SOM 823 - Variational Inequalities and Projected Dynamical Systems -- Theory and Applications

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The course is scheduled to meet on Thursdays from 12:00-2:00.

The topics to be covered are listed below and will consist of approximately a single two hour lecture for each numbered topic below.

The course is interdisciplinary in nature and should be of interest to students in applied mathematics, who are interested in exploring new applications, and to students in regional planning, transportation, and economics, who are interested in learning and applying new methodologies. The course should also be of interest to engineering students not only in transportation but those who work on large-scale systems such as telecommunication systems. Due to the coverage of algorithms for large-scale equilibrium and disequilibrium problems, as well as discussions of parallel computing, the course may also be appropriate to computer scientists with the appropriate mathematics background.

COURSE DESCRIPTION AND SYLLABUS

This course provides an introduction to variational inequality theory and to projected dynamical systems. The first half of the course focuses on the former, emphasizing a variety of equilibrium problems drawn from operations research and economics, whereas the second half of the course focuses on the latter, and emphasizes dynamics and the study of disequilibrium behavior.

The course first introduces and develops variational inequality problems as the unifying framework for mathematical programming problems, including nonlinear programming problems and complementarity problems. It then through the use of applications such as network equilibrium problems motivates the need for variational inequality theory, the foundations of which are given in this course. The applications that are covered include traffic network and spatial price equilibrium problems. Other problems that will be studied as variational inequality problems, include oligopolistic market equilibrium problems and general financial equilibrium problems, in order to explore different behavioral assumptions underlying the equilibrium concept. The course covers a plethora of algorithms which take advantage of the underlying structure (often-times in the form of a network) of these problems. The algorithms are suitable for large-scale problems. The use of advanced computers, including parallel processing systems to solve variational inequalities will also be discussed.

The course then covers the fundamentals of projected dynamical systems theory, in terms of both qualitative properties, as well as iterative procedures. One of the notable features of a projected dynamical system is that its set of stationary points coincides with the set of solutions to a finite-dimensional variational inequality problem. Models which were studied at the equilibrium state in the first part of the course are then extended to the dynamic domain. In particular, we will develop dynamic traffic models, dynamic spatial price models, as well as oligopolistic market models and dynamic financial models. Stability analysis of such models will be covered as well as algorithms with convergence results. Special features of the problems which enable the implementation of algorithms on massively parallel architectures will also be revealed.

It is assumed that the student has some background in optimization theory and mathematical programming.

There will be two books utilized for the course which have been put on reserve in the SOM Library.

Textbook: Network Economics: A Variational Inequality Approach,
1993, Anna Nagurney, Kluwer Academic Publishers, Boston, MA

Textbook: Projected Dynamical Systems and Variational Inequalities with Applications, 1996, Anna Nagurney and Ding Zhang, Kluwer
Academic Publishers, Boston, MA

Outline of Topics to be Covered

1. Introduction to the Material

2. The Variational Inequality Problem
Basic Qualitative Theory
Relationship to Other Mathematical Programming Problems
Sensitivity Analysis

3. Algorithms for the Solution of Variational Inequality Problems The General Iterative Scheme - Projection and Relaxation Methods Decomposition Methods - Serial and Parallel

4. Basic Models of Traffic Assignment and Solution Procedures
The Standard Model
The Extended Model
Multimodal Models
Elastic Demand Models

5. Spatial Price Equilibrium Models and Solution Procedures
The Classical Model
Asymmetric and Multicommodity Models
Models with Policy Interventions
Traffic Network and Spatial Price Equilibrium Problems

6. Other Applications of Variational Inequalities
Oligopolistic Market Equilibrium Problems
Financial Equilibrium Problems

7. Introduction to Projected Dynamical System Theory
Qualitative Properties
Stability Analysis
General Iterative Scheme

8. Dynamic Traffic Network Models and Algorithms
Elastic Demand Models
Fixed Demand Models

9. Dynamic Spatial Price Models and Algorithms
Perfectly Competitive Models
Oligopolistic Market Models

10. Dynamic General Financial Models and Algorithms
Single Country Models
International Financial Models and Policy Interventions

The students are required to satisfactorily complete the homework assignments, which will be regularly distributed throughout the semester. The homeworks will be graded and returned to the students. The students are also required to complete a written report of a class project (of about 15 pages) and to present their results to the class. The class presentations will take place during the final class meeting times. The topic of the project should be on a topic directly related to the course but of the student's interest. For example, for those students interested in computing, a computer project may be selected. Other students may elect to develop a new application through the use of the methodologies studied in the course. Finally, some students may wish to complete an empirical study. The project topic must be approved by the instructor before the end of March. The students are expected to actively participate in the course.

The homework is worth 60% and the project and class presentation are worth 40%.