see Schulz (2007)).

Schulz (2007) applied *N-Q network efficiency measure to a German highway system in order to identify the critical road elements* and found that this measure provided more reasonable results than the measure of Taylor and DEste (2007).

The N-Q measure can also be used to assess which links should be added to improve efficiency. *This measure was used for the evaluation of the proposed North Dublin (Ireland) Metro system* (October 2009 Issue of *ERCIM News*).

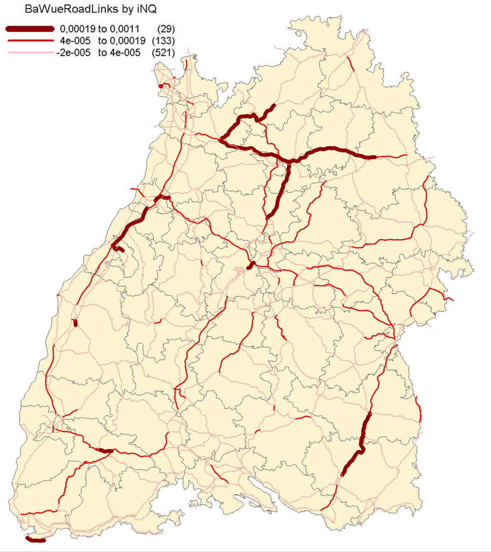


Figure 3: Comparative Importance of the links for the Baden - Wurttemberg Network – Modelling and analysis of transportation networks in earthquake prone areas via the N-Q measure, Tyagunov et al.

Which Nodes and Links Matter Environmentally?

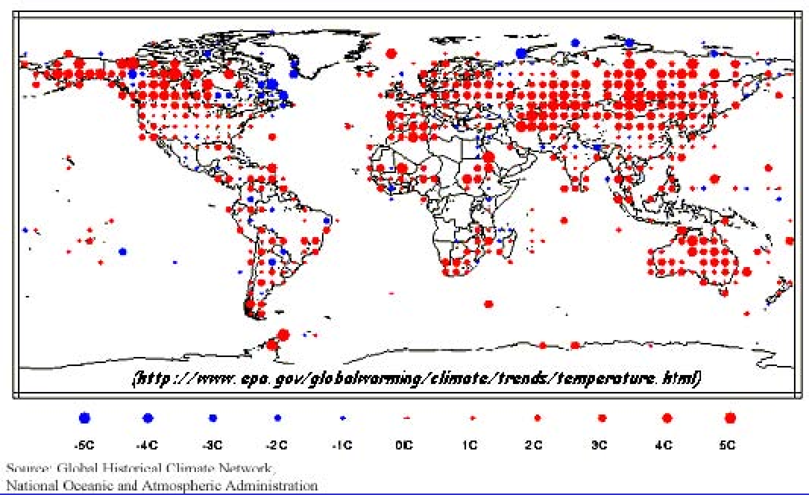


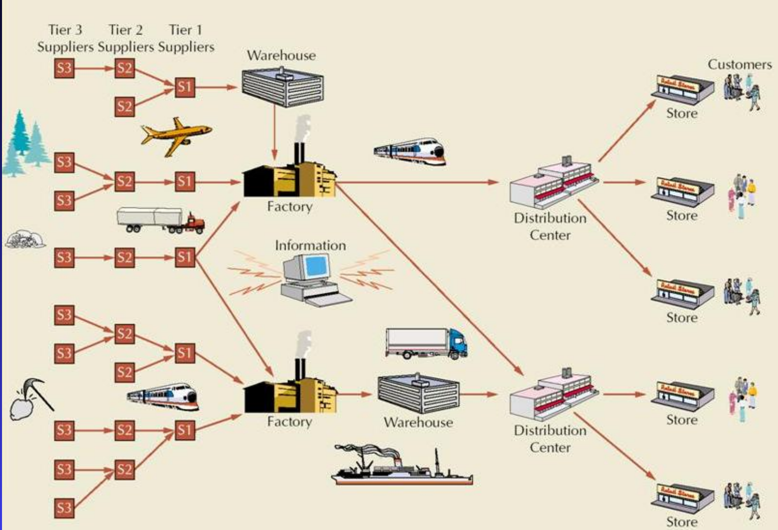
Figure 4: Global Annual Mean Temperature Trend 1950–1999

Figure 5: Impacts of climate change on transportation infrastructure

We have also extended our measures to construct *environmental impact assessment indices and environmental link importance identifiers under either U-O or S-O behaviors*.

What About Transportation's Role in Disaster Relief?

A General Supply Chain



Haiti Earthquake in 2010

Delivering the humanitarian relief supplies (water, food, medicines, etc.) to the victims was a major logistical challenge.

Humanitarian Relief

The period between 2000-2004 experienced an average annual number of disasters that was 55% higher than the period of 1995-1999 with 33% more people affected in the more recent period.

Humanitarian Relief

Humanitarian Supply Chains

The supply chain is a critical component not only of corporations but also of humanitarian organizations and their logistical operations.

At least 50 cents of each dollar's worth of food aid is spent on transport, storage and administrative costs.

Vulnerability of Humanitarian Supply Chains

Extremely poor logistical infrastructures: Modes of transportation include trucks, barges, donkeys in Afghanistan, and elephants in Cambodia.

To ship the humanitarian goods to the affected area in the first 72 hours after disasters is crucial. The successful execution is not just a question of money but a difference between life and death.

Corporations expertise with logistics could help public response efforts for nonprofit organizations.

In the humanitarian sector, organizations are 15 to 20 years behind, as compared to the commercial arena, regarding supply chain network development.

It is clear that better-designed supply chain networks in which transportation plays a pivotal role would have facilitated and enhanced various emergency preparedness and relief efforts and would have resulted in less suffering and lives lost.

Critical Needs Products

Critical needs products are those that are *essential to the survival of the population*, and can include, for example, vaccines, medicine, food, water, etc., depending upon the particular application.

The demand for the product should be met as nearly as possible since otherwise there may be additional loss of life.

In times of crises, a *system-optimization* approach is mandated since the demands for critical supplies should be met (as nearly as possible) at minimal total cost.

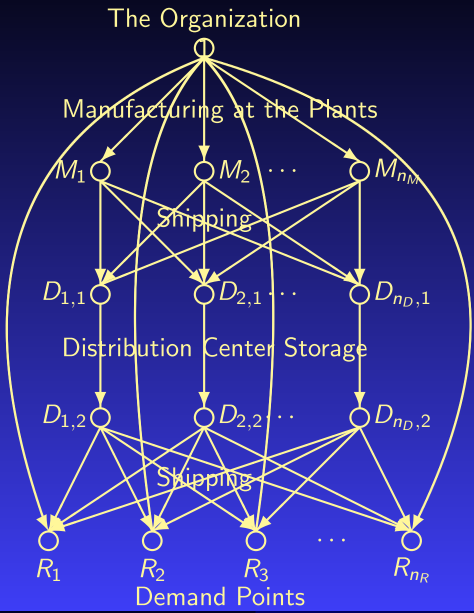
We have now developed a framework for the optimal design of critical needs product supply chains:

"Supply Chain Network Design for Critical Needs with Outsourcing,"

A. Nagurney, M. Yu, and Q. Qiang, *Papers in Regional Science*, in press,

where additional background as well as references can be found.

Supply Chain Network Topology with Outsourcing



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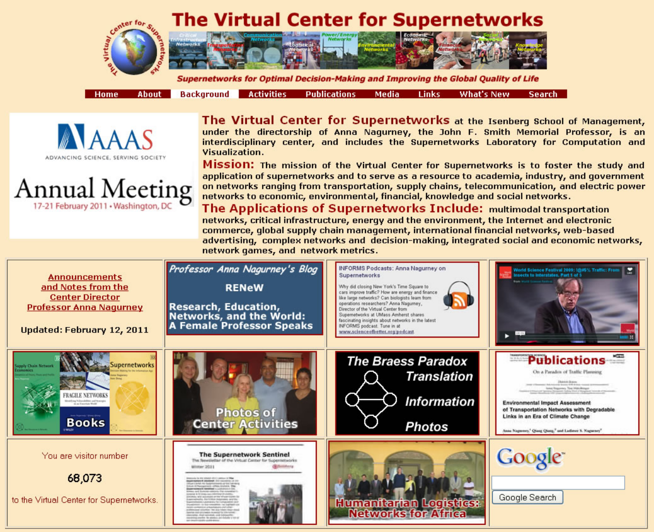

For more information, see: http://supernet.som.umass.edu

An Overview of Some of the Relevant Literature Chronologically

- M. J. Beckmann, C. B. McGuire, and C. B. Winsten (1956) *Studies in the Economics of Transportation*, Yale University Press, New Haven, Connecticut.
- D. Braess (1968) "Uber ein Paradoxon von Verkehrsplanung," *Unternehmenforschung* **12**, 258-286.
- S. C. Dafermos and F. T. Sparrow (1969) "The Traffic Assignment Problem for a General Network," *Journal of Research of the National Bureau of Standards* **73B**, 91-118.
- P. Dupuis and A. Nagurney (1993) "Dynamical Systems and Variational Inequalities," *Annals of Operations Research* **44**, 9-42.
- D. E. Boyce, H. S. Mahmassani, and A. Nagurney (2005) "A Retrospective on Beckmann, McGuire, and Winsten's *Studies in the Economics of Transportation*," *PiRS* **84**, 85-103.

An Overview of Some of the Relevant Literature Chronologically (cont'd.)

- D. Braess, A. Nagurney, and T. Wakolbinger (2005) "On a Paradox of Traffic Planning, Translation of the 1968 Article by Braess, *Transportation Science* **39**, 446-450.

- A. Nagurney, D. Parkes, and P. Daniele (2007) "The Internet, Evolutionary Variational Inequalities and the Time-Dependent Braess Paradox," *Computational Management Science* **4**, 355-375.

- A. Nagurney and Q. Qiang (2009) *Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain World*, John Wiley & Sons, Hoboken, New Jersey.

An Overview of Some of the Relevant Literature Chronologically (cont'd.)

- Q. Qiang, A. Nagurney, and J. Dong (2009) "Modeling of Supply Chain Risk Under Disruptions with Performance Measurement and Robustness Analysis," in *Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers*, T. Wu and J. Blackhurst, Editors, Springer, London, England, 91-111.
- Q. Qiang and A. Nagurney (2010) "A Bi-Criteria Measure to Assess Supply Chain Network Performance for Critical Needs Under Capacity and Demand Disruptions."
- A. Nagurney (2010) "On the Negation of the Braess Paradox as Demand Increases: The Wisdom of Crowds in Transportation Networks," *Europhysics Letters* **91**, 48002, p1-p5.