Topic 3: Fresh Produce Supply Chains

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The first part of this lecture is based primarily on the paper:

Yu, M., Nagurney, A., 2013. Competitive food supply chain networks with application to fresh produce. *European Journal of Operational Research* **224(2)**, 273-282,

where a full list of references can be found, along with some additional results.

Outline

- Background and Motivation
- Literature Review
- The Fresh Produce Supply Chain Network Oligopoly Model
- Case Study
- Relationship of the Model to Others in the Literature
- Summary

Motivation

The fundamental difference between **food supply chains** and other supply chains is the **continuous and significant change in the quality** of food products throughout the entire supply chain until the points of final consumption.

Globalization of Food Supply Chains

Consumers' **expectation of year-around availability** of fresh food products has encouraged the globalization of food markets.

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- The consumption of **fresh vegetables** has increased at a much faster pace than the demand for traditional crops such as wheat and other grains (USDA (2011)).
- In the US alone, consumers now spend over 1.6 **trillion dollars** annually on food (Plunkett Research (2011)).
- The United States is ranked **number one as both importer and exporter** in the international trade of horticultural commodities (Cook (2002)).

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The growing **global competition**, coupled with the associated **greater distances** between food production and consumption locations, creates new challenges for food supply chain management.

Food Waste/Loss

- It is estimated that approximately **one third** of the global food production is wasted or lost annually (Gustavsson et al. (2011)).
- In any country, 20%–60% of the total amount of agricultural **fresh products** has been wasted or lost (Widodo et al. (2006)).
  - In **developed countries**, the overall average losses of fruits and vegetables during **post-production supply chain activities** are approximately 12% of the initial production.
  - The corresponding losses in **developing regions** are even **severer**.

Product Differentiation

Given the **thin profit margins** in the food industries, **product differentiation strategies** are increasingly used in food markets (Lowe and Preckel (2004), Lusk and Hudson (2004), and Ahumada and Villalobos (2009)) with **product freshness** considered one of the differentiating factors (Kärkkäinen (2003) and Lütke Entrup et al. (2005)).

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- One successful example is **fresh-cut produce**, including bagged salads, washed baby carrots, and fresh-cut melons (Cook (2002)).

- Retailers, such as Globus, a German retailer, are also now realizing that **food freshness can be a competitive advantage** (Lütke Entrup et al. (2005)).

Relevant Literature

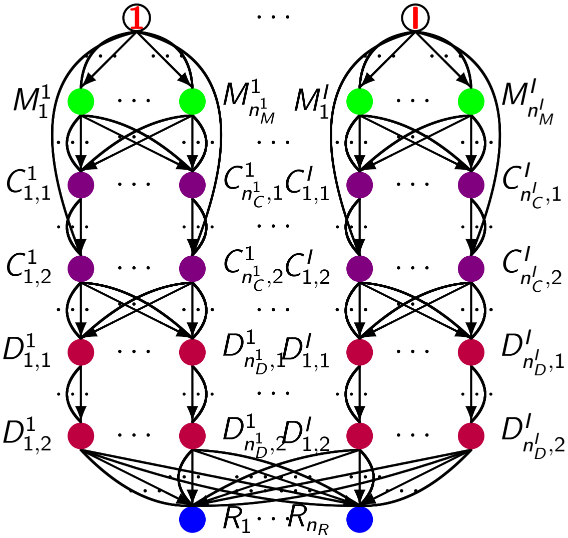
- Nahmias (1982, 2011) and Silver, Pyke, and Peterson (1998); Glen (1987) and Lowe and Preckel (2004); Lucas and Chhajed (2004); Lütke Entrup (2005); Akkerman, Farahani, and Grunow (2010); Ahumada and Villalobos (2009)
- Zhang, Habenicht, and Spieß (2003), Widodo et al. (2006), Monteiro (2007), Blackburn and Scudder (2009), Ahumada and Villalobos (2011), Rong, Akkerman, and Grunow (2011), Kopanos, Puigjaner, and Georgiadis (2012), and Liu and Nagurney (2012)
- Nagurney and Aronson (1989), Masoumi, Yu, and Nagurney (2012), Nagurney, Masoumi, and Yu (2012), Nagurney and Masoumi (2012), and Nagurney and Nagurney (2011)

The Fresh Produce Supply Chain Model

This model focuses on **fresh produce items**, such as **vegetables and fruits**.

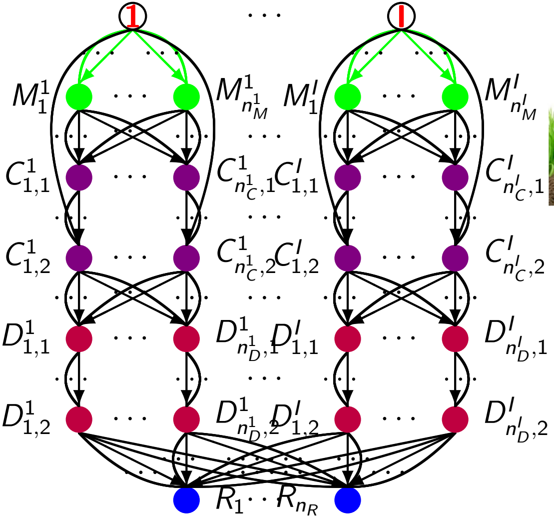
- They require simple or limited processing.
- The life cycle can be measured in days.

The Fresh Produce Supply Chain Topology



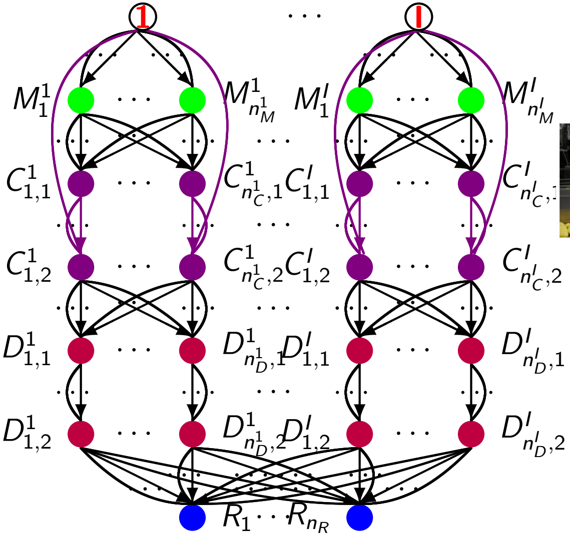
- The I food firms compete **noncooperatively** in an **oligopolistic** manner.
- The products may be **differentiated**, due to product **freshness** and **food safety** concerns.

Food Production



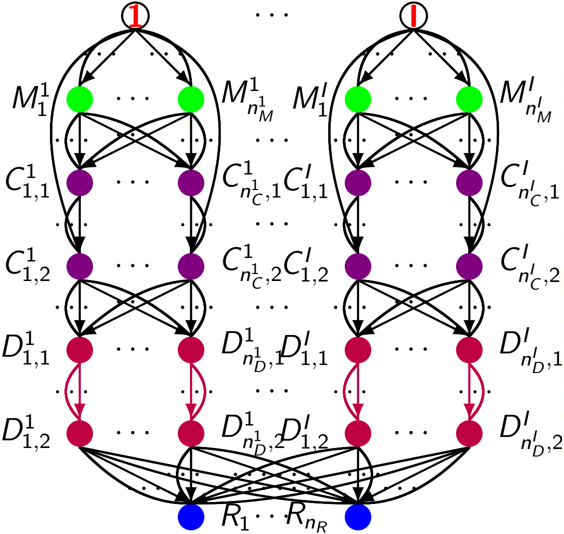
- Soil agitation
- Sowing
- Pest control
- Nutrient
- Water management

Food Processing

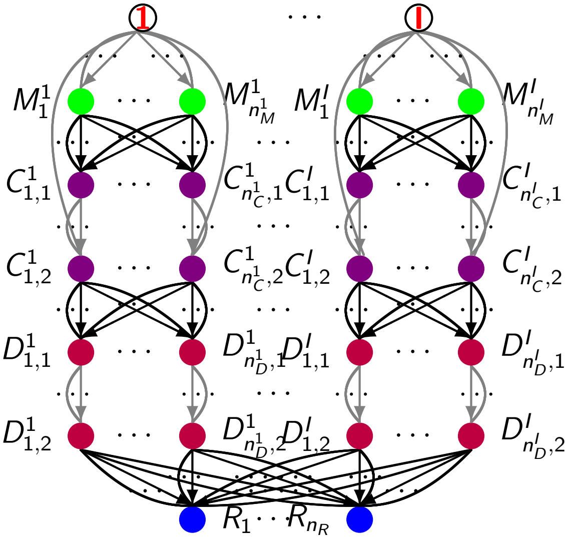


- Cleaning
- Sorting
- Labeling
- Packaging

Food Storage



Food Shipment/Distribution



How to Handle Food Deterioration

- Most of fresh produce items reach their peak quality at the time of production, and then **deteriorate substantially over** time.
- The **decay rate** varies significantly
  - With **different temperatures**, and
  - Under **other environmental conditions**.

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- Most of fresh produce items reach their peak quality at the time of production, and then **deteriorate substantially over** time.
- The **decay rate** varies significantly
  - With **different temperatures**, and
  - Under **other environmental conditions**.

The food products **deteriorate over time** even **under optimal conditions**.

How to Handle Food Deterioration

Microbiological decay is one of the major causes of the food quality degradation, especially for the fresh produce.

Therefore, food deterioration usually follows the first-order reactions with **exponential time decay**.

- **The decrease in quantity** represents the number of units of decayed products (e.g. vegetables and fruits).
- **The degradation in quality** emphasizes that all the products deteriorate at the same rate simultaneously (e.g. meat, dairy, and bakery products).

How to Handle Food Deterioration

The model adopts **exponential time decay** so as to capture **the discarding of spoiled products** associated with **the post-production** supply chain activities.

Each unit has a probability of $e^{-\lambda t}$ to survive another t units of time, where λ is the decay rate, which is given and fixed. Let N_0 denote the quantity at the beginning of the time interval (link). Hence, **the expected quantity surviving at the end of the time interval** (specific link), denoted by $N(t)$, can be expressed as:

$$N(t) = N_0 e^{-\lambda t}. \quad (1)$$

How to Handle Food Deterioration

Let α_a denote **the throughput factor** associate with every link a in the supply chain network, which lies in the range of $(0, 1]$.

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$$\alpha_a = 1, \tag{2a}$$

How to Handle Food Deterioration

Let α_a denote **the throughput factor** associate with every link a in the supply chain network, which lies in the range of $(0, 1]$.

- For a **production link**:

$$\alpha_a = 1, \tag{2a}$$

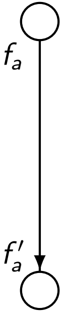
- For a **post-production link**:

$$\alpha_a = e^{-\lambda_a t_a}, \tag{2b}$$

where λ_a and t_a are the decay rate and the time duration associated with the link a, respectively, which are given and fixed.

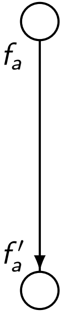
In rare cases, food deterioration follows the zero order reactions with linear decay. Then, $\alpha_a = 1 - \lambda_a t_a$ for a post-production link.

How to Handle Food Deterioration



Let f_a denote the (initial) flow of product on link a; and f'_a denote the final flow on link a.

How to Handle Food Deterioration



Let f_a denote the (initial) flow of product on link a; and f'_a denote the final flow on link a.

$$f'_a = \alpha_a f_a, \quad \forall a \in L. \tag{3}$$

The Number of Units of the Spoiled Fresh Produce on Link a

$$f_a - f'_a = (1 - \alpha_a) f_a, \quad \forall a \in L. \tag{4}$$

How to Handle Food Deterioration

Total Discarding Cost Functions

$$\hat{z}_a = \hat{z}_a(f_a), \quad \forall a \in L, \tag{5}$$

which is assumed to be convex and continuously differentiable.

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- It is imperative to **remove the spoiled fresh food products** from the supply chain network.
  - For instance, fungi are the common post-production diseases of fresh fruits and vegetables, which can **colonize** the fruits and vegetables rapidly.
- The model mainly focuses on the disposal of the decayed food products at **the processing, storage, and distribution stages**.

How to Handle Food Deterioration

Multiplier α_{ap}

$$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{b \in \{a' < a\}_p} \alpha_b, & \text{if } \{a' < a\}_p \neq \emptyset, \\ \delta_{ap}, & \text{if } \{a' < a\}_p = \emptyset, \end{cases} \quad (6)$$

where $\{a' < a\}_p$ denotes the set of the links preceding link a in path p, and $\emptyset$ denotes the null set.

Relationship between Link Flows, f_a, and Path Flows, x_p

$$f_a = \sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} x_p \alpha_{ap}, \quad \forall a \in L. \quad (7)$$

How to Handle Food Deterioration

Path Multiplier μ_p

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \quad \forall p \in P_k^i;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R. \tag{8}$$

How to Handle Food Deterioration

Path Multiplier μ_p

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \quad \forall p \in P_k^i;\ i = 1, \dots, I;\ k = 1, \dots, n_R. \qquad (8)$$

Relationship between Path Flows, x_p, and demands, d_{ik}

$$\sum_{p \in P_k^i} x_p \mu_p = d_{ik}, \quad i = 1, \dots, I;\ k = 1, \dots, n_R. \qquad (9)$$

d_{ik} can capture **production differentiation**, due to **food safety and health concerns**.

Demand Price Functions

$$\rho_{ik} = \rho_{ik}(d), \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \tag{10}$$

which captures the **demand-side competition**. These demand price functions are assumed to be continuous, continuously differentiable, and monotone decreasing.

Total Operational Cost Functions

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L, \tag{11}$$

where f is the vector of all the link flows. Such cost functions can capture the **supply-side competition**. The total cost on each link is assumed to be convex and continuously differentiable.

The Profit Function of Firm i

$$U_i = \sum_{k=1}^{n_R} \rho_{ik}(d) d_{ik} - \sum_{a \in L^i} \Big(\hat{c}_a(f) + \hat{z}_a(f_a) \Big). \tag{12}$$

In this oligopoly competition problem, **the strategic variables are the path flows.**

- X_i: the vector of path flows associated with firm i; $i = 1, \ldots, I$.
- X: the vector of all the firm' strategies, that is, $X \equiv \{\{X_i\} | i = 1, \ldots, I\}$.

Supply Chain Network Cournot-Nash Equilibrium

A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ *is said to constitute a supply chain network Cournot-Nash equilibrium if for each firm* i*;* $i = 1, \ldots, I$*:*

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \tag{13}$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ *and* $K_i \equiv \{X_i | X_i \in R_+^{n_{P^i}}\}$.

An equilibrium is established if NO firm can **unilaterally** improve its profit, given other firms' decisions.

Variational Inequality Formulation

Variational Inequality (Path Flows)

Determine $x^* \in K^1$ *such that:*

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p}+\frac{\partial \hat{Z}_p(x^*)}{\partial x_p}-\hat{\rho}_{ik}(x^*)\mu_p\right.$$

$$\left.-\sum_{l=1}^{n_R}\frac{\partial \hat{\rho}_{il}(x^*)}{\partial x_p}\sum_{p\in P_l^i}\mu_p x_p^*\right]\times[x_p-x_p^*]\geq 0,\quad \forall x\in K^1,\quad (14)$$

where $K^1 \equiv \{x | x \in R_+^{n_P}\}$.

Variational Inequality (Link Flows)
Determine $(f^*, d^*) \in K^2$, *such that:*

$$\sum_{i=1}^{I} \sum_{a \in L^i} \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$

$$+ \sum_{i=1}^{I} \sum_{k=1}^{n_R} \left[-\rho_{ik}(d^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d^*)}{\partial d_{ik}} d_{il}^* \right] \times [d_{ik} - d_{ik}^*] \geq 0,$$

$$\forall (f, d) \in K^2, \quad (15)$$

where $K^2 \equiv \{(f, d) | x \geq 0, \text{and} \, (7) \, \text{and} \, (9) \, \text{hold}\}$.

Existence

There exists at least one solution to variational inequality (14) (equivalently, to (15)), since there exists a $b > 0$, *such that variational inequality*

$$\langle F(X^b), X - X^b \rangle \geq 0, \quad \forall X \in \mathcal{K}_b, \tag{16}$$

admits a solution in $\mathcal{K}_b$ *with*

$$x^b \leq b. \tag{17}$$

Uniqueness

With existence, variational inequality (16) and, hence, variational inequality (16) admits at least one solution. Moreover, if the function $F(X)$ *of variational inequality (15) is strictly monotone on* $\mathcal{K} \equiv K^2$*, that is,*

$$\langle (F(X^1) - F(X^2)), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \, X^1 \neq X^2, \tag{18}$$

then the solution to variational inequality (15) is unique, that is, the equilibrium link flow pattern and the equilibrium demand pattern are unique.

Algorithm – Euler Method

Recall that an iteration of the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993), is:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})),$$

In the Euler method, the sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \to 0$, as $\tau \to \infty$.

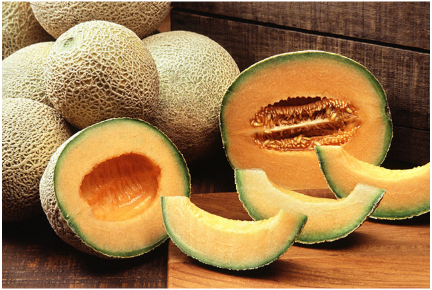
Closed Form Expression for Fresh Produce Path Flows

$$x_p^{\tau+1} = \max\{0, x_p^{\tau} + a_{\tau}(\hat{\rho}_{ik}(x^{\tau})\mu_p + \sum_{l=1}^{n_R} \frac{\partial \hat{\rho}_{il}(x^{\tau})}{\partial x_p} \sum_{q \in P_l^i} \mu_q x_q^{\tau} - \frac{\partial \hat{C}_p(x^{\tau})}{\partial x_p}$$

$$- \frac{\partial \hat{Z}_p(x^{\tau})}{\partial x_p})\}, \quad \forall p \in P_k^i;\ i = 1, \ldots, I;\ k = 1, \ldots, n_R. \qquad (19)$$

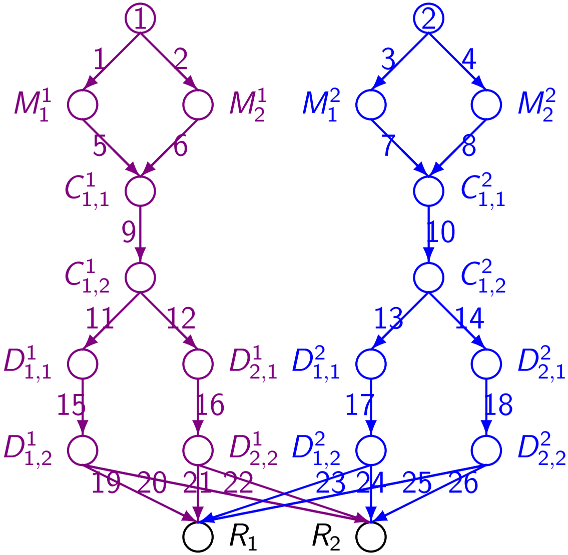
Case Study

Most of cantaloupes consumed in the United States are originally produced in **California**, **Mexico**, and in some countries in **Central America**.



- Typically, cantaloupes can be stored for 12–15 days at 2.2° to 5°C (36° to 41°F).
- It has been noticed that the decay of cantaloupes may result from such **post-production disease**, depending on the **season**, the **region**, and the **handling technologies** utilized between production and consumption.

Supply Chain Topology



- **Firm** 1 is located in **California**.
- **Firm** 2 is located in **Central America**.
- All the distribution centers and the demand markets are located in the United States.

Case 1 Data and Equilibrium Solution

- Consumers at the demand markets are essentially **indifferent** between cantaloupes of Firm 1 and Firm 2.
- Consumers at demand market R_2 are willing to pay relatively more as compared to those at demand market R_1.

The Demand Price Functions

$$\rho_{11} = -.0001 d_{11} - .0001 d_{21} + 4, \quad \rho_{12} = -.0001 d_{12} - .0001 d_{22} + 6;$$

$$\rho_{21} = -.0001 d_{21} - .0001 d_{11} + 4, \quad \rho_{22} = -.0001 d_{22} - .0001 d_{12} + 6.$$

Link a	λ_a	t_a	α_a	$\hat{c}_a(f)$	$\hat{z}_a(f_a)$	f_a^*
1	–	–	1.00	$.005f_1^2 + .03f_1$	0.00	76.32
2	–	–	1.00	$.006f_2^2 + .02f_2$	0.00	75.73
3	–	–	1.00	$.001f_3^2 + .02f_3$	0.00	103.74
4	–	–	1.00	$.001f_4^2 + .02f_4$	0.00	105.62
5	.150	0.20	.970	$.003f_5^2 + .01f_5$	0.00	76.32
6	.150	0.25	.963	$.002f_6^2 + .02f_6$	0.00	75.73
7	.150	0.30	.956	$.001f_7^2 + .02f_7$	0.00	103.74
8	.150	0.30	.956	$.001f_8^2 + .01f_8$	0.00	105.62
9	.040	0.50	.980	$.002f_9^2 + .05f_9$	$.001f_9^2 + 0.02f_9$	147.01
10	.060	0.50	.970	$.001f_{10}^2 + .02f_{10}$	$.001f_{10}^2 + 0.02f_{10}$	200.14
11	.015	1.50	.978	$.005f_{11}^2 + .01f_{11}$	0.00	65.98
12	.015	3.00	.956	$.01f_{12}^2 + .01f_{12}$	0.00	78.12
13	.025	2.00	.951	$.005f_{13}^2 + .02f_{13}$	0.00	96.47
14	.025	4.00	.905	$.01f_{14}^2 + .01f_{14}$	0.00	97.76
15	.010	3.00	.970	$.004f_{15}^2 + .01f_{15}$	$.001f_{15}^2 + 0.02f_{15}$	64.51

Table: Data and Equilibrium Solution for Case 1

Link a	λ_a	t_a	α_a	$\hat{c}_a(f)$	$\hat{z}_a(f_a)$	f_a^*
16	.010	3.00	.970	$.004f_{16}^2 + .01f_{16}$	$.001f_{16}^2 + 0.02f_{16}$	74.68
17	.015	3.00	.956	$.004f_{17}^2 + .01f_{17}$	$.001f_{17}^2 + 0.02f_{17}$	91.77
18	.015	3.00	.956	$.004f_{18}^2 + .01f_{18}$	$.001f_{18}^2 + 0.02f_{18}$	88.45
19	.015	1.00	.985	$.005f_{19}^2 + .01f_{19}$	$.001f_{19}^2 + 0.02f_{19}$	7.98
20	.015	3.00	.956	$.015f_{20}^2 + .1f_{20}$	$.001f_{20}^2 + 0.02f_{20}$	54.62
21	.015	3.00	.956	$.015f_{21}^2 + .1f_{21}$	$.001f_{21}^2 + 0.02f_{21}$	0.00
22	.015	1.00	.985	$.005f_{22}^2 + .01f_{22}$	$.001f_{22}^2 + 0.02f_{22}$	72.48
23	.020	1.00	.980	$.005f_{23}^2 + .01f_{23}$	$.001f_{23}^2 + 0.02f_{23}$	27.74
24	.020	3.00	.942	$.015f_{24}^2 + .1f_{24}$	$.001f_{24}^2 + 0.02f_{24}$	59.99
25	.020	3.00	.942	$.015f_{25}^2 + .1f_{25}$	$.001f_{25}^2 + 0.02f_{25}$	0.00
26	.020	1.00	.980	$.005f_{26}^2 + .01f_{26}$	$.001f_{26}^2 + 0.02f_{26}$	84.56

Table: Data and Equilibrium Solution for Case 1 (continued)

- There is no shipment from distribution centers D_2^1 and D_2^2 to demand market R_1.
- The volume of product flows on distribution link 22 (or link 26) is higher than that of distribution link 20 (or link 24), which indicates that it is more cost-effective to provide fresh fruits from the nearby distribution centers.

The Equilibrium Demands

$$d_{11}^* = 7.86, \quad d_{12}^* = 123.62, \quad d_{21}^* = 27.19, \quad \text{and} \quad d_{22}^* = 139.38.$$

The Equilibrium Prices

$$\rho_{11} = 4.00, \quad \rho_{12} = 5.97, \quad \rho_{21} = 4.00, \quad \text{and} \quad \rho_{22} = 5.97.$$

The Profits of Two Firms

$$U_1 = 370.46 \quad \text{and} \quad U_2 = 454.72.$$

- Since consumers do not differentiate the cantaloupes produced by these two firms, the prices of these two firms' cantaloupes at each demand market are identical.

The Equilibrium Demands

$$d_{11}^* = 7.86, \quad d_{12}^* = 123.62, \quad d_{21}^* = 27.19, \quad \text{and} \quad d_{22}^* = 139.38.$$

The Equilibrium Prices

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The Profits of Two Firms

$$U_1 = 370.46 \quad \text{and} \quad U_2 = 454.72.$$

- Due to the difference in consumers' willingness to pay, the price at demand market R_1 is relatively lower than the price at demand market R_2.

The Equilibrium Demands

$$d_{11}^* = 7.86, \quad d_{12}^* = 123.62, \quad d_{21}^* = 27.19, \quad \text{and} \quad d_{22}^* = 139.38.$$

The Equilibrium Prices

$$\rho_{11} = 4.00, \quad \rho_{12} = 5.97, \quad \rho_{21} = 4.00, \quad \text{and} \quad \rho_{22} = 5.97.$$

The Profits of Two Firms

$$U_1 = 370.46 \quad \text{and} \quad U_2 = 454.72.$$

- As a result of its lower operational costs, Firm 2 dominates both of these two demand markets, leading to a substantially higher profit.

Case 2 Data and Equilibrium Solution

- The CDC reported **a multi-state cantaloupe-associated outbreak**.
- Due to food safety and health concerns, the regular consumers of cantaloupes switched to other fresh fruits.

The Demand Price Functions

$$\rho_{11} = -.001d_{11} - .001d_{21} + .5, \quad \rho_{12} = -.001d_{12} - .001d_{22} + .5;$$

$$\rho_{21} = -.001d_{21} - .001d_{11} + .5, \quad \rho_{22} = -.001d_{22} - .001d_{12} + .5.$$

Link a	λ_a	t_a	α_a	$\hat{c}_a(f)$	$\hat{z}_a(f_a)$	f_a^*
1	–	–	1.00	$.005f_1^2 + .03f_1$	0.00	4.43
2	–	–	1.00	$.006f_2^2 + .02f_2$	0.00	4.40
3	–	–	1.00	$.001f_3^2 + .02f_3$	0.00	5.94
4	–	–	1.00	$.001f_4^2 + .02f_4$	0.00	6.94
5	.150	0.20	.970	$.003f_5^2 + .01f_5$	0.00	4.43
6	.150	0.25	.963	$.002f_6^2 + .02f_6$	0.00	4.40
7	.150	0.30	.956	$.001f_7^2 + .02f_7$	0.00	5.94
8	.150	0.30	.956	$.001f_8^2 + .01f_8$	0.00	6.94
9	.040	0.50	.980	$.002f_9^2 + .05f_9$	$.001f_9^2 + 0.02f_9$	8.53
10	.060	0.50	.970	$.001f_{10}^2 + .02f_{10}$	$.001f_{10}^2 + 0.02f_{10}$	12.31
11	.015	1.50	.978	$.005f_{11}^2 + .01f_{11}$	0.00	4.82
12	.015	3.00	.956	$.01f_{12}^2 + .01f_{12}$	0.00	3.54
13	.025	3.00	.928	$.005f_{13}^2 + .02f_{13}$	0.00	6.86
14	.025	5.00	.882	$.01f_{14}^2 + .01f_{14}$	0.00	5.09
15	.010	3.00	.970	$.004f_{15}^2 + .01f_{15}$	$.001f_{15}^2 + 0.02f_{15}$	4.72

Table: Data and Equilibrium Solution for Case 2

- The longer time durations associated with shipment links 13 and 14 are caused by more imported food inspections by the U.S. Food and Drug Administration.

Link a	λ_a	t_a	α_a	$\hat{c}_a(f)$	$\hat{z}_a(f_a)$	f_a^*
16	.010	3.00	.970	$.004f_{16}^2 + .01f_{16}$	$.001f_{16}^2 + 0.02f_{16}$	3.38
17	.015	3.00	.956	$.004f_{17}^2 + .01f_{17}$	$.001f_{17}^2 + 0.02f_{17}$	6.36
18	.015	3.00	.956	$.004f_{18}^2 + .01f_{18}$	$.001f_{18}^2 + 0.02f_{18}$	4.49
19	.015	1.00	.985	$.005f_{19}^2 + .01f_{19}$	$.001f_{19}^2 + 0.02f_{19}$	4.58
20	.015	3.00	.956	$.015f_{20}^2 + .1f_{20}$	$.001f_{20}^2 + 0.02f_{20}$	0.00
21	.015	3.00	.956	$.015f_{21}^2 + .1f_{21}$	$.001f_{21}^2 + 0.02f_{21}$	0.00
22	.015	1.00	.985	$.005f_{22}^2 + .01f_{22}$	$.001f_{22}^2 + 0.02f_{22}$	3.28
23	.020	1.00	.980	$.005f_{23}^2 + .01f_{23}$	$.001f_{23}^2 + 0.02f_{23}$	6.08
24	.020	3.00	.942	$.015f_{24}^2 + .1f_{24}$	$.001f_{24}^2 + 0.02f_{24}$	0.00
25	.020	3.00	.942	$.015f_{25}^2 + .1f_{25}$	$.001f_{25}^2 + 0.02f_{25}$	0.00
26	.020	1.00	.980	$.005f_{26}^2 + .01f_{26}$	$.001f_{26}^2 + 0.02f_{26}$	4.29

Table: Data and Equilibrium Solution for Case 2 (continued)

- The distribution links: 20, 21, 24, and 25, have zero product flows, since the extremely low demand price cannot cover the costs associated with long-distance distribution.

The Equilibrium Demands

$$d^*_{11} = 4.51, \quad d^*_{12} = 3.24, \quad d^*_{21} = 5.96, \quad \text{and} \quad d^*_{22} = 4.21.$$

The Equilibrium Prices

$$\rho_{11} = 0.49, \quad \rho_{12} = 0.49, \quad \rho_{21} = 0.49, \quad \text{and} \quad \rho_{22} = 0.49.$$

The Profits of Two Firms

$$U_1 = 1.16 \quad \text{and} \quad U_2 = 1.63.$$

- The demand for cantaloupes is battered by the cantaloupe-associated outbreak, with significant decreases in demand prices at demand markets R_1 and R_2.
- Both Firm 1 and Firm 2, in turn, experience dramatic declines in their profits.

Case 3 Data and Equilibrium Solution

- Firm 1 would like to **regain consumers' confidence** in its own product after the cantaloupe-associated outbreak.
- Firm 1 had its label of cantaloupes redesigned.
- The label incorporates the guarantee of food safety.
- The label also causes additional expenditures associated with its processing activities.

The Demand Price Functions

$$\rho_{11} = -.001d_{11} - .0005d_{21} + 2.5, \quad \rho_{12} = -.0003d_{12} - .0002d_{22} + 3;$$

$$\rho_{21} = -.001d_{21} - .001d_{11} + .5, \quad \rho_{22} = -.001d_{22} - .001d_{12} + .5.$$

Link a	λ_a	t_a	α_a	$\hat{c}_a(f)$	$\hat{z}_a(f_a)$	f_a^*
1	–	–	1.00	$.005f_1^2 + .03f_1$	0.00	36.92
2	–	–	1.00	$.006f_2^2 + .02f_2$	0.00	36.64
3	–	–	1.00	$.001f_3^2 + .02f_3$	0.00	5.43
4	–	–	1.00	$.001f_4^2 + .02f_4$	0.00	6.44
5	.150	0.20	.970	$.003f_5^2 + .01f_5$	0.00	36.92
6	.150	0.25	.963	$.002f_6^2 + .02f_6$	0.00	36.64
7	.150	0.30	.956	$.001f_7^2 + .02f_7$	0.00	5.43
8	.150	0.30	.956	$.001f_8^2 + .01f_8$	0.00	6.44
9	.040	0.50	.980	$.003f_9^2 + .06f_9$	$.001f_9^2 + 0.02f_9$	71.11
10	.060	0.50	.970	$.001f_{10}^2 + .02f_{10}$	$.001f_{10}^2 + 0.02f_{10}$	11.35
11	.015	1.50	.978	$.005f_{11}^2 + .01f_{11}$	0.00	36.33
12	.015	3.00	.956	$.01f_{12}^2 + .01f_{12}$	0.00	33.38
13	.025	3.00	.928	$.005f_{13}^2 + .02f_{13}$	0.00	6.68
14	.025	5.00	.882	$.01f_{14}^2 + .01f_{14}$	0.00	4.33
15	.010	3.00	.970	$.004f_{15}^2 + .01f_{15}$	$.001f_{15}^2 + 0.02f_{15}$	35.52

Table: Data and Equilibrium Solution for Case 3

Link a	λ_a	t_a	α_a	$\hat{c}_a(f)$	$\hat{z}_a(f_a)$	f_a^*
16	.010	3.00	.970	$.004f_{16}^2 + .01f_{16}$	$.001f_{16}^2 + 0.02f_{16}$	31.91
17	.015	3.00	.956	$.004f_{17}^2 + .01f_{17}$	$.001f_{17}^2 + 0.02f_{17}$	6.20
18	.015	3.00	.956	$.004f_{18}^2 + .01f_{18}$	$.001f_{18}^2 + 0.02f_{18}$	3.82
19	.015	1.00	.985	$.005f_{19}^2 + .01f_{19}$	$.001f_{19}^2 + 0.02f_{19}$	17.78
20	.015	3.00	.956	$.015f_{20}^2 + .1f_{20}$	$.001f_{20}^2 + 0.02f_{20}$	16.69
21	.015	3.00	.956	$.015f_{21}^2 + .1f_{21}$	$.001f_{21}^2 + 0.02f_{21}$	0.00
22	.015	1.00	.985	$.005f_{22}^2 + .01f_{22}$	$.001f_{22}^2 + 0.02f_{22}$	30.96
23	.020	1.00	.980	$.005f_{23}^2 + .01f_{23}$	$.001f_{23}^2 + 0.02f_{23}$	5.93
24	.020	3.00	.942	$.015f_{24}^2 + .1f_{24}$	$.001f_{24}^2 + 0.02f_{24}$	0.00
25	.020	3.00	.942	$.015f_{25}^2 + .1f_{25}$	$.001f_{25}^2 + 0.02f_{25}$	0.00
26	.020	1.00	.980	$.005f_{26}^2 + .01f_{26}$	$.001f_{26}^2 + 0.02f_{26}$	3.65

Table: Data and Equilibrium Solution for Case 3 (continued)

The Equilibrium Demands

$$d_{11}^* = 17.52, \quad d_{12}^* = 46.46, \quad d_{21}^* = 5.81, \quad \text{and} \quad d_{22}^* = 3.58.$$

The Equilibrium Prices

$$\rho_{11} = 2.48, \quad \rho_{12} = 2.99, \quad \rho_{21} = 0.48, \quad \text{and} \quad \rho_{22} = 0.45.$$

The Profits of Two Firms

$$U_1 = 84.20 \quad \text{and} \quad U_2 = 1.38.$$

- Consumers differentiate cantaloupes due to food safety and health concerns.
- With the newly designed label, Firm 1 has managed to encourage the consumption of its cantaloupes at both of these two demand markets.

The Equilibrium Demands

$$d_{11}^* = 17.52, \quad d_{12}^* = 46.46, \quad d_{21}^* = 5.81, \quad \text{and} \quad d_{22}^* = 3.58.$$

The Equilibrium Prices

$$\rho_{11} = 2.48, \quad \rho_{12} = 2.99, \quad \rho_{21} = 0.48, \quad \text{and} \quad \rho_{22} = 0.45.$$

The Profits of Two Firms

$$U_1 = 84.20 \quad \text{and} \quad U_2 = 1.38.$$

- Practicing product differentiation may be an effective strategy for a food firm to maintain its profit at an acceptable level.
- Considering the cantaloupe-associated outbreak, it is certainly not easy to reclaim the same profit level as in Case 1.

The Equilibrium Demands

$$d_{11}^* = 17.52, \quad d_{12}^* = 46.46, \quad d_{21}^* = 5.81, \quad \text{and} \quad d_{22}^* = 3.58.$$

The Equilibrium Prices

$$\rho_{11} = 2.48, \quad \rho_{12} = 2.99, \quad \rho_{21} = 0.48, \quad \text{and} \quad \rho_{22} = 0.45.$$

The Profits of Two Firms

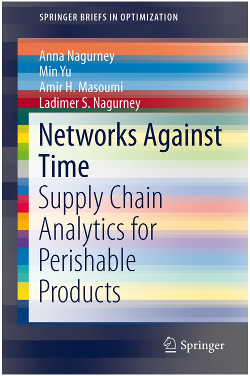
$$U_1 = 84.20 \quad \text{and} \quad U_2 = 1.38.$$

- The demand for Firm 1's product at demand market R_1 in Case 3 is even higher than that of Case 1, which is probably caused by the remarkable decrease in the price as well as the introduced guarantee of food safety.

A Multidisciplinary Perspective for Perishable Product Supply Chains

In our research on perishable and time-sensitive product supply chains, we utilize results from physics, chemistry, biology, and medicine in order to capture the perishability of various products over time from food to healthcare products such as blood, medical nucleotides, and pharmaceuticals.

A variety of perishable product supply chain models, computational procedures, and applications can be found in our book:



Supply Chain Networks – Optimization Models

Blood Supply Chains for the Red Cross

A. Nagurney, A. H. Masoumi, and M. Yu, "Supply Chain Network Operations Management of a Blood Banking System with Cost and Risk Minimization," *Computational Management Science* **9(2)** (2012), pp 205-231.

Blood Supply Chains for the Red Cross

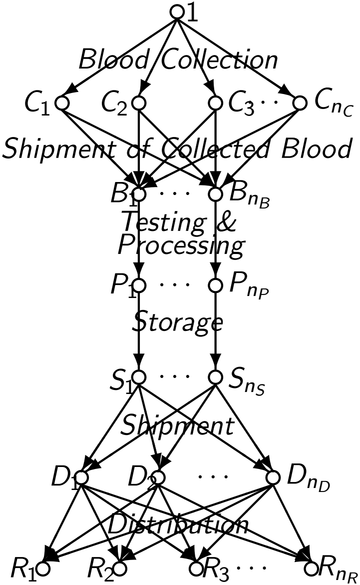
The American Red Cross is the major supplier of blood products to hospitals and medical centers satisfying about **45%** of the demand for blood components nationally.

American Red Cross
Together, we can save a life

Blood Supply Chains for the Red Cross

- The shelf life of platelets is **5 days** and of red blood cells is **42**.
- Over **39,000** donations are needed everyday in the US, and the blood supply was frequently reported to be just **2 days** away from running out (American Red Cross (2010)).
- Some hospitals have delayed surgeries due to blood shortages on **120** days in a year (Whitaker et al. (2007)).
- The national estimate for the number of units blood products outdated by blood centers and hospitals was **1,276,000** out of 15,688,000 units.
- As of February 1, 2016, the American Red Cross was facing **an emergency need for blood and platelet donors** because of severe winter weather in January.

Supply Chain Network Topology for a Regionalized Blood Bank



ARC Regional Division

Blood Collection Sites

Blood Centers

Component Labs

Storage Facilities

Distribution Centers

Demand Points

Blood Supply Chains for the Red Cross

We developed a supply chain network optimization model for the management of the procurement, testing and processing, and distribution of a perishable product – that of human blood.

Novel features of the model include:

- It captures **perishability of this life-saving product** through the use of arc multipliers;
- It contains **discarding costs** associated with waste/disposal;
- It handles **uncertainty** associated with demand points;
- It assesses **costs associated with shortages/surpluses at the demand points**, and
- It quantifies the **supply-side risk** associated with procurement.

Medical Nuclear Supply Chains

We developed a medical nuclear supply chain network design model which captures the decay of the radioisotope molybdenum; see "Medical Nuclear Supply Chain Design: A Tractable Network Model and Computational Approach," A. Nagurney and L. S. Nagurney, *International Journal of Production Economics* **140(2)** (2012), pp 865-874.

Medical Nuclear Supply Chains

Medical nuclear supply chains are essential supply chains in healthcare and provide the conduits for products used in nuclear medical imaging, which is routinely utilized by physicians for diagnostic analysis for both cancer and cardiac problems.

Such supply chains have unique features and characteristics due to the products' time-sensitivity, along with their hazardous nature.

Salient Features:

- complexity
- economic aspects
- underlying physics of radioactive decay
- importance of considering both waste management and risk management.

Medical Nuclear Supply Chains

Over **100,000** hospitals in the world use radioisotopes (World Nuclear Association (2011)).

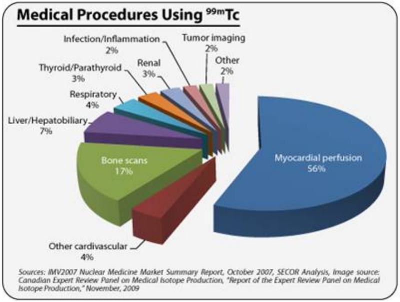
Technetium, ^{99m}Tc, which is a decay product of Molybdenum-99, ^{99}Mo, is the most commonly used medical radioisotope, used in more than **80%** of the radioisotope injections, with more than **30 million** procedures worldwide each year.

The half-life of Molybdenum-99 is 66 hours.

Each day, **41,000** nuclear medical procedures are performed in the United States using Technetium-99m.

Medical Nuclear Supply Chains

A **radioactive isotope** is bound to a pharmaceutical that is injected into the patient and travels to the site or organ of interest in order to construct an image for **medical diagnostic** purposes.



Medical Nuclear Supply Chains

For over two decades, all of the Molybdenum necessary for US-based nuclear medical diagnostic procedures has come from **foreign** sources.

Medical Nuclear Supply Chains

^{99}Mo Supply Chain Challenges:

- The majority of the reactors are between **40 and 50 years old**. Several of the reactors currently used are due to be retired by the end of this decade (Seeverens (2010) and OECD Nuclear Energy Agency (2010a)).

Medical Nuclear Supply Chains

^{99}Mo Supply Chain Challenges:

- The majority of the reactors are between **40 and 50 years old**. Several of the reactors currently used are due to be retired by the end of this decade (Seeverens (2010) and OECD Nuclear Energy Agency (2010a)).
- **Limitations in processing capabilities** make the world critically vulnerable to Molybdenum supply chain disruptions.

Medical Nuclear Supply Chains

^{99}Mo Supply Chain Challenges:

- The majority of the reactors are between **40 and 50 years old**. Several of the reactors currently used are due to be retired by the end of this decade (Seeverens (2010) and OECD Nuclear Energy Agency (2010a)).
- **Limitations in processing capabilities** make the world critically vulnerable to Molybdenum supply chain disruptions.
- The number of generator manufacturers is **under a dozen** (OECD Nuclear Energy Agency (2010b)).

Medical Nuclear Supply Chains

^{99}Mo Supply Chain Challenges:

- The majority of the reactors are between **40 and 50 years old**. Several of the reactors currently used are due to be retired by the end of this decade (Seeverens (2010) and OECD Nuclear Energy Agency (2010a)).
- **Limitations in processing capabilities** make the world critically vulnerable to Molybdenum supply chain disruptions.
- The number of generator manufacturers is **under a dozen** (OECD Nuclear Energy Agency (2010b)).
- **Long-distance transportation** of the product raises safety and security risks, and also results in greater decay of the product.

Medical Nuclear Supply Chains

In 2015, NorthStar Medical Radioisotopes LLC has received approval to begin routine production of molybdenum-99 (Mo-99) at the University of Missouri Research Reactor (MURR) facility in Columbia, Missouri. LEU rather than HEU will be used there.

This transitioning of NorthStar's Mo-99 line at MURR from a development process to a routine production process is another significant step toward establishing a domestic source of Mo-99.

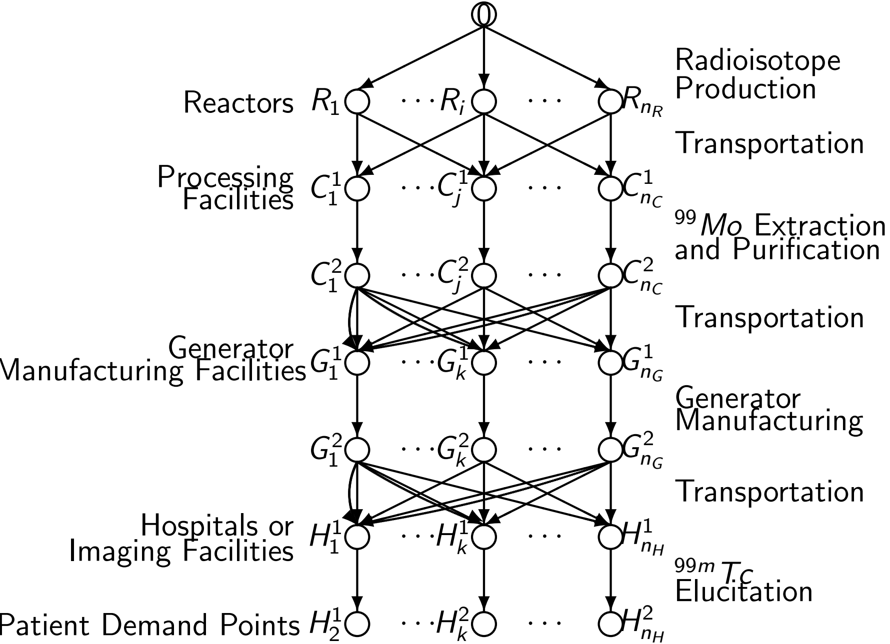


Figure: The Medical Nuclear Supply Chain Network Topology

Arc Multipliers

Because of the exponential decay of molybdenum, we have that the quantity of the radioisotope:

$$N(t) = N_0 e^{-\lambda t}$$

so that an arc multiplier on a link a that takes t_a hours of time corresponds to:

$$\alpha_a = e^{-\frac{ln2}{66.7} t_a}.$$

Supply Chain Networks
Additional Game Theory Models

Relationship of the Model to Others

The above model is now related to several models in the literature. If the arc multipliers are all equal to 1, in which case the product is not perishable, then the model is related to the sustainable fashion supply chain network model of Nagurney and Yu in the *International Journal of Production Economics* **135** (2012), pp 532-540. In that model, however, the other criterion, in addition to the profit maximization one, was emission minimization, rather than waste cost minimization, as in the model in this paper.

Relationship of the Model to Others

If the product is homogeneous, and all the arc multipliers are, again, assumed to be equal to 1, and the total costs are assumed to be separable, then the above model collapses to the supply chain network oligopoly model of Nagurney (2010) in which synergies associated with mergers and acquisitions were assessed.

The Original Supply Chain Network Oligopoly Model

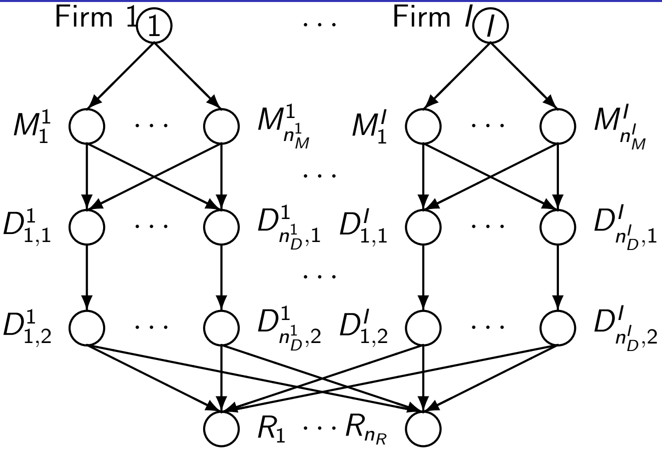


Figure: Supply Chain Network Structure of the Oligopoly Without Perishability; Nagurney, *Computational Management Science* **7**(2010), pp 377-401.

Mergers Through Coalition Formation

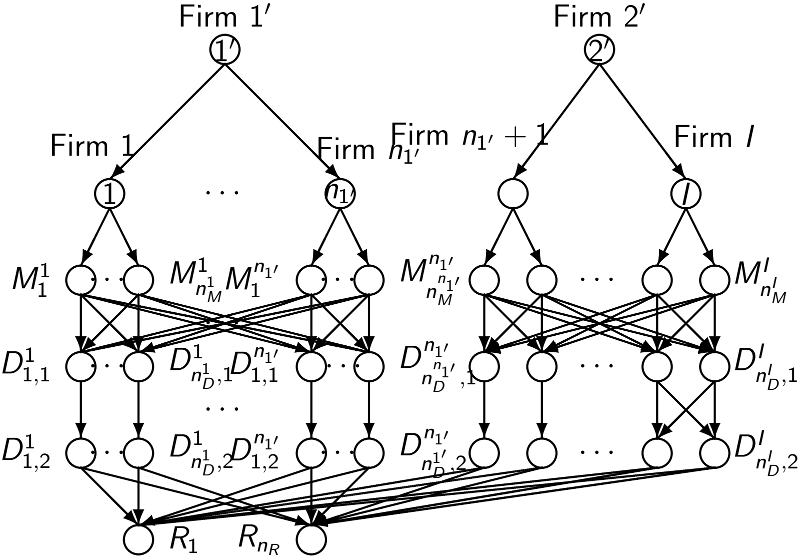


Figure: Mergers of the First $n_{1'}$ Firms and the Next $n_{2'}$ Firms

A Pharmaceutical Oligopoly Model

References can be found in our paper, "A Supply Chain Generalized Network Oligopoly Model for Pharmaceuticals Under Brand Differentiation and Perishability," A.H. Masoumi, M. Yu, and A. Nagurney, *Transportation Research E* **48** (2012), pp 762-780.

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

We consider I pharmaceutical firms, with a typical firm denoted by i.

The firms compete noncooperatively, in an oligopolistic manner, and the consumers can differentiate among the products of the pharmaceutical firms through their individual product brands.

The supply chain network activities include manufacturing, shipment, storage, and, ultimately, the distribution of the brand name drugs to the demand markets.

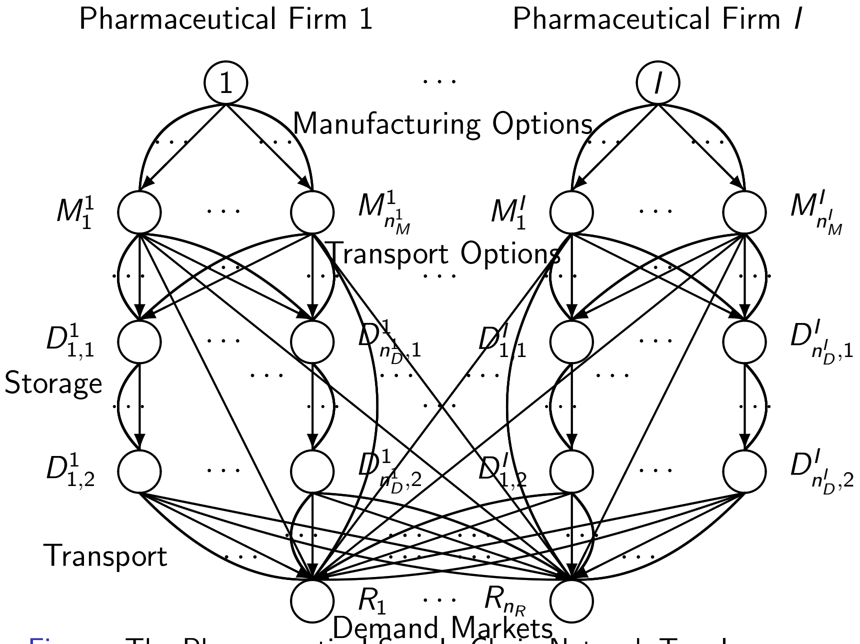


Figure: The Pharmaceutical Supply Chain Network Topology

Our recent research has returned to food supply chains in which we are also capturing explicit quality deterioration in fresh produce using chemical formulae that capture time and temperature of various supply chain network activities. Some of our applications are to farmers' markets.

Summary

With a focus on such **fresh produce items**,

- We adopted **exponential time decay** for the calculation of arc multipliers, so as to handle **the discarding of spoiled food products** associated with the post-production supply chain activities;

- We considered **product differentiation** due to **product freshness and food safety concerns**; and

- We also allowed for the **assessment of alternative technologies** involved in each supply chain activity, which could affect **the time durations** and **environmental conditions** associated with that activity.

- We related the model to several others in the literature.

Further Results on Quality Deterioration in Fresh Produce

In the second part of this lecture, we focus not on perishability in the form of **discarding** of fresh produce but on **the quality deterioration over time based on temperature kinetics**.

We then apply the formalism to an application to Farmers' markets with a case study on apples in western Massachusetts.

This part of the lecture is based on the paper, "Quality in Competitive Fresh Produce Supply Chains with Application to Farmers' Markets," Deniz Besik and Anna Nagurney, Isenberg School of Management, UMass Amherst, 2016.

Further Results on Quality Deterioration in Fresh Produce

The numbering of the equations, for self-containment in this part of the lecture is: (1), (2), etc.

Background

Knowledgeable modern consumers are increasingly demanding **high quality** in their food products, yet, they may be unaware of the great distances the food has traveled through intricate supply chains and the length of time from the initial production or "picking" of the fruits and vegetables to the ultimate delivery and consumption.

Motivation

Even though the transformation of food supply chains from **local to global** is remarkable, there may be some drawbacks.

- Consumers are facing **information asymmetry**.
- The great distances traveled create issues associated with **environmental impact, sustainability, and quality** since fresh produce is perishable (Nahmias (2011) and Nagurney et al. (2013)).

Motivation

We focus on **quality deterioration** through **kinetics** in food supply chains, direct to consumer chains, and, specifically **farmers' markets**.

- Consumers tend to connect the terms **'fresh,' 'good quality,' and 'tasty'** with **locally** produced foods.
- Farmers' markets in **Norway**, have the potential to reduce both physical and social distances between producers and consumers, and, hence, contribute to the sustainability of local food production (Acebo et al.(2007)).
- There were **8,268 farmers' markets** in the United States in 2014, with the number having increased by **180%** since 2006 (USDA(2014)).

Relevant Literature

- Various authors have emphasized quality; see Sloof, Tijskens, and Wilkinson (1996), Van der Vorst (2000), Lowe and Preckel (2004), Ahumada and Villalobos (2009, 2011), Blackburn and Scudder (2009), Akkerman, Farahani, and Grunow (2010), and Aiello, La Scalia, and Micale (2012).
- Yu and Nagurney (2013) propose a game theory model for oligopolistic competition in brand differentiated fresh produce supply chains with perishability.
- Tong, Ren, and Mack (2012) propose an optimal site selection model for farmers' markets in Arizona.
- There is limited research on quality decay through kinetics in direct-to-consumer food supply chains.

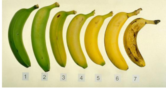
What is Quality Decay?

It is difficult to make a globally accepted definition of quality of fresh produce.

Quality of fresh foods can be defined over the combination of their physical attributes such as: **color and appearance, flavor, texture, and nutritional value.**

An understanding of the biochemical/physicochemical reactions can explain the quality deterioration.

Taoukis and Labuza (1989) explain the rate of quality deterioration of the quality attributes as a function of **microenvironment, gas composition, relative humidity, and temperature.**



Quality as a function of time and temperature

Taoukis and Labuza (1989) and Labuza (1984) show the quality decay of a food attribute Q, over time t, through the differential equation:

$$\frac{-d[Q]}{dt} = k[Q]^n = Ae^{(-E/RT)}[Q]^n, \tag{1}$$

- where k is the reaction rate defined by the **Arrhenius formula**:
$$Ae^{(-E/RT)}[Q]^n,$$
- A is the pre-exponential constant, T is temperature, E is activation energy and R is universal gas constant,
- n is the reaction order that belongs to the set $\mathrm{Z}^* = \{0\} \cup \mathrm{Z}^+$.

Types of Quality Decay Functions

The deterioration function changes with respect to the reaction order of the attribute.
When the initial quality is Q_0, Tijskens and Polderdijk (1996) categorize the decay functions as:

Reaction Order	Type	Quality at Time t
0	Linear	$Q_0 - kt$
1	Exponential	$Q_0 e^{-kt}$

Table: Reaction Kinetics and Quality at Time t

Some Fruits, Vegetables, and Quality Decay

Attribute	Fresh Produce	Reaction Order	Reference
Color Change	**Peaches**	**First**	**Toralles et al. (2005)**
Color Change	Raspberries	First	Ochoa et al. (2001)
Color Change	Blueberries	First	Zhang, Guo, and Ma (2012)
Nutritional (Vitamin C)	Strawberries	First	Castro et al. (2004)
Color Change	Watermelons	Zero	Dermesonlouoglou, Giannakourou, and Taoukis (2007)
Moisture Content	Tomatoes	First	Krokida et al. (2003)
Color Change	Cherries	First	Ochoa et al. (2001)
Texture Softening	**Apples**	**First**	**Tijskens (1979)**
Nutritional (Vitamin C)	Pears	First	Mrad et al. (2012)
Texture Softening	Avocados	First	Maftoonazad and Ramaswamy (2008)
Nutritional (Vitamin C)	Pineapples	First	Karim and Adebowale (2009)
Color Change	Spinach	Zero	Aamir et al. (2013)
Color Change	Asparagus	First	Aamir et al. (2013)
Color Change	Peas	First	Aamir et al. (2013)
Texture Softening	Beans	First	Aamir et al. (2013)
Texture Softening	Brussel Sprouts	First	Aamir et al. (2013)
Texture Softening	Carrots	First	Aamir et al. (2013)
Texture Softening	Peas	First	Aamir et al. (2013)
Color Change	Coriander Leaves	First	Aamir et al. (2013)

Table: Fresh Produce Attributes and Decay Kinetics

Integration of Quality Decay Into the Supply Chain

Let β_a denote the quality decay incurred on link a, which depends on the reaction order n, reaction rate k_a and time t_a on link a, as:

$$\beta_a \equiv \begin{cases} -k_a t_a, , & \text{if } n = 0, \forall a \in L \\ e^{-k_a t_a}, & \text{if } n \neq 0, \forall a \in L. \end{cases} \quad (2)$$

where

$$k_a = Ae^{(-E_A/RT_a)}. \quad (3)$$

Integration of Quality Decay Into the Supply Chain

The quality q_p, over a path p, joining the origin destination farm, i, with a destination node farmers' market, j, can also be shown as:

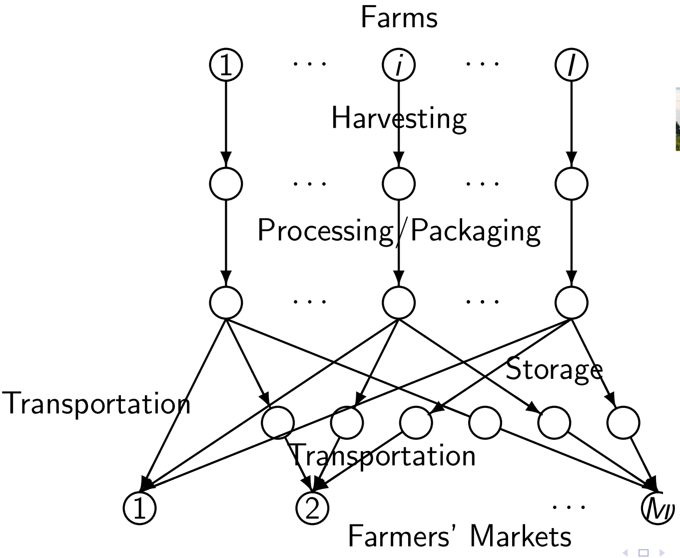
$$q_p \equiv \begin{cases} q_{0i} + \sum_{a \in p} \beta_a, & \text{if} \quad n = 0, \forall a \in L, \; p \in P_j^i, \forall i, j, \\ q_{0i} \prod_{a \in p} \beta_a, & \text{if} \quad n = 1, \forall a \in L, \; p \in P_j^i, \forall i, j, \end{cases} \tag{4}$$

- where q_{0i} is the initial quality of food product at farm i,
- P_j^i represents the set of all paths that have origin i and destination j.

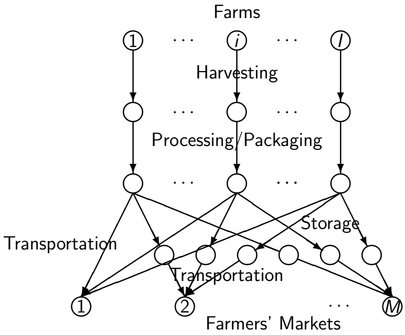
Competition

The I farms compete **noncooperatively** in an **oligopolistic** manner and the products are differentiated based on **quality** at the farmers' markets.

The Fresh Produce Supply Chain Topology



The Fresh Produce Supply Chain Topology



1. **Fixed time horizon** in a given season of the fresh fruit or vegetable, **typically a week,** is assumed.
2. **The demand points** are selected **farmers' markets**.
3. **Picking is made right before the time horizon**, so that there is **no storage** for the first farmers' market of the week.
4. Consumers can buy products that are substitutes within or across the demand points.

The Uncapacitated Fresh Produce Problem

Nonnegativity constraint of the path flows

The flow on the path, joining the farm i to the farmers markets k, is denoted by x_p and it should be nonnegative:

$$x_p \geq 0, \quad \forall p \in P_k^i; \; i = 1, \ldots, I; \; k = 1, \ldots, n_R. \qquad (5)$$

Link flows

The flow on a link a is equal to the sum of the path flows x_p, on paths that include the link a, expressed as:

$$f_a = \sum_{p \in P_k^i} x_p \delta_{ap}, \quad \forall a \in L. \qquad (6)$$

Demand

The demand at the farmers' market j for the fresh produce product of farmer i is given by:

$$\sum_{p \in P_j^i} x_p = d_{ij}, \quad p \in P_j^i; i = 1, \ldots, I; \; j = 1, \ldots, M. \qquad (7)$$

The Uncapacitated Fresh Produce Problem

Demand Price

The demand price function ρ_{ij} for farm i's product at the farmers' market j, is:

$$\rho_{ij} = \rho_{ij}(d, q), \quad i = 1, \ldots, I; \; j = 1, \ldots, M. \tag{8}$$

Link cost

The total operational cost of each link a, denoted by $\hat{c}_a$, depends on the flows on all the links in the fresh produce supply chain network, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L, \tag{9}$$

Profit/Utility

The profit/utility function of farm i, denoted by U_i, is given by:

$$U_i = \sum_{j=1}^{M} \rho_{ij}(d, q) d_{ij} - \sum_{a \in L^i} \hat{c}_a(f). \tag{10}$$

The Uncapacitated Fresh Produce Problem

Definition: Fresh Produce Supply Chain Network Cournot-Nash Equilibrium for Farmers' Markets in the Uncapacitated Case

A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ *constitutes a fresh produce supply chain network Cournot-Nash equilibrium if for each farm* i; $i = 1, \ldots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \tag{11}$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ *and*
$K_i \equiv \{X_i | X_i \in R_+^{n_{P^i}}\}$.

- A Cournot-Nash Equilibrium is established if no farm can unilaterally improve its profit by changing its product flows throughout its supply chain network, given the product flow decisions of the other farms.

The Uncapacitated Fresh Produce Problem

Theorem: Variational Inequality Formulations of the Uncapacitated Model

$X^* \in K$ is a fresh produce supply chain network Cournot-Nash equilibrium for famers' markets according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \qquad (12)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i.

The Uncapacitated Fresh Produce Problem

The variational inequality for our uncapacitated model is equivalent to the variational inequality that determines the vector of equilibrium path flows $x^* \in K^1$ such that:

$$\sum_{i=1}^{I}\sum_{j=1}^{M}\sum_{p\in P_j^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \hat{\rho}_{ij}(x^*,q) - \sum_{l=1}^{M}\frac{\partial \hat{\rho}_{il}(x^*,q)}{\partial x_p}\sum_{r\in P_l^i} x_r^*\right]\times[x_p - x_p^*] \geq 0, \quad \forall x \in K^1, \tag{13}$$

where $K^1 \equiv \{x | x \in R_+^{n_P}\}$, and for each path p; $p \in P_j^i$; $i = 1,\dots,I$; $j = 1,\dots,M$,

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a\in L^i}\sum_{b\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\delta_{ap} \text{ and } \frac{\partial \hat{\rho}_{il}(x,q)}{\partial x_p} \equiv \frac{\partial \rho_{il}(d,q)}{\partial d_{ij}}. \tag{14}$$

The Uncapacitated Fresh Produce Problem

The variational inequality can also be rewritten in terms of link flows as: determine the vector of equilibrium link flows and the vector of equilibrium demands $(f^*, d^*) \in K^2$, such that:

$$\sum_{i=1}^{I}\sum_{a\in L^i}\left[\sum_{b\in L^i}\frac{\partial \hat{c}_b(f^*)}{\partial f_a}\right]\times[f_a - f_a^*]$$

$$+\sum_{i=1}^{I}\sum_{j=1}^{M}\left[-\rho_{ij}(d^*,q)-\sum_{l=1}^{M}\frac{\partial \rho_{il}(d^*,q)}{\partial d_{ik}}d_{il}^*\right]\times[d_{ij}-d_{ij}^*]\geq 0, \forall (f,d) \in \tag{15}$$

where $K^2 \equiv \{(f,d)|x \geq 0, \textit{and}\,(6)\,\textit{and}\,(7)\,\textit{hold}\}$.

- **Proof:** (12) follows from Gabay and Moulin (1980); see, also, Masoumi, Yu, and Nagurney (2012). (13) and (15) then follow using algebraic substitutions. □

The Uncapacitated Fresh Produce Problem

Variational inequalities (13) and (15) can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{16}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space with $N = n_P$ in our model. Let $X \equiv x$ and

$$F(X) \equiv \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} - \hat{\rho}_{ij}(x, q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x, q)}{\partial x_p} \sum_{r \in P_l^i} x_r; \right.$$

$$\left. p \in P_j^i;\ i = 1, \ldots, I;\ j = 1, \ldots, M \right], \tag{17}$$

and $\mathcal{K} \equiv K^1$, then (10) can be re-expressed as (13).

Theorem: Existence

There exists at least one solution to variational inequality (13) (equivalently, to (15)), since there exists a $c > 0$, such that variational inequality (17) admits a solution in $\mathcal{K}_c$ with

$$x^c \leq c. \tag{18}$$

Theorem: Uniqueness

With the existence Theorem, the variational inequalities admit at least one solution. Moreover, if the function $F(X)$ is strictly monotone on $\mathcal{K} \equiv K^2$, that is,

$$\langle (F(X^1) - F(X^2)), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \, X^1 \neq X^2, \tag{19}$$

then the solution to variational inequality is unique, that is, the equilibrium link flow pattern and the equilibrium demand pattern are unique.

The Capacitated Fresh Produce Problem

- Labor shortages, weather conditions, disruptions to storage or transportation can limit the supply chain activities.
- The objective function, the constraints, with conservation of flow equations stay the same.

Link capacity constraint

$$f_a \leq u_a, \quad \forall a \in L, \tag{20a}$$

$$\sum_{p \in P} x_p \delta_{ap} \leq u_a, \quad \forall a \in L, \tag{20b}$$

where $K_i^3 \equiv \{X_i | X_i \in R_+^{n_{P^i}}$ and (20b) holds for $a \in L^i\}$ and $K^3 \equiv \prod_{i=1}^{I} K_i^3$.

The Capacitated Fresh Produce Problem

The variational inequality is equivalent to the variational inequality problem: determine $(x^*, \lambda^*) \in K^4$, where $K^4 \equiv \{x \in R_+^{n_P}, \lambda \in R_+^{n_L}\}$, such that:

$$\sum_{i=1}^{I}\sum_{j=1}^{M}\sum_{p\in P_j^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \hat{\rho}_{ij}(x^*, q) - \sum_{l=1}^{M}\frac{\partial \hat{\rho}_{il}(x^*, q)}{\partial x_p}\sum_{r\in P_l^i} x_r^* + \sum_{a\in L}\lambda_a^*\delta_{ap}\right]\times[x_p - x_p^*]$$

$$+\sum_{a\in L}\left[u_a - \sum_{p\in P} x_p^*\delta_{ap}\right]\times[\lambda_a - \lambda_a^*] \geq 0, \quad \forall (x,\lambda)\in K^4, \qquad (21)$$

where $\frac{\partial \hat{C}_p(x)}{\partial x_p}$ and $\frac{\partial \hat{\rho}_{il}(x,q)}{\partial x_p}$ are as defined in (14).

The Euler Method Explicit Formulae for the Uncapacitated Model

Closed form expressions for the fresh produce path flows, for each path $p \in P_j^i$, $\forall i, j$:

$$x_p^{\tau+1} = \max\{0, x_p^{\tau} + a_{\tau}(\hat{\rho}_{ij}(x^{\tau}, q) + \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x^{\tau}, q)}{\partial x_p} \sum_{r \in P_l^i} x_r^{\tau} - \frac{\partial \hat{C}_p(x^{\tau})}{\partial x_p})\}, \quad (23)$$

$$\forall p \in P_j^i;\ i = 1, \ldots, I;\ j = 1, \ldots, M.$$

The Euler Method Explicit Formulae for the Capacitated Model

For each path $p \in P^i_j$, $\forall i, j$, compute:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\hat{\rho}_{ij}(x^\tau, q) + \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x^\tau, q)}{\partial x_p} \sum_{r \in P^i_l} x_r^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \sum_{a \in L} \lambda_a^\tau \delta_{ap})\},$$

$$\forall p \in P^i_j;\ i = 1, \ldots, I;\ j = 1, \ldots, M. \tag{24}$$

The Lagrange multipliers for each link $a \in L^i; i = 1, \ldots, I$, compute:

$$\lambda_a^{\tau+1} = \max\{0, \lambda_a^\tau + a_\tau(\sum_{p \in P} x_p^\tau \delta_{ap} - u_a)\}, \ \forall a \in L. \tag{25}$$

Case Study of Apple Orchards in Western Massachusetts

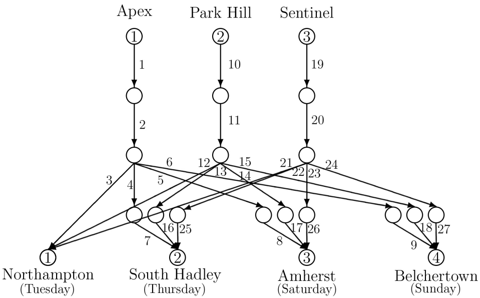
Orchard/farms:

- Apex Orchards are located in Shelburne Falls.
- Park Hill Orchard is located in Easthampton.
- Sentinel Farm is located in Belchertown.

Farmers' markets:

- Northampton Farmers' Market is open on Tuesdays.
- South Hadley Farmers' Market is open on Thursdays.
- Amherst Farmers' Market is open on Saturdays.
- Belchertown Farmers' Market is open on Sundays.

Scenario 1 - Some Information



- Picking is made **on Monday**; therefore, there are **no storage links for the Northampton Farmers' Market**.
- Golden Delicious apples follow **first order** quality decay.
- Harvesting is made between **September and October**, with average temperatures **19-22** C°.

Scenario 1 - Some Information

- Apex Orchards have the largest land size (170 acres), followed by Park Hill Orchard (127 acres) and Sentinel Farm (8 acres).
- Apex is located in a higher altitude, so that the average harvesting temperature at the orchard is lower than others.
- Apex uses controlled atmosphere storage which maintains the optimal temperature, 0 C°.
- We assume that orchard/farm i; $i = 1, 2, 3$, in the supply chain network has initial quality, respectively, of: $q_{01} = 1$, $q_{02} = 0.8$, and $q_{03} = 0.7$.
- Uncapacitated model is used.

Scenario 1- Quality Decay

Operations	Link a	Hours	Temp (C°)	β_a
harvesting	**1**	**4.00**	**19**	**0.992**
processing	2	3.00	19	0.994
transportation	3	2.50	19	0.999
storage (2 days)	4	**48.00**	0	**0.994**
storage (4 days)	5	96.00	0	0.988
storage (5 days)	6	120.00	0	0.985
transportation	7	4.00	19	0.993
transportation	8	3.25	19	0.994
transportation	9	4.00	19	0.993
harvesting	**10**	**3.00**	**22**	**0.992**
processing	11	3.00	22	0.992
transportation	12	2.5	19	0.999
storage (2 days)	**13**	**48.00**	**9**	**0.978**
storage (4 days)	14	96.00	9	0.957
storage (5 days)	15	120.00	9	0.947

Scenario 1 - Quality Decay

Operations	**Link** a	**Hours**	**Temp** (C°)	β_a
transportation	16	3.75	19	0.993
transportation	17	5.16	19	0.990
transportation	18	3.00	19	0.992
harvesting	**19**	**5.00**	**22**	**0.986**
processing	20	5.00	22	0.986
transportation	21	2.50	22	0.998
storage (2 days)	**22**	**48.00**	**12**	**0.967**
storage (4 days)	23	96.00	12	0.936
storage (5 days)	24	120.00	12	0.921
transportation	25	3.75	22	0.990
transportation	26	5.16	22	0.986
transportation	27	3.00	22	0.992

Scenario 1- Demand Price Functions

Demand Price Functions of Apex Orchards:

$$\rho_{11}(d,q) = -0.04d_{11} - 0.01d_{21} - 0.01d_{31} + 8q_{p_1} - 4q_{p_5} - 3q_{p_9} + 30,$$

$$\rho_{12}(d,q) = -0.02d_{12} - 0.01d_{22} - 0.01d_{32} + 3q_{p_2} - 2q_{p_6} - 2q_{p_{10}} + 25,$$

$$\rho_{13}(d,q) = -0.04d_{13} - 0.02d_{23} - 0.01d_{33} + 8q_{p_3} - 4q_{p_7} - 3q_{p_{11}} + 30,$$

$$\rho_{14}(d,q) = -0.04d_{14} - 0.02d_{24} - 0.02d_{34} + 3q_{p_4} - q_{p_8} - 2q_{p_{12}} + 25,$$

Demand Price Functions of Park Hill Orchard:

$$\rho_{21}(d,q) = -0.04d_{21} - 0.02d_{11} - 0.02d_{31} + 3q_{p_5} - 2q_{p_1} - q_{p_9} + 27,$$

$$\rho_{22}(d,q) = -0.04d_{22} - 0.01d_{12} - 0.02d_{32} + 3q_{p_6} - 2q_{p_2} - q_{p_{10}} + 28,$$

$$\rho_{23}(d,q) = -0.04d_{23} - 0.02d_{13} - 0.02d_{33} + 4q_{p_7} - 2q_{p_3} - q_{p_{11}} + 27,$$

$$\rho_{24}(d,q) = -0.02d_{24} - 0.01d_{14} - 0.01d_{34} + 2q_{p_8} - q_{p_4} - q_{p_{12}} + 28,$$

Demand Price Functions of Sentinel Farm:

$$\rho_{31}(d,q) = -0.04d_{31} - 0.02d_{11} - 0.02d_{21} + 4q_{p_9} - q_{p_1} - 2q_{p_5} + 25,$$

$$\rho_{32}(d,q) = -0.04d_{32} - 0.01d_{12} - 0.02d_{22} + 4q_{p_{10}} - 3q_{p_2} - q_{p_6} + 28,$$

$$\rho_{33}(d,q) = -0.02d_{23} - 0.01d_{13} - 0.01d_{33} + 4q_{p_{11}} - 2q_{p_3} - q_{p_7} + 25,$$

$$\rho_{34}(d,q) = -0.04d_{34} - 0.02d_{14} - 0.02d_{24} + 3q_{p_{12}} - 2q_{p_4} - 2q_{p_8} + 28.$$

Scenario 1 - Total Link Cost Functions and Equilibrium Link Flows

Operations	**Link** a	$\hat{c}_a(f)$	f_a^*
harvesting	1	$0.02f_1^2 + 3f_1$	165.8395
processing	2	$0.015f_2^2 + 3f_2$	165.8395
transportation	3	$0.01f_3^2 + 3f_3$	111.9827
storage (2 days)	4	$0.01f_4^2 + 3f_4$	0.0000
storage (4 days)	5	$0.015f_5^2 + 4f_5$	53.8568
storage (5 days)	6	$0.03f_6^2 + 5f_6$	0.0000
transportation	7	$0.02f_7^2 + 6f_7$	0.0000
transportation	8	$0.0125f_8^2 + 4f_8$	53.8568
transportation	9	$0.02f_9^2 + 6.6f_9$	0.0000
harvesting	10	$0.0125f_{10}^2 + 6f_{10}$	94.7414
processing	11	$0.0125f_{11}^2 + 6f_{11}$	94.7414
transportation	12	$0.0045f_{12}^2 + f_{12}$	71.7812
storage (2 days)	13	$0.01f_{13}^2 + 1.67f_{13}$	22.9601
storage (4 days)	14	$0.015f_{14}^2 + 6f_{14}$	0.0000
storage (5 days)	15	$0.015f_{15}^2 + 6.6f_{15}$	0.0000

Scenario 1 - Total Link Cost Functions and Equilibrium Link Flows

Operations	**Link** a	$\hat{c}_a(f)$	f_a^*
transportation	16	$0.0075f_{16}^2 + 6f_{16}$	22.9601
transportation	17	$0.01f_{17}^2 + 6f_{17}$	0.0000
transportation	18	$0.02f_{18}^2 + 4f_{18}$	0.0000
harvesting	19	$0.0125f_{19}^2 + 6f_{19}$	98.5294
processing	20	$0.015f_{20}^2 + 4f_{20}$	98.5294
transportation	21	$0.02f_{21}^2 + 4f_{21}$	17.2084
storage (2 days)	22	$0.007f_{22}^2 + 1.67f_{22}$	32.4314
storage (4 days)	23	$0.009f_{23}^2 + 6f_{23}$	0.0000
storage (5 days)	24	$0.01f_{24}^2 + 6f_{24}$	48.8896
transportation	25	$0.005f_{25}^2 + 6f_{25}$	32.4314
transportation	26	$0.005f_{26}^2 + 6f_{26}$	0.0000
transportation	27	$0.0005f_{27}^2 + 0.1f_{27}$	48.8896

Scenario 1 - Equilibrium Path Flows and Path Quality Decay

Farm	Path p	q_p	x_p^*	Farmers' Market
Apex	p_1	**0.9851**	**111.9827**	**Northampton**
Apex	p_2	0.9733	0.0000	South Hadley
Apex	p_3	0.9684	53.8568	Amherst
Apex	p_4	**0.9645**	**0.0000**	**Belchertown**
Park Hill	p_5	**0.7864**	**71.7812**	**Northampton**
Park Hill	p_6	0.7645	22.9602	South Hadley
Park Hill	p_7	0.7458	0.0000	Amherst
Park Hill	p_8	**0.7395**	**0.0000**	**Belchertown**
Sentinel	p_9	**0.6791**	**17.2084**	**Northampton**
Sentinel	p_{10}	0.6514	32.4314	South Hadley
Sentinel	p_{11}	0.6280	0.0000	Amherst
Sentinel	p_{12}	**0.6217**	**48.8896**	**Belchertown**

Apex Orchards' price of apples per peck:

$$\rho_{11} = 27.33, \quad \rho_{12} = 24.53, \quad \rho_{13} = 30.72, \quad \rho_{14} = 25.42,$$

Park Hill Orchard's price of apples per peck:

$$\rho_{21} = 21.25, \quad \rho_{22} = 26.13, \quad \rho_{23} = 26.34, \quad \rho_{24} = 27.40,$$

Sentinel Farm's price of apples per peck:

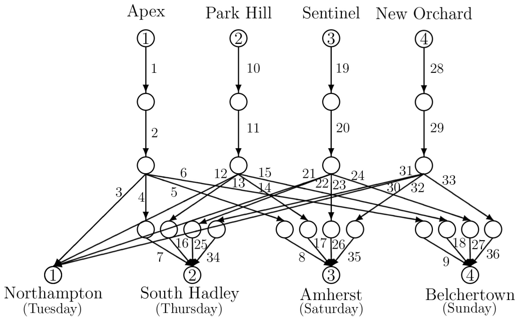
$$\rho_{31} = 20.79, \quad \rho_{32} = 25.16, \quad \rho_{33} = 24.29, \quad \rho_{34} = 24.50.$$

Profits of the orchard/farms, in dollars:

$$\mathbf{U_1(X^*) = 1785.40}, \quad U_2(X^*) = 484.03, \quad U_3(X^*) = 460.15.$$

Scenario 2 - Some Information

- It is assumed that a new orchard, which was solely selling to retailers and wholesalers previously, is attracted by the demand for apples at the farmers' markets.



Scenario 2 - Quality Decay

- It has **similar** orchard characteristics to **Apex Orchards**.
- It is located in **Belchertown**, which has similar seasonal temperatures to the other farm/orchards.
- The **transportation time** from the New Orchard to the farmers' markets is similar to **Sentinel Farm**.

Operations	**Link** a	**Hours**	**Temp** (C°)	β_a
harvesting	28	4.00	19	0.988
processing	29	4.00	19	0.988
transportation	30	0.50	19	0.998
storage (2 days)	**31**	**48.00**	**0**	**0.968**
storage (4 days)	32	**96.00**	**0**	**0.989**
storage (5 days)	**33**	**120.00**	**0**	**0.986**
transportation	34	3.50	19	0.989
transportation	35	3.00	19	0.991
transportation	36	3.00	19	0.991

Scenario 2 - Demand Price Functions

Demand Price Functions of Apex Orchards:

$$\rho_{11}(d,q) = -0.053d_{11} - 0.01d_{21} - 0.01d_{31} - 0.03d_{41} + 8q_{p_1} - 2q_{p_5} - 2q_{p_9} - 4q_{p_{13}} + 30,$$

$$\rho_{12}(d,q) = -0.03d_{12} - 0.01d_{22} - 0.01d_{32} - 0.004d_{42} + 3q_{p_2} - 2q_{p_6} - 2q_{p_{10}} - q_{p_{14}} + 25,$$

$$\rho_{13}(d,q) = -0.053d_{13} - 0.01d_{23} - 0.01d_{33} - 0.03d_{43} + 8q_{p_3} - 2q_{p_7} - 2q_{p_{11}} - 4q_{p_{15}} + 30,$$

$$\rho_{14}(d,q) = -0.03d_{14} - 0.01d_{24} - 0.014d_{34} - 0.004d_{44} + 3q_{p_4} - q_{p_8} - 2q_{p_{12}} - q_{p_{15}} + 25,$$

Demand Price Functions of Park Hill Orchard:

$$\rho_{21}(d,q) = -0.05d_{21} - 0.01d_{11} - 0.01d_{31} - 0.01d_{41} + 3q_{p_5} - q_{p_1} - q_{p_9} - q_{p_{13}} + 27,$$

$$\rho_{22}(d,q) = -0.04d_{22} - 0.01d_{12} - 0.02d_{32} - 0.004d_{42} + 3q_{p_6} - 2q_{p_2} - q_{p_{10}} - q_{p_{14}} + 28,$$

$$\rho_{23}(d,q) = -0.05d_{23} - 0.02d_{13} - 0.01d_{33} - 0.02d_{43} + 4q_{p_7} - 2q_{p_3} - q_{p_{11}} - 2q_{p_{15}} + 27,$$

$$\rho_{24}(d,q) = -0.04d_{24} - 0.01d_{14} - 0.02d_{34} - 0.004d_{44} + 2q_{p_8} - q_{p_4} - q_{p_{12}} - q_{p_{16}} + 28,$$

Scenario 2 - Demand Price Functions

Demand Price Functions of Sentinel:

$$\rho_{21}(d,q) = -0.05d_{21} - 0.01d_{11} - 0.01d_{31} - 0.01d_{41} + 3q_{p_5} - q_{p_1} - q_{p_9} - q_{p_{13}} + 27,$$

$$\rho_{22}(d,q) = -0.04d_{22} - 0.01d_{12} - 0.02d_{32} - 0.004d_{42} + 3q_{p_6} - 2q_{p_2} - q_{p_{10}} - q_{p_{14}} + 28,$$

$$\rho_{23}(d,q) = -0.05d_{23} - 0.02d_{13} - 0.01d_{33} - 0.02d_{43} + 4q_{p_7} - 2q_{p_3} - q_{p_{11}} - 2q_{p_{15}} + 27,$$

$$\rho_{24}(d,q) = -0.04d_{24} - 0.01d_{14} - 0.02d_{34} - 0.004d_{44} + 2q_{p_8} - q_{p_4} - q_{p_{12}} - q_{p_{16}} + 28,$$

Demand Price Functions of New Orchard:

$$\rho_{41}(d,q) = -0.053d_{41} - 0.03d_{11} - 0.01d_{21} - 0.01d_{31} + 5q_{p_{13}} - 2q_{p_1} - q_{p_5} - q_{p_9} + 30,$$

$$\rho_{42}(d,q) = -0.03_{42} - 0.006d_{12} - 0.01d_{22} - 0.01d_{32} + 2q_{p_{14}} - q_{p_2} - q_{p_6} - q_{p_{10}} + 25,$$

$$\rho_{43}(d,q) = -0.053d_{43} - 0.03d_{13} - 0.01d_{23} - 0.01d_{33} + 5q_{p_{15}} - 2q_{p_3} - q_{p_7} - q_{p_{11}} + 30,$$

$$\rho_{44}(d,q) = -0.03d_{44} - 0.006d_{14} - 0.01d_{24} - 0.01d_{34} + 2q_{p_{16}} - q_{p_4} - q_{p_8} - q_{p_{12}} + 25.$$

Scenario 2 - Equilibrium Path Flows and Path Quality Decay

- Initial quality of the apples at the orchards: $q_{01} = 1$, $q_{02} = 0.8$, $q_{03} = 0.7$, and $q_{04} = 1$.

Farm	Path p	q_p	x_p^*	Farmers' Market
Apex	p_1	**0.9851**	**79.5849**	**Northampton**
Apex	p_2	0.9733	0.0000	South Hadley
Apex	p_3	0.9684	44.5036	Amherst
Apex	p_4	0.9645	0.0000	Belchertown
Park Hill	p_5	0.7864	69.2348	Northampton
Park Hill	p_6	**0.7645**	**18.2460**	**South Hadley**
Park Hill	p_7	0.7458	0.0000	Amherst
Park Hill	p_8	0.7395	0.0000	Belchertown
Sentinel	p_9	**0.6791**	**18.3520**	**Northampton**
Sentinel	p_{10}	0.6514	30.9408	South Hadley
Sentinel	p_{11}	0.6280	0.0000	Amherst
Sentinel	p_{12}	0.6217	36.7854	Belchertown
New Orchard	p_{13}	**0.9742**	**82.0895**	**Northampton**
New Orchard	p_{14}	0.9345	0.0000	South Hadley
New Orchard	p_{15}	**0.9567**	**44.0319**	**Amherst**
New Orchard	p_{16}	0.9538	0.0000	Belchertown

Apex Orchards' price of apples per peck:

$$\boldsymbol{\rho_{11} = 23.49}, \quad \rho_{12} = 23.66, \quad \rho_{13} = 27.49, \quad \rho_{14} = 24.44,$$

Park Hill Orchard's price of apples per peck:

$$\rho_{21} = 21.46, \quad \rho_{22} = 25.41, \quad \rho_{23} = 25.49, \quad \rho_{24} = 26.20,$$

Sentinel Farm's price of apples per peck:

$$\rho_{31} = 20.38, \quad \rho_{32} = 24.38, \quad \rho_{33} = 22.91, \quad \rho_{34} = 23.08,$$

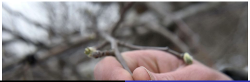
New Orchard's price of apples per peck:

$$\rho_{41} = 23.82, \quad \rho_{42} = 23.99, \quad \rho_{43} = 27.80, \quad \rho_{44} = 24.21.$$

Profits of the orchard/farms, in dollars:
$\mathbf{U_1(X^*) = 1097.39}$, $U_2(X^*) = 471.71$, $U_3(X^*) = 345.45$, $\mathbf{U_4(X^*) = 1142.19}$.

Scenario 3 - Some Information

- This scenario is constructed to illustrate the apple shortage experienced in **western Massachusetts in 2016.**
- According to various news articles, the **cold snap** happened in May damaged the **green apple buds** and **an apple shortage** at the local markets, which includes the farmers' markets, is expected.
- Expected shortage is assumed to **be more for Apex** due to being located in a **higher altitude.**
- The capacities are written according to the **expected damage level of harvest** at the orchard/farms.
- Initial quality of the apples in the orchards is $q_{01} = 0.4$, $q_{02} = 0.5$, and $q_{03} = 0.6$.



Scenario 3 - Link Capacities, Equilibrium Link Flows and Equilibrium Lagrange Multipliers

Operations	**Link** a	**Capacity**	f_a^*	λ_a^*
harvesting	**1**	**20**	**20.0000**	**16.4077**
processing	2	15000	20.0000	0.0000
transportation	3	15000	20.0000	0.0000
storage (2 days)	4	15000	0.0000	0.0000
storage (3 days)	5	15000	0.0000	0.0000
storage (4 days)	6	15000	0.0000	0.0000
transportation	7	15000	0.0000	0.0000
transportation	8	15000	0.0000	0.0000
transportation	9	15000	0.0000	0.0000
harvesting	**10**	**50**	**50.0000**	**6.4906**
processing	11	15000	50.0000	0.0000
transportation	12	15000	50.0000	0.0000
storage (2 days)	13	15000	0.0000	0.0000
storage (3 days)	14	15000	0.0000	0.0000
storage (4 days)	15	15000	0.0000	0.0000

Scenario 3 - Link Capacities, Equilibrium Link Flows, and Equilibrium Lagrange Multipliers

Operations	**Link** a	**Capacity**	f_a^*	λ_a^*
transportation	16	15000	0.0000	0.0000
transportation	17	15000	0.0000	0.0000
transportation	18	15000	0.0000	0.0000
harvesting	**19**	**60**	**60.0000**	**5.6685**
processing	20	15000	60.0000	0.0000
transportation	21	15000	13.1918	0.0000
storage (2 days)	22	15000	18.7448	0.0000
storage (3 days)	23	15000	0.0000	0.0000
storage (4 days)	24	15000	28.0624	0.0000
transportation	25	15000	18.7448	0.0000
transportation	26	15000	0.0000	0.0000
transportation	27	15000	28.0624	0.0000

Scenario 3 - Equilibrium Path Flows and Path Quality Decay

Farm	Path p	q_p	x_p^*	Farmers' Market
Apex	p_1	**0.3940**	**20.0000**	**Northampton**
Apex	p_2	0.3893	0.0000	South Hadley
Apex	p_3	0.3873	0.0000	Amherst
Apex	p_4	0.3858	0.0000	Belchertown
Park Hill	p_5	**0.4915**	**50.0000**	**Northampton**
Park Hill	p_6	0.4778	0.0000	South Hadley
Park Hill	p_7	0.4662	0.0000	Amherst
Park Hill	p_8	0.4622	0.0000	Belchertown
Sentinel	p_9	**0.5821**	**13.1918**	**Northampton**
Sentinel	p_{10}	**0.5584**	**18.7448**	**South Hadley**
Sentinel	p_{11}	0.5383	0.0000	Amherst
Sentinel	p_{12}	**0.5329**	**28.0624**	**Belchertown**

Apex Orchards' price of apples per peck:

$$\rho_{11} = 28.01, \quad \rho_{12} = 23.91, \quad \rho_{13} = 29.62, \quad \rho_{14} = 24.35.$$

Park Hill Orchard's price of apples per peck:

$$\rho_{21} = 24.44, \quad \rho_{22} = 27.72, \quad \rho_{23} = 27.55, \quad \rho_{24} = 27.72,$$

Sentinel Farm's price of apples per peck:

$$\rho_{31} = 24.02, \quad \rho_{32} = 27.84, \quad \rho_{33} = 25.91, \quad \rho_{34} = 26.78.$$

The profits of the orchard/farms in this scenario, in dollars:

$$\mathbf{U_1(X^*) = 362.15}, \quad U_2(X^*) = 498.28, \quad U_3(X^*) = 507.58.$$

Conclusion

- We provided **explicit formulae** for quality deterioration and found the quality associated with every path in the network.
- We focused on **farmers' markets** which are direct to consumer chains.
- We provided a **game theory model** for **supply chain** competition in a network framework for farmers' markets.
- This is the **first work** in the literature with a supply chain game theory model for farmers' markets with **quality deterioration.**

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