Topic 4: Supply Chain Performance Assessment

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This lecture is based on the paper, “Supply Chain Performance Assessment and Supplier and Component Importance Identification in a General Competitive Multitiered Supply Chain Network Model,” co-authored with Dong “Michelle” Li, published in the *Journal of Global Optimization* (2017), **67(1)**, pp 223-250, and also on our book *Competing on Supply Chain Quality*, Springer 2016. References can be found therein.

In this lecture, we capture decision-making associated with different tiers of decision-makers in a supply chain with suppliers and with firms, which you can think of as OEMs. The former compete in prices in Bertrand fashion, whereas the latter compete in product quantities a la Cournot.

The governing concept is still, as in the previous lectures, that of Nash equilibrium.

Outline

- Background and Motivation
- Representation of Supply Chains as Networks
- Methodology - The Variational Inequality Problem
- The Multitiered Supply Chain Network Model with Suppliers
- The Nagurney-Qiang (N-Q) Network Efficiency / Performance Measure
- Supply Chain Network Performance Measures
- The Algorithm and Numerical Examples
- Summary and Conclusions

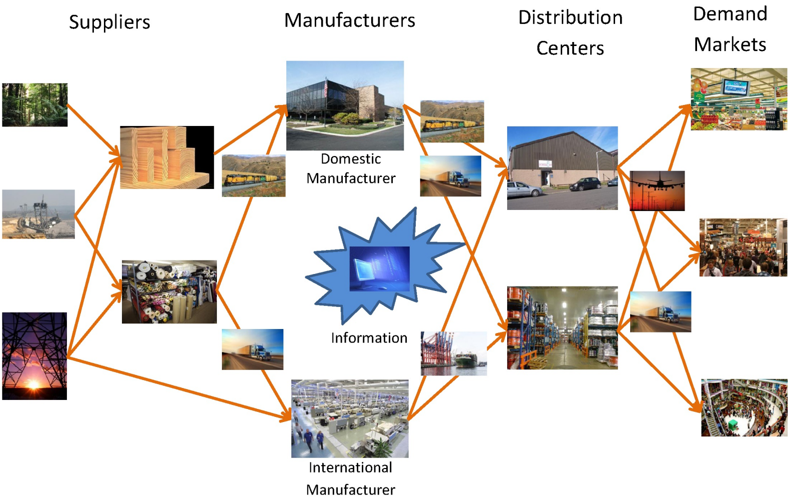
Background and Motivation

Supply chains are the **critical infrastructure and backbones** for the production, distribution, and consumption of goods as well as services in our globalized **Network Economy**.

Supply chains, in their most fundamental realization, **consist of manufacturers and suppliers, distributors, retailers, and consumers at the demand markets**.

Today, supply chains may span thousands of miles across the globe, involve numerous suppliers, retailers, and consumers, and be underpinned by multimodal transportation and telecommunication networks.

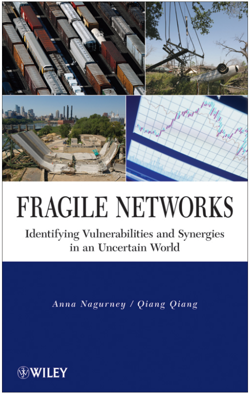
A General Supply Chain

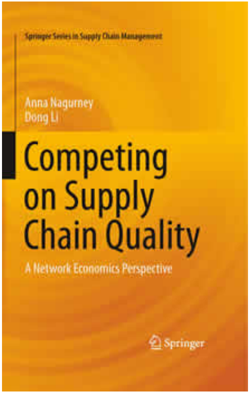


Examples of Supply Chains

- food and food products
- high tech products
- automotive
- energy (oil, electric power, etc.)
- clothing and toys
- healthcare supply chains
- humanitarian relief
- supply chains in nature.

Examples of Supply Chains





We are living in an era of *Fragile Networks* and, yet, at the same time, quality of products is essential.

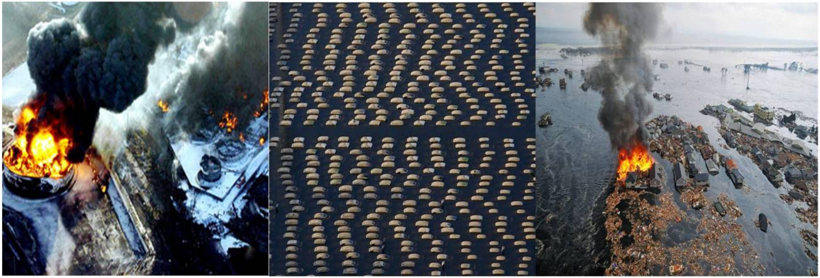
Background and Motivation

Suppliers are critical in providing essential components and resources for finished goods in today's globalized supply chain networks. Even in the case of bread ingredients may travel across the globe as inputs into production processes.

Suppliers are also decision-makers and compete with one another to provide components to downstream

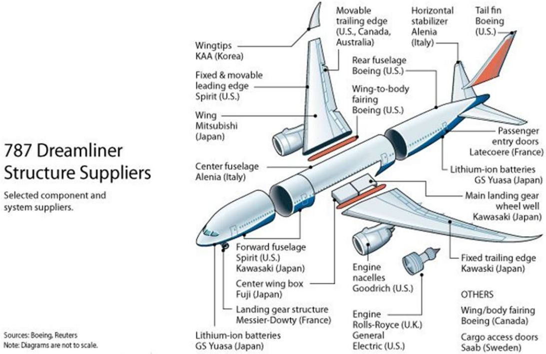
Background and Motivation

When suppliers are faced with **disruptions**, whether due to man-made activities or errors, natural disasters, unforeseen events, or even terrorist attacks, the ramifications and effects may propagate through a supply chain or multiple supply chains.



Examples of Supplier Failures Due to Quality Issues

Boeing, facing challenges with its 787 Dreamliner supply chain design and numerous delays, ended up having to buy two suppliers for $2.4 billion because the units were underperforming in the chain (Tang, Zimmerman, and Nelson (2009)).



Examples of Supplier Failures Due to Quality Issues

• In 2007, the toy giant Mattel recalled 19 million toy cars because of a **supplier's lead paint and small, poorly designed magnets, which could harm children if ingested** (Story and Barboza (2007)).

• In 2010, four Japanese car-makers, including Toyota and Nissan, recalled 3.6 million vehicles sold around the globe, because **the airbags supplied by Takata Corp., were at risk of exploding and causing injury** (Kubota and Klayman (2013)). The recalls are still ongoing and have expanded to other companies.

• In 2013, in the food industry, Taylor Farms, a large vegetable supplier, was under investigation in connection with **an illness outbreak affecting hundreds of people in the US** (Strom (2013)).

Examples of Supplier Failures Due to Quality Issues

- In 2016, Samsung made the unprecedented decision to **recall every single one of the Galaxy Note 7 smartphones sold** because of explosions and fires, suspected from the batteries (Hollister (2016)).

Examples of Supplier Failures Due to Natural Disasters

- **The Royal Philips Electronics cell phone chip manufacturing plant fire**, due to a lightning strike on March 17, 2000, and subsequent water and smoke damage, which adversely affected Ericsson, which, unlike Nokia, did not have a backup, and suffered a second quarter operating loss in 2000 of $200 million in its mobile phone division.
- **The Fukushima triple disaster on March 11, 2011 in Japan** resulted in shortages of memory chips, automotive sensors, silicon wafers, and even certain colors of automotive paints, because of the affected suppliers.

Representation of Supply Chains as Networks

Characteristics of Supply Chains Today

- **large-scale nature** and complexity of network topology;
- **congestion**, which leads to nonlinearities;
- **alternative behavior of users of the networks**, which may lead to paradoxical phenomena;
- **possibly conflicting criteria associated with optimization**;
- **interactions among the underlying networks themselves**, such as the Internet with electric power networks, financial networks, and transportation and logistical networks;
- recognition of **their fragility and vulnerability**;
- policies surrounding networks today may have major impacts.

Supply Chains Are Network Systems

Supply chains are, in fact, **Complex Network Systems**.

Hence, **any formalism that seeks to model supply chains and to provide quantifiable insights and measures must be a system-wide one and network-based.**

Such crucial issues as the stability and resiliency of supply chains, as well as their adaptability and responsiveness to events in **a global environment of increasing risk and uncertainty** can only be rigorously examined from the view of supply chains as network systems.

Representation of Supply Chains as Networks

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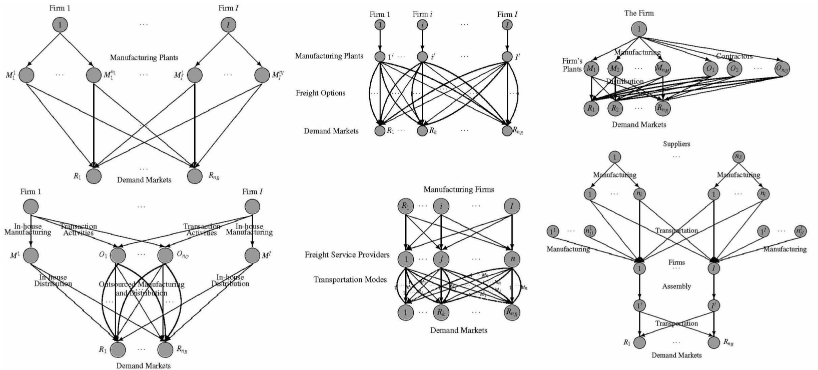
• build meaningful extensions using the graphical/network conceptualization.

In *Competing on Supply Chain Quality*, we present supply chain network models and tools to investigate: information asymmetry, impacts of outsourcing on quality, **minimum quality standards**, applications to industries such as **pharma and high tech**, freight services and quality, and the identification of which suppliers matter the most to both individual firms' supply chains and to that of the supply chain network economy.



Methodology - The Variational Inequality Problem

Methodology - The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized supply chain network problems, which is one of the foci of this seminar.

Definition: The Variational Inequality Problem
The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$*, is to determine a vector* $X^* \in \mathcal{K}$*, such that:*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F *is a given continuous function from* $\mathcal{K}$ *to* R^N*,* $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* R^N*.*

Methodology - The Variational Inequality Problem

The vector X consists of the decision variables – typically, the flows (products, prices, etc.).

$\mathcal{K}$ is the feasible set representing how the decision variables are constrained – for example, the flows may have to be nonnegative; budget constraints may have to be satisfied; similarly, quality and/or time constraints may have to be satisfied.

The function F that enters the variational inequality represents functions that capture the behavior in the form of the functions such as costs, profits, risk, etc.

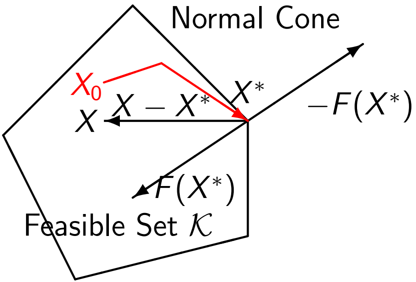
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- game theory problems, operating under Nash equilibrium,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of supply chain network problems from centralized to decentralized ones as well as to design problems.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides natural underlying dynamics associated with travel (and other) behavior to the equilibrium.

To model the *dynamic behavior of complex networks*, including supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in

evolutionary games (Sandholm (2005, 2011)),

ecological predator-prey networks (Nagurney and Nagurney (2011a, b)), and

even neuroscience (Girard et al. (2008)

dynamic spectrum model for cognitive radio networks (Setoodeh, Haykin, and Moghadam (2012)).

Overview

We present a **multitiered competitive supply chain network game theory model**, which includes the supplier tier.

- The firms are **differentiated** by brands and can produce their **own** components, as reflected by their **capacities**, and/or obtain components from one or more suppliers, who also are **capacitated**.
- The firms compete in **Cournot-Nash** fashion, whereas
- the suppliers compete a la **Bertrand**.
- All decision-makers seek to **maximize their profits**.
- Consumers reflect their preferences through the **demand price functions** associated with the demand markets for the firms' products.

Overview

- We propose **supply chain network** performance measures, on the **full supply chain** and on the **individual firm levels**, that assess the efficiency of the supply chain or firm, respectively, and also allow for the identification and ranking of the importance of **suppliers** as well as the **components of suppliers** with respect to the full supply chain or individual firm.
- Our framework adds to the growing literature on **supply chain disruptions** by providing metrics that allow individual firms, industry overseers or regulators, and/or government policy-makers to identify the importance of suppliers and the components that they produce for various product supply chains.

The Multitiered Supply Chain Network Model

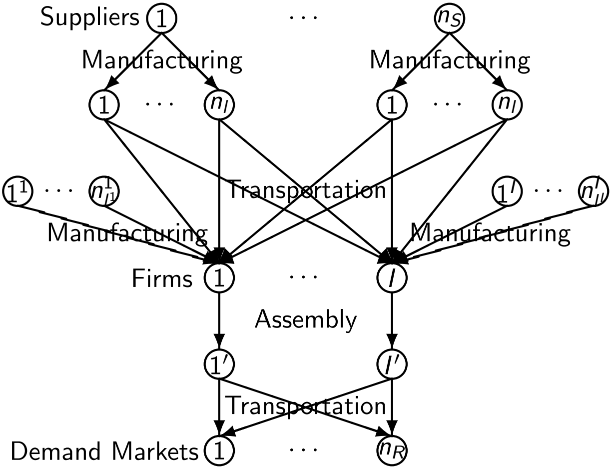


Figure: The Multitiered Supply Chain Network Topology

Notation

Q^S_{jil}: the nonnegative amount of firm i's component l produced by supplier j; $j = 1, \ldots, n_S$; $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$.

Q^F_{il}: the nonnegative amount of firm i's component l produced by firm i itself.

Q_{ik}: the nonnegative shipment of firm i's product from firm i to demand market k; $k = 1, \ldots, n_R$.

π_{jil}: the price charged by supplier j for producing one unit of firm i's component l.

d_{ik}: the demand for firm i's product at demand market k.

θ_{il}: the amount of component l needed by firm i to produce one unit product i.

Notation

$f_{il}^F(Q^F)$: firm i's production cost for producing its component l.

$f_i(Q)$: firm i's cost for assembling its product using the components needed.

$tc_{ik}^F(Q)$: firm i's transportation cost for shipping its product to demand market k.

$c_{ijl}(Q^S)$: the transaction cost paid by firm i for transacting with supplier j for its component l.

$\rho_{ik}(d)$: the demand price for firm i's product at demand market k.

All the $\{Q_{jil}^S\}$ elements are grouped into $Q^S \in R_+^{n_S \sum_{i=1}^{I} n_{l^i}}$; the $\{Q_{il}^F\}$ elements into $Q^F \in R_+^{\sum_{i=1}^{I} n_{l^i}}$; the $\{Q_{ik}\}$ elements into $Q \in R_+^{In_R}$, and the $\{d_{ik}\}$ elements into $d \in R_+^{In_R}$.

The Behavior of the Firms

$$\text{Maximize}_{Q_i, Q_i^F, Q_i^S} \quad U_i^F = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - f_i(Q) - \sum_{l=1}^{n_{l^i}} f_{il}^F(Q^F) - \sum_{k=1}^{n_R} tc_{ik}^F(Q)$$

$$- \sum_{j=1}^{n_S} \sum_{l=1}^{n_{l^i}} \pi_{jil}^* Q_{jil}^S - \sum_{j=1}^{n_S} \sum_{l=1}^{n_{l^i}} c_{ijl}(Q^S) \qquad (1)$$

subject to:

$$Q_{ik} = d_{ik}, \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \qquad (2)$$

$$\sum_{k=1}^{n_R} Q_{ik} \theta_{il} \leq \sum_{j=1}^{n_S} Q_{jil}^S + Q_{il}^F, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \qquad (3)$$

$$Q_{ik} \geq 0, \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \qquad (4)$$

$$CAP_{jil}^S \geq Q_{jil}^S \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}, \qquad (5)$$

$$CAP_{il}^F \geq Q_{il}^F \geq 0, \quad i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \qquad (6)$$

Group firm i's $\{Q_{jil}^S\}$ elements into $Q_i^S \in R_+^{n_S n_{l^i}}$, its $\{Q_{il}^F\}$ elements into $Q_i^F \in R_+^{n_{l^i}}$, and its $\{Q_{ik}\}$ elements into $Q_i \in R_+^{n_R}$.

The Behavior of the Firms

We define $\overline{K}_i^F \equiv \{(Q_i, Q_i^F, Q_i^S) | (3)$ - (6) are satisfied$\}$. All $\overline{K}_i^F$; $i = 1, \ldots, I$, are closed and convex. We also define the feasible set $\overline{\mathcal{K}}^F \equiv \Pi_{i=1}^I \overline{K}_i^F$.

Definition 1: A Cournot-Nash Equilibrium *A product shipment, in-house component production, and contracted component production pattern* $(Q^*, Q^{F^*}, Q^{S^*}) \in \overline{\mathcal{K}}^F$ *is said to constitute a Cournot-Nash equilibrium if for each firm* i; $i = 1, \ldots, I$,

$$U_i^F(Q_i^*, \hat{Q}_i^*, Q_i^{F^*}, \hat{Q}_i^{F^*}, Q_i^{S^*}, \hat{Q}_i^{S^*}, \pi^*) \geq U_i^F(Q_i, \hat{Q}_i^*, Q_i^F, \hat{Q}_i^{F^*}, Q_i^S, \hat{Q}_i^{S^*}, \pi^*),$$

$$\forall (Q_i, Q_i^F, Q_i^S) \in \overline{K}_i^F, \tag{7}$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*),$$

$$\hat{Q}_i^{F^*} \equiv (Q_1^{F^*}, \ldots, Q_{i-1}^{F^*}, Q_{i+1}^{F^*}, \ldots, Q_I^{F^*}),$$

$$\hat{Q}_i^{S^*} \equiv (Q_1^{S^*}, \ldots, Q_{i-1}^{S^*}, Q_{i+1}^{S^*}, \ldots, Q_I^{S^*}).$$

Variational Inequality Formulation of the Cournot-Nash Equilibrium

Theorem 1: Variational Inequality Formulations *Assume that, for each firm i; $i = 1, \ldots, I$, the utility function $U_i^F(Q, Q^F, Q^S, \pi^*)$ is concave with respect to its variables in Q_i, Q_i^F, and Q_i^S, and is continuous and continuously differentiable. Then $(Q^*, Q^{F^*}, Q^{S^*}) \in \overline{\mathcal{K}}^F$ is a Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:*

$$
-\sum_{i=1}^{I}\sum_{k=1}^{n_R}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*)
$$

$$
-\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{il}^F} \times (Q_{il}^F - Q_{il}^{F^*})
$$

$$
-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{jil}^S} \times (Q_{jil}^S - Q_{jil}^{S^*}) \geq 0, \quad \forall (Q, Q^F, Q^S) \in \overline{\mathcal{K}}^F, \tag{8}
$$

Theorem 1 (continued) *with notice that: for* $i = 1, \ldots, I$; $k = 1, \ldots, n_R$:

$$-\frac{\partial U_i^F}{\partial Q_{ik}} = \left[\frac{\partial f_i(Q)}{\partial Q_{ik}} + \sum_{h=1}^{n_R} \frac{\partial tc_{ih}^F(Q)}{\partial Q_{ik}} - \sum_{h=1}^{n_R} \frac{\partial \hat{\rho}_{ih}(Q)}{\partial Q_{ik}} Q_{ih} - \hat{\rho}_{ik}(Q) \right],$$

for $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$:

$$-\frac{\partial U_i^F}{\partial Q_{il}^F} = \left[\sum_{m=1}^{n_{l^i}} \frac{\partial f_{im}^F(Q^F)}{\partial Q_{il}^F} \right],$$

for $j = 1, \ldots, n_S$; $i = 1, \ldots, I$; $l = 1, \ldots, n_{l^i}$:

$$-\frac{\partial U_i^F}{\partial Q_{jil}^S} = \left[\pi_{jil}^* + \sum_{g=1}^{n_S} \sum_{m=1}^{n_{l^i}} \frac{\partial c_{igm}(Q^S)}{\partial Q_{jil}^S} \right].$$

Theorem 1 (continued) *Equivalently,* $(Q^*, Q^{F^*}, Q^{S^*}, \lambda^*) \in \mathcal{K}^F$ *is a vector of the equilibrium product shipment, in-house component production, contracted component production pattern, and Lagrange multipliers if and only if it satisfies the variational inequality*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[\frac{\partial f_i(Q^*)}{\partial Q_{ik}} + \sum_{h=1}^{n_R}\frac{\partial tc^F_{ih}(Q^*)}{\partial Q_{ik}} - \sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*)}{\partial Q_{ik}}Q^*_{ih} - \hat{\rho}_{ik}(Q^*) + \sum_{l=1}^{n_{l^i}}\lambda^*_{il}\theta_{il}\right]$$

$$\times(Q_{ik} - Q^*_{ik}) + \sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f^F_{im}(Q^{F^*})}{\partial Q^F_{il}} - \lambda^*_{il}\right]\times(Q^F_{il} - Q^{F^*}_{il})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\pi^*_{jil} + \sum_{g=1}^{n_S}\sum_{m=1}^{n_{l^i}}\frac{\partial c_{igm}(Q^{S^*})}{\partial Q^S_{jil}} - \lambda^*_{il}\right]\times(Q^S_{jil} - Q^{S^*}_{jil})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{j=1}^{n_S}Q^{S^*}_{jil} + Q^{F^*}_{il} - \sum_{k=1}^{n_R}Q^*_{ik}\theta_{il}\right]\times(\lambda_{il} - \lambda^*_{il}) \geq 0, \quad \forall(Q, Q^F, Q^S, \lambda) \in \mathcal{K}^F, \tag{9}$$

where $\mathcal{K}^F \equiv \Pi^I_{i=1}K^F_i$ *and* $K^F_i \equiv \{(Q_i, Q^F_i, Q^S_i, \lambda_i)|\lambda_i \geq 0$ *with (4) - (6) satisfied*$\}$.

Notation for the Suppliers

$f_{jl}^S(Q^S)$: supplier j's production cost for producing component l; $l = 1, \ldots, n_l$.

$tc_{jil}^S(Q^S)$: supplier j's transportation cost for shipping firm i's component l.

$oc_j(\pi)$: supplier j's opportunity cost.

We group all the $\{\pi_{jil}\}$ elements into the vector $\pi \in R_+^{n_S \sum_{i=1}^{I} n_{l^i}}$.

The Behavior of the Suppliers

$$\text{Maximize}_{\pi_j} \quad U_j^S = \sum_{i=1}^{I} \sum_{l=1}^{n_{l^i}} \pi_{jil} Q_{jil}^{S^*} - \sum_{l=1}^{n_l} f_{jl}^S (Q^{S^*}) - \sum_{i=1}^{I} \sum_{l=1}^{n_{l^i}} tc_{jil}^S (Q^{S^*}) - oc_j(\pi) \tag{11}$$

subject to:

$$\pi_{jil} \geq 0, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{12}$$

For supplier j, we group its $\{\pi_{jil}\}$ elements into the vector $\pi_j \in R_+^{\sum_{i=1}^{I} n_{l^i}}$.

The Behavior of the Suppliers

We define the feasible sets $K_j^S \equiv \{\pi_j | \pi_j \in R_+^{\sum_{i=1}^{I} n_{I^i}}\}$, $\mathcal{K}^S \equiv \Pi_{j=1}^{n_S} K_j^S$, and $\overline{\mathcal{K}} \equiv \overline{\mathcal{K}}^F \times \mathcal{K}^S$.

Definition 2: A Bertrand Equilibrium *A price pattern $\pi^* \in \mathcal{K}^S$ is said to constitute a Bertrand equilibrium if for each supplier j; $j = 1, \ldots, n_S$,*

$$U_j^S(Q^{S^*}, \pi_j^*, \hat{\pi}_j^*) \geq U_j^S(Q^{S^*}, \pi_j, \hat{\pi}_j^*), \quad \forall \pi_j \in K_j^S, \tag{13}$$

where

$$\hat{\pi}_j^* \equiv (\pi_1^*, \ldots, \pi_{j-1}^*, \pi_{j+1}^*, \ldots, \pi_{n_S}^*).$$

Variational Inequality Formulation of Bertrand Equilibrium

Theorem 2: Variational Inequality Formulation *Assume that, for each supplier j; $j = 1, \ldots, n_S$, the profit function $U_j^S(Q^{S^*}, \pi)$ is concave with respect to the variables in π_j, and is continuous and continuously differentiable. Then $\pi^* \in \mathcal{K}^S$ is a Bertrand equilibrium according to Definition 2 if and only if it satisfies the variational inequality:*

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}} \frac{\partial U_j^S(Q^{S^*}, \pi^*)}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*) \geq 0,$$

$$\forall \pi \in \mathcal{K}^S, \tag{14}$$

with notice that: for $j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}$:

$$-\frac{\partial U_j^S}{\partial \pi_{jil}} = \frac{\partial oc_j(\pi)}{\partial \pi_{jil}} - Q_{jil}^{S^*}.$$

Equilibrium Conditions for the Multitiered Supply Chain Network

Definition 3: Multitiered Supply Chain Network Equilibrium with Suppliers *The equilibrium state of the multitiered supply chain network with suppliers is one where both variational inequalities (8) (or (9)) and (14) hold simultaneously.*

Theorem 3: Variational Inequality Formulations of the Multitiered Supply Chain
The equilibrium conditions governing the multitiered supply chain network model with suppliers are equivalent to the solution of the variational inequality problem: determine $(Q^*, Q^{F^*}, Q^{S^*}, \pi^*) \in \overline{\mathcal{K}}$, *such that:*

$$-\sum_{i=1}^{I}\sum_{k=1}^{n_R}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{ik}} \times (Q_{ik} - Q_{ik}^*) - \sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{il}^F}$$

$$\times (Q_{il}^F - Q_{il}^{F^*}) - \sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\frac{\partial U_i^F(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)}{\partial Q_{jil}^S} \times (Q_{jil}^S - Q_{jil}^{S^*})$$

$$-\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\frac{\partial U_j^S(Q^{S^*}, \pi^*)}{\partial \pi_{jil}} \times (\pi_{jil} - \pi_{jil}^*) \geq 0, \quad \forall (Q, Q^F, Q^S, \pi) \in \overline{\mathcal{K}}. \qquad (15)$$

Theorem 3 (continued) *Equivalently: determine* $(Q^*, Q^{F^*}, Q^{S^*}, \lambda^*, \pi^*) \in \mathcal{K}$, *such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\left[\frac{\partial f_i(Q^*)}{\partial Q_{ik}} + \sum_{h=1}^{n_R}\frac{\partial tc_{ih}^F(Q^*)}{\partial Q_{ik}} - \sum_{h=1}^{n_R}\frac{\partial \hat{\rho}_{ih}(Q^*)}{\partial Q_{ik}}Q_{ih}^* - \hat{\rho}_{ik}(Q^*) + \sum_{l=1}^{n_{l^i}}\lambda_{il}^*\theta_{il}\right]$$

$$\times(Q_{ik} - Q_{ik}^*) + \sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{m=1}^{n_{l^i}}\frac{\partial f_{im}^F(Q^{F^*})}{\partial Q_{il}^F} - \lambda_{il}^*\right] \times (Q_{il}^F - Q_{il}^{F^*})$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\pi_{jil}^* + \sum_{g=1}^{n_S}\sum_{m=1}^{n_{l^i}}\frac{\partial c_{igm}(Q^{S^*})}{\partial Q_{jil}^S} - \lambda_{il}^*\right] \times (Q_{jil}^S - Q_{jil}^{S^*})$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\sum_{j=1}^{n_S}Q_{jil}^{S^*} + Q_{il}^{F^*} - \sum_{k=1}^{n_R}Q_{ik}^*\theta_{il}\right] \times (\lambda_{il} - \lambda_{il}^*)$$

$$+\sum_{j=1}^{n_S}\sum_{i=1}^{I}\sum_{l=1}^{n_{l^i}}\left[\frac{\partial oc_j(\pi^*)}{\partial \pi_{jil}} - Q_{jil}^{S^*}\right]\times(\pi_{jil} - \pi_{jil}^*) \geq 0, \quad \forall (Q, Q^F, Q^S, \lambda, \pi) \in \mathcal{K}, \tag{16}$$

where $\mathcal{K} \equiv \mathcal{K}^F \times \mathcal{K}^S$.

Standard Variational Inequality Form Determine $X^* \in \mathcal{K}$ where X is a vector in R^N, $F(X)$ is a continuous function such that $F(X) : X \mapsto \mathcal{K} \subset R^N$, and

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{17}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in the N-dimensional Euclidean space, $N = In_R + 2n_S \sum_{i=1}^{I} n_{l^i} + 2 \sum_{i=1}^{I} n_{l^i}$, and $\mathcal{K}$ is closed and convex. We define the vector $X \equiv (Q, Q^F, Q^S, \lambda, \pi)$ and the vector $F(X) \equiv (F^1(X), F^2(X), F^3(X), F^4(X), F^5(X))$,

$$F^1(X) = \left[\frac{\partial f_i(Q)}{\partial Q_{ik}} + \sum_{h=1}^{n_R} \frac{\partial tc^F_{ih}(Q)}{\partial Q_{ik}} - \sum_{h=1}^{n_R} \frac{\partial \hat{\rho}_{ih}(Q)}{\partial Q_{ik}} Q_{ih} - \hat{\rho}_{ik}(Q) + \sum_{l=1}^{n_{l^i}} \lambda_{il}\theta_{il}; \right.$$

$$\left. i = 1, \ldots, I; k = 1, \ldots, n_R \right], \tag{18a}$$

$$F^2(X) = \left[\sum_{m=1}^{n_{l^i}} \frac{\partial f^F_{im}(Q^F)}{\partial Q^F_{il}} - \lambda_{il}; i = 1, \ldots, I; l = 1, \ldots, n_{l^i} \right], \tag{18b}$$

$$F^3(X) = \left[\pi_{jil} + \sum_{g=1}^{n_S} \sum_{m=1}^{n_{l^i}} \frac{\partial c_{igm}(Q^S)}{\partial Q^S_{jil}} - \lambda_{il}; \right.$$

$$\left. j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i} \right], \tag{18c}$$

$$F^4(X) = \left[\sum_{j=1}^{n_S} Q^S_{jil} + Q^F_{il} - \sum_{k=1}^{n_R} Q_{ik}\theta_{il}; i = 1, \ldots, I; l = 1, \ldots, n_{l^i} \right], \tag{18d}$$

$$F^5(X) = \left[\frac{\partial oc_j(\pi)}{\partial \pi_{jil}} - Q^S_{jil}; j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i} \right]. \tag{18e}$$

Qualitative Properties

It is reasonable to expect that the price charged by each supplier j for producing one unit of firm i's component l, π_{jil}, is bounded by a sufficiently large value, since, in practice, each supplier cannot charge unbounded prices to the firms.

Assumption 1 *Suppose that in our supply chain network model with suppliers there exists a sufficiently large* Π*, such that,*

$$\pi_{jil} \leq \Pi, \quad j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \tag{19}$$

Theorem 4: Existence *With Assumption 1 satisfied, there exists at least one solution to variational inequalities (17); equivalently, (16) and (15).*

Qualitative Properties

Theorem 5: Uniqueness *If Assumption 1 is satisfied, the equilibrium product shipment, in-house component production, contracted component production, and suppliers' price pattern* $(Q^*, Q^{F^*}, Q^{S^*}, \pi^*)$ *in variational inequality (17), is unique under the following conditions:*

(i). one of the two families of convex functions $f_i(Q)$; $i = 1, \ldots, I$, *and* $tc^F_{ik}(Q)$; $k = 1 \ldots .n_R$, *is strictly convex in* Q_{ik};

(ii). the $f^F_{il}(Q^F)$; $i = 1, \ldots, I, l = 1, \ldots .n_{l^i}$, *are strictly convex in* Q^F_{il};

(iii). the $c_{ijl}(Q^S)$; $j = 1, \ldots, n_S$, $i = 1, \ldots, I, l = 1, \ldots .n_{l^i}$, *are strictly convex in* Q^S_{jil};

(iv). the $oc_j(\pi)$; $j = 1, \ldots, n_S$, *are strictly convex in* π_{jil};

(v). the $\rho_{ik}(d)$; $i = 1, \ldots, I, k = 1, \ldots .n_R$, *are strictly monotone decreasing of* d_{ik}.

The Nagurney-Qiang (N-Q) Network Efficiency / Performance Measure

The Nagurney and Qiang (N-Q) Network Efficiency / Performance Measure

Since our supply chain performance assessment measure and the various importance indicators are inspired by the Nagurney and Qiang network efficiency measure and associated important indicators and, without loss of generality, we even make use of similar notation, we now briefly recall the N-Q measure and a few of its applications world-wide.

The reference is: A. Nagurneyand Q. Qiang (2008), "A network efficiency measure with application to critical infrastructure networks." *Journal of Global Optimization*, **40**, pp 261-275.

The Nagurney and Qiang (N-Q) Network Efficiency / Performance Measure

Definition: A Unified Network Performance Measure
The network performance/efficiency measure, $\mathcal{E}(\mathcal{G}, d)$, for a given network topology $\mathcal{G}$ and the equilibrium (or fixed) demand vector d, is:

$$\mathcal{E} = \mathcal{E}(\mathcal{G}, d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where recall that n_W is the number of O/D pairs in the network, and d_w and λ_w denote, for simplicity, the equilibrium (or fixed) demand and the equilibrium disutility for O/D pair w, respectively.

The Importance of Nodes and Links

Definition: Importance of a Network Component
The importance of a network component $g \in \mathcal{G}$, $I(g)$, is measured by the relative network efficiency drop after g is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(\mathcal{G}, d) - \mathcal{E}(\mathcal{G} - g, d)}{\mathcal{E}(\mathcal{G}, d)}$$

where $\mathcal{G} - g$ is the resulting network after component g is removed from network $\mathcal{G}$.

The Approach to Identifying the Importance of Network Components

The elimination of a link is treated in the N-Q network efficiency measure by removing that link while the removal of a node is managed by removing the links entering and exiting that node.

In the case that the removal results in no path connecting an O/D pair, we simply assign the demand for that O/D pair to an abstract path with a cost of infinity.

The N-Q measure is well-defined even in the case of disconnected networks.

The Advantages of the N-Q Network Efficiency Measure

- The measure captures **demands, flows, costs, and behavior of users**, in addition to **network topology**.
- The resulting importance definition of network components is applicable and **well-defined even in the case of disconnected networks**.
- It can be used to identify the **importance (and ranking) of either nodes, or links, or both**.
- It can be applied to **assess the efficiency/performance of a wide range of network systems, including financial systems and supply chains under risk and uncertainty**.
- It is applicable also to **elastic demand networks**and also to **dynamic networks**, **including the Internet**.

Some Applications of the N-Q Measure

The Sioux Falls Network

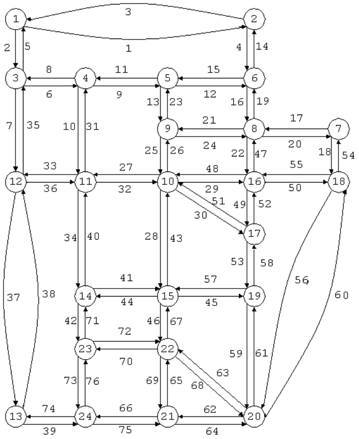
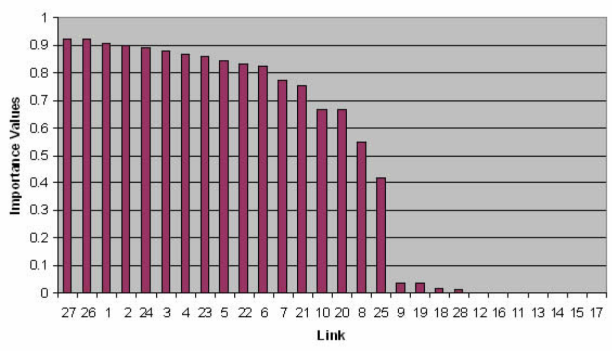


Figure: The Sioux Falls network with 24 nodes, 76 links, and 528 O/D pairs of nodes.

Importance of Links in the Sioux Falls Network

The computed network efficiency measure $\mathcal{E}$ for the Sioux Falls network is $\mathcal{E} = 47.6092$. Links 27, 26, 1, and 2 are the most important links, and hence special attention should be paid to protect these links accordingly, while the removal of links 13, 14, 15, and 17 would cause the least efficiency loss.



According to the European Environment Agency (2004), **since 1990, the annual number of extreme weather and climate related events has doubled, in comparison to the previous decade**. These events account for approximately 80% of all economic losses caused by catastrophic events. In the course of climate change, catastrophic events are projected to occur more frequently (see Schulz (2007)).

Schulz (2007) applied **N-Q network efficiency measure to a German highway system in order to identify the critical road elements** and found that this measure provided more reasonable results than the measure of Taylor and D'Este (2007).

The N-Q measure can also be used to assess which links should be added to improve efficiency. **This measure was used for the evaluation of the proposed North Dublin (Ireland) Metro system** (October 2009 Issue of *ERCIM News*).

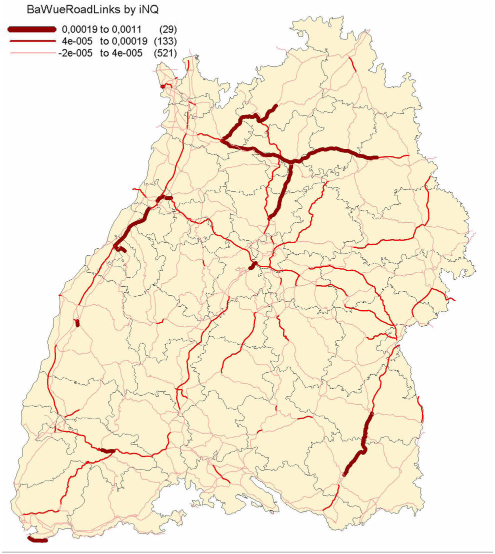
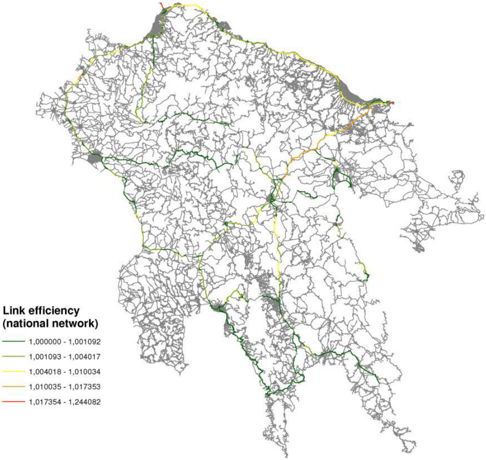
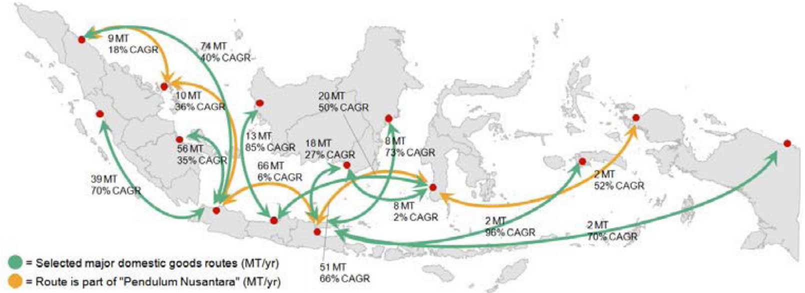


Figure: Comparative Importance of the links for the Baden - Wurttemberg Network – Modelling and analysis of transportation networks in earthquake prone areas via the N-Q measure, Mitsakis et al. (2014) applied the N-Q measure to identify the importance of links in Peloponessus, Greece. The work was inspired by the immense fires that hit this region in 2007.



The N-Q measure is noted in the "Guidebook for Enhancing Resilience of European Road Transport in Extreme Weather

The N-Q measure has also been used to assess new shipping routes in Indonesia in a report, "State of Logistics - Indonesia 2015."



An Application to the Braess Paradox

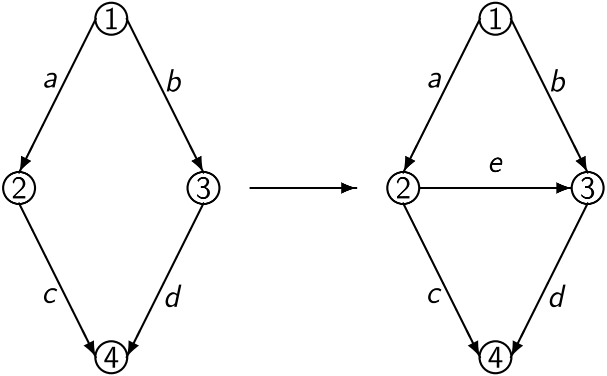


Figure: The Braess Network Example

The importance of behavior will now be illustrated through a famous example known as the Braess paradox which demonstrates what can happen under **U-O** as opposed to **S-O** behavior.

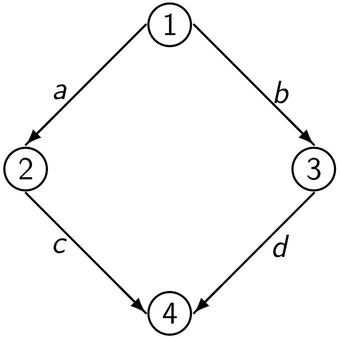
Although the paradox was presented in the context of transportation networks, it is relevant to other network systems in which decision-makers act in a noncooperative (competitive) manner.

The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand of **6,** the equilibrium path flows are $x^*_{p_1} = x^*_{p_2} = 3$ and

The equilibrium path travel cost is
$C_{p_1} = C_{p_2} = 83.$



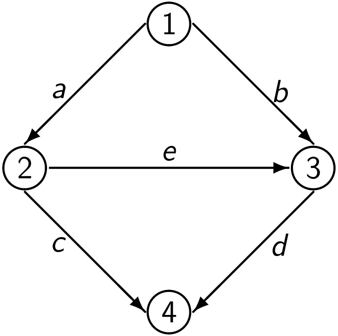
$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path $p_3 = (a, e, d)$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, $C_{p_3} = 70$.



The new equilibrium flow pattern network is $x^*_{p_1} = x^*_{p_2} = x^*_{p_3} = 2$.

$$c_e(f_e) = f_e + 10$$

The equilibrium path travel cost: $C_{p_1} = C_{p_2} = C_{p_3} = 92$.

The 1968 Braess article has been translated from German to English and appears as: "On a Paradox of Traffic Planning,"

D. Braess, A. Nagurney, and T. Wakolbinger (2005) Transportation Science **39**, pp 446-450.

Über ein Paradoxon aus der Verkehrsplanung

Von D. Braess, Münster [1])

Eingegangen am 28. März 1968

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

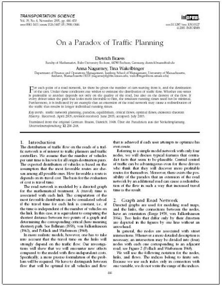
1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist,, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. Dietrich Braess, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.



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On a Paradox of Traffic Planning

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1. Introduction

The distribution of traffic flow on the roads of a traffic network is of interest to traffic planners and traffic controllers. We assume that the number of vehicles per unit time is known for all origin-destination pairs. The expected distribution of vehicles is based on the assumption that the most favorable routes are chosen among all possible ones. How favorable a route is depends on its travel cost. The basis for the evaluation of cost is travel time.

The road network is modeled by a directed graph for the mathematical treatment. A (travel) time is associated with each link. The computation of the most favorable distribution can be considered solved if the travel time for each link is constant, i.e., if the time is independent of the number of vehicles on the link. In this case, it is equivalent to computing the shortest distance between two points of a graph and determining the corresponding critical (here meaning shortest) path. See Bellman (1958), von Falkenhausen (1963), and Pollack and Wiebenson (1960).

In more realistic models, however, one has to take into account that the travel time on the links will strongly depend on the traffic flow. Our investigations will show that we will encounter new effects compared to the model with flow-independent costs. Specifically, a more precise formulation of the problem will be required. We have to distinguish between flow that will be optimal for all vehicles and flow that is achieved if each user attempts to optimize his own route.

Referring to a simple model network with only four nodes, we will discuss typical features that contradict facts that seem to be plausible. Central control of traffic can be advantageous even for those drivers who think that they will discover more profitable routes for themselves. Moreover, there exists the possibility of the paradox that an extension of the road network by an additional road can cause a redistribution of the flow in such a way that increased travel time is the result.

2. Graph and Road Network

Directed graphs are used for modeling road maps, and the links, the connections between the nodes, have an orientation (Berge 1958, von Falkenhausen 1966). Two links that differ only by their direction are depicted in the figures by one line without an arrowhead.

In general, the nodes are associated with street intersections. Whenever a more detailed description is necessary, an intersection may be divided into (four) nodes with each one corresponding to an adjacent road; see Figure 2 (Pollack and Wiebenson 1960).

We will use the following notation for the nodes, links, and flows. The indices belong to finite sets. Because we use each index only in connection with one variable, we do not write the range of the indices.

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An Application to the Braess Paradox

We now apply the unified network efficiency measure $\mathcal{E}$ to the Braess network with the link e to identify the importance and ranking of nodes and links. The results are reported in the Tables.

Table: Link Results for the Braess Network

Link	$\mathcal{E}$ Measure Importance Value	$\mathcal{E}$ Measure Importance Ranking
a	.2069	1
b	.1794	2
c	.1794	2
d	.2069	1
e	-.1084	3

An Application to the Braess Paradox

Table: Nodal Results for the Braess Network

Node	$\mathcal{E}$ Measure Importance Value	$\mathcal{E}$ Measure Importance Ranking
1	1.0000	1
2	.2069	2
3	.2069	2
4	1.0000	1

Supply Chain Network Performance Measures

Supply Chain Network Performance Measures

We now present the supply chain network performance measure for the whole competitive supply chain network G and that for the supply chain network of each individual firm i, G_i; $i = 1, \ldots, I$, under competition.

- Such measures capture the efficiency of the supply chains in that the higher the demand to price ratios normalized over associated firm and demand market pairs, the higher the efficiency.
- Hence, a supply chain network is deemed to perform better if it can satisfy higher demands, on the average, relative to the product prices.

Supply Chain Network Performance Measures

Definition 4.1: The Supply Chain Network Performance Measure for the Whole Competitive Supply Chain Network G
The supply chain network performance/efficiency measure, $\mathcal{E}(G)$, for a given competitive supply chain network topology G and the equilibrium demand vector d^*, is defined as follows:

$$\mathcal{E} = \mathcal{E}(G) = \frac{\sum_{i=1}^{I} \sum_{k=1}^{n_R} \frac{d_{ik}^*}{\rho_{ik}(d^*)}}{I \times n_R}. \tag{20}$$

Definition 4.2: The Supply Chain Network Performance Measure for an Individual Firm under Competition
The supply chain network performance/efficiency measure, $\mathcal{E}_i(G_i)$, for the supply chain network topology of a given firm i, G_i, under competition and the equilibrium demand vector d^*, is defined as:

$$\mathcal{E}_i = \mathcal{E}_i(G_i) = \frac{\sum_{k=1}^{n_R} \frac{d_{ik}^*}{\rho_{ik}(d^*)}}{n_R}, \quad i = 1, \ldots, I. \tag{21}$$

Definition 5.1: Importance of a Supplier for the Whole Competitive Supply Chain Network G *The importance of a supplier j, corresponding to a supplier node $j \in G$, $I(j)$, for the whole competitive supply chain network, is measured by the relative supply chain network efficiency drop after j is removed from the whole supply chain:*

$$I(j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - j)}{\mathcal{E}(G)}, \quad j = 1, \ldots, n_S, \tag{22}$$

where $G - j$ is the resulting supply chain after supplier j is removed from the competitive supply chain network G.

We also can construct using an adaptation of (22) a robustness-type measure for the whole competitive supply chain by evaluating how the supply chain is impacted if all the suppliers are eliminated due to a major disruption. Specifically, we let:

$$I(\sum_{j=1}^{n_S} j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - \sum_{j=1}^{n_S} j)}{\mathcal{E}(G)}, \qquad (23)$$

measure how the whole supply chain can respond if all of its suppliers are unavailable.

Definition 5.2: Importance of a Supplier for the Supply Chain Network of an Individual Firm under Competition *The importance of a supplier* j*, corresponding to a supplier node* $j \in G_i$*,* $I_i(j)$*, for the supply chain network of a given firm* i *under competition, is measured by the relative supply chain network efficiency drop after* j *is removed from* G_i*:*

$$I_i(j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I; j = 1, \ldots, n_S. \quad (24)$$

The corresponding robustness measure for the supply chain of firm i if all the suppliers are eliminated is:

$$I_i(\sum_{j=1}^{n_S} j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - \sum_{j=1}^{n_S} j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I. \quad (25)$$

Definition 5.3: Importance of a Supplier's Component for the Whole Competitive Supply Chain Network G *The importance of a supplier j's component* l_j; $l_j = 1_j, \ldots, n_{l_j}$, *corresponding to j's component node* $l_j \in G$, $I(l_j)$, *for the whole competitive supply chain network, is measured by the relative supply chain network efficiency drop after* l_j *is removed from G:*

$$I(l_j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - l_j)}{\mathcal{E}(G)}, \quad j = 1, \ldots, n_S; l_j = 1_j, \ldots, n_{l_j}. \quad (26)$$

where $G - l_j$ *is the resulting supply chain after supplier j's component* l_j *is removed from the whole competitive supply chain network.*

The corresponding robustness measure for the whole competitive supply chain network if all suppliers' component l_j; $l_j = 1_j, \ldots, n_{l_j}$, are eliminated is:

$$I(\sum_{j=1}^{n_S} l_j) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - \sum_{j=1}^{n_S} l_j)}{\mathcal{E}(G)}, \quad l_j = 1_j, \ldots, n_{l_j}. \quad (27)$$

Definition 5.4: Importance of a Supplier's Component for the Supply Chain Network of an Individual Firm under Competition
The importance of supplier j's component I_j; $I_j = 1_j, \ldots, n_{I_j}$, *corresponding to a component node* $I_j \in G_i$, $I_i(I_j)$, *for the supply chain network of a given firm i under competition, is measured by the relative supply chain network efficiency drop after* I_j *is removed from* G_i:

$$I_i(I_j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - I_j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I; j = 1, \ldots, n_S; I_j = 1_j, \ldots, n_{I_j} \tag{28}$$

The corresponding robustness measure for the supply chain network of firm i if all suppliers' component I_j, $I_j = 1_j, \ldots, n_{I_j}$, are eliminated is:

$$I_i(\sum_{j=1}^{n_S} I_j) = \frac{\triangle \mathcal{E}_i}{\mathcal{E}_i} = \frac{\mathcal{E}_i(G_i) - \mathcal{E}_i(G_i - \sum_{j=1}^{n_S} I_j)}{\mathcal{E}_i(G_i)}, \quad i = 1, \ldots, I; I_j = 1_j, \ldots, n_{I_j}. \tag{29}$$

The Algorithm and Numerical Examples

The Algorithm - The Euler Method

Iteration τ of the Euler method is:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{30}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (17).

For convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \rightarrow 0$, as $\tau \rightarrow \infty$.

$$Q_{ik}^{\tau+1} = \max\{0, Q_{ik}^{\tau} + a_{\tau}(-\frac{\partial f_i(Q^{\tau})}{\partial Q_{ik}} - \sum_{h=1}^{n_R} \frac{\partial tc_{ih}^F(Q^{\tau})}{\partial Q_{ik}} + \sum_{h=1}^{n_R} \frac{\partial \hat{\rho}_{ih}(Q^{\tau})}{\partial Q_{ik}} Q_{ih}^{\tau} + \hat{\rho}_{ik}(Q^{\tau})$$

$$-\sum_{l=1}^{n_{l^i}} \lambda_{il}^{\tau}\theta_{il})\}; i = 1, \dots, I; k = 1, \dots, n_R. \quad (31a)$$

$$Q_{il}^{F^{\tau+1}} = \min\{CAP_{il}^F, \max\{0, Q_{il}^{F^{\tau}} + a_{\tau}(-\sum_{m=1}^{n_{l^i}} \frac{\partial f_{im}^F(Q^{F^{\tau}})}{\partial Q_{il}^F} + \lambda_{il}^{\tau})\}\};$$

$$i = 1, \dots, I; l = 1, \dots, n_{l^i}. \quad (31b)$$

$$Q_{jil}^{S^{\tau+1}} = \min\{CAP_{jil}^S, \max\{0, Q_{jil}^{S^{\tau}} + a_{\tau}(-\pi_{jil}^{\tau} - \sum_{g=1}^{n_S} \sum_{m=1}^{n_{l^i}} \frac{\partial c_{igm}(Q^{S^{\tau}})}{\partial Q_{jil}^S} + \lambda_{il}^{\tau})\}\};$$

$$j = 1, \dots, n_S; i = 1, \dots, I; l = 1, \dots, n_{l^i}. \quad (31c)$$

$$\lambda_{il}^{\tau+1} = \max\{0, \lambda_{il}^{\tau} + a_{\tau}(-\sum_{j=1}^{n_S} Q_{jil}^{S^{\tau}} - Q_{il}^{F^{\tau}} + \sum_{k=1}^{n_R} Q_{ik}^{\tau}\theta_{il})\}; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \quad (31d)$$

$$\pi_{jil}^{\tau+1} = \max\{0, \pi_{jil}^{\tau} + a_{\tau}(-\frac{\partial oc_j(\pi^{\tau})}{\partial \pi_{jil}} + Q_{jil}^{S^{\tau}})\}; j = 1, \ldots, n_S; i = 1, \ldots, I; l = 1, \ldots, n_{l^i}. \quad (31e)$$

Numerical Examples

We implemented the Euler method using Matlab on a Lenovo Z580. The convergence tolerance is 10^{-6}, so that the algorithm is deemed to have converged when the absolute value of the difference between each successive quantities, prices, and Lagrange multipliers is less than or equal to 10^{-6}. The sequence $\{a_\tau\}$ is set to: $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$.

We initialize the algorithm by setting the product and component quantities equal to 50 and the prices and the Lagrange multipliers equal to 0.

Numerical Examples - Example 1

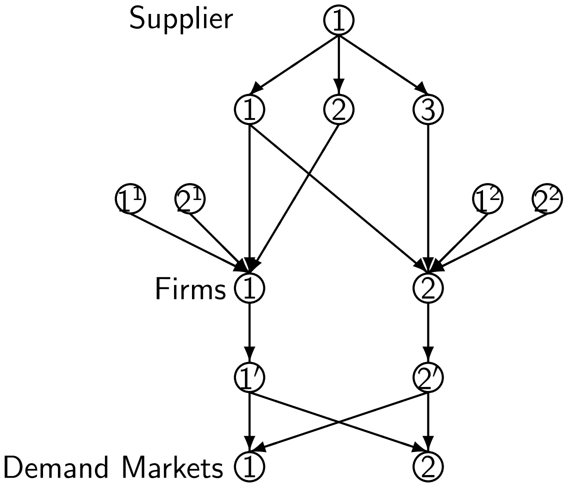

Figure: Example 1

Numerical Examples - Example 1

The product of firm 1 requires two components: 1^1 and 2^1. 2 units of component 1^1 and 3 units of component 2^1 are needed for producing one unit of firm 1's product.

The product of firm 2 requires two components, 1^2 and 2^2. To produce one unit of firm 2's product, 2 units of component 1^2 and 2 units of component 2^2 are needed. Therefore,

$$\theta_{11} = 2, \quad \theta_{12} = 3, \quad \theta_{21} = 2, \quad \theta_{22} = 2.$$

Components 1^1 and 1^2 are the same component, which corresponds to node 1 in the second tier in Figure 6. Components 2^1 and 2^2 correspond to nodes 2 and 3, respectively.

Numerical Examples - Example 1

The capacities of the suppliers are:

$$CAP^S_{111} = 80, \quad CAP^S_{112} = 90, \quad CAP^S_{121} = 80, \quad CAP^S_{122} = 50,$$

The firms are not capable of producing components 1^1 or 1^2, so their capacities are:

$$CAP^F_{11} = 0, \quad CAP^F_{12} = 20, \quad CAP^F_{21} = 0, \quad CAP^F_{22} = 30.$$

The supplier's production costs are:

$$f^S_{11}(Q^S_{111}, Q^S_{121}) = 2(Q^S_{111} + Q^S_{121}), \quad f^S_{12}(Q^S_{112}) = 3Q^S_{112}, \quad f^S_{13}(Q^S_{122}) = Q^S_{122}.$$

The supplier's transportation costs are:

$$tc^S_{111}(Q^S_{111}, Q^S_{112}) = 0.75Q^S_{111} + 0.1Q^S_{112}, \quad tc^S_{112}(Q^S_{112}, Q^S_{111}) = 0.1Q^S_{112} + 0.05Q^S_{111},$$

$$tc^S_{121}(Q^S_{121}, Q^S_{122}) = Q^S_{121} + 0.2Q^S_{122}, \quad tc^S_{122}(Q^S_{122}, Q^S_{121}) = 0.6Q^S_{122} + 0.25Q^S_{121}.$$

The opportunity cost of the supplier is:

$$oc_1(\pi_{111}, \pi_{112}, \pi_{121}, \pi_{122}) = 0.5(\pi_{111} - 10)^2 + (\pi_{112} - 5)^2 + 0.5(\pi_{121} - 10)^2 + 0.75(\pi_{122} - 7)^2.$$

Numerical Examples - Example 1

The firms' assembly costs are:

$$f_1(Q_{11}, Q_{12}, Q_{21}, Q_{22}) = 2(Q_{11} + Q_{12})^2 + 2(Q_{11} + Q_{12}) + (Q_{11} + Q_{12})(Q_{21} + Q_{22}),$$

$$f_2(Q_{11}, Q_{12}, Q_{21}, Q_{22}) = 1.5(Q_{21} + Q_{22})^2 + 2(Q_{21} + Q_{22}) + (Q_{11} + Q_{12})(Q_{21} + Q_{22}).$$

The firms' production costs for producing their components are:

$$f^F_{11}(Q^F_{11}, Q^F_{21}) = 3Q^{F^2}_{11} + Q^F_{11} + 0.5Q^F_{11}Q^F_{21}, \quad f^F_{12}(Q^F_{12}) = 2Q^{F^2}_{12} + 1.5Q^F_{12},$$

$$f^F_{21}(Q^F_{11}, Q^F_{21}) = 3Q^{F^2}_{21} + 2Q^F_{21} + 0.75Q^F_{11}Q^F_{21}, \quad f^F_{22}(Q^F_{22}) = 1.5Q^{F^2}_{22} + Q^F_{22}.$$

The firms' transportation costs for shipping their products to the demand markets are:

$$tc^F_{11}(Q_{11}, Q_{21}) = Q^2_{11} + Q_{11} + 0.5Q_{11}Q_{21}, \quad tc^F_{12}(Q_{12}, Q_{22}) = 2Q^2_{12} + Q_{12} + 0.5Q_{12}Q_{22},$$

$$tc^F_{21}(Q_{21}, Q_{11}) = 1.5Q^2_{21} + Q_{21} + 0.25Q_{11}Q_{21}, \quad tc^F_{22}(Q_{12}, Q_{22}) = Q^2_{22} + 0.5Q_{22} + 0.25Q_{12}Q_{22}.$$

Numerical Examples - Example 1

The transaction costs of the firms are:

$$c_{111}(Q^S_{111}) = 0.5Q^{S^2}_{111} + 0.25Q^S_{111}, \quad c_{112}(Q^S_{112}) = 0.25Q^{S^2}_{112} + 0.3Q^S_{112},$$

$$c_{211}(Q^S_{121}) = 0.3Q^{S^2}_{121} + 0.2Q^S_{121}, \quad c_{212}(Q^S_{122}) = 0.2Q^{S^2}_{122} + 0.1Q^S_{122}.$$

The demand price functions are:

$$\rho_{11}(d_{11}, d_{21}) = -1.5d_{11} - d_{21} + 500, \quad \rho_{12}(d_{12}, d_{22}) = -2d_{12} - d_{22} + 450,$$

$$\rho_{21}(d_{11}, d_{21}) = -2d_{21} - 0.5d_{11} + 500, \quad \rho_{22}(d_{12}, d_{22}) = -2d_{22} - d_{12} + 400.$$

Numerical Examples - Example 1

The Euler method converges to the following equilibrium solution.

$$Q^*_{11} = 13.39, \quad Q^*_{12} = 4.51, \quad Q^*_{21} = 18.62, \quad Q^*_{22} = 5.87.$$

$$d^*_{11} = 13.39, \quad d^*_{12} = 4.51, \quad d^*_{21} = 18.62, \quad d^*_{22} = 5.87.$$

$$\rho_{11} = 461.30, \quad \rho_{12} = 435.11, \quad \rho_{21} = 456.07, \quad \rho_{22} = 383.75.$$

$$Q^{F^*}_{11} = 0.00, \quad Q^{F^*}_{12} = 11.50, \quad Q^{F^*}_{21} = 0.00, \quad Q^{F^*}_{22} = 14.35.$$

$$Q^{S^*}_{111} = 35.78, \quad Q^{S^*}_{112} = 42.18, \quad Q^{S^*}_{121} = 48.99, \quad Q^{S^*}_{122} = 34.64.$$

$$\lambda^*_{11} = 81.82, \quad \lambda^*_{12} = 47.48, \quad \lambda^*_{21} = 88.58 \quad \lambda^*_{22} = 44.05.$$

$$\pi^*_{11} = 45.78, \quad \pi^*_{12} = 26.09, \quad \pi^*_{21} = 58.99, \quad \pi^*_{22} = 30.09.$$

The profits of the firms are, respectively, 2,518.77 and 3,485.51. The profit of the supplier is 3,529.19.

Numerical Examples - Example 1 - Performance Measures

Table: Supply Chain Network Performance Measure Values for Example 1

Chain	$\mathcal{E}(G)$	$\mathcal{E}(G-1)$	$\mathcal{E}(G-1_1)$	$\mathcal{E}(G-2_1)$	$\mathcal{E}(G-3_1)$
Whole	0.0239	0	0	0.0181	0.0183
	$\mathcal{E}_i(G_i)$	$\mathcal{E}_i(G_i-1)$	$\mathcal{E}_i(G_i-1_1)$	$\mathcal{E}_i(G_i-2_1)$	$\mathcal{E}_i(G_i-3_1)$
Firm 1's	0.0197	0	0	0.0071	0.0203
Firm 2's	0.0281	0	0	0.0292	0.0163

Numerical Examples - Example 1 - Importance Measures

Table: Importance and Rankings of Supplier 1's Components 1, 2, and 3 for Example 1

	Importance for the Whole Chain	Ranking
Supplier 1	1	
Component 1	1	1
Component 2	0.2412	2
Component 3	0.2331	3

	Importance for Firm 1's Chain	Ranking	Importance for Firm 2's Chain	Ranking
Supplier 1	1		1	
Component 1	1	1	1	1
Component 2	0.6401	2	−0.0387	3
Component 3	−0.0329	3	0.4197	2

Discussion of Results for Example 1

Because supplier 1's component 2 is produced exclusively for firm 1, it is more important for firm 1 than supplier 1's component/node 3, but not as important as component 1. After removing it from the supply chain, firm 1's profit decreases, but firm 2's profit increases because of competition. The supply chain performance of firm 2's supply chain also increases after the removal. In addition, component 2 is more important for firm 1 than for firm 2 and for the whole supply chain network.

For a similar reason, since supplier 1's component/node 3 is made exclusively for firm 2, it is more important than supplier 1's component 2 for firm 2.

Numerical Examples - Example 2

Example 2 is the same as Example 1 except that supplier 1 is no longer the only entity that can produce components 1^1 and 1^2. **The firms have recovered some capacity and can produce the components**.

The capacities of the firms are now:

$$CAP^F_{11} = 20, \quad CAP^F_{12} = 20, \quad CAP^F_{21} = 20, \quad CAP^F_{22} = 30.$$

Table: Equilibrium Solution and Incurred Demand Prices for Example 2

Q^*	$Q^*_{11} = 14.43$	$Q^*_{121} = 5.13$	$Q^*_{21} = 19.60$	$Q^*_{22} = 7.02$
Q^{F^*}	$Q^{F^*}_{11} = 10.23$	$Q^{F^*}_{12} = 12.50$	$Q^{F^*}_{21} = 11.28$	$Q^{F^*}_{22} = 15.47$
Q^{S^*}	$Q^{S^*}_{111} = 28.89$	$Q^{S^*}_{112} = 46.19$	$Q^{S^*}_{121} = 41.97$	$Q^{S^*}_{122} = 37.78$
λ^*	$\lambda^*_{11} = 68.04$	$\lambda^*_{12} = 51.49$	$\lambda^*_{21} = 77.35$	$\lambda^*_{22} = 47.40$
π^*	$\pi^*_{111} = 38.89$	$\pi^*_{112} = 28.10$	$\pi^*_{121} = 51.97$	$\pi^*_{122} = 32.19$
d^*	$d^*_{11} = 14.43$	$d^*_{12} = 5.13$	$d^*_{21} = 19.60$	$d^*_{22} = 7.02$
ρ	$\rho_{11} = 458.75$	$\rho_{12} = 432.72$	$\rho_{21} = 453.58$	$\rho_{22} = 380.83$

The profits of the firms are now 2,968.88 and 4,110.89, and the profit of the supplier is now 3,078.45.

Numerical Examples - Example 2

With recovered capacities, the firms' profits increase but that of the supplier decreases.

If there are costs for capacity investment for each firm, and if the costs are less than the associated profit increment, it is profitable for the firms to recover their capacities and produce more components.

If not, purchasing from the supplier would be a wise choice.

Numerical Examples - Example 2 - Performance Measures

Table: Supply Chain Network Performance Measure Values for Example 2

Chain	$\mathcal{E}(G)$	$\mathcal{E}(G-1)$	$\mathcal{E}(G-1_1)$	$\mathcal{E}(G-2_1)$	$\mathcal{E}(G-3_1)$
Whole	0.0262	0.0086	0.0105	0.0197	0.0195
	$\mathcal{E}_i(G_i)$	$\mathcal{E}_i(G_i-1)$	$\mathcal{E}_i(G_i-1_1)$	$\mathcal{E}_i(G_i-2_1)$	$\mathcal{E}_i(G_i-3_1)$
Firm 1's	0.0217	0.0067	0.0106	0.0071	0.0226
Firm 2's	0.0308	0.0105	0.0105	0.0324	0.0163

Numerical Examples - Example 2 - Importance Measures

Table: Importance and Rankings of Supplier 1 and its Components 1, 2, and 3 for Example 2

	Importance for the Whole Supply Chain	Ranking
Supplier 1	0.6721	
Component 1	0.5984	1
Component 2	0.2476	3
Component 3	0.2586	2

	Importance for Firm 1's Chain	Ranking	Importance for Firm 2's Chain	Ranking
Supplier 1	0.6897		0.6598	
Component 1	0.5121	2	0.6590	1
Component 2	0.6721	1	−0.0505	3
Component 3	−0.0438	3	0.4710	2

Discussion of Results for Example 2

With firms' recovered capacities for producing components 1^1 and 1^2, **supplier 1's component 1 is still the most important component for the whole supply chain network and for firm 2**, compared to the other components. **However, for firm 1's supply chain, component 2 is now the most important component**.

In addition, supplier 1 is now most important for firm 1. Therefore, in the case of a disruption on the supplier's side, firm 1's supply chain will be affected the most. Moreover, components 1 and 3 are most important for firm 2, and component 2 is most important for firm 1.

Numerical Examples - Example 3

Example 3 is the same as Example 2, except that two more suppliers are now available to the firms in addition to supplier 1.

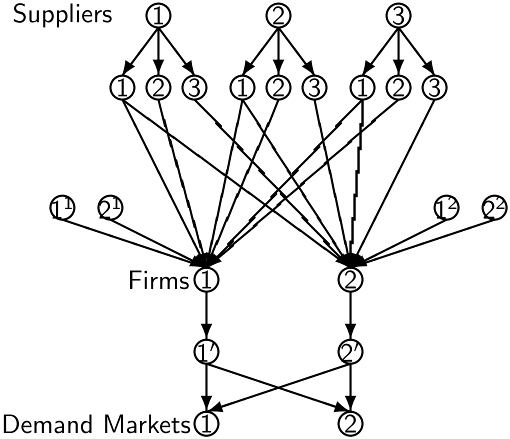

Figure: Example 3

Numerical Examples - Example 3

The data associated with suppliers 2 and 3 are following.

The capacities of suppliers 2 and 3 are:

$$CAP^S_{211} = 60, \quad CAP^S_{212} = 70, \quad CAP^S_{221} = 50, \quad CAP^S_{222} = 60,$$

$$CAP^S_{311} = 50, \quad CAP^S_{312} = 80, \quad CAP^S_{321} = 80, \quad CAP^S_{322} = 60.$$

The production costs of the suppliers are:

$$f^S_{21}(Q^S_{211}, Q^S_{221}) = Q^S_{211} + Q^S_{221}, \quad f^S_{22}(Q^S_{212}) = 3Q^S_{212}, \quad f^S_{23}(Q^S_{222}) = 2Q^S_{222},$$

$$f^S_{31}(Q^S_{311}, Q^S_{321}) = 10(Q^S_{311} + Q^S_{321}), \quad f^S_{32}(Q^S_{312}) = Q^S_{312}, \quad f^S_{33}(Q^S_{322}) = 2.5Q^S_{322}.$$

The transportation costs are:

$$tc^S_{211}(Q^S_{211}, Q^S_{212}) = 0.5Q^S_{211} + 0.2Q^S_{212}, \quad tc^S_{212}(Q^S_{212}, Q^S_{211}) = 0.3Q^S_{212} + 0.1Q^S_{211},$$

$$tc^S_{221}(Q^S_{221}, Q^S_{222}) = 0.8Q^S_{221} + 0.2Q^S_{222}, \quad tc^S_{222}(Q^S_{222}, Q^S_{221}) = 0.75Q^S_{222} + 0.1Q^S_{221},$$

$$tc^S_{311}(Q^S_{311}, Q^S_{312}) = 0.4Q^S_{311} + 0.05Q^S_{312}, \quad tc^S_{312}(Q^S_{312}, Q^S_{311}) = 0.4Q^S_{312} + 0.2Q^S_{311},$$

$$tc^S_{321}(Q^S_{321}, Q^S_{322}) = 0.7Q^S_{321} + 0.1Q^S_{322}, \quad tc^S_{322}(Q^S_{322}, Q^S_{321}) = 0.6Q^S_{322} + 0.1Q^S_{321}.$$

Numerical Examples - Example 3

The opportunity costs are:

$$oc_2(\pi_{211}, \pi_{212}, \pi_{221}, \pi_{222}) = (\pi_{211}-6)^2+0.75(\pi_{212}-5)^2+0.3(\pi_{221}-8)^2+0.5(\pi_{222}-4)^2,$$

$$oc_3(\pi_{311}, \pi_{312}, \pi_{321}, \pi_{322}) = 0.5(\pi_{311}-5)^2+1.5(\pi_{312}-5)^2+0.5(\pi_{321}-3)^2+0.5(\pi_{322}-4)^2.$$

The transaction costs of the firms now become:

$$c_{121}(Q^S_{211}) = 0.5Q^{S^2}_{211} + Q^S_{211}, \quad c_{122}(Q^S_{212}) = 0.25Q^{S^2}_{212} + 0.3Q^S_{212},$$

$$c_{221}(Q^S_{221}) = Q^{S^2}_{221} + 0.1Q^S_{221}, \quad c_{222}(Q^S_{222}) = Q^{S^2}_{222} + 0.5Q^S_{222},$$

$$c_{131}(Q^S_{311}) = 0.2Q^{S^2}_{311} + 0.3Q^S_{311}, \quad c_{132}(Q^S_{312}) = 0.5Q^{S^2}_{312} + 0.2Q^S_{312},$$

$$c_{231}(Q^S_{321}) = 0.1Q^{S^2}_{321} + 0.1Q^S_{321}, \quad c_{232}(Q^S_{322}) = 0.5Q^{S^2}_{322} + 0.1Q^S_{322}.$$

Numerical Examples - Example 3

The Euler method converges in 563 iterations.

Table: Equilibrium Solution and Incurred Demand Prices for Example 3

Q^*	$Q^*_{11} = 21.82$	$Q^*_{12} = 9.61$	$Q^*_{21} = 24.23$	$Q^*_{22} = 12.41$
Q^{F^*}	$Q^{F^*}_{11} = 5.57$	$Q^{F^*}_{12} = 9.11$	$Q^{F^*}_{21} = 6.48$	$Q^{F^*}_{22} = 12.94$
Q^{S^*}	$Q^{S^*}_{111} = 13.71$	$Q^{S^*}_{112} = 32.64$	$Q^{S^*}_{121} = 21.77$	$Q^{S^*}_{122} = 30.68$
	$Q^{S^*}_{211} = 20.45$	$Q^{S^*}_{212} = 27.98$	$Q^{S^*}_{221} = 10.07$	$Q^{S^*}_{222} = 11.78$
	$Q^{S^*}_{311} = 23.13$	$Q^{S^*}_{312} = 24.56$	$Q^{S^*}_{321} = 34.94$	$Q^{S^*}_{322} = 17.86$
λ^*	$\lambda^*_{11} = 37.68$	$\lambda^*_{12} = 37.94$	$\lambda^*_{21} = 45.03$	$\lambda^*_{22} = 39.83$
π^*	$\pi^*_{111} = 23.71$	$\pi^*_{112} = 21.32$	$\pi^*_{121} = 31.77$	$\pi^*_{122} = 27.45$
	$\pi^*_{211} = 16.23$	$\pi^*_{212} = 23.65$	$\pi^*_{221} = 24.79$	$\pi^*_{222} = 15.78$
	$\pi^*_{311} = 28.13$	$\pi^*_{312} = 13.19$	$\pi^*_{321} = 37.94$	$\pi^*_{322} = 21.86$
d^*	$d^*_{11} = 21.82$	$d^*_{12} = 9.61$	$d^*_{21} = 24.23$	$d^*_{22} = 12.41$
ρ	$\rho_{11} = 443.04$	$\rho_{12} = 418.38$	$\rho_{21} = 440.64$	$\rho_{22} = 365.58$

Numerical Examples - Example 3

The profits of the firms are now 4,968.67 and 5,758.13, and the profits of the suppliers are 1,375.22, 725.17, and 837.44, respectively.

With more competition on the supplier's side, the prices of supplier 1 decrease, and its profit also decreases, compared to the values in Example 2.

However, the profits of the firms increase. In addition, with more products made, the prices at the demand markets decrease.

Numerical Examples - Example 3 - Performance Measures

Table: Supply Chain Network Performance Measure Values for Example 3

Chain	$\mathcal{E}(G)$	$\mathcal{E}(G-1)$	$\mathcal{E}(G-2)$	$\mathcal{E}(G-3)$	$\mathcal{E}(G-\sum_{j=1}^{n_S} j)$
Whole	0.0403	0.0334	0.0361	0.0332	0.0
	$\mathcal{E}_i(G_i)$	$\mathcal{E}_i(G_i-1)$	$\mathcal{E}_i(G_i-2)$	$\mathcal{E}_i(G_i-3)$	$\mathcal{E}_i(G_i-\sum_{j=1}^{n_S} j)$
Firm 1's	0.0361	0.0309	0.0303	0.0309	0.0067
Firm 2's	0.0445	0.0358	0.0419	0.0355	0.0105

Numerical Examples - Example 3 - Importance Measures

Table: Importance and Rankings of Suppliers for Example 3

	Importance for the Whole Supply Chain	Ranking
Supplier 1	0.1717	2
Supplier 2	0.1035	3
Supplier 3	0.1760	1
All Suppliers	0.7864	

	Importance for Firm 1's Chain	Ranking	Importance for Firm 2's Chain	Ranking
Supplier 1	0.1443	2	0.1939	2
Supplier 2	0.1612	1	0.0566	3
Supplier 3	0.1438	3	0.2021	1
All Suppliers	0.8139		0.7641	

Discussion of Results for Example 3

As shown in the Table, **supplier 2 is the most important supplier for firm 1's supply chain, and supplier 3 is the most important supplier for firm 2 and the whole supply chain network**, compared to the other suppliers. In addition, suppliers 1 and 3 are most important for firm 2.

The group of suppliers, including suppliers 1, 2, and 3, is most important for firm 1. If a major disaster occurs and all the suppliers are unavailable to the firms, firm 1's supply chain will be affected the most.

Summary and Conclusions

- We provided background and motivation for the need for general multitiered supply chain models with suppliers.
- The behaviors of both suppliers and firms are captured in order to be able to assess both supply chain network performance as well as vulnerabilities.
- The firms have the option of producing the components needed in-house.
- A unified variational inequality is constructed, whose solution yields the equilibrium quantities of the components, produced in-house and/or contracted for, the quantities of the final products, the prices charged by the suppliers, as well as the Lagrange multipliers.
- The model is used for the introduction of supply chain network performance measures for the entire supply chain network economy consisting of all the firms as well as for that of an individual firm.

Summary and Conclusions

- Importance indicators are constructed that allow for the ranking of suppliers for the whole supply chain or that of an individual firm, as well as for the supplier components.