Supply Chain Network Models for Humanitarian Logistics: Identifying Synergies and Vulnerabilities

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Ethiopia’s Food Crisis

Source: BBC News
Flooding in Kenya

Source: www.alertnet.org
Famine in Southern Africa

Source: BBC News
Humanitarian Relief

- In 2001 the total U.S. expenditure for humanitarian economic assistance was $1.46B, of which 9.7% represents a special supplement for victims of floods and typhoons in southern Africa (Tarnoff and Nowels (2001)).

- The period between 2000-2004 experienced an average annual number of disasters that was 55% higher than the period of 1995-1999 with 33% more people affected in the more recent period (Balcik and Beamon (2008)).

- According to ISDR (2006) 157 million people required immediate assistance due to disasters in 2005 with approximately 150 million requiring assistance the year prior (Balcik and Beamon (2008)).
Humanitarian Supply Chain

- The supply chain is a critical component not only of corporations but also of humanitarian organizations and their logistical operations.

- At least 50 cents of each dollar’s worth of food aid is spent on transport, storage and administrative costs (Dugger (2005)).

- The costs of provision may be divided among different products (food, clothing, fuel, medical supplies, shelter, etc.), which may increase efficiencies and enhance the organizations’ operational effectiveness.
Humanitarian organizations may not only benefit from multiproduct supply chains (cf. Perea-Lopez, Ydtsil, and Grossman (2003)), but also from the integrated management and control of the entire supply chain (Thomas and Griffin (1996)).

Coordination enables the sharing of information, which, according to Cachon and Fisher (2000), can reduce supply chain costs by approximately 2.2%.

By offering services to enhance the World Food Programs existing logistics, the humanitarian organization, TPG, reduced operating and delivery costs enabling WFP to feed more people (Shister (2004)).
Vulnerability of Humanitarian Supply Chains

- Extremely poor logistic infrastructures: Modes of transportation include trucks, barges, donkeys in Afghanistan, and elephants in Cambodia (Shister (2004)).

- To ship the humanitarian goods to the affected area in the first 72 hours after disasters is crucial. The successful execution is not just a question of money but a difference between life and death (Van Wassenhove (2006)).

- Corporations’ expertise with logistics could help public response efforts for nonprofit organizations (Sheffi (2002), Samii et al. (2002)).

- In the humanitarian sector, organizations are 15 to 20 years behind, as compared to the commercial arena, regarding supply chain network development (Van Wassenhove (2006)).
Literature

- Min and Zhou (2002)
- Nagurney (2006b)
- Dafermos (1972, 1982)
- Cheng and Wu (2006)
- Davis and Wilson (2006)
- Soylu et al. (2006)
- Xu (2007)
- Beamon (2004)
- Thomas and Kopczak (2005)
- Wassenhove (2006)
Humanitarian Supply Chain Literature

- Haghani and Oh (1996), Balcik and Beamon (2008)
- Clark and Culkin (2007)
- Thomas (2003)
- Van Wassenhove (2006)
- Thomas and Mizushima (2005)
- Qiang and Nagurney (2008), Nagurney and Qiang (2008), Nagurney and Qiang (2007a, 2007b)
Contributions

- We build on supply chain network models with nonlinear costs that can also capture the reality of congestion, which may occur in humanitarian disaster relief operations.

- We built on the recent work of Nagurney (2007) who developed a system-optimization perspective for supply chain network integration in the case of horizontal mergers.
Contributions

- We also focus on supply chain network integration and we extend the contributions in Nagurney (2007) to include multiple products and with a humanitarian logistics perspective.

- We analyze the synergy effects associated with the “merging” of two humanitarian organizations, in the form of the integration of their supply chain networks, in terms of the operational synergy. Here we consider not only monetary costs but rather, generalized, costs that can include risk, environmental impacts associated with the humanitarian operations, etc.
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4 Synergy Measure

5 Vulnerability Analysis

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Supply Chains of Humanitarian Organizations $A$ and $B$
Prior to the Integration
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

Let $G_i = [N_i, L_i]$ denote the graph consisting of nodes $[N_i]$ and directed links $[L_i]$ representing the activities associated with each organization $i; i = A, B$.

Let $L^0$ denote the links: $L_A \cup L_B$.

We assume that each organization can provide multiple, homogeneous products to the deserving populations at the respective demand points.

The demands for the products are assumed as given and are associated with each product, and each organization and demand pair.
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

Let $d^j_{R^i_k}$ denote the demand for product $j; j = 1, \ldots, J$, at demand point $R^i_k$ associated with organization $i; i = A, B; k = 1, \ldots, n^i_R$.

Let $x^j_p$ denote the nonnegative flow of product $j$, on path $p$. A path consists of a sequence of supply chain activities comprising supply/manufacturing, storage, and distribution of the products.

Let $P^0_{R^i_k}$ denote the set of paths joining an origin node $i$ with (destination) demand node $R^i_k$.

Clearly, since we are first considering the two humanitarian organizations prior to any integration, the paths associated with a given organization have no links in common with paths of the other organization.
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

This changes (see also Nagurney (2007), Nagurney and Woolley (2008)) when the integration occurs, in which case the number of paths and the sets of paths also change, as do the number of links and the sets of links.

The following conservation of flow equations must hold for each organization $i$, each demand point $k$, and each product $j$:

$$
\sum_{p \in P^0_{R_k} i} x^j_p = d^j_{R_k} i, \quad i = A, B; \quad j = 1, \ldots, J; \quad k = 1, \ldots, n^i_R
$$

that is, the demand for each product must be satisfied at each demand point.
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

Links are denoted by $a$, $b$, etc. Let $f^j_a$ denote the flow of product $j$ on link $a$.

We must have the following conservation of flow equations satisfied:

$$f^j_a = \sum_{p \in P^0} x^j_p \delta_{ap}, \quad j = 1 \ldots, J; \quad \forall a \in L^0$$

where $\delta_{ap} = 1$ if link $a$ is contained in path $p$ and $\delta_{ap} = 0$, otherwise.

Here $P^0$ denotes the set of all paths in Figure 1, that is,

$$P^0 = \bigcup_{i=A,B;k=1,\ldots,n^i_R} P^0_{R^i_k}.$$
The path flows must be nonnegative, that is,

$$x^j_p \geq 0, \quad j = 1, \ldots, J; \quad \forall p \in P^0.$$ 

We group the path flows into the vector $x$.

Assume that there is a total cost associated with each link (cf. Figure 1) of the network corresponding to each organization $i$; $i = A, B$, and each product, $j$; $j = 1, \ldots, J$.

We assume that this cost is a generalized cost and includes not only the monetary cost but also such costs as risk and even, if appropriate, the environmental cost, with all the associated costs weighted accordingly to produce the generalized cost.
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

We denote the total (generalized) cost on a link $a$ associated with product $j$ by $\hat{c}^j_a$. The total cost of a link associated with a product, be it a supply/manufacturing link, a shipment/distribution link, or a storage link is assumed to be a function of the flow of all the products on the link; see, for example, Dafermos (1972).

Hence, we have that

$$\hat{c}^j_a = \hat{c}^j_a(f^1_a, \ldots, f^J_a), \quad j = 1, \ldots, J; \quad \forall a \in L^0.$$
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

We assume that the total cost on each link is convex, continuously differentiable, and has a bounded second order partial derivative. Since the humanitarian organizations, pre-integration, have no links in common, their individual cost minimization problems can be formulated jointly as follows:

\[
\text{Minimize} \quad \sum_{j=1}^{J} \sum_{a \in L^0} \hat{c}_a(f_{a}^{1}, \ldots, f_{a}^{J})
\]

subject to the constraints and capacity constraints (presented on the following slide).
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

The capacity constraints are as follows:

\[ \sum_{j=1}^{J} \alpha_j f_j \leq u_a, \quad \forall a \in L^0. \]

The term \(\alpha_j\) denotes the volume taken up by product \(j\), whereas \(u_a\) denotes the nonnegative capacity of link \(a\).
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

Observe that this problem is, as is well-known in the transportation literature (cf. Beckmann, McGuire, and Winsten (1956), Dafermos and Sparrow (1969), and Dafermos (1972)), a system-optimization problem but in capacitated form.

Under the above imposed assumptions, the optimization problem is a convex optimization problem.

If we further assume that the feasible set underlying the problem represented by all of the constraints is non-empty, then it follows from the standard theory of nonlinear programming (cf. Bazaraa, Sherali, and Shetty (1993)) that an optimal solution exists.
The Pre-integration Multiproduct Decision-making Optimization Problem (Case 0)

Let $\mathcal{K}^0$ denote the set where $\mathcal{K}^0 \equiv \{ f | \exists x, \text{ and the constraints hold} \}$, where $f$ is the vector of link flows. We associate the Lagrange multiplier $\beta_a$ with the link upper bound constraint on link $a \ \forall a \in L^0$.

We denote the associated optimal Lagrange multiplier by $\beta^*_a$. These terms may be interpreted as the price or value of an additional unit of capacity on link $a$. 
Theorem

The vector of link flows $f^* \in \mathcal{K}^0$ is an optimal solution to the pre-integration problem if and only if it satisfies the following variational inequality problem with the vector of nonnegative Lagrange multipliers $\beta^*$:

$$
\sum_{j=1}^J \sum_{k=1}^J \sum_{a \in L^0} \left[ \frac{\partial \hat{c}_k(f_a^{1*}, \ldots, f_a^{J*})}{\partial f_a^j} + \alpha_j \beta_a^* \right] \times [f_a^j - f_a^{j*}]
$$

$$
+ \sum_{a \in L^0} [u_a - \sum_{j=1}^J \alpha_j f_a^{j*}] \times [\beta_a - \beta_a^*] \geq 0,
$$

$$
\forall f \in \mathcal{K}^0, \forall \beta \geq 0.
$$

Supply Chain Network after Humanitarian Organizations $A$ and $B$ Integrate their Supply Chains

SCNM Models for Humanitarian Logistics

University of Massachusetts Amherst
The Integrated Multiproduct Decision-making Optimization Problem (Case 1)

As in the pre-integration case, the post-integration optimization problem is also concerned with total cost minimization, with the emphasis that the total cost is reflective of the generalized total cost.

We refer to the network underlying this integration as $G^1 = [N^1, L^1] \equiv \bigcup_{i=1}^{NA,B}[N_i, L_i]$. We associate total cost functions with the new links, for each product $j$.

We assume, for simplicity, that the corresponding functions on the links emanating from the supersource node are equal to zero.
The Integrated Multiproduct Decision-making Optimization Problem (Case 1)

A path $p$ now originates at the node 0 and is destined for one of the bottom demand nodes.

The set of paths $P^1 \equiv \bigcup_{i=A,B; k=1,\ldots,n_R^i} R_k^i$.  

Let $x^j_p$, in the integrated network configuration, denote the flow of product $j$ on path $p$ joining (origin) node 0 with a (destination) demand node.
The Integrated Multiproduct Decision-making Optimization Problem (Case 1)

The following conservation of flow equations must hold:

\[ \sum_{p \in P_{R_k}^1} x^j_p = d^j_{R_k}, \quad i = A, B; \quad j = 1, \ldots, J; \quad k = 1, \ldots, n^j_R, \]

where \( P_{R_k}^1 \) denotes the set of paths connecting node 0 with demand node \( R_k \).

Due to the integration, the demand points can obtain each product \( j \) from any supply plant, and any distributor.
The Integrated Multiproduct Decision-making Optimization Problem (Case 1)

In addition, as before, let $f_a^j$ denote the flow of product $j$ on link $a$. Hence, we must also have the following conservation of flow equations satisfied:

$$f_a^j = \sum_{p \in P^1} x_p^j \delta_{ap}, \quad j = 1, \ldots, J; \quad \forall a \in L^1.$$

Of course, we also have that the path flows must be nonnegative for each product $j$, that is,

$$x_p^j \geq 0, \quad j = 1, \ldots, J; \quad \forall p \in P^1.$$
The Integrated Multiproduct Decision-making Optimization Problem (Case 1)

We assume, again, that the supply chain network activities have nonnegative capacities, denoted as $u_a$, $\forall a \in L^1$, with $\alpha_j$ representing the volume factor for product $j$.

Hence, the following constraints must be satisfied:

$$\sum_{j=1}^{J} \alpha_j f_a^j \leq u_a, \quad \forall a \in L^1.$$
The Integrated Multiproduct Decision-making Optimization Problem (Case 1)

Consequently, the following optimization problem for the integrated supply chain network needs now to be solved:

$$\text{Minimize } \sum_{j=1}^{J} \sum_{a \in L^1} \hat{c}^j_a(f_a^1, \ldots, f_a^J)$$

subject to the constraints.
The Integrated Multiproduct Decision-making Optimization Problem (Case 1)

The solution to the optimization problem subject to the constraints can also be obtained as a solution to a variational inequality problem akin to the pre-integrated optimization case, but where now $a \in L^1$.

The vectors: $f$, $x$, and $\beta$ have identical definitions as before, but are re-dimensioned/expanded accordingly.

Finally, instead of the feasible set $\mathcal{K}^0$ we now have $\mathcal{K}^1 \equiv \{ f | \exists x, \text{ and the constraints hold} \}$.

We denote the solution to the variational inequality problem governing Case 1 by $(f^{1*}, \beta^{1*})$. 
The vector of link flows $f_{1*}^1 \in \mathcal{K}^1$ is an optimal solution to the post-integration problem if and only if it satisfies the following variational inequality problem with the vector of nonnegative Lagrange multipliers $\beta_{1*}^k$:

$$
\sum_{j=1}^{J} \sum_{k=1}^{J} \sum_{a \in L^1} \left[ \frac{\partial \hat{c}_a^k(f_{1*}^k, \ldots, f_{J*}^k)}{\partial f_a^j} + \alpha_j \beta_{a}^* \right] \times [f_a^j - f_{a*}^j] + \sum_{a \in L^1} [u_a - \sum_{j=1}^{J} \alpha_j f_{a*}^j] \times [\beta_a - \beta_{a*}^a] \geq 0,
$$

$$
\forall f \in \mathcal{K}^1, \forall \beta \geq 0.
$$
Motivation

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Models

Synergy Measure

Vulnerability Analysis

Numerical Examples

Conclusions
Quantifying Synergy Associated with Multiproduct Decision-Making Organizations with Integration

We let $TC^0$ denote the total cost generated under solution $f^{0*}$ and we let $TC^1$ denote the total cost generated under solution $f^{1*}$.

Due to the similarity of the variational inequalities, the same computational procedure can be utilized to compute the solutions.

We measure the synergy by analyzing the total costs pre and post the supply chain network integration (cf. Eccles, Lanes, and Wilson (1999) and Nagurney (2007)).
Quantifying Synergy Associated with Multiproduct Decision-Making Organizations with Integration

The synergy based on total costs and proposed by Nagurney (2007), but now in a multiproduct context, which we denote here by $S^{TC}$, can be calculated as the percentage difference between the total cost pre vs the total cost post the integration:

$$S^{TC} \equiv \left[ \frac{TC^0 - TC^1}{TC^0} \right] \times 100\%.$$


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3. Models
4. Synergy Measure
5. Vulnerability Analysis
6. Numerical Examples
7. Conclusions
Uncertainties in Supply Chain Network

- There are two types of uncertainties, namely, demand-side uncertainty and supply-side uncertainty (Snyder 2003). The demand-side uncertainty is related to the demand fluctuation while the supply-side uncertainty deals with disruptions and unreliable suppliers.

- Most supply chain studies focus on the demand-side uncertainty and supply-side uncertainty has not been adequately addressed.

- In humanitarian logistics supply chain networks, it is crucial to have reliable suppliers and distributors to meet demands at demand points. Recall that to be able to deliver the humanitarian goods to the affected area in the first 72 hours after a disaster is crucial. Oftentimes, the successful execution means a difference between life and death (Van Wassenhove (2006)).
Because the complex and broad nature of the supply-side uncertainty issue, most studies in this area discuss managerial issues such as the supply chain risk mitigation strategies (see, e.g., Tang (2006a) and (2006b), Kleindorfer and Saad (2005)).

Most supply-disruption studies analyze the local effect, that is, a single-supplier problem (see, e.g., Gupta (1996), Parlar (1997)) or a two-supplier problem (see, e.g., Parlar and Perry (1996)).

Very few papers have examined the supply chain disruption management topic in an environment with multiple entities (Tomline (2006)).

Challenges in Humanitarian Logistics Vulnerability Study

- Humanitarian supply chains are exposed to more uncertainty and risk comparing to commercial supply chain networks (Van Wassenhove (2006)).

- Different goals with that of commercial supply chain networks. “A successful humanitarian operation mitigates the urgent needs of a population with a sustainable reduction of their vulnerability in the shortest amount of time and with the least amount of resources” (Tomasini and Van Wassenhove (2004)).

- A comprehensive and consistent approach is needed to study the performance of a humanitarian logistics network in order to address the vulnerability issue.

- Possible reasonable performance measures include: percentage of satisfied demand, total generalized cost, average response time, etc.
Numerical Examples

We present five numerical examples for which we compute the synergy measure.

We use single product cases for illustration purposes.

We apply the modified projection method of Korpelevich (1977) embedded with the equilibration algorithm of Dafermos and Sparrow (1969) (see also Nagurney (1993)) to solve the numerical examples.

We implemented the algorithm in FORTAN and utilized a Unix system at the University of Massachusetts for the computations.
The Pre-integration Supply Chain Network Topology for the Numerical Examples
The Integrated Supply Chain Network Topology for the Numerical Examples
Definition of the Links and the Associated Cost Functions for the Numerical Examples

<table>
<thead>
<tr>
<th>Link a</th>
<th>From Node</th>
<th>To Node</th>
<th>Ex. 1: $\hat{c}_a(f_a)$</th>
<th>Ex. 2: $\hat{c}_a(f_a)$</th>
<th>Ex. 3,4: $\hat{c}_a(f_a)$</th>
<th>Ex. 5: $\hat{c}_a(f_a)$</th>
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Numerical Example 1

Example 1 served as the baseline example.

The capacities on all the links in all the examples were set to: $u_a = 15$ for all links $a$.

The demands at each demand market, except were noted, was: $d_{R_1}^A = 5$, $d_{R_2}^A = 5$, and $d_{R_1}^B = 5$, $d_{R_2}^B = 5$.

The strategic advantage or synergy was approximately 15.1% when the demand markets could obtain the product from any supplier and from any distribution center.
Numerical Example 2

Example 2 was constructed from Example 1 as follows.

The demands were the same at each demand market, as in Example 1, as were the capacities and total cost functions for all links except for the total cost functions associated with the storage links, link 5 and link 12, representing the total costs associated with storing the product at the distribution centers associated with Organization A and Organization B, respectively.

Rather than having these total costs be given by $\hat{c}_5 = f_5^2 + 2f_5$ and $\hat{c}_{12} = f_{12} + 2f_{12}$ as they were in Example 1, they were now reduced to: $\hat{c}_5 = .5f_5^2 + f_5$ and $\hat{c}_{12} = .5f_{12} + f_{12}$.

The strategic advantage now increased.
Numerical Example 3

Example 3 was constructed from Example 2 and had the same data except that now we reduced the total cost associated with the supply plants belonging to Organization A.

Specifically, we changed $\hat{c}_1 = f_1^2 + 2f_1$ and $\hat{c}_2 = f_2 + 2f_2$ to:

$\hat{c}_1 = .5f_1^2 + f_1$ and $\hat{c}_2 = .5f_2 + f_2$.

The computed strategic advantage was greater than Example 2, although not substantially so.
Example 4 was identical to Example 3 except that now the demand \( d_{R_1}^A = 10 \), that is, the demand for the product doubled at the first demand market associated with Organization A.

Note that the integration yielded a strategic advantage of 24.1% which was higher than that obtained in Example 3.
Numerical Example 5

Example 5 was constructed from Example 4 and here we considered an idealized version in that the total cost functions associated with shipment links which are added after the respective integration are all equal to zero.

The strategic advantage was now quite significant, at 57.5%.

This example demonstrates that significant cost reductions can occur with integration in which the costs associated with distribution between the associated suppliers and distribution centers and the distribution centers and the demand markets are very low.
### Synergy Values for the Numerical Examples

<table>
<thead>
<tr>
<th>Example</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TC^0$</td>
<td>660.00</td>
<td>540.00</td>
<td>505.00</td>
<td>766.25</td>
<td>766.25</td>
</tr>
<tr>
<td>$TC^1$</td>
<td>560.00</td>
<td>432.00</td>
<td>389.80</td>
<td>581.30</td>
<td>320.20</td>
</tr>
<tr>
<td>$S$</td>
<td>15.1%</td>
<td>20%</td>
<td>20.8%</td>
<td>24.1%</td>
<td>57.5%</td>
</tr>
</tbody>
</table>
Conclusions

- We presented the pre-integration and the post-integration multiproduct humanitarian logistics supply chain network models, derived their variational inequality formulations, and then defined a total generalized cost synergy measure.

- The framework is based on a supply chain network perspective, in a system-optimization context, that captures the activities of a humanitarian organization such as manufacturing/production, storage, as well as distribution.

- We are the first to analyze the synergy effects of integration as related to humanitarian efforts.
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