

Topic 8: Supply Chain Network Competition in Time-Sensitive Markets

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Outline

- ▶ Motivation and Background
- ▶ The Supply Chain Network Model for Time-Sensitive Markets
- ▶ Illustrative Examples and Additional Applications
- ▶ Algorithm and Case Study
- ▶ Summary and Conclusions

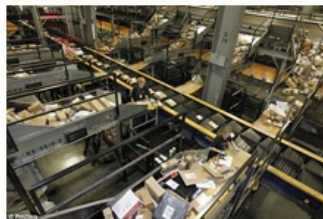
Motivation

Timely deliveries of products are essential not only to consumers but to a company's reputation and bottom line.

- ▶ The December 2013 holiday season shipping fiasco in the United States
- ▶ Late delivery of Christmas trees in 1999 in Sweden.

The unfulfilled demands may result in

- ▶ A tremendous loss of good will, anger, frustration;
- ▶ Economic losses due to perished and spoiled products; and
- ▶ The potential loss of future business.



Time-Sensitive Markets

Markets in which consumers are willing to pay **a higher price** for **lower delivery times** are referred to as being **time-sensitive**.



Information Asymmetry

Large demands at a demand market may be supplied by a firm that has **multiple** manufacturing plants, **multiple** distribution centers, and uses **multiple** modes of transportation.



Consumers may not always be aware of

- ▶ The precise manufacturing plant,
- ▶ The distribution center,
- ▶ The transport mode; nor
- ▶ The time associated with supply chain network activities.

There may exist **information asymmetry**. Consumers at demand markets are aware of the **average delivery time** of the ordered products, and respond accordingly through the prices that they are willing to pay.

Average Delivery Time

- ▶ The **average time** from the placement of an order to a product's delivery
 - ▶ indicates how effective and efficient a supply chain network is;
 - ▶ provides a valuable metric as to the time a product spends in the system; and
 - ▶ has emerged as a key performance indicator (KPI).
- ▶ In the 1990s, the average time to fulfill customer orders was measured in **weeks**.
- ▶ Today, delivery times are being measured in **days**, or, in some cases, in **hours**.

Average Delivery Time

- ▶ **Intel** introduced the average delivery time, the **Order Fulfillment Lead Time** (OFLT), as a KPI.
- ▶ **The Logistics Management Institute** refers to this KPI as the **Logistics Response Time** (LRT), measured, typically, in **days**.
- ▶ This supply chain performance measure is one of the four that has been utilized by **the U.S. Department of Defense**.

Average Delivery Time

- ▶ In **humanitarian relief chains**, the **average delivery time** of **critical needs supplies**, such as water, food, medicines, and even shelter, in disasters, is also an important performance measure.
- ▶ For **pharmaceuticals**, more responsive time-efficient supply chains can save lives.
 - ▶ For the deliveries of HIV/AIDS drugs to Kenyans, the Kenya pharma supply chain improved the average time for shipments to clear customs:
 - ▶ **Sea** shipments: from **21.8** days to **5.7** days;
 - ▶ **Air** shipments: from **2.5** days to **1.8** days.
- ▶ The World Bank Logistics Performance Index shows that the median import lead time is more than **3.5 times longer** in **low performing countries** than in **high performing countries**.

Product Differentiation

Because of the recognized competitive advantages associated with speed and timely deliveries, firms are increasingly **differentiating** their products to include **time for delivery** and consumers are responding.

- ▶ **Online retailers**, such as Amazon, have fulfillment/delivery options that often trade-off cost and speed of delivery.
- ▶ **Package delivery services**, such as the U.S. Post Office, UPS, FEDEX, and DHL, routinely offer multiple delivery options with reduced delivery time coming at increased shipping cost.
- ▶ Similar development has long existed in **passenger transport**, where, for example, both regular and high speed rail services are offered to the same destinations but at different prices.

Fashion Apparel

- ▶ In order to adapt to the increasingly rapid fashion changes, the U.S. apparel industry moves production from **lower wage locations in Asia** to **higher wage locations** in Mexico and the Caribbean that are closer to the U.S. consumer, thus trading **cost** for **time**.



- ▶ Benetton uses a **postponement strategy** by delaying the dying of garments during production to be able to respond faster to trends.

Fashion Apparel

- ▶ A recent vivid example is the demand for apparel and related products based on the top-grossing Disney animated film *Frozen*.
- ▶ Due to the **demand** and **willingness of consumers to pay higher prices**, the shipments from manufacturing plants in China were increasingly **airlifted**.



The Supply Chain Network Model for Time-Sensitive Markets

- ▶ The **profit-maximizing** firms compete in a noncooperative manner in an **oligopolistic** fashion to provide their products to the demand markets.
 - ▶ Food
 - ▶ Tobacco
 - ▶ Vaccine production
 - ▶ Fashion and luxury goods
- ▶ Firms produce **substitutable**, but **differentiated**, products.
- ▶ Consumers respond to each firm's product as a separate brand.

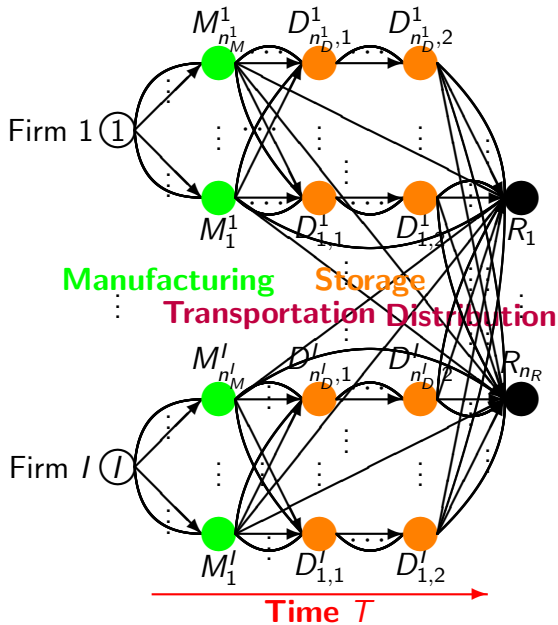


Figure: Supply Chain Network Topology with Progression in Time

The Supply Chain Network Model for Time-Sensitive Markets

- ▶ Each firm can possess **multiple** manufacturing plants, may have access to **multiple** distribution centers, serves **multiple** demand markets and can avail itself of **multiple** transport options.
- ▶ Each of the supply chain activities is **time-consuming**.
- ▶ The **firms** have **perfect information** available as to the product flows on the paths as well as the associated times.
- ▶ The **consumers** are only aware of the **average delivery time** of the products at the demand markets.

Notation	Definition
$x_p; p \in P_k^i$	the nonnegative flow of firm i 's product to demand market k ; $i = 1, \dots, I$; $k = 1, \dots, n_R$. We group the $\{x_p\}$ elements for firm i into the vector $X_i \in R_+^{n_{pi}}$ and all the firms' product flows into the vector $x \in R_+^{np}$.
f_a	the nonnegative flow of the product on link a . We group the link flows into the vector $f \in R_+^{n_L}$.
d_{ik}	the demand for the product of firm i at demand market k ; $i = 1, \dots, I$; $k = 1, \dots, n_R$. We group the demands into the vector $d \in R_+^{I \times n_R}$.
$\hat{c}_a(f)$	the total cost associated with link a .
$t_a(f_a)$	the unit product time consumption function for the activity associated with link a .
T_p	the unit product time to complete the activities associated with path p .
T_{ik}^{ave}	the average time to complete all activities associated with firm i ; $i = 1, \dots, I$ and demand market k ; $k = 1, \dots, n_R$; we group the T_{ik}^{ave} into the vector $T^{ave} \in R_+^{I \times n_R}$.
$\rho_{ik}(d, T^{ave})$	the demand price of the product of firm i at demand market k ; $i = 1, \dots, I$; $k = 1, \dots, n_R$.

Time Consumption

The **unit link time consumption** functions depend on the volume of link flow.

$$t_a = t_a(f_a) = g_a f_a + h_a, \quad \forall a \in L, \quad (1)$$

where $h_a > 0$ and $g_a \geq 0$.

The **unit path time consumption** is the sum of the link consumption times on links that comprise the path.

$$T_p = \sum_{a \in L} t_a \delta_{ap}, \quad \forall p \in P_k^i, \forall i, \forall k, \quad (2)$$

where $\delta_{ap} = 1$, if link a is contained in path p , and 0, otherwise.

Time Consumption

The **average delivery time** T_{ik}^{ave} associated with firm i 's product at demand market k

$$T_{ik}^{ave} = \frac{\sum_{p \in P_k^i} T_p x_p}{\sum_{p \in P_k^i} x_p}, \quad \forall i, \forall k. \quad (3)$$

The average delivery time reflects the completion of *ALL* the associated activities as represented by the links that comprise the paths.

Conservation of Flow Equations

Relationship between path flows and demands

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad \forall i, \forall k. \quad (4)$$

Relationship between link flows and path flows

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L. \quad (5)$$

Nonnegativity

$$x_p \geq 0, \quad \forall p \in P. \quad (6)$$

Demand Price Functions

$$\hat{\rho}_{ik} = \hat{\rho}_{ik}(x) \equiv \rho_{ik}(d, T^{ave}), \quad \forall i, \forall k. \quad (7)$$

- ▶ The consumers respond to the **average delivery time** through the demand price functions.
- ▶ The functions are assumed to be continuous, continuously differentiable, and monotone decreasing in both the product demand at the specific demand market and the average time.

Total Operational Cost Functions

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L. \quad (8)$$

- ▶ The total cost expressions capture **competition** among the firms for resources used in the manufacture, transportation, storage, and distribution of their products.
- ▶ The total cost on each link is assumed to be convex, continuous, and continuously differentiable.

The Profit Function of Firm i

$$U_i = \sum_{k=1}^{n_R} \rho_{ik}(d, T^{ave}) d_{ik} - \sum_{a \in L^i} \hat{c}_a(f). \quad (9)$$

In this oligopoly competition problem, **the strategic variables are the path flows**.

- ▶ X_i : the vector of path flows associated with firm i ;
 $i = 1, \dots, I$, where $X_i \equiv \{\{x_p\} | p \in P^i\} \in R_+^{n_{P^i}}$.
- ▶ X is the vector of all the firms' strategies, that is,
 $X \equiv \{\{X_i\} | i = 1, \dots, I\}$.

Supply Chain Network Cournot-Nash Equilibrium

A path flow pattern $X^* \in K = \prod_{i=1}^l K_i$ constitutes a supply chain network Cournot-Nash equilibrium if for each firm i ; $i = 1, \dots, l$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \quad (10)$$

where $\hat{X}_i^* \equiv (X_1^*, \dots, X_{i-1}^*, X_{i+1}^*, \dots, X_l^*)$ and $K_i \equiv \{X_i | X_i \in R_+^{n_{pi}}\}$.

An equilibrium is established if *NO* firm can unilaterally improve its profit by changing its product flows throughout its supply chain network, given the product flow decisions of the other firms.

Variational Inequality Formulation

Determine the vector of equilibrium path flows $x^* \in K^1$ such that:

$$\sum_{i=1}^l \sum_{k=1}^{n_R} \sum_{p \in P_k^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \hat{\rho}_{ik}(x^*) - \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p} \sum_{q \in P_l^i} x_q^* \right] \times [x_p - x_p^*] \geq 0,$$
$$\forall x \in K^1, \quad (11)$$

where $K^1 \equiv \{x | x \in R_+^{n_P}\}$, and for each path p ; $p \in P_k^i$;
 $i = 1, \dots, l$; $k = 1, \dots, n_R$,

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \delta_{ap}. \quad (12)$$

Existence

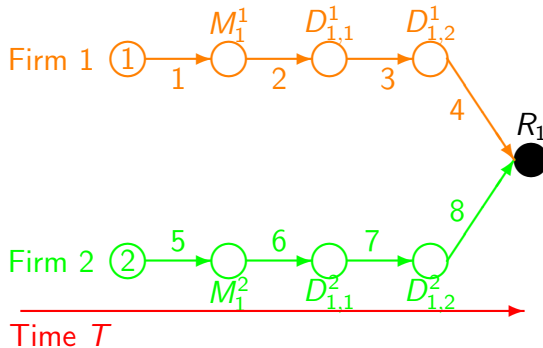
There exists at least one solution to variational inequality (11), since there exists a $b > 0$, such that variational inequality

$$\langle F(X^b), X - X^b \rangle \geq 0, \quad \forall X \in \mathcal{K}_b, \quad (13)$$

admits a solution in \mathcal{K}_b with

$$x^b \leq b. \quad (14)$$

An Illustrative Example



- ▶ **Firm 1** is based in **the U.S.**, and its manufacturing plant M_1^1 is also located in the U.S., as is its distribution center.
- ▶ **Firm 2** is based in **Asia**, where its manufacturing plant M_1^2 is located; however, its distribution center is in the U.S.
- ▶ **The demand market R_1** is located in the U.S.

An Illustrative Example

Demand Price Functions

$$\rho_{11}(d, T^{ave}) = -2d_{11} - d_{21} - 3T_{11}^{ave} + 3T_{21}^{ave} + 100,$$

$$\rho_{21}(d, T^{ave}) = -3d_{21} - d_{11} - 2T_{21}^{ave} + 2T_{11}^{ave} + 100.$$

Link a	\hat{c}_a	$t_a(f_a)$	f_a^*
1	$5f_1^2 + 10f_1$	$.5f_1 + 10$	3.31
2	$2f_2$	$f_2 + 1$	3.31
3	$f_3^2 + f_3$	$f_3 + 1$	3.31
4	$3f_4$	$f_4 + 2$	3.31
5	$f_5^2 + 5f_5$	$.1f_5 + 4$	4.64
6	$8f_6$	$f_6 + 7$	4.64
7	$f_7^2 + f_7$	$f_7 + 1$	4.64
8	$2f_8$	$f_8 + 2$	4.64

An Illustrative Example

	Firm 1	Firm 2
Demand	3.31	4.64
Average Delivery Time	25.59	28.37
Price	97.10	77.20
Profit	202.74	240.79

Firm 2 compensates for its greater distance from the demand market by **lower manufacturing times** as well as **lower manufacturing costs**.

Variant 1

Firm 1 has enhanced its manufacturing process, which has resulted in greater efficiency and time reduction with the consequence that its unit time function on link 1 has been reduced to:

$$t_1(f_1) = .5f_1 + 5.$$

	Firm 1	Firm 2
Demand	3.64	4.28
Average Delivery Time	21.72	27.62
Price	105.06	72.47
Profit	244.47	204.91

Firm 1's profit has increased and now surpasses that of Firm 2's.

Variant 2

Firm 1 has reduced its operational cost associated with the manufacturing link 1 so that

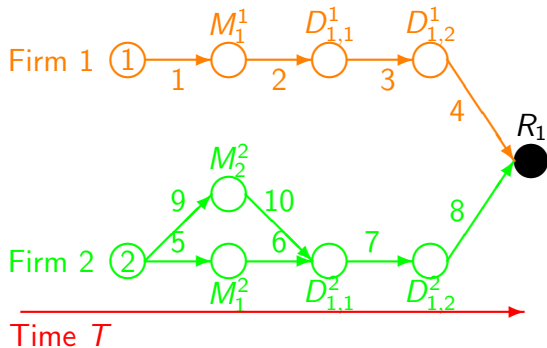
$$\hat{c}_a(f) = 2.5f_1^2 + 10f_1.$$

	Firm 1	Firm 2
Demand	4.25	4.44
Average Delivery Time	23.86	27.77
Price	98.79	74.62
Profit	288.40	220.87

Firm 1's profit has further **increased**, whereas that of **Firm 2** has also **increased**, but not as significantly.

Variant 3

The Asian firm, **Firm 2**, is very concerned by the **increasing success of Firm 1** and has acquired a second manufacturing plant, M_2^2 , which is located in the U.S.



Variant 3

Link a	\hat{c}_a	$t_a(f_a)$	f_a^*
1	$2.5f_1^2 + 10f_1$	$.5f_1 + 5$	4.01
2	$2f_2$	$f_2 + 1$	4.01
3	$f_3^2 + f_3$	$f_3 + 1$	4.01
4	$3f_4$	$f_4 + 2$	4.01
5	$f_5^2 + 5f_5$	$.1f_5 + 4$	2.78
6	$8f_6$	$f_6 + 7$	2.78
7	$f_7^2 + f_7$	$f_7 + 1$	5.01
8	$2f_8$	$f_8 + 2$	5.01
9	$3f_9^2 + 10f_9$	$.5f_9 + 5$	2.23
10	$2f_{10}$	$f_{10} + 2$	2.23

Variant 3

	Firm 1	Firm 2
Demand	4.01	5.01
Average Delivery Time	23.03	25.44
Price	94.18	76.13
Profit	257.21	255.97

- ▶ **Firm 2**, by adding a new manufacturing plant that is closer to its distribution center and demand market has **reduced its average time** and now **enjoys higher profit**.
- ▶ **Firm 1**, in turn, because of the increased competition from Firm 2, now **suffers a decrease in profits**.

Variant 4

Firm 2 engages in additional marketing to inform U.S. consumers that it has invested in a plant in the U.S., with the result that the demand price function for its product has now been changed to:

$$\rho_{21}(d, T^{ave}) = -3d_{21} - d_{11} - 2T_{21}^{ave} + 2T_{11}^{ave} + 200.$$

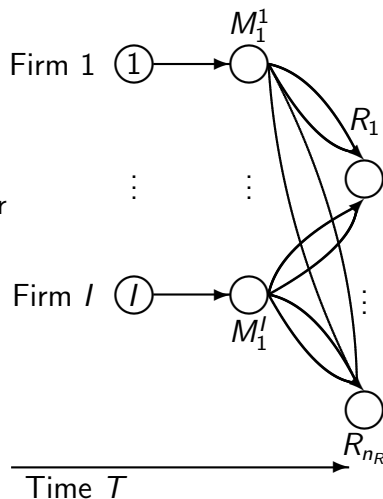
	Firm 1	Firm 2
Demand	5.17	10.32
Average Delivery Time	27.11	39.63
Price	116.89	138.83
Profit	428.28	1,076.26

- ▶ The demand for Firm 2's product has more than doubled as compared to the demand in Variant 3, whereas its profit has more than quadrupled.

Additional Applications

► Delay Tolerant Networks

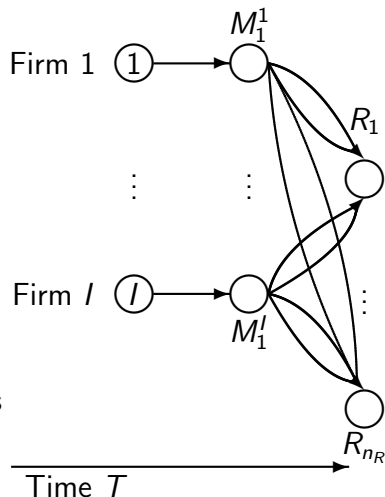
- Each firm has costs associated with the operation of its telecommunication infrastructure.
- The transport is provided by either a real-time connection or by a physical mechanical backhaul option.
- Consumers respond to the average time associated with the delivery of the data and information.
- Firms are profit-maximizers.



Additional Applications

► Web Hosting

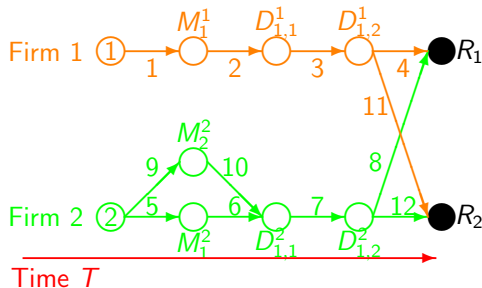
- Consumers would select the web-host(s) that provide the appropriate time-average responsiveness (and price) for the desired information, whether that of videos, news, and the like.
- The links terminating in the demand markets would correspond to the aggregations of telecommunication options.



Closed Form Expressions for Product Path Flows

$$x_p^{\tau+1} = \max\left\{0, x_p^{\tau} + a_{\tau}(\hat{\rho}_{ik}(x^{\tau})) + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^{\tau})}{\partial x_p} \sum_{q \in P_l^i} x_q^{\tau} - \frac{\partial \hat{C}_p(x^{\tau})}{\partial x_p}\right\},$$
$$\forall p \in P_k^i; i = 1, \dots, l; k = 1, \dots, n_R. \quad (15)$$

Case Study



- ▶ The U.S. firm, **Firm 1**, has one manufacturing plant and one distribution center, both of which are located in the U.S.
- ▶ The Asian firm, **Firm 2**, has two manufacturing plants. M_1^2 is located in Asia; while M_2^2 is located in the U.S., as is its distribution center.
- ▶ Each firm serves two geographically separated demand markets in the U.S.

Case Study Example 1

The consumers at demand market R_1 are more sensitive to the average delivery times than those at demand market R_2 are. The demand price functions are:

$$\rho_{11}(d, T^{ave}) = -2d_{11} - d_{21} - 3T_{11}^{ave} + 3T_{21}^{ave} + 100,$$

$$\rho_{12}(d, T^{ave}) = -3d_{12} - d_{22} - 2T_{12}^{ave} + 2T_{22}^{ave} + 100,$$

$$\rho_{21}(d, T^{ave}) = -2d_{21} - d_{11} - 3T_{21}^{ave} + 3T_{11}^{ave} + 100,$$

$$\rho_{22}(d, T^{ave}) = -3d_{22} - d_{12} - 2T_{22}^{ave} + 2T_{12}^{ave} + 100.$$

Case Study Example 1

Link a	\hat{c}_a	$t_a(f_a)$	f_a^*
1	$2.5f_1^2 + 10f_1$	$.5f_1 + 5$	5.06
2	$2f_2$	$f_2 + 1$	5.06
3	$f_3^2 + f_3$	$f_3 + 1$	5.06
4	$3f_4$	$f_4 + 2$	2.33
5	$f_5^2 + 5f_5$	$.1f_5 + 4$	3.83
6	$8f_6$	$f_6 + 7$	3.83
7	$f_7^2 + f_7$	$f_7 + 1$	6.65
8	$2f_8$	$f_8 + 2$	3.26
9	$3f_9^2 + 10f_9$	$.5f_9 + 5$	2.81
10	$2f_{10}$	$f_{10} + 2$	2.81
11	$3f_{11}$	$f_{11} + 2$	2.73
12	$2f_{12}$	$f_{12} + 2$	3.39

Case Study Example 1

The **equilibrium demands** at the demand markets are:

$$\text{Firm 1: } d_{11}^* = 2.33, \quad d_{12}^* = 2.73,$$

$$\text{Firm 2: } d_{21}^* = 3.26, \quad d_{22}^* = 3.39.$$

The **average delivery times** are:

$$\text{Firm 1: } T_{11}^{ave} = 23.98, \quad T_{12}^{ave} = 24.38,$$

$$\text{Firm 2: } T_{21}^{ave} = 24.67, \quad T_{22}^{ave} = 28.25.$$

The incurred **demand prices** are:

$$\text{Firm 1: } \rho_{11} = 94.17, \quad \rho_{12} = 96.17,$$

$$\text{Firm 2: } \rho_{21} = 89.06, \quad \rho_{22} = 79.36.$$

The **profits** of Firms 1 and 2 are:

$$U_1 = 311.33, \quad U_2 = 372.94.$$

Since Firm 2 is capable of providing competitive delivery service at a significantly lower price, **Firm 2 dominates both demand markets**, leading to a higher profit.

Case Study Example 2

The consumers at demand market R_1 are now more sensitive with respect to the average delivery times.

The demand price functions associated with demand market R_1 are now:

$$\begin{aligned}\rho_{11}(d, T^{ave}) &= -2d_{11} - d_{21} - 4T_{11}^{ave} + 4T_{21}^{ave} + 100, \\ \rho_{21}(d, T^{ave}) &= -2d_{21} - d_{11} - 4T_{21}^{ave} + 4T_{11}^{ave} + 100.\end{aligned}$$

Case Study Example 3

The consumers at the demand market R_1 are even more sensitive with respect to the average delivery times, with the demand price functions associated with demand market R_1 given by:

$$\begin{aligned}\rho_{11}(d, T^{ave}) &= -2d_{11} - d_{21} - 5T_{11}^{ave} + 5T_{21}^{ave} + 100, \\ \rho_{21}(d, T^{ave}) &= -2d_{21} - d_{11} - 5T_{21}^{ave} + 5T_{11}^{ave} + 100.\end{aligned}$$

		Example 1	Example 2	Example 3
Demands for Firm 1's Product	d_{11}^*	2.33	1.93	1.66
	d_{12}^*	2.73	2.79	2.82
Demands for Firm 2's Product	d_{21}^*	3.26	2.65	2.37
	d_{22}^*	3.39	3.53	3.48
Firm 1's Average Delivery Times	T_{11}^{ave}	23.98	22.72	21.86
	T_{12}^{ave}	24.38	23.58	23.02
Firm 2's Average Delivery Times	T_{21}^{ave}	24.67	22.79	21.77
	T_{22}^{ave}	28.25	27.61	27.15
Demand Prices of Firm 1's Product	ρ_{11}	94.17	93.80	93.85
	ρ_{12}	96.17	96.17	96.32
Demand Prices of Firm 2's Product	ρ_{21}	89.06	92.48	94.06
	ρ_{22}	79.36	78.56	78.49
Profit of Firm 1		311.33	295.64	285.51
Profit of Firm 2		372.94	354.39	341.51

Case Study

- ▶ Due to the consumers' increased sensitivity to the average delivery times, the **average delivery times** from both Firms 1 and 2 **to the demand market R_1 decline significantly**.
- ▶ The **average delivery times** to demand market R_2 also **decrease**, but **slightly**.
- ▶ Both firms **charge higher prices for the timely delivery**.
- ▶ In order to fulfill the high requirement for timely delivery,
 - ▶ Firm 2 mainly relies on its manufacturing plant in the U.S., M_2^2 , to satisfy the demand at demand market R_1 ;
 - ▶ Its Asian manufacturing plant M_1^2 covers all the demand at demand market R_2 .
- ▶ The firms are **trading cost for time** when reacting to the changed customer preferences.

		Example 1		Example 2		Example 3	
Firm	Path	x_p^*	T_p	x_p^*	T_p	x_p^*	T_p
Firm 1	p_1	2.33	23.98	1.93	22.72	1.66	21.86
	p_2	2.73	24.38	2.79	23.58	2.82	23.02
Firm 2	p_3	0.44	28.12	0.01	26.72	0.00	26.04
	p_4	2.81	24.13	2.64	22.78	2.37	21.77
	p_5	3.39	28.25	3.53	27.61	3.48	27.15
	p_6	0.00	24.26	0.00	23.67	0.00	22.88

- ▶ **Firm 2's path p_6 is never used** at equilibrium in our three case study examples.
- ▶ **Firm 2's equilibrium path flow on its path p_3 decreases** in Case Study Examples 2 and 3, and reaches the value of 0 in the latter.
- ▶ **The fastest delivery time** is achieved by Firm 2 on its path p_4 (21.77) in Case Study Example 3; and
- ▶ **The slowest delivery time** is achieved by Firm 2 on its path p_5 (28.25) in Case Study Example 1.

Summary and Conclusions

- ▶ In this supply chain network model, multiple firms compete with one another in both **quantities of their product** and in **the average delivery time** to the demand markets.
- ▶ The model contributes to the literature in **information asymmetry** .
- ▶ The new model is relevant to **time-sensitive product markets**, in which consumers are willing to pay a higher price for lower average times of delivery.

Summary and Conclusions

- ▶ The is the first time that such a general model for **time-sensitive markets** with such **a general network topology** has been constructed and formulated.
- ▶ We utilize **variational inequality theory** for the formulation of the governing equilibrium conditions under Cournot-Nash, and also provide qualitative properties of the equilibrium state.
- ▶ Our proposed computational procedure has nice features for implementation.
- ▶ The numerical results reveal that **the firms are trading cost for time when reacting to the changed customer preferences**.
- ▶ Future work may include further disaggregating the supply chain networks of the firms in order to further detail the production and transportation choices. Supply chain network design for time-sensitive products would also be of interest.