Topic 7: More on Cybersecurity

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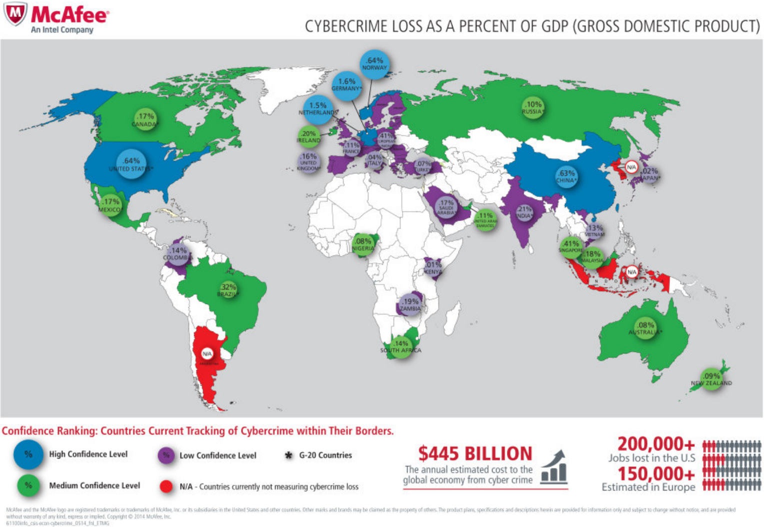
Introduction

- An increasingly connected world may amplify the effects of a disruption.
- **Cyber threat management** is more than a strategic imperative, it is **fundamental to business**.
- Breaches are inevitable:
  - (i) **Tangible costs** - lost funds, regulatory and legal fines, compensation, recovery - information and infrastructure rehabilitation.
  - (ii) **Intangible costs** - loss of reputation, business, competitive advantage, intellectual property, personal information.

Cost of Cybercrime

- Cybercrime climbs to 2nd most reported economic crime affecting **32% of organizations** globally (PwC Survey, 2016).
- Cost of data breaches to increase to **$2.1 trillion** globally by 2019 - four times the estimated cost of breaches in 2015 (Forbes, 2016).
- "Cyber threats are not just increasing, but **mutating**" (Forrester Research, 2016).

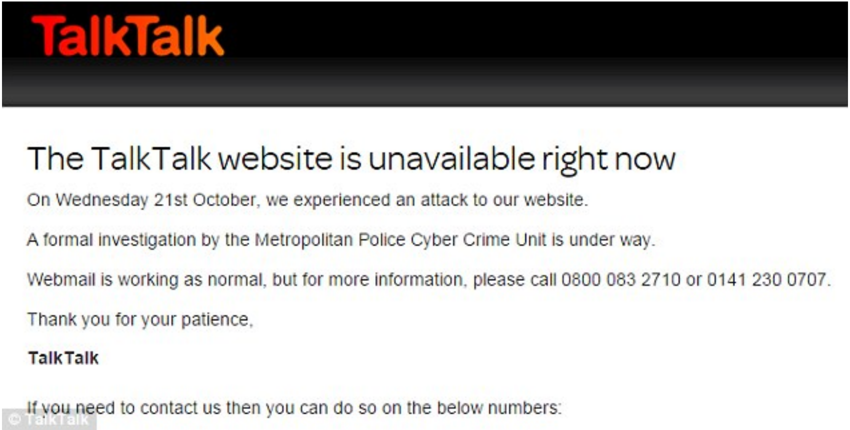
Cyber Loss as a Percent of GDP (2014)



Major Cyberattacks

- **Hilton Worldwide**(2015) - POS terminals hacked, credit card holders' names, numbers, expiry date, and security codes stolen. Hackers shopped online (SecurityWeek, 2016).
- **TalkTalk** (2015) - Nearly 157,000 had data breached. Cost of crime was £60 m, customers chose to leave, bonuses slashed (The Guardian, 2016).
- **Sony Pictures** (2014) - 100 terabytes of sensitive data leaked, 5 Sony films put online for free, private emails, salary information of top executives, medical documents, and Sony's Twitter account also leaked. Cost of crime could be $100 m (Reuters, 2014).

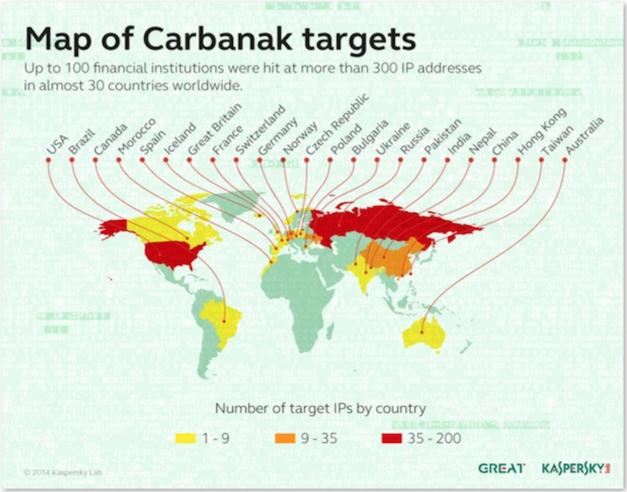
Major Cyberattacks



Major Cyberattacks

- **JD Wetherspoon**(2015) - Names, email ids, birthdates and contact numbers of 656,723 customers hacked. Company became aware of the attack almost 5 months later (Telegraph, 2015).
- Kaspersky Lab reported a cyber heist **(Carbanak)** of $1 bn when hackers infiltrated 100 banks across 30 countries over a period of 2 years.
- Other notable attacks - Target, Home Depot, Michaels Stores, Staples, eBay.

Major Cyberattacks



Motivation

The **median number of days that attackers stay dormant** within a network before detection is **over 200** (Microsoft, 2015)

The majority of data breach victims surveyed, 81 percent, report they had **neither a system nor a managed security service** in place to ensure they could self-detect data breaches, **relying instead on notification from an external party**.

This was the case despite the fact that self-detected breaches take just 14.5 days to contain from their intrusion date, whereas **breaches detected by an external party take an average of 154 days to contain** (Trustwave, 2015).

Motivation

- Growing interest in the development of **rigorous scientific tools**.
- As reported in Glazer (2015), JPMorgan was expected to double its cybersecurity spending in 2015 to $500 million from $250 million in 2014.
- According to Purnell (2015), the research firm Gartner reported in January 2015 that the global information security spending would increase by 7.6% in 2015 to $790 billion.
- It is clear that making the best **cybersecurity investments is a very timely problem and issue**.

Approach

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- **Nonlinear budget constraints** are considered, Nash equilibrium conditions discussed, and variational inequality formulations presented.
- We also discuss how to measure the **vulnerability of a firm to cyberattacks and that of the supply chain network**, as a whole.

Important References:

Nagurney, A. (2015). A multiproduct network economic model of cybercrime in financial services. *Service Science*, **7(1)**, 70-81.

Nagurney, A., Nagurney, L.S., Shukla, S. (2015). A supply chain game theory framework for cybersecurity investments under network vulnerability. In *Computation, Cryptography, and Network Security*, Daras, Nicholas J., Rassias, Michael Th. (Eds.), Springer, 381-398.

Nagurney, A., Nagurney, L. S. (2015). A game theory model of cybersecurity investments with information asymmetry. *NETNOMICS: Economic Research and Electronic Networking*, **16(1-2)**, 127-148.

The Supply Chain Game Theory Model

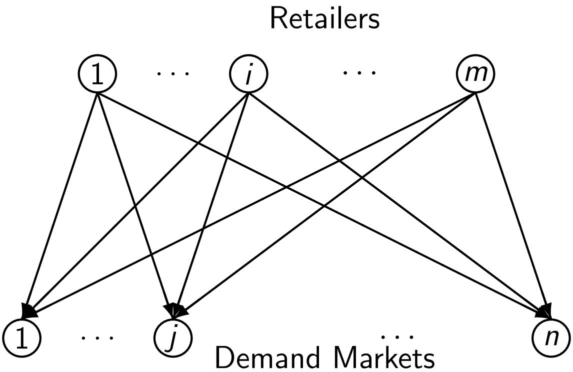

Figure: The Bipartite Structure of the Supply Chain Network Game Theory Model

Network Security Level of firm i; s_i:

$$0 \leq s_i \leq u_{s_i}; \quad i = 1, \ldots, m.$$

$u_{s_i} < 1$: Upper bound on security level of firm i.

Average Network Security of the Chain, $\bar{s}$:

$$\bar{s} = \frac{1}{m} \sum_{i=1}^{m} s_i.$$

Probability of a Successful Cyberattack on i, p_i:

$$p_i = (1 - s_i)(1 - \bar{s}), \quad i = 1, \ldots, m.$$

Vulnerability of firm i, v_i: $v_i = (1 - s_i); i = 1, \ldots, m.$

Vulnerability of network, $\bar{v} = (1 - \bar{s})$.

The Supply Chain Game Theory Model

Investment Cost Function to Acquire Security s_i, $h_i(s_i)$:

$$h_i(s_i) = \alpha_i(\frac{1}{\sqrt{(1-s_i)}} - 1), \quad \alpha_i > 0, \qquad i = 1, \ldots, m.$$

α_i quantifies size and needs of retailer i; $h_i(0) = 0 =$ insecure retailer, and $h_i(1) = \infty =$ complete security at infinite cost.

Nonlinear Budget Constraint:

$$\alpha_i(\frac{1}{\sqrt{(1-s_i)}} - 1) \leq B_i, \qquad i = 1, \ldots, m.$$

Each retailer cannot exceed his allocated cybersecurity budget, B_i.

The Supply Chain Game Theory Model

Incurred financial damage if attack successful: D_i.
Expected Financial Damage after Cyberattack for Firm i; $i = 1, ..., m$**:**

$$D_i p_i, \quad D_i \geq 0.$$

The **demand for the product at demand market** j must satisfy the following conservation of flow equation:

$$d_j = \sum_{i=1}^{m} Q_{ij}, \quad j = 1, ..., n,$$

where

$$0 \leq Q_{ij} \leq \bar{Q}_{ij}, \quad i = 1, ..., m; j = 1, ..., n.$$

The Supply Chain Game Theory Model

In view of the demand, we can define **demand price functions**

$$\hat{\rho}_j(Q, s) \equiv \rho_j(d, \bar{s}), \forall j.$$

The consumers reflect their preferences through vector of demands and supply chain network security.

Profit of Retailer $i; i = 1, ..., m$**, in absence of cyberattack and investments,** f_i**:**

$$f_i(Q, s) = \sum_{j=1}^{n} \hat{\rho}_j(Q, s) Q_{ij} - c_i \sum_{j=1}^{n} Q_{ij} - \sum_{j=1}^{n} c_{ij}(Q_{ij}),$$

Q_{ij} : Quantity from i to j; c_i : Cost of processing at i; c_{ij} : Cost of transactions from i to j.

The Supply Chain Game Theory Model

Expected Utility $i; i = 1, ..., m$**:**

$$E(U_i) = (1 - p_i)f_i(Q, s) + p_i(f_i(Q, s) - D_i) - h_i(s_i).$$

Each $E(U_i(s))$ is strictly concave with respect to s_i and each $h_i(s_i)$ is strictly convex.

Feasible Set: $K \equiv \prod_{i=1}^{m} K^i$, where
$K^i \equiv \{(Q_i, s_i) | 0 \leq Q_i \leq \bar{Q}_{ij}; 0 \leq s_i \leq u_{s_i}$, and budget constraint$\}$

Definition 1: A Supply Chain Nash Equilibrium in Product Transactions and Security Levels

We seek to determine a nonnegative product transaction and security level pattern $(Q^*, s^*) \in K$ for which the m retailers will be in a state of equilibrium as defined below.

Definition 1: Nash Equilibrium in Cybersecurity Levels

A product transaction and security level pattern $(Q^*, s^*) \in K$ *K is said to constitute a supply chain Nash equilibrium if for each retailer* i; $i = 1, \ldots, m$:

$$E(U_i(Q_i^*, s_i^*, \hat{Q}_i^*, \hat{s}_i^*)) \geq E(U_i(Q_i, s_i, \hat{Q}_i^*, \hat{s}_i^*)), \quad \forall (Q_i, s_i) \in K_i^1,$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_m^*); \hat{s}_i^* \equiv (s_1^*, \ldots, s_{i-1}^*, s_{i+1}^*, \ldots, s_m^*).$$

Nonlinear Budget Constraints in the Feasible Set

In our model, unlike in many network equilibrium problems from congested urban transportation networks to supply chains and financial networks, the feasible set contains nonlinear constraints.

Lemma 1
Let h_i be a convex function for all retailers i; $i = 1, \ldots, m$. The feasible set K is then convex.

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation
$(Q^*, s^*) \in K$ *is a Nash equilibrium if and only if it satisfies the variational inequality,*

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*)$$

$$-\sum_{i=1}^{m}\frac{\partial E(U_i(Q^*, s^*))}{\partial s_i} \times (s_i - s_i^*) \geq 0, \forall (Q, s) \in K,$$

or, equivalently,

Variational Inequality Formulation

$(Q^*, s^*) \in K$ *is a Nash equilibrium if and only if it satisfies the variational inequality,*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}[c_i + \frac{\partial c_{ij}(Q_{ij}^*)}{\partial Q_{ij}} - \hat{\rho}_j(Q^*, s^*) - \sum_{k=1}^{n}\frac{\hat{\rho}_k(Q^*, s^*)}{\partial Q_{ij}}Q_{ik}^*] \times (Q_{ij} - Q_{ij}^*)$$

$$+\sum_{i=1}^{m}[\frac{\partial h_i(s_i^*)}{\partial s_i} - \sum_{k=1}^{n}\frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i}Q_{ik}^*$$

$$-(1 - \sum_{k=1}^{m}\frac{s_k^*}{m} + \frac{1 - s_i^*}{m})D_i)] \times (s_i - s_i^*) \geq 0, \forall (Q, s) \in K.$$

Existence

Theorem 2: Existence
A solution (Q^*, s^*) *to the variational inequality is guaranteed to exist.*

The result follows from the classical theory of variational inequaliities (see Kinderlehrer and Stampacchia (1980)) since the feasible set K is compact, and the function that enters the variational inequality is continuous.

Uniqueness

We define the $(mn + m)$-dimensional column vector $X \equiv (Q, s)$ and the $(mn + m)$-dimensional column vector $F(X) = (F^1(X), F^2(X))$ with the (i,j)-th component, F^1_{ij} of $F^1(X)$ is $\frac{\partial E(U_i(Q^*,s^*))}{\partial Q_{ij}}$, and i-th component F^2_i of $F^2(X)$ is $\frac{\partial E(U_i(Q^*,s^*))}{\partial s_i}$.

Theorem 3: Uniqueness
A solution (Q^*, s^*) to the variational inequality is unique if $F(X)$ and $X \equiv (Q, s)$ is strictly monotone (see Kinderlehrer and Stampacchia (1980)).

Variational Inequality Formulation with Lagrange Multipliers

Feasible set: $\mathcal{K} \equiv \prod_{i=1}^m \mathcal{K}_i^1 \times R_+^m$
where $\mathcal{K}_i^1 \equiv \{(Q_i, s_i) | 0 \leq Q_{ij} \leq \bar{Q}_{ij}, \forall j; 0 \leq s_i \leq u_{s_i}\}$.

Theorem 4: Alternative Variational Inequality Formulation
A vector (Q^*, s^*, λ^*) *in feasible set,* $\mathcal{K}$*, containing non-negativity constraints is an equilibrium solution if and only if it satisfies the following variational inequality,*

$$\sum_{i=1}^m \sum_{j=1}^n [c_i + \frac{\partial c_{ij}(Q_{ij}^*)}{\partial Q_{ij}} - \hat{\rho}_j(Q^*, s^*) - \sum_{k=1}^n \frac{\hat{\rho}_k(Q^*, s^*)}{\partial Q_{ij}} Q_{ik}^*] \times (Q_{ij} - Q_{ij}^*)$$

$$+ \sum_{i=1}^m [\frac{\partial h_i(s_i^*)}{\partial s_i} - \sum_{k=1}^n \frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i} Q_{ik}^*$$

$$-(1 - \sum_{k=1}^m \frac{s_k^*}{m} + \frac{1 - s_i^*}{m})D_i) + \frac{\lambda_i^*}{2} \alpha_i (1 - s_i^*)^{-\frac{3}{2}}] \times (s_i - s_i^*)$$

$$+ \sum_{i=1}^m [B_i - \alpha_i (\frac{1}{\sqrt{1 - s_i}} - 1)] \times (\lambda_i - \lambda_i^*) \geq 0, \quad \forall (Q, s, \lambda) \in \mathcal{K}.$$

Proof

Proof: Each retailer i; $i = 1, \ldots, m$, according to the Definition, seeks to determine his strategy vector (Q_i, s_i) so as to

$$\text{Maximize}_{(Q_i, s_i)} E(U_i) = (1 - p_i) f_i(Q, s) + p_i(f_i(Q, s) - D_i) - h_i(s_i)$$

subject to:

$$\alpha_i\left(\frac{1}{\sqrt{(1 - s_i)}} - 1\right) - B_i \leq 0,$$

$$0 \leq Q_{ij} \leq \bar{Q}_{ij}, \quad j = 1, \ldots, n,$$

$$0 \leq s_i \leq u_{s_i},$$

where $f_i(Q, s)$, $h_i(s_i)$, are as above, and $p_i = (1 - s_i)(1 - \bar{s})$.

Proof, continued

Simplifying the terms in the objective function and converting the Maximization problem into a Minimization problem, the above optimization problem with the newly defined feasible set $\mathcal{K}_i^1$ becomes:

$$\text{Minimize} \quad -f_i(Q,s) + D_i(1-s_i)(1-\bar{s}) + h_i(s_i)$$

subject to:

$$\alpha_i(\frac{1}{\sqrt{(1-s_i)}} - 1) - B_i \leq 0,$$

$$(Q_i, s_i) \in \mathcal{K}_i^1.$$

Proof, continued

If we now let $X_i \equiv (Q_i, s_i)$, $\hat{X}_i \equiv (X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_m)$, and $\hat{f}_i(X_i, \hat{X}_i) \equiv -f_i(Q, s) + D_i(1 - s_i)(1 - \bar{s}) + h_i(s_i)$, we can rewrite retailer i's optimization problem, where $\hat{X}_i^*$ denotes the other retailers' optimal solutions, as:

$$\text{Minimize} \quad \hat{f}_i(X_i, \hat{X}_i^*)$$

subject to:

$$g_i(X_i) \leq 0,$$

$$X_i \in \mathcal{K}_i^1.$$

Note that $g_i(X_i) = \alpha_i(\frac{1}{\sqrt{(1-s_i)}} - 1) - B_i$.

Proof, continued

We now form the Lagrangian
$\mathcal{L}(X_i, \hat{X}_i^*, \lambda_i) = \hat{f}_i(X_i, \hat{X}_i^*) + \lambda_i g_i(X_i)$.

Also, we make the following assumption:

Assumption: (Slater Condition). There exists a Slater vector $\tilde{X}_i \in \mathcal{K}_i^1$ for each $i = 1, \ldots, m$, such that $g_i(\tilde{X}_i) < 0$.
This is easy to verify.

Proof, continued

Then, according to Koshal, Nedic, and Shanbhag (2011), pages 1049-1051, since $\hat{f}_i$ is convex in X_i and is continuously differentiable and g_i is also convex and continuously differentiable, and $\mathcal{K}_i^1$ is nonempty, closed and convex, $(X_i^*, \lambda_i^*) \in \mathcal{K}_i^1 \times R_+$ is a solution to the above optimization problem, if and only if it is a solution to the variational inequality:

$$\nabla_{X_i} \mathcal{L}(X_i^*, \hat{X}_i^*, \lambda_i^*) \times (X_i - X_i^*) + (-g_i(X_i^*)) \times (\lambda_i - \lambda_i^*) \geq 0,$$

$$\forall (X_i, \lambda_i) \in \mathcal{K}_i^1 \times R_+,$$

with $\nabla_{X_i} \mathcal{L}$ representing the gradient with respect to X_i of the Lagrangian $\mathcal{L}$.

Proof, continued

Expanding the above by using the definitions of our functions and vectors and making the appropriate substitutions, we obtain that $X_i^* \in \mathcal{K}_i^1$ is a solution if and only if $(Q_i^*, s_i^*, \lambda_i^*) \in \mathcal{K}_i^1$ is a solution to the variational inequality:

$$\sum_{j=1}^{n}\left[c_i + \frac{\partial c_{ij}(Q_{ij}^*)}{\partial Q_{ij}} - \hat{\rho}_j(Q^*, s^*) - \sum_{k=1}^{n}\frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial Q_{ij}} \times Q_{ik}^*\right] \times (Q_{ij} - Q_{ij}^*)$$

$$+\left[\frac{\partial h_i(s_i^*)}{\partial s_i} - (1 - \sum_{k=1}^{m}\frac{s_k^*}{m} + \frac{1-s_i^*}{m})D_i - \sum_{k=1}^{n}\frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i} \times Q_{ik}^* + \frac{\lambda_i^*}{2}\alpha_i(1-s_i^*)^{-\frac{3}{2}}\right] \times (s_i - s_i^*)$$

$$+\left[B_i - \alpha_i(\frac{1}{\sqrt{(1-s_i^*)}} - 1)\right] \times (\lambda_i - \lambda_i^*) \geq 0, \quad \forall (Q_i, s_i, \lambda_i) \in \mathcal{K}_i^1.$$

But the above inequality holds for each i; $i = 1, \ldots, m$. Hence, since we are dealing with a Nash equilibrium problem, summation of the above inequality over all i; $i = 1, \ldots, m$, yields the desired variational inequality. □

The Algorithm

The Euler Method: At each iteration τ, one solves the following problem:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})),$$

where $P_{\mathcal{K}}$ is the projection operator and F is the function that enters the Variational Inequality, $\langle F(X^*), X - X^* \rangle \geq 0$, where $X \equiv (Q, s, \lambda)$.

As established in Dupuis and Nagurney (1993), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

Explicit Formulae for the Euler Method

Closed form expression for the product transactions, $i = 1, ..., m; j = 1, ..., n$:

$$Q_{ij}^{\tau+1}$$

$$= max\{0, min\{\bar{Q}_{ij}, Q_{ij}^{\tau} + a_\tau(\hat{\rho}_j(Q^\tau, s^\tau) + \sum_{k=1}^{n} \frac{\partial \hat{\rho}_k(Q^\tau, s^\tau)}{\partial Q_{ij}} Q_{ik}^{\tau} - c_i - \frac{\partial c_{ij}(Q_{ij}^{\tau})}{\partial Q_{ij}})\}\}$$

Closed form expression for security levels and Lagrange multipliers for $i = 1, ..., m$:

$$s_i^{\tau+1} = max\{0, min\{u_{s_i}, s_i^\tau + a_\tau(\sum_{k=1}^{n} \frac{\partial \hat{\rho}_k(Q^\tau, s^\tau)}{\partial s_i} Q_{ik}^\tau - \frac{\partial h_i(s_i^\tau)}{\partial s_i^\tau}$$

$$+(1 - \sum_{j=1}^{m} \frac{s_j^\tau}{m} + \frac{1 - s_i}{m})D_i) - \frac{\lambda_i^\tau}{2} \alpha_i (1 - s_i^\tau)^{\frac{-3}{2}}\}\},$$

$$\lambda_i^{\tau+1} = max\{0, \lambda_i^\tau + a_\tau(B_i + \alpha_i(\frac{1}{\sqrt{1 - s_i^\tau}} - 1))\}.$$

Numerical Examples

Convergence Criterion: $\epsilon = 10^{-4}$.
The Euler method was considered to have converged if, at a given iteration, the absolute value of the difference of each product transaction and each security level differed from its respective value at the preceding iteration by no more than ϵ.
Sequence a_τ: $.1(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots)$.

Initial Values: We initialized the Euler method by setting each product transaction $Q_{ij} = 1.00, \forall i, j$, the security level of each retailer $s_i = 0.00, \forall i$, and the Lagrange multiplier for each retailer's budget constraint $\lambda_i = 0.00, \forall i$. The capacities $\bar{Q}_{ij}$ were set to 100 for all i, j.

Example 1

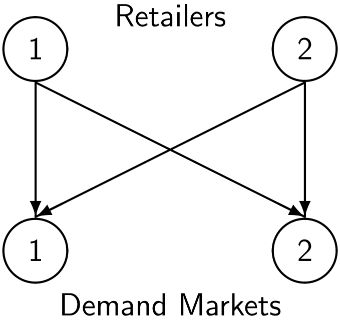

Figure: Network Topology for Examples 1 and 2

Example 1

Cost functions:

$$c_1 = 5, \quad c_2 = 10,$$

$$c_{11}(Q_{11}) = .5Q_{11}^2 + Q_{11}, \quad c_{12}(Q_{12}) = .25Q_{12}^2 + Q_{12},$$

$$c_{21}(Q_{21}) = .5Q_{21}^2 + 2, \quad c_{22}(Q_{22}) = .25Q_{22}^2 + Q_{22}.$$

Demand price functions:

$$\rho_1(d, \bar{s}) = -d_1 + .1(\frac{s_1 + s_2}{2}) + 100, \rho_2(d, \bar{s}) = -.5d_2 + .2(\frac{s_1 + s_2}{2}) + 200.$$

Damage parameters: $D_1 = 50, D_2 = 70$. Budgets: $B_1 = B_2 = 2.5$.

Example 1

Investment cost functions:

$$h_1(s_1) = \frac{1}{\sqrt{(1 - s_1)}} - 1, \quad h_2(s_2) = \frac{1}{\sqrt{(1 - s_2)}} - 1.$$

Example 1

Results:

Solution	Ex.1
Q_{11}^*	24.27
Q_{12}^*	98.34
Q_{21}^*	21.27
Q_{22}^*	93.34
d_1^*	45.55
d_2^*	191.68
s_1^*	.91
s_2^*	.91
$\bar{s}^*$	.91
λ_1^*	0.00
λ_2^*	0.00
$\rho_1(d_1^*, \bar{s}^*)$	54.55
$\rho_2(d_2^*, \bar{s}^*)$	104.34
$E(U_1)$	8137.38
$E(U_2)$	7213.49

Example 2

Example 2 was constructed from Example 1, except that the investment cost function of Retailer 1 was changed to: $h_1(s_1) = 10\frac{1}{\sqrt{(1-s_1)}} - 1$.

Solution	Ex.2
Q_{11}^*	24.27
Q_{12}^*	98.31
Q_{21}^*	21.27
Q_{22}^*	93.31
d_1^*	45.53
d_2^*	191.62
s_1^*	.36
s_2^*	.91
$\bar{s}^*$	.63
λ_1^*	3.68
λ_2^*	1.06
$\rho_1(d_1^*, \bar{s}^*)$	54.53
$\rho_2(d_2^*, \bar{s}^*)$	104.32
$E(U_1)$	8122.77
$E(U_2)$	7207.47

Example 3

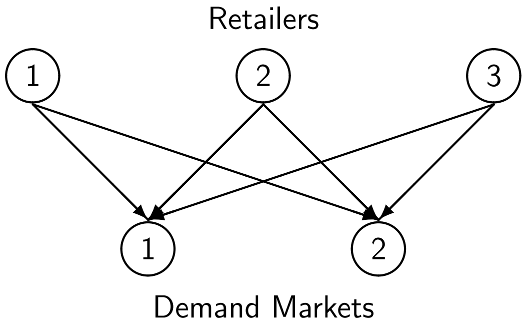

Figure: Network Topology for Examples 3 and 4

Example 3

Example 3 was constructed from Example 1 with the following data added for Retailer 3. Cost functions:

$$c_3 = 3,$$

$$c_{31}(Q_{31}) = Q_{31}^2 + 2Q_{31}, \quad c_{32}(Q_{32}) = Q_{32}^2 + 4Q_{32}.$$

Damage parameter: $D_3 = 80$. Budget: $B_3 = 3.0$.
Added investment cost function:

$$h_3(s_3) = 3(\frac{1}{\sqrt{(1-s_3)}} - 1).$$

.

Example 3

Results:

Q_{11}^*	20.80
Q_{12}^*	89.48
Q_{21}^*	17.80
Q_{22}^*	84.48
Q_{31}^*	13.87
Q_{32}^*	35.40
d_1^*	52.48
d_2^*	209.36
s_1^*	.90
s_2^*	.91
s_3^*	.74
$\bar{s}^*$	.85
λ_1^*	0.00
λ_2^*	0.00
λ_3^*	0.00
$\rho_1(d_1^*, \bar{s}^*)$	47.61
$\rho_2(d_2^*, \bar{s}^*)$	95.49
$E(U_1)$	6655.13
$E(U_2)$	5828.82
$E(U_3)$	2262.26

Example 4

Example 4 constructed from Example 3. All damages at 0.00.

Q_{11}^*	20.80
Q_{12}^*	89.48
Q_{21}^*	17.80
Q_{22}^*	84.47
Q_{31}^*	13.87
Q_{32}^*	35.40
d_1^*	52.47
d_2^*	209.30
s_1^*	.82
s_2^*	.81
s_3^*	.34
$\bar{s}^*$	.66
λ_1^*	0.00
λ_2^*	0.00
λ_3^*	0.00
$\rho_1(d_1^*, \bar{s}^*)$	47.60
$\rho_2(d_2^*, \bar{s}^*)$	95.48
$E(U_1)$	6652.45
$E(U_2)$	5828.10
$E(U_3)$	2264.24

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- Various data instances are evaluated through the algorithm, with relevant managerial insights and sensitivity analysis.
- The generalized framework of cybersecurity investments in a supply chain network game theory context with nonlinear budget constraints is a **novel contribution to the literature of both variational inequalities and game theory, and cybersecurity investments**.