SOM 825 – Management Science Seminar: Advances in Variational Inequalities, Networks, and Game Theory

Spring 2018

Class Time: Tuesdays: 1:00-3:45PM (Time may be moved earlier to accommodate students)
Classroom: ISOM Room 306

Instructor: Dr. Anna Nagurney
John F. Smith Memorial Professor of Operations Management
Director – Virtual Center for Supernetworks

Office: ISOM Room 316
Phone: 545-5635
E-Mail: nagurney@isenberg.umass.edu
Office Hours: Tuesdays: 10:30-11:30AM; Thursdays: 11:00AM-noon, and by appointment

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Course Description:

This course is a seminar course with a focus on advances in variational inequalities, networks, and game theory with a driver being novel applications.

Since students may come to the seminar with different backgrounds and the seminar can be taken multiple times, Professor Nagurney, as needed, will begin with a series of lectures on the powerful methodology of variational inequalities, providing highlights and a review of the fundamentals. Theoretical foundations will be reviewed so that students can analyze a spectrum of equilibrium models qualitatively. In addition, a variety of algorithms will be presented, their advantages and disadvantages discussed, along with convergence conditions. Algorithms that exploit network structure are also reviewed since many of the applications of variational inequality theory from congested urban transportation networks to spatial price equilibrium problems to supply chain networks and the Internet are network-based.

The basics of networks and of game theory are covered, within the scope of variational inequalities, along with advanced topics on different applications.

Specifically, game theory models based on the Nash equilibrium concept are discussed with applications to oligopolies even in the context of novel supply chains. Also, both user-optimized and system-optimized networks are discussed.

The lecture component of the course also provides the fundamentals of projected dynamical systems theory and its relationships to variational inequality theory. Projected dynamical systems theory has evolved as a powerful methodology for capturing the dynamics underlying complex network systems and the associated disequilibrium behavior since one of the notable features of a projected dynamical system is that its set of stationary points coincides with the set of solutions to a finite-dimensional variational inequality problem.

Projected dynamical systems have been used, to-date, to formulate a variety of dynamic models in operations research / management science, operations management, engineering, and economics. The theory has been used in evolutionary game theory, ecology, and even neuroscience and is a powerful complement to variational inequalities.

Specific applications that will be highlighted in this seminar include: a variety of transportation network problems, both fixed demand and elastic demand ones, spatial price equilibrium problems (single and multiple commodity ones), and oligopolies and Nash equilibrium. In addition, there will be several lectures on special topics ranging from supply chains to the Internet as well as cybersecurity in order to illustrate recent applications of both variational inequality theory and projected dynamical systems theory.

An essential part of this seminar is the reading of primary scientific literature sources in the form of journal articles. In the latter part of the seminar we also hope to discuss students' research projects, as appropriate.

Professor Nagurney will provide the students with copies of all the lecture materials as well as the journal articles. Each set of lecture notes contains references.

There will be two books utilized as supplements for the course which will be made available to students in ISOM G11. These books are:

Network Economics: A Variational Inequality Approach, second and revised edition, 1999, Anna Nagurney, Kluwer Academic Publishers, Boston, Massachusetts.

Projected Dynamical Systems and Variational Inequalities with Applications, 1996, Anna Nagurney and Ding Zhang, Kluwer Academic Publishers, Boston, Massachusetts.

Outline of Proposed Course Topics

Fundamental and Advanced Topics

Variational Inequality Theory
- Definition
- Relationship to Systems of Equations, Optimization Problems, Complementarity

and Fixed Point Problems
Existence and Uniqueness Results

Algorithms
General Iterative Scheme
Projection Method
Relaxation Method
Modified Projection Method
Decomposition Algorithms
Equilibration Algorithms – User-Optimization and System-Optimization

Traffic Network Equilibrium
Single-Modal Models (Fixed and Elastic Demand)
Multimodal Models (Fixed and Elastic Demand)
Qualitative Analysis
Algorithms

Spatial Price Equilibrium
Classical Models (Quantity and Price Formulations)
Model on a General Network
Qualitative Analysis
Relationship to Traffic Network Equilibrium
Computations

Oligopolies and Game Theory
Definition of Nash Equilibrium
Fundamental Game Theory Model
Variational Inequality Formulation
Classical Oligopoly Models
Special-Purpose Algorithm
Application of the General Iterative Scheme
A Spatial Oligopoly Model
Relationship to the Spatial Price Equilibrium Model

Projected Dynamical Systems
Fundamental Theory
Relationship to the Variational Inequality Problem
Interpretation as a Tatonnment Process
General Iterative Scheme – Euler Method and the Heun Method
Convergence Results

Special Advanced Topics
Supply Chain Network Problems (including Perishable Product Ones)
Game Theory and Disaster Relief
Future Internet Architectures – ChoiceNet
Cybersecurity and Network Vulnerability

Requirements

There will be regular written homework assignments given out in class that will be graded and returned.

There will also be a midterm exam given in class.

In addition, there will be a research project, consisting of a paper and class presentation, to be discussed further in class. The students are required to attend the classes. If a student cannot attend class, please notify Professor Nagurney via email or by phone prior to the class absence.

Grading

Class participation:	**10%**
Homework:	**20%**
Midterm Exam:	**35%**
Research project and presentation:	**35%**