

Lecture 4: Fundamental Issues in Humanitarian Logistics

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SCH-MGMT 597LG
Humanitarian Logistics and Healthcare
Spring 2019

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Humanitarian Logistical Operations

In this lecture, we will be discussing not only fundamental questions and issues regarding humanitarian logistics but we will also be quantifying different modes of network management and supply chain organization (including centralized versus decentralized).

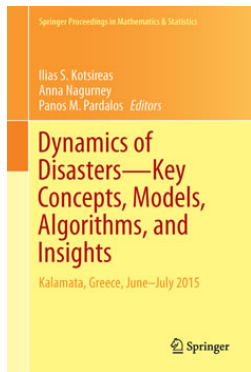
Humanitarian Logistics and Dynamics of Disasters

In 2015, I co-organized the Dynamics of Disasters conference in Kalamata, Greece, along with Professor Ilias S. Kotsireas of Wilfrid Laurier University and Professor Panos M. Pardalos of the University of Florida.



Humanitarian Logistics and Dynamics of Disasters

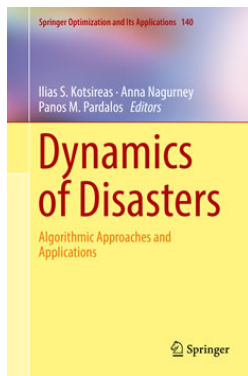
One of the outcomes of the conference was the edited book, several chapters of which we are using in this course.



Click [here](#) for information on the 2015 conference.

Humanitarian Logistics and Dynamics of Disasters

We also co-organized another Dynamics of Disasters conference in Kalamata, Greece in 2017 and the proceedings volume is now also published with Springer, again, as the publisher.



Click here for information on the 2017 conference.

Humanitarian Logistical Operations

According to humanitarian standards local communities are estimated to be self-reliant only for 48 hours, after which humanitarian organizations must be operational on the ground (Tomasini and Van Wassenhove (2009)).

A Case Study – International Federation of Red Cross and Red Crescent Societies (IFRC)

The International Federation of Red Cross and Red Crescent Societies (IFRC) was created in 1919.

It is the largest humanitarian organization in existence today, regrouping 186 autonomous National Societies (NSs) which it assists in responding to humanitarian crises around the globe.

[Click here for video on the IFRC and disaster relief in Syria.](#)

[Click here for video on the IFRC helping out refugees in Bangladesh where, as of August 2017, more than half a million people have crossed the border fleeing violence in northern Rakhine state in Myanmar.](#)

A Case Study – International Federation of Red Cross and Red Crescent Societies (IFRC)

In the IFRC centralized supply chain, critical needs items were usually transported in large batches via transcontinental flights. This is not only expensive, but it also leads to logistical bottlenecks in receiving and storing the supplies at local airports. or other nearby warehouses.

The new decentralized supply chain (implemented over 10 years) with its prepositioned stocks and supplies allowed for a quicker regional operation by air during the emergency phase.

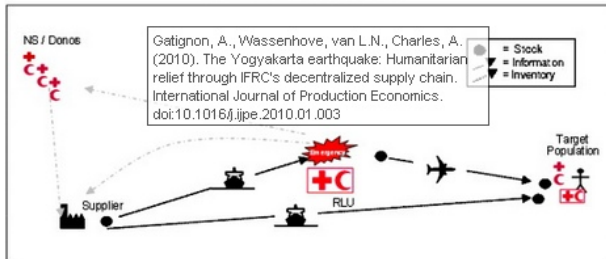
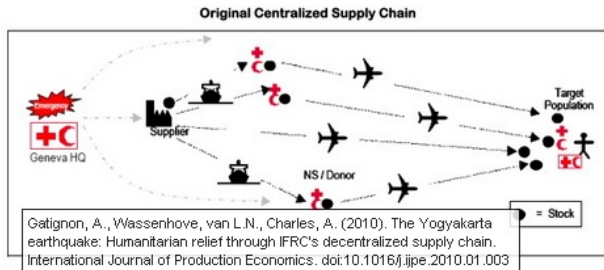
In this way, a faster response is possible during the crucial ramp-up phase of operations.

A Case Study – International Federation of Red Cross and Red Crescent Societies (IFRC)

Researchers evaluated the decentralized supply chain's performance in responding to humanitarian crises through an analysis of the IFRC's operations during the Yogyakarta earthquake in Java in May 2006.

This was the first operation to benefit from the support of Regional Logistics Units (RLUs).

Centralized vs. Decentralized Supply Chains



Centralized vs. Decentralized Supply Chains

The IFRC selected three locations for their Regional Logistics Units (RLUs) on the basis of their strategic positioning in terms of the IFRCs global supply chain and operations.

RLUs are charged with providing regional support for the IFRCs members, 186 National Societies (NSs), by assisting them in managing the supply chain of goods from donors and suppliers to the field of operations.

- Dubai was chosen to cover Europe, the Middle East, and Africa,
- Kuala Lumpur was chosen to cover Asia and Australia,
- Panama was chosen to cover America.

Centralized vs. Decentralized Supply Chains

The Yogyakarta relief operations case showed that the IFRC's decentralized supply chain is effective with:

- Faster service, fully operational in three days.
- Better service to the disaster victims over a longer period of time.
- Much cheaper than previous interventions.

According to Gatignon, Van Wassenhove, and Charles, 2010, one reason why the decentralized approach worked so well was because the RLU is nearer to the field of operations than its headquarters in Geneva, Switzerland. Communications with donors were more fluid in the same time zone. **In addition, prepositioning of supplies at multiple locations enabled better disaster preparedness.**

Centralized vs. Decentralized Supply Chains

Table 1 – Better	Indian Ocean Tsunami 100,000 families	Pakistan EQ 95,000 families	Yogyakarta EQ 65,000 families
Families receiving at least partial package by 2 months	28,021	29,229	53,112
Average number of families served per day	445	555	613
% goods delivered from the region	13%	68%	100%

Table 2 – Faster	Indian Ocean Tsunami	Pakistan EQ	Yogyakarta EQ
Days to activate end to end supply chain	18	10	3
Order lead time (requisition to delivery) in days	30	23	16
% of appeal items mobilized & delivered at 2 months	55%	38%	74%
Average distance of relief items (km) to families	11,805	2,962	1,617

Table 3 - Cheaper	Indian Ocean Tsunami	Pakistan EQ	Yogyakarta EQ
Operations total costs at 2 months	Not available	55,944,027	10,505,962
% logistics cost at 8 months (items + transport + storage value)	-	86%	87%
Cost to deliver relief package per family at 2 months	-	824	142
Cost to deliver relief package per family at 8 months	-	450	142

Source: Justin Cuckow, The effect of the IFRC Regional Logistics concept on the efficiency of relief item delivery for the population affected by the Yogyakarta earthquake, Internal IFRC case study, August 24th 2006

Fundamental Questions in Humanitarian Logistics

When disasters strike, stakeholders mobilize, but not necessarily in a coordinated manner, and the degree to which a humanitarian operation is successful (or not) depends on how well multifaceted issues and questions have been addressed.

During a disaster, devastated infrastructure, incomplete information, powerful emotions, and even chaos may rein.

Hence, it is essential to harness all the available tools, **including methodological and technological ones**, in order to address all phases of the disaster cycle.

Fundamental Questions in Humanitarian Logistics

Recent disasters, from the triple disaster in Japan in March 2011, to Hurricane Katrina in August 2005, or the earthquake that devastated Haiti in January 2010, are just several examples of disasters with immense impacts on the affected populations (and beyond). Moreover, **numerous slow-onset disasters** have led to refugee crises because of events in Syrian, Myanmar, and Yemen.

The challenges faced by local, regional, national, and international relief agencies and governments to prepare for and manage the response activities have emerged as a common factor in all of these events. **From these experiences, there is a clear need for major improvements in the performance of humanitarian logistics in terms of efficiency, with efficiency broadly and appropriately defined.**

Fundamental Questions in Humanitarian Logistics

Framing and synthesizing knowledge derived from both theory and practice, which can be quantitative as well as qualitative, in humanitarian logistics has now begun and the importance of this new branch of logistics and supply chain management needs to be emphasized.

Only by working together, learning from one another, and documenting our research and findings in the field, will we be able to gain valuable insights and information for best practices and fundamental tools, methods, and metrics in order to drive both theory and practice forward.

With the number of disasters growing as well as the number of people affected by them, the challenges cannot be set aside for a time in the future but must be tackled now.

Research and Education in Humanitarian Logistics

Research on and education in humanitarian logistics in academic institutions is taking place at INSEAD in France, Cranfield University in England, Vienna University in Economics and Business in Austria, Hanken in Finland, Griffith University in Australia, among others outside the US, and RPI, Georgia Tech, Northwestern, Isenberg School of Management at UMass Amherst, Naval Postgraduate School, MIT, in the US, to start.

Fundamental Questions in Humanitarian Logistics

Some fundamental questions/issues that need addressing in a disaster, which can serve as a guide for more efficient and effective humanitarian supply chains:

- What is the status of the infrastructure: transportation, healthcare, telecommunications, housing, emergency services, such as police and fire departments, and even governmental?
- Who is impacted after a disaster?
- What is the severity/degree of impact?
- What are the needs of the population impacted?
- Who is going to help?

Fundamental Questions in Humanitarian Logistics

- How are the aid operations performed?
- What types of plans are needed for the relief operations?
- What are the preparedness actions required?
- What types of critical supplies: water, food, medicines, shelter, are needed and in what amounts?
- What are the mechanisms and processes for distribution?
- How should the influx of personnel, information and materiel be handled, processed, and delivered?

A Fundamental Performance Metric

The previous questions address all the major issues necessary to ensure an efficient and effective humanitarian supply chain.

The performance of the humanitarian supply chain is evaluated in terms of the minimization of human suffering and the minimization of “logistical” costs.

Nonlinear Network Optimization Models for Humanitarian Operations

In this course, we will be studying a spectrum of rigorous models for humanitarian logistics with the goal of providing a framework from which the models can be acquired, adapted, and applied.

We will, first, construct basic nonlinear network optimization models. The presentation is done at a high level, in order to establish theoretical foundations from which to build from.

We will also describe how the models, in various settings, can be extended, as well as applied.

Nonlinear Network Optimization Models for Humanitarian Operations

Haghani and Oh (1996), in their much-cited paper on network flow modeling for disaster relief operations, recognized the reality of congestion in many humanitarian operations and the importance of including nonlinearities.

Nonlinear network optimization models capture congestion, which can occur at ports, airport runways, warehouses roads, or other transportation and logistical network locations during disaster relief operations.

Here we focus on distinct behavioral principles, both of which are useful and relevant, in different humanitarian logistic scenarios.

System-Optimization Versus User-Optimization

The basic network models are now reviewed, under distinct assumptions of their operation and behavior of the users of the network. The models are classical and were developed in the context of transportation. They are due to Beckmann, McGuire, and Winsten (1956) and Dafermos and Sparrow (1969). We also link these models to humanitarian operations.

For definiteness, and for easy reference, we first present the classical system-optimized network model and then the classical user-optimized network model.

System-Optimization Versus User-Optimization

Although these models were first developed for transportation networks, here they are presented in the broader setting of network systems, including humanitarian operations and logistical networks. Indeed, the cost on a link can capture time, and, in a generalized setting, also risk, monetary cost, etc.

Under system-optimization, the total “cost” associated with delivering the flow (product, materiel, or even personnel) is minimized, subject to the demands being satisfied.

System-Optimization Versus User-Optimization

Under user-optimization, the flows are such that for each pair of origin/destination nodes, there is no path/route that provides a lower individual (average) “cost.”

Hence, user-optimization, in a humanitarian setting, may be interpreted as a fair or equitable distribution policy, since the product, in question, uses only the most “efficient” routes. Thus, the time for delivery will not be route-dependent, between a supply-demand point pair.

The System-Optimized Problem

Consider a general network $G = [N, L]$, where N denotes the set of nodes, and L the set of directed links.

Let a denote a link of the network connecting a pair of nodes, and let p denote a path consisting of a sequence of links connecting an O/D pair.

In transportation networks, nodes correspond to origins and destinations, as well as to intersections. Links, on the other hand, correspond to roads/streets in the case of urban transportation networks and to railroad segments in the case of train networks.

The System-Optimized Problem

A path in its most basic setting, thus, is a sequence of “roads” which comprise a route from an origin to a destination.

In the logistical setting, a path may consist of procurement links, storage or warehousing links, in addition to the transportation/shipment/distribution links. Hence, a path represents a series/sequence of supply chain activities from an origin to a destination.

The System-Optimized Problem

Let P_w denote the set of paths connecting the origin/destination (O/D) pair of nodes w . Let P denote the set of all paths in the network and assume that there are J origin/destination pairs of nodes in the set W .

Let x_p represent the flow on path p and let f_a denote the flow on link a . The path flows on the network are grouped into the column vector x with n_P components, where n_P denotes the number of paths in the network. The link flows, in turn, are grouped into the column vector f , with n components, where n denotes the number of links in the network.

The System-Optimized Problem

The following conservation of flow equation must hold:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \quad (1)$$

where $\delta_{ap} = 1$, if link a is contained in path p , and 0, otherwise. Expression (1) states that the flow on a link a is equal to the sum of all the path flows on paths p that contain (traverse) link a .

Let d_w denote the demand associated with O/D pair w , then one must have that

$$d_w = \sum_{p \in P_w} x_p, \quad \forall w \in W, \quad (2)$$

where $x_p \geq 0$, $\forall p \in P$; that is, the sum of all the path flows between an origin/destination pair w must be equal to the given demand d_w .

The System-Optimized Problem

Let c_a denote the (individual/unit) link cost associated with traversing link a , and let C_p denote the user cost associated with traversing the path p .

Assume that the link cost function is given by the *separable* function

$$c_a = c_a(f_a), \quad \forall a \in L, \quad (3)$$

where c_a is assumed to be an increasing function of the link flow f_a in order to model the effect of the link flow on the cost.

The total cost on link a , denoted by $\hat{c}_a(f_a)$, hence, is given by:

$$\hat{c}_a(f_a) = c_a(f_a) \times f_a, \quad \forall a \in L, \quad (4)$$

that is, the total cost on a link is equal to the link cost on the link times the flow on the link.

The System-Optimized Problem

Here the cost is interpreted in a generalized sense. From a transportation engineering perspective the cost on a link is assumed to coincide with the travel time on a link. We have the cost be a generalized cost (including financial, time, risk, etc.).

In the system-optimized (S-O) problem, there exists a central controller who seeks to minimize the total cost in the network system, where the total cost is expressed as

$$\sum_{a \in L} \hat{c}_a(f_a), \quad (5)$$

where the total cost on a link is given by expression (4).

The System-Optimized Problem

The system-optimization problem is, thus, given by:

$$\text{Minimize } \sum_{a \in L} \hat{c}_a(f_a) \quad (6)$$

subject to:

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w \in W, \quad (7)$$

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \quad (8)$$

$$x_p \geq 0, \quad \forall p \in P. \quad (9)$$

The System-Optimized Problem

The constraints (7) and (8), along with (9), are commonly referred to in network terminology as *conservation of flow equations*. In particular, they guarantee that the flow in the network, that is, the users (whether these are personnel or humanitarian supplies, for example) do not “get lost.”

The System-Optimized Problem

The total cost on a path, denoted by \hat{C}_p , is the user cost on a path times the flow on a path, that is,

$$\hat{C}_p = C_p x_p, \quad \forall p \in P, \quad (10)$$

where the individual cost on a path, C_p , is given by the sum of the individual costs on the links that comprise the path, that is,

$$C_p = \sum_{a \in L} c_a(f_a) \delta_{ap}, \quad \forall a \in L. \quad (11)$$

The System-Optimized Problem

In view of (8), one may express the cost on a path p as a function of the path flow variables and, hence, an alternative version of the above system-optimization problem can be stated in path flow variables only, where one has now the problem:

$$\text{Minimize } \sum_{p \in P} C_p(x) x_p \quad (12)$$

subject to constraints (7) and (9).

The System-Optimality Conditions

Under the assumption of increasing link cost functions, the objective function in the S-O problem is convex, and the feasible set consisting of the linear constraints is also convex.

Therefore, the optimality conditions, that is, the Kuhn-Tucker conditions are: For each O/D pair $w \in W$, and each path $p \in P_w$, the flow pattern x (and link flow pattern f), satisfying (7)–(9) must satisfy:

$$\hat{C}'_p \begin{cases} = \mu_w, & \text{if } x_p > 0 \\ \geq \mu_w, & \text{if } x_p = 0, \end{cases} \quad (13)$$

where \hat{C}'_p denotes the marginal of the total cost on path p , given by:

$$\hat{C}'_p = \sum_{a \in \mathcal{L}} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}, \quad (14)$$

and in (13) it is evaluated at the solution.

The System-Optimality Conditions

Note that in the S-O problem, according to the optimality conditions (13), it is the marginal of the total cost on each used path connecting an O/D pair which is equalized and minimal. Indeed, conditions (13) state that a system-optimized flow pattern is such that for each origin/destination pair the incurred marginals of the total costs on all used paths are equal and minimal.

Relevance to Humanitarian Operations

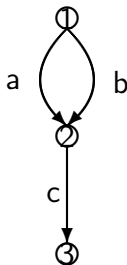
The system-optimized model can be applied to humanitarian operations in many different scenarios.

1. It can be used to assess existing supply chains from a total cost perspective subject to demands being met at the demand points.
2. It can be used to determine where supplies should be positioned. This can be done by defining different origins and then origin/destination pairs with which the demand is associated. Hence, it enables the planner to assess different network configurations.
3. Other applications include the evaluation of synergies associated with the teaming of organizations for humanitarian relief, as we will illustrate later.

4. Extensions to multiple products as well as the use of link capacity variables can enable optimal supply chain network design (see Nagurney, Yu, and Qiang (2011)).

A Numerical System-Optimized Example

Consider the network in the Figure in which there are three nodes: 1, 2, 3; three links: a , b , c ; and a single O/D pair $w_1 = (1, 3)$ with demand $d_{w_1} = 15$. Let path $p_1 = (a, c)$ and path $p_2 = (b, c)$.



The total link cost functions are: $\hat{c}_a(f) = f_a^2 + 15f_a$,
 $\hat{c}_b(f) = f_b^2 + 25f_b$, and $\hat{c}_c(f) = f_c^2 + 5f_c$.

A Numerical System-Optimized Example

The demand $d_{w_1} = 15$.

The system-optimized (S-O) path flow pattern is:

$$x_{p_1} = 10, \quad x_{p_2} = 5,$$

which induces the S-O link flow pattern:

$$f_a = 10, \quad f_b = 5, \quad f_c = 15.$$

Indeed, under this flow pattern the marginal total costs on both paths, and both paths are used are:

$$\hat{C}'_{p_1} = \hat{C}'_{p_2} = 70$$

The S-O conditions (13) are satisfied.

The total cost of supplying the demand point from the origin point is: 750.

Other Applications of System-Optimization to Humanitarian Logistics and Operations

This model can also be applied to the teaming of organizations in the case of humanitarian operations.

Bellagio Conference on Humanitarian Logistics

Humanitarian Logistics: Networks for Africa



Rockefeller Foundation Bellagio Center Conference, Bellagio, Lake Como, Italy

May 5-9, 2008

**Conference Organizer: Anna Nagurney, John F. Smith Memorial Professor
University of Massachusetts at Amherst**

Click here for the conference website presentations, etc.



Teaming in Humanitarian Operations for Network Synergies

A successful team in humanitarian operations depends on the ability to measure the anticipated synergy of the proposed team and its utilization of its supply chain network resources (procurement/manufacturing, storage, and distribution/transportation).

- ◇ A. Nagurney, 2009. A system-optimization perspective for supply chain network integration: The horizontal merger case, *Transportation Research E* **45**, 1-15.
- ◇ A. Nagurney and Q. Qiang, 2009. *Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain World*, John Wiley & Sons, Hoboken, New Jersey.

Teaming in Humanitarian Operations for Network Synergies

We consider two humanitarian organizations A and B each of which has access to its own supply chain network consisting of manufacturing links, transportation/shipment links, storage links, and final shipment links to the points of demand.

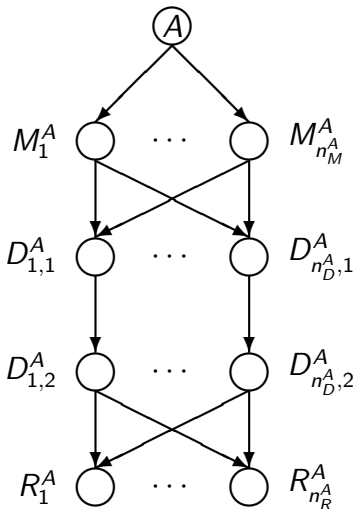
Each organization minimizes its total costs associated with its humanitarian supply chain network and distributes the critical needs product flows accordingly. Each has a total cost associated with its optimization.

Teaming in Humanitarian Operations for Network Synergies

In the evaluation of teaming, the organizations may elect to share one another's resources in terms of the procurement/manufacturing facilities (or staging areas), as well as the distribution/storage facilities.

The graphical representation is given in the next set of slides.

Organization A



Organization B

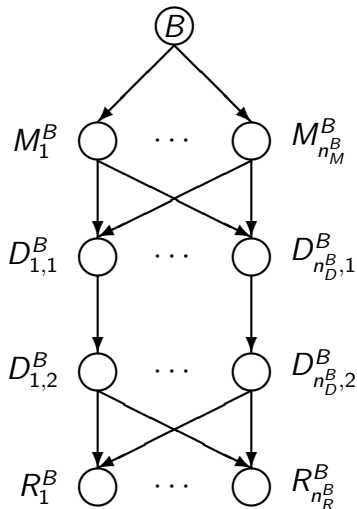


Figure: Case 0: Humanitarian Organizations A and B Prior to a Horizontal Merger

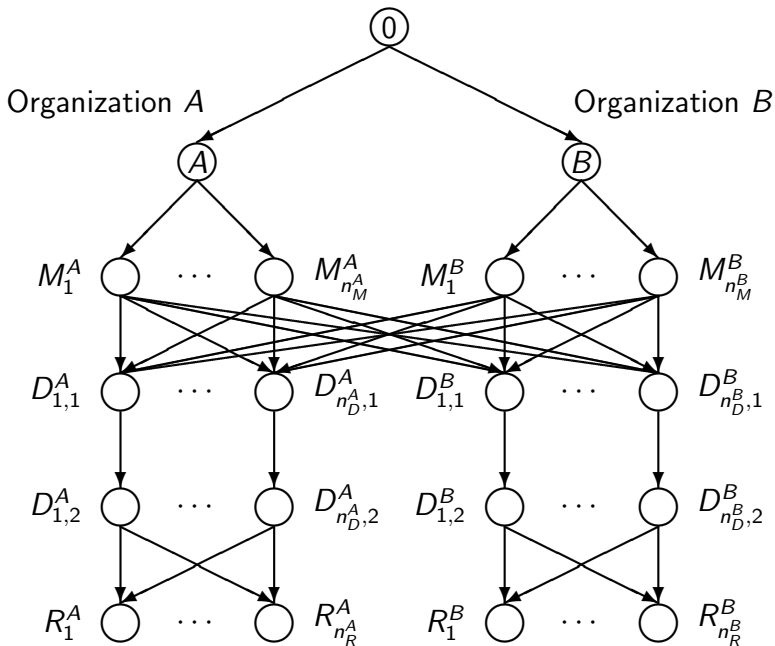


Figure: Case 1: Organizations A and B Merge/Team Up

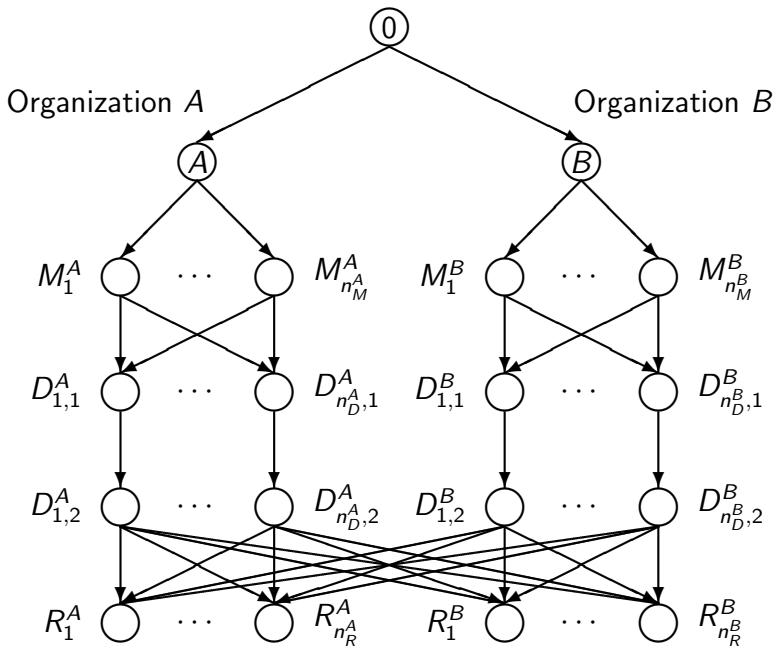


Figure: Case 2: Organizations A and B Merge/Team Up

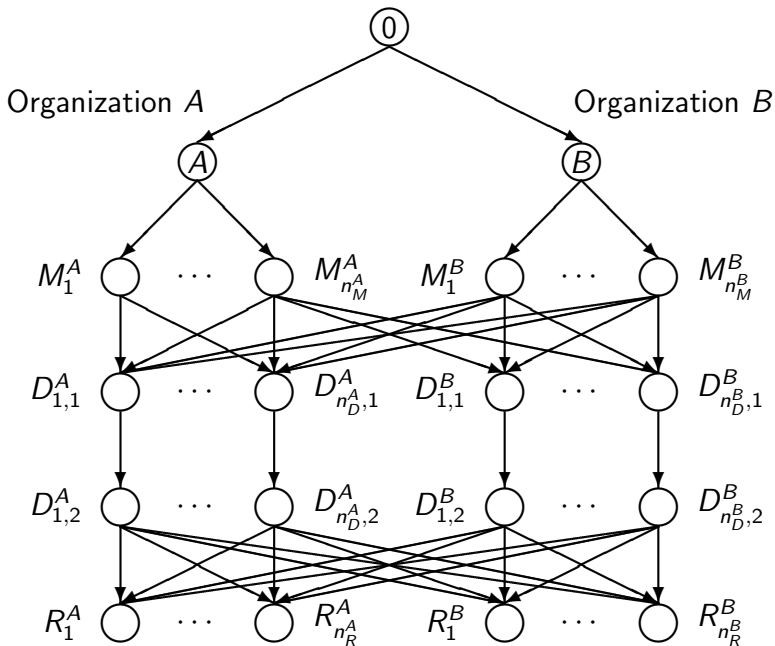


Figure: Case 3: Organizations A and B Merge/Team Up

Synergy Measure

The measure that we utilized in Nagurney (2009) to capture the gains, if any, associated with a horizontal merger Case i ; $i = 1, 2, 3$ is as follows:

$$\mathcal{S}^i = \left[\frac{TC^0 - TC^i}{TC^0} \right] \times 100\%,$$

where TC^i is the total cost associated with the value of the objective function $\sum_{a \in L^i} \hat{c}_a(f_a)$ for $i = 0, 1, 2, 3$ evaluated at the optimal solution for Case i . Note that \mathcal{S}^i ; $i = 1, 2, 3$ may also be interpreted as *synergy*.

The User-Optimized Problem

We now describe the user-optimized network problem, also commonly referred to in the transportation literature as the *assignment* problem or the *network equilibrium* problem.

As in the system-optimized problem, the network $G = [N, L]$, the demands associated with the origin/destination pairs, as well as the user link cost functions are assumed as given.

The User-Optimized Problem

The U-O or Network Equilibrium Conditions

One seeks to determine the path flow pattern x^* (and link flow pattern f^*) which satisfies the conservation of flow equations (7), (8), and the nonnegativity assumption on the path flows (9), and which also satisfies the network equilibrium conditions given by the following statement.

The User-Optimized Problem For each O/D pair $w \in W$ and each path $p \in P_w$:

$$C_p \begin{cases} = \lambda_w, & \text{if } x_p^* > 0 \\ \geq \lambda_w, & \text{if } x_p^* = 0. \end{cases} \quad (15)$$

The User-Optimized Problem

Hence, in the user-optimization problem there is no explicit optimization concept, since now “users” of the network system (which can correspond to routed products) act independently, in a non-cooperative manner, until they cannot improve on their situations unilaterally and, thus, an equilibrium is achieved, governed by the above equilibrium conditions.

Conditions (15) are simply a restatement of Wardrop's (1952) first principle mathematically and mean that only those paths connecting an O/D pair will be used which have equal and minimal user costs. Otherwise, a user of the network could improve upon his situation by switching to a path with lower cost. User-optimization represents decentralized decision-making, whereas system-optimization represents centralized decision-making.

The User-Optimized Problem

In order to obtain a solution to the above problem, Beckmann, McGuire, and Winsten (1956) established that the solution to the equilibrium problem, in the case of user link cost functions (cf. (3)) in which the cost on a link only depends on the flow on that link could be obtained by solving the following optimization problem:

$$\text{Minimize} \quad \sum_{a \in L} \int_0^{f_a} c_a(y) dy \quad (16)$$

subject to:

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w \in W, \quad (17)$$

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \quad (18)$$

$$x_p \geq 0, \quad \forall p \in P. \quad (19)$$

The User-Optimized Problem

Note that the conservation of flow equations are identical in both the user-optimized network problem (see (17)–(19)) and the system-optimized problem (see (7) – (9)). The behavior of the individual decision-makers termed “users,” however, is different. Users of the network system, which generate the flow on the network now act independently, and are not controlled by a centralized controller.

The User-Optimized Problem

The objective function given by (16) is simply a device constructed to obtain a solution using general purpose convex programming algorithms. It does not possess the economic meaning of the objective function encountered in the system-optimization problem given by (6), equivalently, by (12).

Algorithms

Algorithms for the solution of the S-O and U-O problems can be found in the Transportation & Logistics, OIM413 course materials of Professor Anna Nagurney on [the link to the course](#).

Interpretation of the U-O Conditions for Humanitarian Logistics

Although U-O behavior has been interpreted as being *selfish* as opposed to *unselfish*, in disaster operations it has relevance for the following reason:

The U-O solution guarantees that the cost on each used path (that is one with positive flow) for each O/D pair of nodes is equal and minimal.

Hence, if the flow corresponds to a critical needs product (water, food, medicine, etc.) then routing of the flow according to the U-O principle is fair and equitable. All those in need will be receiving their products from an origin point to a specific demand point at the same cost (and recall that cost can also include time).

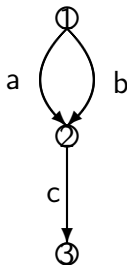
Interpretation of the U-O Conditions for Humanitarian Logistics

U-O can also be used as a routing mechanism (but would require a central controller and appropriate communication mechanisms) for evacuation networks.

Evacuation networks are relevant in emergency and disaster preparedness, for example, in the case of hurricanes, [ending floods, etc. as well as during quick-onset disasters.

A Numerical User-Optimized Example

Consider the network in the S-O numerical example in which there are three nodes: 1, 2, 3; three links: a, b, c; and a single O/D pair $w_1 = (1, 3)$ with demand $d_{w_1} = 15$. Let path $p_1 = (a, c)$ and path $p_2 = (b, c)$.



The individual/user link cost functions are: $c_a(f) = f_a + 15$, $c_b(f) = f_b + 25$, and $c_c(f) = f_c + 5$.

A Numerical User-Optimized Example

The demand $d_{w_1} = 15$.

The user-optimized (U-O) path flow pattern is:

$$x_{p_1}^* = 12\frac{1}{2}, \quad x_{p_2}^* = 2\frac{1}{2},$$

which induces the U-O link flow pattern:

$$f_a^* = 12\frac{1}{2}, \quad f_b^* = 2\frac{1}{2}, \quad f_c^* = 15.$$

Indeed, under this flow pattern the individual/user costs on both paths, and both paths are used are:

$$C_{p_1} = C_{p_2} = 47\frac{1}{2}.$$

The U-O conditions (15) are satisfied.

An Interesting Comparison

Let us now evaluate the individual path costs under the S-O flow pattern for this network example.

Recall that the S-O path flow pattern was: $x_{p_1} = 10$, $x_{p_2} = 5$, with $f_a = 10$, $f_b = 5$, and $f_c = 15$.

The individual path costs under this S-O flow pattern would, hence, be: $C_{p_1} = 55$, and $C_{p_2} = 50$, which implies that some of those in need at the demand point would wait 55 units of time for the critical product, whereas others would wait 50 units of time.

Recall that, under the U-O solution, all those at the demand point would received the good in $47\frac{1}{2}$ units of time, which is not only faster, but, also, more equitable.

References

- ⇒ M. J. Beckmann, C. B. McGuire, and C. B. Winsten, 1956. *Studies in the Economics of Transportation*, Yale University Press, New Haven, Connecticut; also published as Rand-RM-1488-PR, Rand Corporation, Santa Monica, CA, May 12, 1955
<http://cowles.econ.yale.edu/archive/reprints/specpub-BMW.pdf>
- ⇒ S. C. Dafermos and F. T. Sparrow, 1969. The traffic assignment problem for a general network, *Journal of Research of the National Bureau of Standards* **73B**, 91-118.
- ⇒ A. Gatignon, L. N. Van Wassenhove, and A. Charles, 2010. The Yogyakarta earthquake: Humanitarian relief through IFRC's decentralized supply chain, *International Journal of Production Economics* **126**, 102-110.
- ⇒ A. Haghani and S.-C. Oh, 1996. Formulation and solution of a multi-commodity, multi-modal network flow model for disaster relief operations, *Transportation Research A* **30**, 231-250.
- ◇ A. Nagurney, 2009. A system-optimization perspective for supply chain network integration: The horizontal merger case, *Transportation Research E* **45**, 1-15.

References

- ⇒ A. Nagurney and Q. Qiang, 2009. *Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain World*, John Wiley & Sons, Hoboken, New Jersey.
- ⇒ A. Nagurney, M. Yu, and Q. Qiang, 2011. Multiproduct humanitarian healthcare supply chains: A network modeling and computational framework, *Proceedings of the 23rd Annual POMS Conference*, Chicago, Illinois, April 20-23, 2012.
- ⇒ R. Tomasini and L. N. Van Wassenhove, 2009. *Humanitarian Logistics*, Palgrave, Macmillan, London, England.
- ⇒ J. G. Wardrop, 1952. Some theoretical aspects of road traffic research, *Proceedings of the Institute of Civil Engineers*, Part II, pp. 325-378.