SOM 825 – Management Science Seminar: Variational Inequalities, Networks, and Game Theory

Spring 2014

Class Time: Tuesdays: 1:00-4:00PM
Classroom: ISOM Room 128

Instructor: Dr. Anna Nagurney
John F. Smith Memorial Professor of Operations Management
Director – Virtual Center for Supernetworks

Office: ISOM Room 316
Phone: 545-5635
E-Mail: nagurney@isenberg.umass.edu
Office Hours: Tuesdays: 11:00AM-12:00PM; Thursdays: 12:00-1:30PM, and by appointment

---

Course Description:

This course is an advanced mathematical programming course with a focus on variational inequalities, networks, and game theory. As such, it is assumed that the students are familiar with the fundamentals of optimization theory.

The course begins with a series of lectures given by Professor Anna Nagurney on the powerful methodology of variational inequalities. Theoretical foundations are given so that students can analyze equilibrium models qualitatively. In addition, a variety of algorithms are presented, along with convergence conditions. Algorithms that exploit network structure are also highlighted since many of the applications of variational inequality theory from congested urban transportation networks to spatial price equilibrium problems to supply chain networks and the Internet are network-based.

In addition, the basics of networks and of game theory are provided, within the scope of variational inequalities. Specifically, game theory models based on the Nash equilibrium concept are discussed with applications to oligopolies. Also, both user-optimized and system-optimized networks are formulated.

The lecture component of the course also provides the fundamentals of projected dynamical systems theory and its relationships to variational inequality theory. Projected

dynamical systems theory has evolved as a powerful methodology for capturing the dynamics underlying complex network systems and the associated disequilibrium behavior since one of the notable features of a projected dynamical system is that its set of stationary points coincides with the set of solutions to a finite-dimensional variational inequality problem.

Projected dynamical systems have been used, to-date, to formulate a variety of dynamic models in operations research / management science, operations management, engineering, and economics. It has been used in evolutionary game theory, ecology, and even neuroscience and is a powerful complement to variational inequalities.

Specific applications that will be covered in the course include: a variety of transportation network problems, both fixed demand and elastic demand ones, spatial price equilibrium problems (single and multiple commodity ones), and oligopolies and Nash equilibrium. In addition, there will be several lectures on special topics ranging from supply chains to the Internet in order to illustrate recent applications of both variational inequality theory and projected dynamical systems theory.

An essential part of this seminar is the reading of primary scientific literature sources in the form of journal articles.

Professor Nagurney will provide the students with copies of all the lecture materials as well as the journal article. Each set of lecture notes contains references.

There will be two books utilized as supplements for the course which will be made available to students in ISOM G28. These books are:

***Network Economics: A Variational Inequality Approach*, second and revised edition, 1999, Anna Nagurney, Kluwer Academic Publishers, Boston, Massachusetts.**

***Projected Dynamical Systems and Variational Inequalities with Applications*, 1996, Anna Nagurney and Ding Zhang, Kluwer Academic Publishers, Boston, Massachusetts.**

Outline of Course Topics

Variational Inequality Theory
- **Definition**
- **Relationship to Systems of Equations, Optimization Problems, Complementarity and Fixed Point Problems**
- **Existence and Uniqueness Results**

Algorithms
- **General Iterative Scheme**
  - **Projection Method**
  - **Relaxation Method**
- **Modified Projection Method**
- **Decomposition Algorithms**
- **Equilibration Algorithms – User-Optimization and System-Optimization**

Traffic Network Equilibrium
- **Single-Modal Models (Fixed and Elastic Demand)**
- **Multimodal Models (Fixed and Elastic Demand)**
- **Qualitative Analysis**

Algorithms

Spatial Price Equilibrium
- Classical Models (Quantity and Price Formulations)
- Model on a General Network
- Qualitative Analysis
- Relationship to Traffic Network Equilibrium
- Computations

Oligopolies and Game Theory
- Definition of Nash Equilibrium
- Fundamental Game Theory Model
- Variational Inequality Formulation
- Classical Oligopoly Models
- Special-Purpose Algorithm
- Application of the General Iterative Scheme
- A Spatial Oligopoly Model
- Relationship to the Spatial Price Equilibrium Model

Projected Dynamical Systems
- Fundamental Theory
- Relationship to the Variational Inequality Problem
- Interpretation as a Tatonnment Process
- General Iterative Scheme – Euler Method and the Heun Method
- Convergence Results

Special Topics
- **Supply Chain Network problems**
- **Future Internet**

Requirements

There will be regular written homework assignments given out in class that will be graded and returned.

There will also be a midterm exam given in class.

In addition, there will be two individual short projects, consisting only of class presentations, and a research paper project, consisting of a paper and class presentation, to be discussed further in class. The students are required to attend the classes. If a student cannot attend class, please notify Professor Nagurney via email or by phone prior to the class absence.

Grading

Class participation:	**10%**
Homework:	**20%**
Midterm Exam:	**20%**
Individual class presentations:	**20%**
Research project and presentation:	**30%**