

Program of the Workshop at the University of Catania, Italy

"Complex Networks - Equilibrium and Vulnerability Analysis with Applications"

Monday, March 10, 2008:

9:30 – 10:00 Registration

10:00 – 11:15 Anna Nagurney, Isenberg School of Management, University of Massachusetts at Amherst, "*Vulnerability Analysis of Complex Networks from Transportation Networks to the Internet and Electric Power Supply Chains.*"

Lunch

Fundamental Network Paradigms and Applications

14:45 – 15:15 Registration

15:15 – 15:30 Opening Ceremony

15:30 – 16:10 Patrizia Daniele, Department of Mathematics and Computer Sciences, University of Catania, "*Variational Inequalities and Applications to Complex Dynamic Multi-level Models.*"

16:10 – 16:50 Laura Scrimali, Department of Mathematics and Computer Sciences, University of Catania, "*Quasi-variational Inequalities and Applications to Complex Networks.*"

Coffee Break

17:10 – 17:50 Fabio Raciti, Department of Mathematics and Computer Sciences, University of Catania, "*Random Variational Inequalities and Applications to Network Equilibrium Problems.*"

17:50 – 18:30 Salvo Scellato, Scuola Superiore, University of Catania, "*Epcast: Controlled Dissemination in Human-based Wireless Networks by Means of Epidemic Spreading Models.*"

Tuesday, March 11, 2008: **Dynamic Networks**

9:00 – 9:40 Antonino Maugeri and Fabio Raciti, Department of Mathematics and Computer Sciences, University of Catania, "*On Existence Theorems for Variational Inequalities.*"

9:40 – 10:20 Stephane Pia, D.I.M.E.T., Faculty of Engineering, University of Reggio Calabria, "*Recent Results in Hilbert Spaces for PDS and VI when Duality Appears.*"

Coffee Break

10:40 – 11:20 Annamaria Barbagallo and Stephane Pia; respectively, Department of Mathematics and Computer Sciences, University of Catania, and D.I.M.E.T., Faculty of Engineering, University of Reggio Calabria, "*Weighted Variational Inequalities in Non-pivot Hilbert Spaces: Existence and Regularity Results and Applications.*"

11:20 – 12:00 Sofia Giuffrè, Giovanna Idone, and Stephane Pia, D.I.M.E.T., Faculty of Engineering, University of Reggio Calabria, *"Some Classes of Projected Dynamical Spaces in Banach Spaces and Equivalence Results."*

Lunch

15:30 – 16.10 Alessio Cardillo, I.N.F.N., University of Catania, *"Structural Properties of Urban Street Patterns and the Multiple Centrality Assessment."*

16.10 General Discussion

Wednesday, March 12, 2008: New Perspectives for Complex Networks and Vulnerability Analysis

9:00 – 9:40 Anna Nagurney and Trisha Woolley, Isenberg School of Management, University of Massachusetts at Amherst, *"Environmental Cost Synergy in Supply Chain Network Integration in Mergers and Acquisitions."*

9:40 – 10:20 Giuseppe Mangioni and Vincenzo Nicosia, D.I.I.T., Faculty of Engineering, University of Catania, *"Extension of Modularity for Overlapping Communities Detection in Complex Directed Graphs."*

Coffee Break

10:40 – 11:20 Maria Bernadette Donato, Department of Mathematics, University of Messina, *"A Competitive Equilibrium for a Pure Exchange Walrasian Economy: A Quasi-variational Approach."*

11:20 – 12:00 Monica Milasi, Department of Mathematics, University of Messina, *"Existence of Equilibrium for a Walrasian Pure Exchange Economy with Utility Functions: Duality and Lagrangean Theory."*

The co-organizers, Professors Patrizia Daniele and Anna Nagurney, thank the Fulbright Commission and the Department of Mathematics and Computer Sciences at the University of Catania for making this workshop possible. Special thanks also go to Professor Antonino Maugeri of the Department of Mathematics and Computer Sciences at the University of Catania.

Weighted Variational Inequalities in Non-pivot Hilbert Spaces: Existence and Regularity Results and Applications

Annamaria Barbagallo - Stephane Pia

The aim of the talk is to consider weighted variational inequalities in non-pivot Hilbert spaces and to establish some existence and regularity results. In particular we prove an existence result under general assumptions, we show a continuity result for the solution to strongly pseudo-monotone weighted variational inequalities. Moreover, using a penalization procedure, we are able to generalize the previous regularity result in the case of strictly pseudo-monotone weighted variational inequalities. Finally, we present the dynamic weighted traffic equilibrium problem and we apply the established results to this problem. At last, we use an extension version of the Solodov-Sveiter algorithm to solve the equilibrium problem using a discretization procedure.

Structural Properties of Urban Street Patterns and the Multiple Centrality Assessment

Alessio Cardillo

We investigate the basic structural properties of urban street patterns. Samples of different world cities are turned into spatial graphs. In such graphs, the nodes are embedded in the two-dimensional plane and represent street intersections, the edges represent streets, and the edge values are equal to the street lengths.

We evaluate the local properties of the graphs by measuring the meshedness coefficient and counting short cycles from three to five edges, and the global properties by measuring global efficiency and cost. The results obtained are compared to extreme cases, such as minimal spanning trees (MST) and greedy triangulations (GT) induced by the same spatial distribution of nodes. We also present a comprehensive study of centrality distributions over networks of urban streets. Five different measures of centrality, namely degree, closeness, betweenness, straightness and information, are compared over samples of different world cities.

The results indicate that a spatial analysis, that we term Multiple Centrality Assessment (MCA), grounded not on a single but on a set of different centrality indices, allows an extended comprehension of the city structure, nicely capturing the skeleton of most central routes and the location of commercial activities and services in a city.

Variational Inequalities and Applications to Complex Dynamic Multi-level Models

Patrizia Daniele

After a brief introduction on variational inequalities, time-dependent variational inequalities and their applications, we present a complex dynamic model where multi-level networks of decision-makers (manufacturers, retailers, consumers) are studied in a time interval $[0, T]$. Specifically, we consider a multi-level supply chain supernetwork with electronic commerce where supply and demand excesses can occur. Using recent tools of infinite dimensional Lagrange theory and functional analysis, we characterize the optimality conditions for the representatives of each tier and furnish existence results, regularity theorems and computational procedures.

A Competitive Equilibrium for a Pure Exchange Walrasian Economy: A Quasi-variational Approach

Maria Bernadette Donato

A competitive equilibrium problem for a pure exchange economy consisting of l goods and n agents is considered and for each commodity a nonnegative price is associated. Each agent is endowed with a positive quantity of at least one commodity. Agents' preferences for consuming different goods are given by a utility function; the aim of the agent is to maximize his utility function by performing the exchange of the goods. The choice is subjected to the constraint that the amount that the agent pays for acquiring the goods is at most the amount that the agent receives for his initial endowment. Assumptions made on the utility function guarantee that this market is regulated by Walras' law: if there is a positive excess demand for any one commodity, then there must be a corresponding negative excess demand for at least one other commodity. The equilibrium conditions for such model are given and the equivalent formulation in terms of a quasi-variational inequality is discussed.

Some Classes of Projected Dynamical Systems in Banach Spaces and Equivalence Results

Sofia Giuffrè, Giovanna Idone, and Stephane Pia

It is well known that in Hilbert spaces critical points of projected dynamical systems are equivalent to equilibrium points of a variational inequality. This theory allows the study of the behaviour of a system before the equilibrium. We present a generalization of this equivalence result to strictly convex and smooth (SCS) Banach spaces by means of the metric and generalized projection operator introduced by Alber and by means of the generalization of the Moreau decomposition theorem in SCS Banach spaces. Moreover the relationship between projected dynamical systems and differential inclusions are investigated.

Extension of Modularity for Overlapping Communities Detection in Complex Directed Graphs

Giuseppe Mangioni - Vincenzo Nicosia

Complex networks are a really valuable model to describe and study many kinds of phenomena, such as the structure and properties of social relationships, functional dependencies of parts in complex systems, cooperation among individuals, communication and information interchange in technological environments and so on. In the last few years, a growing number of researchers has focused on the problem of finding interesting decomposition in such networks in modules or community, where a module usually contains all the nodes which share given properties. Many solutions to the problem of structural modularity of graphs, i.e. the problem of finding topological communities of a networks, have been recently proposed and perform reasonably well, at least in the case of undirected graphs with known sharp community decomposition. The main issue with those methods is that real networks are rarely decomposed into sharp and separate communities: in

those networks a node can easily belong to more than just one community, with a certain degree. This contribution reviews some concepts and metrics used for community detection and extends them to the case of directed networks with overlapping community structure, proposing a generic analytical reformulation of the modularity function given by Girvan and Newman which works for generic graphs where overlap among communities is allowed.

On Existence Theorems for Variational Inequalities

Antonino Maugeri - Fabio Raciti

Let E be a real topological vector space, let K be a convex and nonempty subset of E , and let $C: K \rightarrow E^*$ be given. Let us consider the variational inequality

$$\text{Find } x \in K : \langle C(x), y-x \rangle \geq 0, \forall y \in K. \quad (1)$$

The following assumptions are used in order to ensure existence theorems for problem (1):

- (i) Pseudomonotonicity in the sense of Brezis,
- (ii) Pseudomonotonicity in the sense of Karamardian,
- (iii) Pseudomonotonicity (hemicontinuity) in the sense of D.M.O.

Moreover, the following kinds of continuity are sometimes added to one of the previous kind of pseudomonotonicity:

- (i) Hemicontinuity along line segments in the sense of D.M.O. namely, for all $x, y \in K$, the function $\xi \rightarrow \langle C(\xi), x - y \rangle$ is lower semicontinuous on the line segment $[x, y]$,
- (ii) Continuity on finite-dimensional subspaces,
- (iii) Hemicontinuity along line segments namely, for all $x, y \in K$, the function $\xi \rightarrow \langle C(\xi), x - y \rangle$ is continuous on the line segment $[x, y]$.

The aim of the talk is to make a comparison among the various existence theorems and to find necessary conditions for the existence of solutions to problem (1).

Existence of Equilibrium for a Walrasian Pure Exchange Economy with Utility Functions: Duality and Lagrangean Theory

Monica Milasi

A Walrasian equilibrium problem for a pure exchange economy is studied. To this end the equilibrium conditions are expressed in terms of a quasi-variational inequality. Firstly the existence of the equilibrium is investigated: the quasi-variational inequality that characterizes the competitive equilibrium does not satisfy the assumptions of existence classical theorems, but these difficulties are overcome thanks to convergence in the sense of Mosco. Moreover, the Generalized Lagrangean and Duality theories and, as an interesting consequence, Lagrangean variables are obtained. This theory allows us to describe the behaviour of the market. Moreover, in order to support the theoretical results a numerical example is provided.

Environmental and Cost Synergy in Supply Chain Network Integration in Mergers and Acquisitions

Anna Nagurney - Trisha Woolley

In this paper, we focus on synergy rather than vulnerability, and we quantify and assess, from a supply chain network perspective, the environmental effects resulting when a merger of two firms or an acquisition of one firm by another occurs. We develop a multicriteria decision-making supply chain network framework that captures the economic activities of manufacturing, storage, and distribution pre and post the merger. The variational inequality-based models yield the system optima associated with the minimization of total costs and the total emissions under firm-specific weights. We propose a synergy measure that captures the total generalized cost. We then apply the new mathematical framework to quantify the synergy obtained for specific numerical examples. This work generalizes the recent system-optimization models for supply chain network integration and associated synergies of Nagurney (2007), forthcoming in *Transportation Research E*, to the environmental and multicriteria dimensions.

Recent Results and Applications in Hilbert Spaces for PDS and VI when Duality Appears

Stephane Pia

We present some recent results for both Projected Dynamical System and Variational Inequality theory. We show how the standard framework of Hilbert spaces can be extended to non-pivot spaces and then how this viewpoint opens the doors to enhancements in strictly convex and smooth Banach spaces. We present also as an application the weighted traffic problem in non-pivot Hilbert spaces.

Random Variational Inequalities and Applications to Network Equilibrium Problems

Fabio Raciti

We study a class of monotone random variational inequalities on random sets and extend some previous results obtained in the case of a random bilinear form in a Hilbert space setting. The new results allow us to treat a larger class of applications and are used to model nonlinear traffic networks in presence of random data.

Epcast: Controlled Dissemination in Human-based Wireless Networks by Means of Epidemic Spreading Models

Salvo Scellato

Epidemics-inspired techniques have received huge attention in recent years from the distributed systems and networking communities. These algorithms and protocols rely on probabilistic message replication and redundancy to ensure reliable communication. Moreover, they have been successfully exploited to support group communication in distributed systems, broadcasting, multicasting and information dissemination in fixed and mobile networks. However, in most of the existing work, the probability of infection is determined heuristically, without relying on any analytical model. This often leads to unnecessarily high transmission overheads. Models of epidemic spreading in complex networks can be applied to the problem of tuning and controlling

the dissemination of information in wireless ad hoc networks composed of devices carried by individuals, i.e., human-based networks. The novelty of this idea resides in the evaluation and exploitation of the structure of the underlying human network for the automatic tuning of the dissemination process in order to improve the protocol performance. We evaluate the results using synthetic mobility models and real human contacts traces.

Quasi-variational Inequalities and Applications to Complex Networks

Laura Scrimali

Recently there has been a sharp increase in interest in the field of quasi-variational inequalities, which has led to a remarkable development in the theory of existence and regularity of solutions; algorithms; and applications to network-based problems. This contribution focuses on quasivariational inequalities modelling mixed equilibrium strategies on networks, with particular regard to existence and sensitivity analysis of solutions.

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