Topic 8: Disaster Relief Supply Chains

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Outline

- Background and Motivation
- A Mean-Variance Disaster Relief Supply Chain Network Model for Risk Reduction
- A Game Theory Model for Post-Disaster Humanitarian Relief

Disaster Relief Supply Chains

This first part of this lecture is based on the paper, "A Mean-Variance Disaster Relief Supply Chain Network Model for Risk Reduction with Stochastic Link Costs, Time Targets, and Demand Uncertainty," Anna Nagurney and Ladimer S. Nagurney, in *Dynamics of Disasters: Key Concepts, Models, Algorithms, and Insights*, I.S. Kotsireas, A. Nagurney, and P.M. Pardalos, editors, Springer International Publishing Switzerland, 2016, pp. 231-255, where many additional references can be found.

The second part of this lecture is based on the paper, "A Generalized Nash Equilibrium Network Model for Post-Disaster Humanitarian Relief," Anna Nagurney, Emilio Alvarez-Flores, and Ceren Soylu, *Transportation Research E*, (2016), **95**, pp. 1-18.

Background and Motivation

Supply chains are the **fundamental critical infrastructure** for the production and distribution of goods and services in our globalized **Network Economy**.

Supply chain networks also serve as the primary conduit for **disaster preparedness, response, recovery, and reconstruction.**

Some Recent Disasters

- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 – one in the UK and the other in Italy and Switzerland;
- The Indonesian tsunami (and earthquake), December 26, 2004;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge collapse, August 1, 2007;
- The Sichuan earthquake on May 12, 2008;
- The Haiti earthquake that struck on January 12, 2010 and the Chilean one on February 27, 2010;
- The triple disaster in Japan on March 11, 2011;
- Superstorm Sandy, October 29, 2012.

Hurricane Katrina in 2005

Hurricane Katrina has been called an **"American tragedy,"** in which essential services failed completely.

The Haitian and Chilean Earthquakes

The Triple Disaster in Japan on March 11, 2011

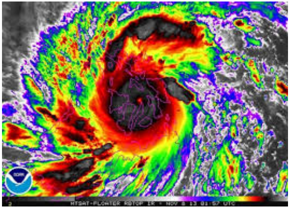
The world reeled from the aftereffects of the triple disaster in Japan with death tolls in the Fukushina area alone over 8,000, and with disruptions in the high tech, automotive, and even food industries.

Superstorm Sandy and Power Outages

Manhattan without power October 30, 2012 as a result of the devastation wrought by Superstorm Sandy.

Haiyan Typhoon in the Philippines in 2013

Typhoon Haiyan was a very powerful tropical cyclone that devastated portions of Southeast Asia, especially the Philippines, on November 8, 2013. It is the deadliest Philippine typhoon on record, killing at least 6,190 people in that country alone. Haiyan was also the strongest storm recorded at landfall. As of January 2014, bodies were still being found. The overall economic losses from Typhoon Haiyan totaled $10 billion.



Nepal Earthquake in 2015

The 7.8 magnitude earthquake that struck Nepal on April 25, 2015, and the aftershocks that followed, killed nearly 9,000 people and injured 22,000 others. This disaster also pushed about 700,000 people below the poverty line in the Himalayan nation, which is one of the world's poorest. About 500,000 homes were made unlivable by the quakes, leaving about three million people homeless. According to *The Wall Street Journal*, Nepal needs $6.66 billion to rebuild.

The Ebola Crisis in West Africa

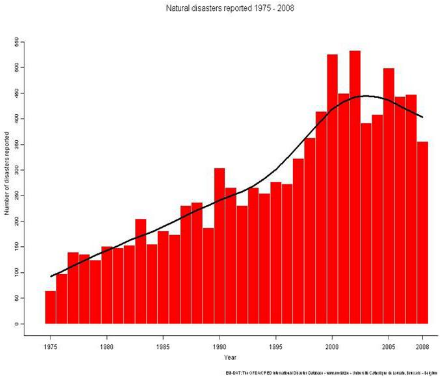
According to bbc.com and the World Health Organization, more than one year from the first confirmed case recorded on March 23, 2014, at least 11,178 people were reported as having died from Ebola in six countries; Liberia, Guinea, Sierra Leone, Nigeria, the US and Mali. The total number of reported cases was more than 27,275. Image thanks to cnn.com.

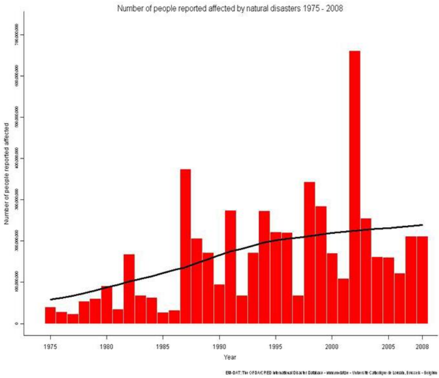
Ms. Debbie Wilson of Doctors Without Borders

On February 4, 2015, the students in my Humanitarian Logistics and Healthcare class at the Isenberg School heard Debbie Wilson, a nurse, who has worked with Doctors Without Borders, speak on her 6 weeks of experiences battling Ebola in Liberia in September and October 2014.

Disasters have a catastrophic effect on human lives and a region's or even a nation's resources.

Natural Disasters (1975–2008)





A Mean-Variance Disaster Relief Supply Chain Network Model for Risk Reduction

Elements of Our Model

Recently, there has been growing interest in constructing **integrated frameworks** that can assist in multiple phases of disaster management.

Network-based models and tools, which allow for a graphical depiction of disaster relief supply chains and provide the flexibility of adding nodes and links, coupled with effective computational procedures, in particular, offer promise.

The Importance of Time in Disaster Relief

The U.S. Federal Emergency Management Agency (FEMA) has identified **key benchmarks to response and recovery, which emphasize time** and they are: to meet the survivors' initial demands within 72 hours, to restore basic community functionality within 60 days, and to return to as normal of a situation within 5 years (Fugate (2012)).

Timely and efficient delivery of relief supplies to the affected population **not only decreases the fatality rate but may also prevent chaos**. In the case of Typhoon Haiyan, slow relief delivery efforts forced people to seek any possible means to survive. Several relief trucks were attacked and had food stolen, and some areas were reported to be on the brink of anarchy (Chicago Tribune (2013) and CBS News (2013)).

Inspiration for the Model

The model that is now presented is inspired by the supply chain network integration model for risk reduction in the case of mergers and acquisitions developed by Liu and Nagurney, in the paper: "Risk Reduction and Cost Synergy in Mergers and Acquisitions via Supply Chain Network Integration," *Journal of Financial Decision-Making*, (2011), **7(2)**, 1-18, coupled with the integrated disaster relief framework of Nagurney, Masoumi, and Yu in the paper, "An Integrated Disaster Relief Supply Chain Network Model with Time Targets and Demand Uncertainty," in *Regional Science Matters: Studies Dedicated to Walter Isard*, (2015), P. Nijkamp, A. Rose, and K. Kourtit, Editors, Springer, 287-318.

Risk Reduction Model of Liu and Nagurney (2011)

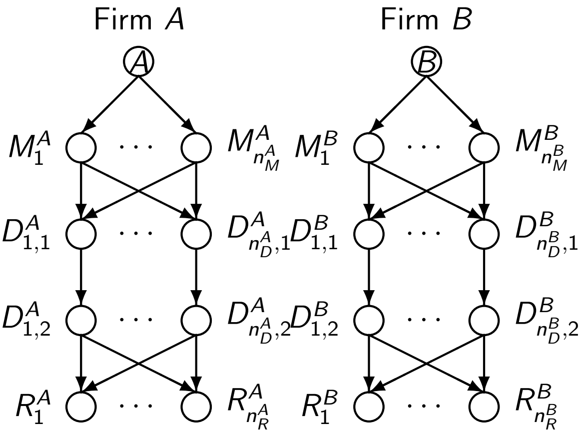


Figure: The Pre-Merger Supply Chain Network

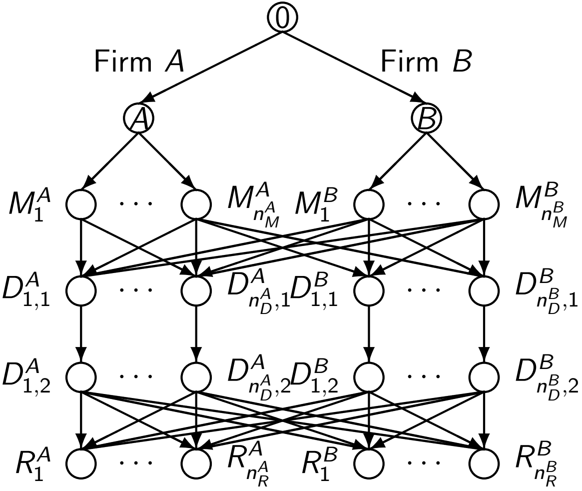


Figure: Firms A and B Merge: Demand Points of Either Firm Can Get the Product from Any Manufacturing Plant Via Any Distribution Center

The synergy measures developed and the framework are also applicable to the teaming of organizations as in horizontal collaboration.

Integrated Disaster Relief Model of Nagurney, Masoumi, and Yu (2015)

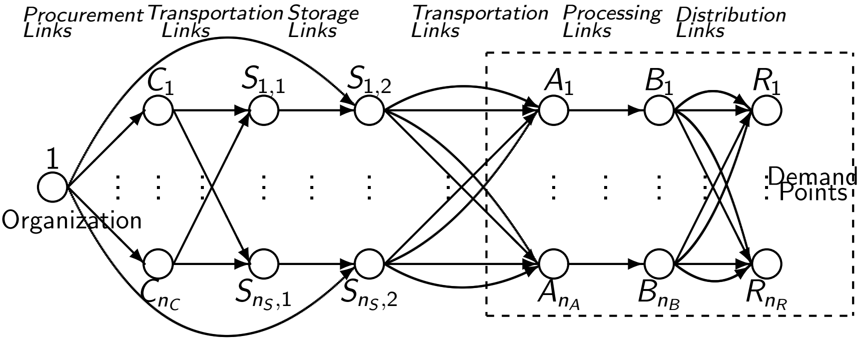


Figure: Network Topology of the Integrated Disaster Relief Supply Chain

Inspiration for the Model

The MV approach to risk reduction dates to the work of the Nobel laureate Harry Markowitz (1952, 1959) and is still relevant in **finance** (Schneeweis, Crowder, and Kazemi (2010)), in **supply chains** (Chen and Federgruen (2000) and Kim, Cohen, and Netessine (2007)), as well as in **disaster relief and humanitarian operations**, where the focus, to-date, has been on inventory management (Ozbay and Ozguven (2007) and Das (2014)).

Inspiration for the Model

The new model constructed here is the **first to integrate preparedness and response in a supply chain network framework** using a **Mean-Variance approach for risk reduction under demand and cost uncertainty** and **time targets plus penalties for shortages and surpluses**.

Bozorgi-Amiri et al. (2013) developed a model with uncertainty on the demand side and also in procurement and transportation using expected costs and variability with associated weights but **did not consider the critical time elements** as well as the **possibility of local versus nonlocal procurement post- or pre-disaster**.

Inspiration for the Model

In addition, Boyles and Waller (2009) developed a MV model for the minimum cost network flow problem with stochastic link costs and emphasized that an MV approach is especially relevant in logistics and distribution problems with critical implications for supply chains.

They noted that a solution that only minimizes expected cost and not variances may not be as reliable and robust as one that does.

What We Seek to Achieve with the Model

• In our model, the **humanitarian organization seeks to minimize its expected total operational costs and the total risk in operations with an individual weight assigned to its valuation of the risk, as well as the minimization of expected costs of shortages and surpluses and tardiness penalties associated with the target time goals at the demand points**.

What We Seek to Achieve with the Model

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• The risk is captured through the variance of the total operational costs, which is of relevance also to the reporting of the proper use of funds to stakeholders, including donors.

What We Seek to Achieve with the Model

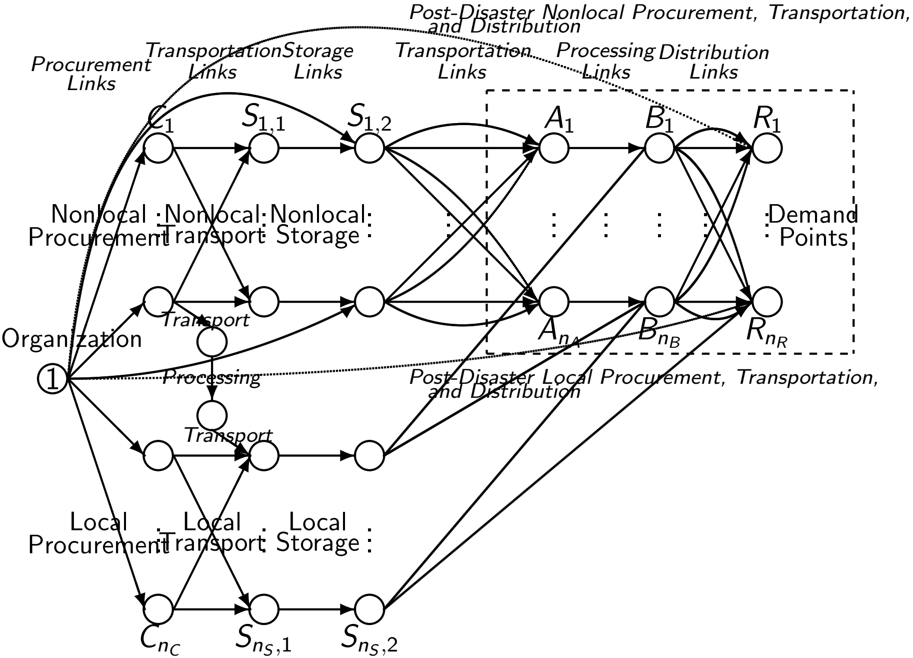
• The time goal targets associated with the demand points enable prioritization of demand points as to the timely delivery of relief supplies.

What We Seek to Achieve with the Model

- This framework handles both the pre-positioning of relief supplies, whether local or nonlocal, as well as the procurement (local or nonlocal), transport, and distribution of supplies post-disaster. **There is growing empirical evidence showing that the use of local resources in humanitarian supply chains can have positive impacts (see Matopoulos, Kovacs, and Hayes (2014)).** Earlier work on procurement with stochastic components did not distinguish between local or nonlocal procurement (see Falasca and Zobel (2011)).

What We Seek to Achieve with the Model

• The time element in our model is captured through **link time completion functions as the relief supplies progress along paths in the supply chain network**. Each path consists of a series of directed links, from the origin node, which represents the humanitarian organization, to the destination nodes, which are the demand points for the relief supplies.


Post-Disaster Nonlocal Procurement, Transportation, and Distribution
Procurement Links
Transportation Links
Storage Links
Transportation Links
Processing Links
Distribution Links
C_1
$S_{1,1}$
$S_{1,2}$
A_1
B_1
R_1
Nonlocal Procurement
Nonlocal Transport
Nonlocal Storage
Demand Points
Transport
Organization
1
Processing
Transport
A_{n_A}
B_{n_B}
R_{n_R}
Post-Disaster Local Procurement, Transportation, and Distribution
Local Procurement
Local Transport
Local Storage
C_{n_C}
$S_{n_S,1}$
$S_{n_S,2}$

Mean-Variance Disaster Relief Supply Chain Model

In the model, the demand is uncertain due to the unpredictability of the actual demand at the demand points. The probability distribution of demand might be derived using census data and/or information gathered during the disaster preparedness phase. Since d_k denotes the actual (uncertain) demand at destination point k, we have:

$$P_k(D_k) = P_k(d_k \leq D_k) = \int_0^{D_k} \mathcal{F}_k(u)du, \qquad k = 1, \ldots, n_R, \tag{1}$$

where P_k and $\mathcal{F}_k$ denote the probability distribution function, and the probability density function of demand at point k, respectively.

Mean-Variance Disaster Relief Supply Chain Model

Here v_k is the "projected demand" for the disaster relief item at demand point k; $k = 1, \ldots, n_R$. The amounts of shortage and surplus at destination node k are calculated according to:

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \qquad k = 1, \ldots, n_R, \qquad (2a)$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \qquad k = 1, \ldots, n_R. \qquad (2b)$$

Mean-Variance Disaster Relief Supply Chain Model

The expected values of shortage and surplus at each demand point are, hence:

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (u - v_k)\mathcal{F}_k(u)du, \qquad k = 1, \ldots, n_R, \qquad (3a)$$

$$E(\Delta_k^+) = \int_{0}^{v_k} (v_k - u)\mathcal{F}_k(u)du, \qquad k = 1, \ldots, n_R. \qquad (3b)$$

The expected penalty incurred by the humanitarian organization due to the shortage and surplus of the relief item at each demand point is equal to:

$$E(\lambda_k^-\Delta_k^- + \lambda_k^+\Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \qquad k = 1, \ldots, n_R. \qquad (4)$$

Mean-Variance Disaster Relief Supply Chain Model

We have the following two sets of conservation of flow equations. The projected demand at destination node k, v_k, is equal to the sum of flows on all paths in the set $\mathcal{P}_k$, that is:

$$v_k \equiv \sum_{p \in \mathcal{P}_k} x_p, \qquad k = 1, \ldots, n_R. \tag{5}$$

The flow on link a, f_a, is equal to the sum of flows on paths that contain that link:

$$f_a = \sum_{p \in \mathcal{P}} x_p \, \delta_{ap}, \qquad \forall a \in L, \tag{6}$$

where δ_{ap} is equal to 1 if link a is contained in path p and is 0, otherwise.

Mean-Variance Disaster Relief Supply Chain Model

Here we consider total operational link cost functions of the form:

$$\hat{c}_a = \hat{c}_a(f_a, \omega_a) = \omega_a \hat{g}_a f_a + g_a f_a, \quad \forall a \in L, \qquad (7)$$

where $\hat{g}_a$ and g_a are positive-valued for all links $a \in L$. We permit ω_a to follow any probability distribution and the ωs of different supply chain links can be correlated with one another.

The term $\hat{g}_a f_a$ represents the part of the total link operational cost that is subject to variation of ω_a with $g_a f_a$ denoting that part of the total cost that is independent of ω_a.

Mean-Variance Disaster Relief Supply Chain Model

The random variables ω_a, $a \in L$ can capture various elements of uncertainty, due, for example, to disruptions because of the disaster, and price uncertainty for storage, procurements, transport, processing, and distribution services.

Mean-Variance Disaster Relief Supply Chain Model

The completion time function associated with the activities on link a is given by:

$$\tau_a(f_a) = \hat{t}_a f_a + t_a, \quad \forall a \in L,$$

where $\hat{t}_a$ and t_a are ≥ 0.

The target for completion of activities on paths corresponding to demand point k is given by T_k and is imposed for each demand point k by the humanitarian organization decision-maker.

The target for a path p to demand point k is then $T_{kp} = T_k - t_p$, where $t_p = \sum_{a \in L} t_a \delta_{ap}$, $\forall p \in \mathcal{P}_k$.

Mean-Variance Disaster Relief Supply Chain Model

The variable z_p is the amount of deviation with respect to the target time T_{kp} associated with the late delivery of relief items to k on path p, $\forall p \in \mathcal{P}_k$. We group the s_ps into the vector $z \in R_+^{n_\mathcal{P}}$.

$\gamma_k(z)$ is the tardiness penalty function corresponding to demand point k; $k = 1, \ldots, n_R$.

Mean-Variance Disaster Relief Supply Chain Model

The objective function faced by the organization's decision-maker, which he seeks to minimize, is the following:

$$E\left[\sum_{a\in L}\hat{c}_a(f_a,\omega_a)\right]+\alpha Var\left[\sum_{a\in L}\hat{c}_a(f_a,\omega_a)\right]+\sum_{k=1}^{n_R}(\lambda_k^- E(\Delta_k^-)+\lambda_k^+ E(\Delta_k^+))$$
$$+\sum_{k=1}^{n_R}\gamma_k(z)$$
$$=\sum_{a\in L}E\left[\hat{c}_a(f_a,\omega_a)\right]+\alpha Var\left[\sum_{a\in L}\hat{c}_a(f_a,\omega_a)\right]+\sum_{k=1}^{n_R}(\lambda_k^- E(\Delta_k^-)+\lambda_k^+ E(\Delta_k^+))$$
$$+\sum_{k=1}^{n_R}\gamma_k(z), \tag{8}$$

where E denotes the expected value, Var denotes the variance, and α represents the risk aversion factor (weight) for the organization that the organization's decision-maker places on the risk.

Mean-Variance Disaster Relief Supply Chain Model

The goal of the decision-maker is, thus, to minimize the following problem, with the objective function in (8), in lieu of (7), taking the form in (9) below:

$$\text{Minimize} \sum_{a \in L} E(\omega_a)\hat{g}_a f_a + \sum_{a \in L} g_a f_a + \alpha Var(\sum_{a \in L} \omega_a \hat{g}_a f_a)$$
$$+ \sum_{k=1}^{n_R} (\lambda_k^+ - E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z) \qquad (9)$$

subject to constraint (6) and the following constraints:

$$x_p \geq 0, \quad \forall p \in \mathcal{P}, \qquad (10)$$

$$z_p \geq 0, \quad \forall p \in \mathcal{P}, \qquad (11)$$

$$\sum_{q \in \mathcal{P}} \sum_{a \in L} \hat{t}_a x_q \delta_{aq} \delta_{ap} - z_p \leq T_{kp}, \quad \forall p \in \mathcal{P}_k; k = 1, \ldots, n_R. \qquad (12)$$

Mean-Variance Disaster Relief Supply Chain Model

In view of constraint (6) we can reexpress the objective function in (9) in path flows (rather than in link flows and path flows) to obtain the following optimization problem:

$$\text{Minimize} \sum_{a \in L} \left[E(\omega_a) \hat{g}_a \sum_{q \in \mathcal{P}} x_q \delta_{aq} + g_a \sum_{q \in \mathcal{P}} x_q \delta_{aq} \right]$$

$$+\alpha Var\Big(\sum_{a \in L} \omega_a \hat{g}_a \sum_{q \in \mathcal{P}} x_q \delta_{aq}\Big) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z) \tag{13}$$

subject to constraints: (10) – (12).

Let K denote the feasible set:

$$K \equiv \{(x, z, \mu) | x \in R_+^{n_{\mathcal{P}}}, z \in R_+^{n_{\mathcal{P}}}, \text{ and } \mu \in R_+^{n_{\mathcal{P}}}\}, \tag{14}$$

where μ is the vector of Lagrange multipliers corresponding to the constraints in (12) with an individual element

Mean-Variance Disaster Relief Supply Chain Model

Before presenting the variational inequality formulation of the optimization problem immediately above, we review the respective partial derivatives of the expected values of shortage and surplus of the disaster relief item at each demand point with respect to the path flows, derived in Dong, Zhang, and Nagurney (2004), Nagurney, Yu, and Qiang (2011), and Nagurney, Masoumi, and Yu (2012). In particular, they are given by:

$$\frac{\partial E(\Delta_k^-)}{\partial x_p} = P_k\left(\sum_{q\in\mathcal{P}_k} x_q\right) - 1, \quad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \tag{15a}$$

and,

$$\frac{\partial E(\Delta_k^+)}{\partial x_p} = P_k\left(\sum_{q\in\mathcal{P}_k} x_q\right), \quad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{15b}$$

Mean-Variance Disaster Relief Supply Chain Model

Theorem: Variational Inequality Formulation

The optimization problem (13), subject to its constraints (10) – (12), is equivalent to the variational inequality problem: determine $(x^*, z^*, \mu^*) \in K$*, such that,* $\forall (x, z, \mu) \in K$*:*

$$\sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[\sum_{a \in L} (E(\omega_a)\hat{g}_a + g_a)\delta_{ap} + \alpha \frac{\partial Var(\sum_{a \in L} \omega_a \hat{g}_a \sum_{q \in \mathcal{P}} x_q^* \delta_{aq})}{\partial x_p} \right.$$

$$\left. + \lambda_k^+ P_k(\sum_{q \in \mathcal{P}_k} x_q^*) \; - \; \lambda_k^-(1 - P_k(\sum_{q \in \mathcal{P}_k} x_q^*)) + \sum_{q \in \mathcal{P}} \sum_{a \in L} \mu_q^* g_a \delta_{aq} \delta_{ap} \right]$$

$$\times [x_p - x_p^*] + \sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[\frac{\partial \gamma_k(z^*)}{\partial z_p} - \mu_p^* \right] \times [z_p - z_p^*]$$

$$+ \sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[T_{kp} + z_p^* - \sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q^* \delta_{aq} \delta_{ap} \right] \times [\mu_p - \mu_p^*] \geq 0. \quad (16)$$

VI (16) can be put into standard form: find $X^* \in \mathcal{K}$:

$$\left\langle F(X^*), X - X^* \right\rangle \geq 0, \quad \forall X \in \mathcal{K}, \qquad (17)$$

with the feasible set $\mathcal{K} \equiv K$, the column vectors $X \equiv (x, z, \mu)$, and $F(X) \equiv (F_1(X), F_2(X), F_3(X))$:

$$F_1(X) = \Bigg[\sum_{a\in L}(E(\omega_a)\hat{g}_a + g_a)\delta_{ap} + \alpha\frac{\partial Var(\sum_{a\in L}\omega_a\hat{g}_a\sum_{q\in\mathcal{P}}x_q\delta_{aq})}{\partial x_p}$$

$$+\lambda_k^+ P_k(\sum_{q\in\mathcal{P}_k}x_q) - \lambda_k^-(1 - P_k(\sum_{q\in\mathcal{P}_k}x_q)) + \sum_{q\in\mathcal{P}}\sum_{a\in L}\mu_q g_a\delta_{aq}\delta_{ap}, p \in \mathcal{P}_k;$$

$$F_2(X) = \left[\frac{\partial\gamma_k(z)}{\partial z_p} - \mu_p, p \in \mathcal{P}_k; k = 1, \ldots, n_R\right],$$

$$F_3(X) = \left[T_{kp} + z_p - \sum_{q\in\mathcal{P}}\sum_{a\in L}g_a x_q\delta_{aq}\delta_{ap}, p \in \mathcal{P}_k; \forall k\right]. \qquad (18)$$

The Algorithm

At an iteration τ of the Euler method (cf. Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{19}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{20}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous.

The Algorithm

As shown in Dupuis and Nagurney (1993); see also Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, among other methods, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \rightarrow 0$, as $\tau \rightarrow \infty$. Specific conditions for convergence of this scheme can be found for a variety of network-based problems, similar to those constructed here, in Nagurney and Zhang (1996) and the references therein.

The Algorithm

Explicit Formulae for the Euler Method

Closed form expressions $\forall p \in \mathcal{P}_k; \forall k$:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^-(1 - P_k(\sum_{q\in\mathcal{P}_k} x_q^\tau)) - \lambda_k^+ P_k(\sum_{q\in\mathcal{P}_k} x_q^\tau)$$

$$- \sum_{a\in L}(E(\omega_a)\hat{g}_a + g_a)\delta_{ap} - \alpha\frac{\partial Var(\sum_{a\in L}\omega_a\hat{g}_a\sum_{q\in\mathcal{P}} x_q^\tau\delta_{aq})}{\partial x_p}$$

$$- \sum_{q\in\mathcal{P}}\sum_{a\in L}\mu_q^\tau g_a\delta_{aq}\delta_{ap})\}; \quad (21)$$

$$z_p^{\tau+1} = \max\{0, z_p^\tau + a_\tau(\mu_p^\tau - \frac{\partial\gamma_k(z^\tau)}{\partial z_p})\}, \quad (22)$$

$$\mu_p^{\tau+1} = \max\{0, \mu_p^\tau + a_\tau(\sum_{q\in\mathcal{P}}\sum_{a\in L} g_a x_q^\tau\delta_{aq}\delta_{ap} - T_{kp} - z_p^\tau\}. \quad (23)$$

The Algorithm

In view of (21), we can define a generalized marginal total cost on path p; $p \in \mathcal{P}$, denoted by $G\hat{C}'_p$, where

$$G\hat{C}'_p \equiv \sum_{a \in L} (E(\omega_a)\hat{g}_a + g_a)\delta_{ap} + \alpha \frac{\partial Var(\sum_{a \in L} \omega_a \hat{g}_a \sum_{q \in \mathcal{P}} x_q \delta_{aq})}{\partial x_p}. \quad (24)$$

Background on the Mexico Case Study

According to the United Nations (2011), **Mexico is ranked as one of the world's thirty most exposed countries to three or more types of natural disasters, notably, storms, hurricanes, floods, as well as earthquakes, and droughts**.

For example, as reported by The International Bank for Reconstruction and Development/The World Bank (2012), **41% of Mexico's national territory is exposed to storms, hurricanes, and floods; 27% to earthquakes, and 29% to droughts**.

Background on the Mexico Case Study

The hurricanes can come from the Atlantic or Pacific oceans or the Caribbean.

As noted by de la Fuente (2011), the single most costly disaster in Mexico were the 1985 earthquakes, followed by the floods in the southern state of Tabasco in 2007, with damages of more than 3.1 billion U.S. dollars.

Mexico Case Study

We consider a humanitarian organization such as the Mexican Red Cross, which is interested in preparing for another possible hurricane, and recalls **the devastation wrought by Hurricane Manuel and Hurricane Ingrid, which struck Mexico within a 24 hour period in September 2013**.

Ingrid caused 32 deaths, primarily, in eastern Mexico, whereas Manuel resulted in at least 123 deaths, primarily in western Mexico (NOAA (2014)). According to Pasch and Zelinsky (2014), the total economic impact of Manuel alone was estimated to be approximately $4.2 billion (U.S.), with the biggest losses occurring in Guerrero.

Mexico Case Study

We assume that the Mexican Red Cross is mainly concerned about the delivery of relief supplies to the Mexico City area and the Acapulco area.

Ingrid affected Mexico City and Manuel affected the Acapulco area and also points northwest.

Photos of Acapulco post Manuel courtesy The Weather Channel.

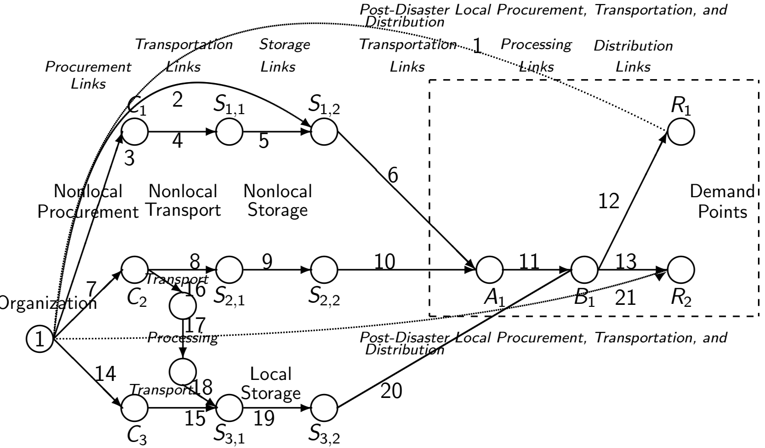


Figure: Disaster Relief Supply Chain Network Topology for Mexico Example and its Variant

Mexico Case Study

- The Mexican Red Cross represents the organization and is denoted by node 1.
- There are two demand points, R_1 and R_2, for the ultimate delivery of the relief supplies. R_1 is situated closer to Mexico City and R_2 is closer to Acapulco.
- Nonlocal procurement is done through two locations in Texas, C_1 and C_2. Because of good relationships with the U.S. and the American Red Cross, there are two nonlocal storage facilities that the Mexican Red Cross can utilize, both located in Texas, and represented by links 5 and 9 emanating from $S_{1,1}$ and $S_{2,1}$, respectively.
- Local storage, on the other hand, is depicted by the link emanating from node $S_{3,1}$, link 19.
- The Mexican Red Cross can also procure locally (see C_3).

Mexico Case Study

Nonlocal procurement, post-disaster, is depicted by link 2, whereas procurement locally, post-disaster, and direct delivery to R_1 and R_2 are depicted by links 1 and 21, respectively.

Link 11 is a processing link to reflect processing of the arriving relief supplies from the U.S. and we assume one portal A_1, which is in southcentral Mexico.

Link 17 is also a processing link but that processing is done prior to storage locally and pre-disaster. Such a link is needed if the goods are procured nonlocally (link 7). The transport is done via road in the disaster relief supply chain network in the figure.

Mexico Case Study

The demand for the relief items at the demand point R_1 (in thousands of units) is assumed to follow a uniform probability distribution on the interval $[20, 40]$. The path flows and the link flows are also in thousands of units. Therefore,

$$P_{R_1}(\sum_{p\in\mathcal{P}_1} x_p) = \frac{\sum_{p\in\mathcal{P}_1} x_p - 20}{40-20} = \frac{\sum_{i=1}^{6} x_{p_i} - 20}{20}.$$

Also, the demand for the relief item at R_2 (in thousands of units) is assumed to follow a uniform probability distribution on the interval $[20, 40]$. Hence,

$$P_{R_2}(\sum_{p\in\mathcal{P}_2} x_p) = \frac{\sum_{p\in\mathcal{P}_1} x_p - 20}{40-20} = \frac{\sum_{i=7}^{12} x_{p_i} - 20}{20}.$$

Mexico Case Study

The time targets for the delivery of supplies at R_1 and R_2, respectively, in hours, are: $T_1 = 48$ and $T_2 = 48$. The penalties at the two demand points for shortages are: $\lambda_1^- = 10,000$ and $\lambda_2^- = 10,000$ and for surpluses: $\lambda_1^+ = 100$ and $\lambda_2^+ = 100$. The tardiness penalty function $\gamma_{R_1}(z) = 3(\sum_{p \in \mathcal{P}_{R_1}} z_p^2)$ and the tardiness penalty function $\gamma_{R_2}(z) = 3(\sum_{p \in \mathcal{P}_{R_2}} z_p^2)$.

We assume that the covariance matrix associated with the link total cost functions $\hat{c}_a(f_a, \omega_a)$, $a \in L$, is a 21×21 matrix $\sigma^2 I$.

Also, $\sigma^2 = 1$ and the risk aversion factor $\alpha = 10$ since the humanitarian organization is risk-averse with respect to its costs associated with its operations.

Mexico Case Study

The additional data are given in the Tables, where we also report the computed optimal link flows via the Euler method, which are calculated from the computed path flows.

Note that the time completion functions, $\tau_a(f_a)$, $\forall a \in L$, are 0.00 if the links correspond to procurement, transport, and storage, pre-disaster, since such supplies are immediately available for shipment once a disaster strikes.

Mexico Case Study

Table: Link Total Cost, Expected Value of Random Link Cost, Marginal Generalized Link Total Cost, and Time Completion Functions and Optimal Link Flows: $\alpha = 10$

Link a	$\hat{c}_a(f_a, \omega_a)$	$E(\omega_a)$	$g\hat{c}'_a$	$\tau_a(f_a)$	f^*_a; $\sigma^2 = 1$
1	$\omega_1 6f_1 + f_1$	2	$\alpha 72\sigma^2 f_1 + 13$	$f_1 + 15$	9.07
2	$\omega_2 3f_2 + f_2$	2	$\alpha 18\sigma^2 f_2 + 7$	$f_2 + 7$	2.54
3	$\omega_3 2f_3 + f_3$	1	$\alpha 8\sigma^2 f_3 + 3$	0.00	2.57
4	$\omega_4 3f_4 + f_4$	1	$\alpha 18\sigma^2 f_4 + 4$	0.00	2.57
5	$\omega_5 2f_5 + f_5$	1	$\alpha 8\sigma^2 f_5 + 3$	0.00	2.57
6	$\omega_6 2f_6 + f_6$	2	$\alpha 8\sigma^2 f_6 + 5$	$2f_6 + 10$	5.11
7	$\omega_7 2f_7 + f_7$	1	$\alpha 8\sigma^2 f_7 + 3$	0.00	8.51
8	$\omega_8 3f_8 + f_8$	1	$\alpha 18\sigma^2 f_8 + 4$	0.00	4.36
9	$\omega_9 2f_9 + f_9$	1	$\alpha 8\sigma^2 f_9 + 3$	0.00	4.36

Mexico Case Study

Table: Table continued

10	$\omega_{10} 2f_{10} + f_{10}$	1	$\alpha 8\sigma^2 f_{10} + 3$	$2f_{10} + 10$	4.36
11	$\omega_{11} f_{11} + f_{11}$	2	$\alpha 2\sigma^2 f_{11} + 3$	$f_{11} + 2$	9.47
12	$\omega_{12} f_{12} + f_{12}$	2	$\alpha 2\sigma^2 f_{12} + 3$	$f_{12} + 6$	17.78
13	$\omega_{13} f_{13} + f_{13}$	2	$\alpha 2\sigma^2 f_{13} + 3$	$f_{13} + 7$	17.64
14	$\omega_{14} f_{14} + f_{14}$	1	$\alpha 2\sigma^2 f_{14} + 2$	0.00	21.79
15	$\omega_{15} f_{15} + f_{15}$	1	$\alpha 2\sigma^2 f_{15} + 2$	0.00	21.79
16	$\omega_{16} f_{16} + f_{16}$	1	$\alpha 2\sigma^2 f_{16} + 2$	0.00	4.15
17	$\omega_{17} .5f_{17} + f_{17}$	1	$\alpha \sigma^2 .5f_{17} + 1.5$	0.00	4.15
18	$\omega_{18} f_{18} + f_{18}$	1	$\alpha 2\sigma^2 f_{18} + 2$	0.00	4.15
19	$\omega_{19} .5f_{19} + f_{19}$	2	$\alpha \sigma^2 .5f_{19} + 1.5$	0.00	25.94
20	$\omega_{20} f_{20} + f_{20}$	2	$\alpha 2\sigma^2 f_{20} + 2$	$2f_{20} + 5$	25.94
21	$\omega_{21} 6f_{21} + f_{21}$	2	$\alpha 72\sigma^2 f_{21} + 13$	$f_{21} + 14$	9.13

Results for Mexico Case Study

Table: Path Definitions, Target Times, Optimal Path Flows, Optimal Path Time Deviations, and Optimal Lagrange Multipliers

	Path Definition (Links)	x_p^*	z_p^*	μ_p^*
$\mathcal{P}_{R_1}$:	$p_1 = (1)$	9.07	0.00	0.00
	$p_2 = (2, 6, 11, 12)$	1.27	34.75	208.53
	$p_3 = (3, 4, 5, 6, 11, 12)$	1.29	25.26	151.56
	$p_4 = (7, 8, 9, 10, 11, 12)$	2.18	23.78	142.69
	$p_5 = (7, 16, 17, 18, 19, 20, 12)$	2.98	50.48	302.85
	$p_6 = (14, 15, 19, 20, 12)$	10.06	50.48	302.85
$\mathcal{P}_{R_2}$:	$p_7 = (2, 6, 11, 13)$	1.27	35.48	212.88
	$p_8 = (3, 4, 5, 6, 11, 13)$	1.29	25.99	155.91
	$p_9 = (7, 8, 9, 10, 11, 13)$	2.18	24.51	147.04
	$p_{10} = (7, 16, 17, 18, 19, 20, 13)$	1.17	51.20	307.19
	$p_{11} = (14, 15, 19, 20, 13)$	11.74	51.20	307.19
	$p_{12} = (21)$	9.13	0.00	0.00

Example - Variant 1

In Variant 1, we kept the data as before, but now we assumed that the humanitarian organization has a better forecast for the demand at the two demand points. The demand for the relief items at the demand point R_1 again follows a uniform probability distribution but on the interval $[30, 40]$ so that:

$$P_{R_1}(\sum_{p \in \mathcal{P}_1} x_p) = \frac{\sum_{p \in \mathcal{P}_1} x_p - 30}{40 - 30} = \frac{\sum_{i=1}^{6} x_{p_i} - 30}{10}.$$

Also, the demand for the relief item at R_2 follows a uniform probability distribution on the interval $[30, 40]$ so that:

$$P_{R_2}(\sum_{p \in \mathcal{P}_2} x_p) = \frac{\sum_{p \in \mathcal{P}_2} x_p - 30}{40 - 30} = \frac{\sum_{i=7}^{12} x_{p_i} - 30}{10}.$$

Results for Example - Variant 1

Table: Path Definitions, Target Times, Optimal Path Flows, Optimal Path Time Deviations, and Optimal Lagrange Multipliers for Variant 1

	Path Definition (Links)	x_p^*	z_p^*	μ_p^*
$\mathcal{P}_{R_1}$:	$p_1 = (1)$	11.30	0.00	0.00
	$p_2 = (2, 6, 11, 12)$	1.37	43.13	258.78
	$p_3 = (3, 4, 5, 6, 11, 12)$	1.49	33.42	200.49
	$p_4 = (7, 8, 9, 10, 11, 12)$	2.58	32.28	193.69
	$p_5 = (7, 16, 17, 18, 19, 20, 12)$	2.81	64.37	386.19
	$p_6 = (14, 15, 19, 20, 12)$	12.29	64.37	386.19
$\mathcal{P}_{R_2}$:	$p_7 = (2, 6, 11, 13)$	1.37	43.92	263.49
	$p_8 = (3, 4, 5, 6, 11, 13)$	1.49	34.20	205.20
	$p_9 = (7, 8, 9, 10, 11, 13)$	2.57	33.07	198.40
	$p_{10} = (7, 16, 17, 18, 19, 20, 13)$	1.96	65.15	390.90
	$p_{11} = (14, 15, 19, 20, 13)$	13.04	65.15	390.90
	$p_{12} = (21)$	11.36	0.00	0.00

Results for Example - Variant 1

The projected demands are: $v_{R_1} = 31.84$ and $v_{R_2} = 31.79$. The greatest percentage increase in path flow volumes occurs on paths p_1 and p_6 for demand point R_1 and on paths p_{11} and p_{12} for demand point R_2, reinforcing the previous results.

For both the Example and its variant the time targets are met for paths p_1 and p_2 since $\mu^*_{p_1}$ and $\mu^*_{p_2} = 0.00$ for both examples. Hence, direct local procurement post-disaster is effective time-wise, and cost-wise. Mexico is a large country and this result is quite reasonable.

A Game Theory Model
for Post-Disaster Humanitarian Relief

Game Theory and Disaster Relief

Although there have been quite a few optimization models developed for disaster relief there are very few game theory models.

Nevertheless, it is clear that humanitarian relief organizations and NGOs compete for financial funds from donors. Within three weeks after the 2010 earthquake in Haiti, there were 1,000 NGOs operating in that country. Interestingly, and, as noted by Ortuño et al. (2013), **although the importance of donations is a fundamental difference of humanitarian logistics with respect to commercial logistics, this topic has "not yet been sufficiently studied by academics and there is a wide field for future research in this context."**

Game Theory and Disaster Relief

Toyasaki and Wakolbinger (2014) developed perhaps the first models of financial flows that captured the strategic interaction between donors and humanitarian organizations using game theory and also included earmarked donations.

Game Theory and Disaster Relief

In this part of the lecture, we construct what we believe is **the first Generalized Nash Equilibrium (GNE) model for post-disaster humanitarian relief, which contains both a financial component and a supply chain component**. The Generalized Nash Equilibrium problem is a generalization of the Nash Equilibrium problem (cf. Nash (1950, 1951)) in that the players' strategies, as defined by the underlying constraints, depend also on their rivals' strategies.

The Network Structure of the Model

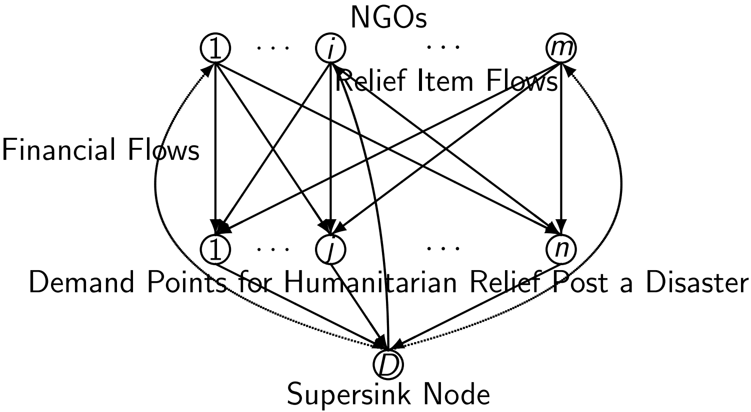


Figure: The Network Structure of the Game Theory Model

The Game Theory Model

We assume that each NGO i has, at its disposal, an amount s_i of the relief item that it can allocate post-disaster. Hence, we have the following conservation of flow equation, which must hold for each i; $i = 1, \ldots, m$:

$$\sum_{j=1}^{n} q_{ij} \leq s_i. \qquad (1)$$

In addition, we know that the product flows for each i; $i = 1, \ldots, m$, must be nonnegative, that is:

$$q_{ij} \geq 0, \quad j = 1, \ldots, n. \qquad (2)$$

Each NGO i encumbers a cost, c_{ij}, associated with shipping the relief items to location j, denoted by c_{ij}, where we assume that

$$c_{ij} = c_{ij}(q_{ij}), \quad j = 1, \ldots, n, \qquad (3)$$

The Game Theory Model

In addition, each NGO i; $i = 1, \ldots, m$, derives satisfaction or utility associated with providing the relief items to j; $j = 1, \ldots, n$, with its utility over all demand points given by $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$. Here γ_{ij} is a positive factor representing a measure of satisfaction/utility that NGO i acquires through its supply chain activities to demand point j. Each NGO i; $i = 1, \ldots, m$, associates a positive weight ω_i with $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$, which provides a monetization of, in effect, this component of the objective function.

The Game Theory Model

Finally, each NGO i; $i = 1, \ldots, m$, based on the media attention and the visibility of NGOs at location j; $j = 1, \ldots, n$, acquires funds from donors given by the expression

$$\beta_i \sum_{j=1}^{n} P_j(q), \tag{4}$$

where $P_j(q)$ represents the financial funds in donation dollars due to visibility of all NGOs at location j. Hence, β_i is a parameter that reflects the proportion of total donations collected for the disaster at demand point j that is received by NGO i. Expression (4), therefore, represents the financial flow on the link joining node D with node NGO i.

The Game Theory Model

Each NGO seeks to maximize its utility with the utility corresponding to the financial gains associated with the visibility through media of the relief item flow allocations, $\beta_i \sum_{j=1}^{n} P_j(q)$, plus the utility associated with the supply chain aspect of delivery of the relief items, $\sum_{j=1}^{n} \gamma_{ij} q_{ij} - \sum_{j=1}^{n} c_{ij}(q_{ij})$. The optimization problem faced by NGO i; $i = 1, \ldots, m$, is, hence,

$$\text{Maximize} \quad \beta_i \sum_{j=1}^{n} P_j(q) + \omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} - \sum_{j=1}^{n} c_{ij}(q_{ij}) \qquad (5)$$

subject to constraints (1) and (2).

The Game Theory Model

We also have that, at each demand point j; $j = 1, \ldots, n$:

$$\sum_{i=1}^{m} q_{ij} \geq \underline{d}_j, \tag{6}$$

and

$$\sum_{i=1}^{m} q_{ij} \leq \bar{d}_j, \tag{7}$$

where $\underline{d}_j$ denotes a lower bound for the amount of the relief items needed at demand point j and $\bar{d}_j$ denotes an upper bound on the amount of the relief items needed post the disaster at demand point j.

The Game Theory Model

We assume that

$$\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} \underline{d}_j, \tag{8}$$

so that the supply resources of the NGOs are sufficient to meet the minimum resource needs at all the demand points following the disaster.

Each NGO i; $i = 1, \ldots, m$, seeks to determine its optimal vector of relief items or strategies, q_i^*, that maximizes objective function (5), subject to constraints (1), (2), and (6), (7).

This is the Generalized Nash Equilibrium problem for our humanitarian relief post disaster problem.

The Game Theory Model

Theorem: Optimization Formulation of the Generalized Nash Equilibrium Model of Financial Flow of Funds
The above Generalized Nash Equilibrium problem, with each NGO's objective function (5) rewritten as:

$$\text{Minimize} \quad -\beta_i \sum_{j=1}^{n} P_j(q) - \omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} + \sum_{j=1}^{n} c_{ij}(q_{ij}) \tag{9}$$

and subject to constraints (1) and (2), with common constraints (6) and (7), is equivalent to the solution of the following optimization problem:

$$\text{Minimize} \quad -\sum_{j=1}^{n} P_j(q) - \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\omega_i \gamma_{ij}}{\beta_i} q_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{1}{\beta_i} c_{ij}(q_{ij}) \tag{10}$$

subject to constraints: (1), (2), (6), and (7).

Variational Inequality Formulation

The solution q^* with associated Lagrange multipliers λ_k^*; $k = 1, \ldots, m$, for the supply constraints, the Lagrange multipliers: λ_l^{1*}; $l = 1, \ldots, n$, for the lower bound demand constraints, and the Lagrange multipliers: λ_l^{2*}; $l = 1, \ldots, n$, for the upper bound demand constraints, can be obtained by solving the variational inequality problem: determine $(q^*, \lambda^*, \lambda^{1*}, \lambda^{2*}) \in R_+^{mn+m+2n}$:

$$\sum_{k=1}^{m} \sum_{l=1}^{n} \left[-\sum_{j=1}^{n} \left(\frac{\partial P_j(q^*)}{\partial q_{kl}} \right) - \frac{\omega_k \gamma_{kl}}{\beta_k} + \frac{1}{\beta_k} \frac{\partial c_{kl}(q_{kl}^*)}{\partial q_{kl}} + \lambda_k^* - \lambda_l^{1*} + \lambda_l^{2*} \right] \times [q_{kl} - q_{kl}^*]$$

$$+ \sum_{k=1}^{m} \left(s_k - \sum_{l=1}^{n} q_{kl}^* \right) \times (\lambda_k - \lambda_k^*) + \sum_{l=1}^{n} \left(\sum_{k=1}^{n} q_{kl}^* - \underline{d}_l \right) \times (\lambda_l - \lambda_l^{1*})$$

$$+ \sum_{l=1}^{n} \left(\bar{d}_l - \sum_{k=1}^{m} q_{kl}^* \right) \times (\lambda_l^2 - \lambda_l^{2*}) \geq 0, \quad \forall (q, \lambda, \lambda^1, \lambda^2) \in R_+^{mn+m+2n}, \tag{11}$$

where λ is the vector of Lagrange multipliers: $(\lambda_1, \ldots, \lambda_m)$, λ^1 is the vector of Lagrange multipliers: $(\lambda_1^1, \ldots, \lambda_n^1)$, and λ^2 is the vector of Lagrange multipliers: $(\lambda_1^2, \ldots, \lambda_n^2)$.

The Algorithm

Explicit Formulae for the Euler Method

We have the following closed form expression for the product flows $k = 1, \ldots, m$; $l = 1, \ldots, n$, at each iteration:

$$q_{kl}^{\tau+1}$$

$$= \max\{0, \{q_{kl}^{\tau} + a_{\tau}(\sum_{j=1}^{n}(\frac{\partial P_j(q^{\tau})}{\partial q_{kl}}) + \frac{\omega_k \gamma_{kl}}{\beta_{kl}} - \frac{1}{\beta_k}\frac{\partial c_{kl}(q_{kl}^{\tau})}{\partial q_{kl}} - \lambda_k^{\tau} + \lambda_l^{1^{\tau}} - \lambda_l^{2^{\tau}})\}\}$$

the following closed form expressions for the Lagrange multipliers associated with the supply constraints, respectively, for $k = 1, \ldots, m$:

$$\lambda_k^{\tau+1} = \max\{0, \lambda_k^{\tau} + a_{\tau}(-s_k + \sum_{l=1}^{n} q_{kl}^{\tau})\}.$$

Explicit Formulae for the Euler Method Applied to the Game Theory Model

The following closed form expressions are for the Lagrange multipliers associated with the lower bound demand constraints, respectively, for $l = 1, \ldots, n$:

$$\lambda_l^{1^{\tau+1}} = \max\{0, \lambda_l^{1^{\tau}} + a_\tau(-\sum_{k=1}^{n} q_{kl}^{\tau} + \underline{d}_l)\}.$$

The following closed form expressions are for the Lagrange multipliers associated with the upper bound demand constraints, respectively, for $l = 1, \ldots, n$:

$$\lambda_l^{2^{\tau+1}} = \max\{0, \lambda_l^{2^{\tau}} + a_\tau(-\bar{d}_l + \sum_{k=1}^{m} q_{kl}^{\tau})\}.$$

Hurricane Katrina Case Study

Making landfall in August of 2005, Katrina caused extensive damages to property and infrastructure, **left 450,000 people homeless, and took 1,833 lives in Florida, Texas, Mississippi, Alabama, and Louisiana (Louisiana Geographic Information Center (2005))**.

Given the hurricane's trajectory, most of the damage was concentrated in Louisiana and Mississippi. In fact, 63% of all insurance claims were in Louisiana, a trend that is also reflected in FEMA's post-hurricane damage assessment of the region (FEMA (2006)).

Hurricane Katrina Case Study

The total damage estimates range from $105 billion (Louisiana Geographic Information Center (2005)) to $150 billion (White (2015)), making Hurricane Katrina not only a far-reaching and costly disaster, but also a very challenging environment for providing humanitarian assistance.

We now present a case study on Hurricane Katrina using available data.

The P_j functions were as follows:

$$P_j(q) = k_j\sqrt{\sum_{i=1}^{m} q_{ij}}.$$

The weights were:

$$\omega_1 = \omega_2 = \omega_3 = 1,$$

Hurricane Katrina Case Study

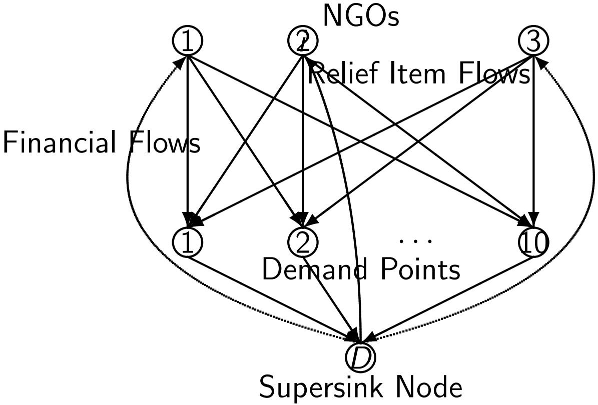


Figure: Hurricane Katrina Relief Network Structure

Hurricane Katrina Case Study

Hurricane Katrina Demand Point Parameters					
Parish	Node j	k_j	$\underline{d}_j$	$\bar{d}_j$	p_j (in %)
St. Charles	1	8	16.45	50.57	2.4
Terrebonne	2	16	752.26	883.82	6.7
Assumption	3	7	106.36	139.24	1.9
Jefferson	4	29	742.86	1,254.89	19.5
Lafourche	5	6	525.53	653.82	1.7
Orleans	6	42	1,303.99	1,906.80	55.9
Plaquemines	7	30	33.28	62.57	57.5
St. Barnard	8	42	133.61	212.43	78.4
St. James	9	9	127.53	166.39	1.2
St. John the Baptist	10	7	19.05	52.59	6.7

Table: Demand Point Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

Hurricane Katrina Case Study

We then estimated the cost of providing aid to the Parishes as a function of the total damage in the area and the supply chain efficiency of each NGO. We assume that these costs follow the structures observed by Van Wassenhove (2006) and randomly generate a number based on his research with a mean of $\hat{p} = .8$ and standard deviation of $s = \sqrt{\frac{.8(.2)}{3}}$.

We denote the corresponding coefficients by π_i. Thus, each NGO i; $i = 1, 2, 3$, incurs costs according the the following functional form:

$$c_{ij}(q_{ij}) = \left(\pi_i q_{ij} + \frac{1}{1 - p_j}\right)^2.$$

Hurricane Katrina Case Study

Data Parameters for NGOs Providing Aid					
NGO	i	π_i	γ_{ij}	β_i	s_i
Others	1	.82	950	.355	1,418
Red Cross	2	.83	950	.55	2,200
Salvation Army	3	.81	950	.095	382

Table: NGO Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

Hurricane Katrina Case Study

Generalized Nash Equilibrium Product Flows			
Demand Point	Others	Red Cross	Salvation Army
St. Charles	17.48	28.89	4.192
Terrebonne	267.023	411.67	73.57
Assumption	49.02	77.26	12.97
Jefferson	263.69	406.68	72.45
Lafourche	186.39	287.96	51.18
Orleans	463.33	713.56	127.1
Plaquemines	21.89	36.54	4.23
St. Barnard	72.31	115.39	16.22
St. James	58.67	92.06	15.66
St. John the Baptist	18.2	29.99	4.40

Table: Flows to Demand Points under Generalized Nash Equilibrium

The total utility obtained through the above flows for the Generalized Nash Equilibrium for Hurricane Katrina is $9,257,899$, with the Red Cross capturing 3,022,705, the Salvation Army 3,600,442.54, and Others 2,590,973. It is interesting to see that, despite having the lowest available supplies, the Salvation Army is able to capture the largest part of the total utility. This is due to the fact that the costs of providing aid grow at a nonlinear rate, so even if the Salvation Army was less efficient and used all of its available supplies, it will not be capable of providing the most expensive supplies.

In addition, we have that the Red Cross, the Salvation Army, and Others receive 2,200.24, 1418.01, and 382.31 million in donations, respectively. Also, notice how the flows meet at least the lower bound, even if doing so is very expensive due to the damages to the infrastructure in the region.

Hurricane Katrina Case Study

Furthermore, the above flow pattern behaves in a way that, after the minimum requirements are met, any additional supplies are allocated in the most efficient way. For example, only the minimum requirements are met in New Orleans Parish, while the upper bound is met for St. James Parish.

The Nash Equilibrium Solution

If we remove the shared constraints, we obtain a Nash Equilibrium solution, and we can compare the outcomes of the humanitarian relief efforts for Hurricane Katrina under the Generalized Nash Equilibrium concept and that under the Nash Equilibrium concept.

The Nash Equilibrium Solution

Nash Equilibrium Product Flows			
Demand Point	Others	Red Cross	Salvation Army
St. Charles	142.51	220.66	38.97
Terrebonne	142.50	220.68	38.93
Assumption	142.51	220.66	38.98
Jefferson	142.38	220.61	38.74
Lafourche	142.50	220.65	38.98
Orleans	141.21	219.59	37.498
Plaquemines	141.032	219.28	37.37
St. Barnard	138.34	216.66	34.59
St. James	142.51	220.65	38.58
St. John the Baptist	145.51	220.66	38.98

Table: Flows to Demand Points under Nash Equilibrium

The Nash Equilibrium Solution

Under the Nash Equilibrium, the NGOs obtain a higher utility than under the Generalized Nash Equilibrium. Specifically, of the total utility $10,346,005.44$, 2,804,650 units are received by the Red Cross, 5,198,685 by the Salvation Army, and 3,218,505 are captured by all other NGOs.

Under this product flow pattern, there are total donations of 3,760.73, of which 2,068.4 are donated to the Red Cross, 357.27 to the Salvation Army, and 1,355 to the other players.

The Nash Equilibrium Solution

It is clear that there is a large contrast between the flow patterns under the Generalized Nash and Nash Equilibria. For example, the Nash Equilibrium flow pattern results in about $500 million less in donations.

While this has strong implications about how collaboration between NGOs can be beneficial for their fundraising efforts, the differences in the general flow pattern highlights a much stronger point.

Additional Insights

Under the Nash Equilibrium, NGOs successfully maximize their utility. Overall, the Nash Equilibrium solution leads to an increase of utility of roughly 21% when compared to the flow patterns under the Generalized Nash Equilibrium. **But they do so at the expense of those in need.** In the Nash Equilibrium, each NGO chooses to supply relief items such that costs can be minimized. On the surface, this might be a good thing, but recall that, given the nature of disasters, it is usually more expensive to provide aid to demand points with the greatest needs.

Additional Insights

With this in mind, one can expect oversupply to the demand points with lower demand levels, and undersupply to the most affected under a purely competitive scheme. This behavior can be seen explicitly in the results summarized in the Tables.

For example, St. Charles Parish receives roughly 795% of its upper demand, while Orleans Parish only receives about 30.5% of its minimum requirements. That means that much of the 21% in 'increased' utility is in the form of waste.

Additional Insights

In contrast, the flows under the Generalized Nash Equilibrium guarantee that minimum requirements will be met and that there will be no waste; that is to say, as long as there is a coordinating authority that can enforce the upper and lower bound constraints, the humanitarian relief flow patterns under this bounded competition will be significantly better than under untethered competition.

Additional Insights

In addition, we found that changes to the values in the functional form result in changes in the product flows, but the general behavioral differences are robust to changes in the coefficients: β_i, γ_{ij}, k_j, $\forall i, j$, and the bounds on upper and lower demand estimates.

Summary and Conclusions

• In this lecture, we first described a Mean-Variance disaster relief supply chain network model for risk reduction with stochastic link costs, uncertain demands for the relief supplies and time targets associated with the demand points. Theoretical results and a case study to hurricanes hitting Mexico were given.

Summary and Conclusions

- We also presented a Generalized Nash Equilibrium model, with a special case being a Nash Equilibrium model, for disaster relief with supply chain and financial fund aspects for each NGO's objective function. Each NGO obtains utility from providing relief to demand points post a disaster and also seeks to minimize costs but can gain in financial donations based on the visibility of the NGOs in terms of product deliveries to the demand points. A case study based on Hurricane Katrina was discussed.

- All the models were network-based and provide new insights in terms of disaster relief and management.