Lecture 7: An Integrated Disaster Relief Model

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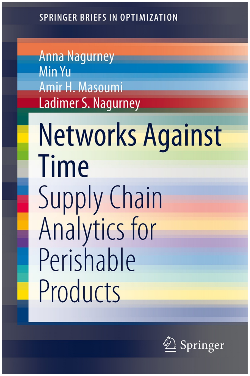
Background: Natural Disasters

This lecture is based on the paper: "An Integrated Disaster Relief Supply Chain Network Model with Time Targets and Demand Uncertainty," Anna Nagurney, Amir H. Masoumi, and Min Yu, *Regional Science Matters: Studies Dedicated to Walter Isard*, P. Nijkamp, A. Rose, and K. Kourtit, Editors, Springer International Publishing Switzerland (2015), pp 287-318, where references can be found.

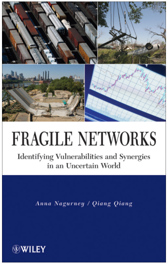
One of Our Recent Books

Our recent book focuses on supply chains for perishable products with chapters also relevant to disaster relief supply chains (on blood supply chains and pharmaceuticals).



Background: Natural Disasters

The number of natural disasters and the sizes of the populations affected by such events have been growing (Schultz, Koenig, and Noji (1996) and Nagurney and Qiang (2009)).



Scientists are warning that we can expect **more frequent extreme weather events** in the future. For example, tropical cyclones, which include **hurricanes** in the United States, are expected to be stronger as a result of global warming (Sheppard (2011) and Borenstein (2012)).

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Hence, being prepared against potential disasters leads to reduced vulnerability and a lower number of fatalities.

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Hence, being prepared against potential disasters leads to reduced vulnerability and a lower number of fatalities.

"During a natural disaster, one has only two options: **to become a victim, or to become a responder**" (Alvendia-Quero (2012)).

Background: Disaster Relief Supply Chains

The *complexity* of disaster relief supply chains originates from *several inherent factors*:

- **Large demands** for relief products,
- Level of **uncertainty**,
- **Irregularities** in the size, the timing, and the location of relief product demand patterns,
- Disaster-driven supply chains are typically **incident-responsive**, and
- may result in new networks of relationships within **days** or even **hours**, and have very **short life-cycles**.

Background: Criticality of *Time*

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FEMA's key benchmarks in response and recovery:

- To meet the survivors initial demands within **72 hours,**
- To restore basic community functionality within **60 days,** and
- To return to as normal of a situation within **5 years.**

Regional Science and Disaster Management

Disasters necessarily affect regions and pose challenges in all phases of disaster management.

Vivid examples of disasters such as Hurricane Katrina in August 2005, the Haiti earthquake in January 2010, Fukushima in March 2011, and Superstorm Sandy in October 2012 have challenged researchers, practitioners, as well as policy-makers and other decision-makers and have yielded multidisciplinary approaches to models, methods, and techniques with major contributors being from regional science (cf. West and Lenze (1994), Israelevich et al. (1997), Rose et al. (1997), Okuyama, Hewings, and Sonis (1999), Cho et al. (2001), Okuyama (2004), Rose and Liao (2005), Ham, Kim, and Boyce (2005), Greenberg, Lahr, and Mantell (2007), Grubesic et al. (2008), Reggiani and Nijkamp (2009), Nagurney and Qiang (2009), and Rose (2009), among others).

Regional Science and Disaster Management

MacKenzie and Barker (2011) integrated a risk-management approach with a Multiregional Input-Output model using ideas from Isard et al. (1998), *Methods of Interregional and Regional Analysis*, Ashgate Publishing, Aldershot, UK, to quantify the regional economic impacts of a supply shortage.

Additional Relevant Literature Chronology

- Okuyama, Y. (2003). Economics of natural disasters: A critical review. Paper presented at the 50th North American Meeting, Regional Science Association International Philadelphia, Pennsylvania.

- Beamon, B., Kotleba, S. (2006) Inventory management support systems for emergency humanitarian relief operations in South Sudan. *International Journal of Logistics Management* **17(2)**, 187–212.

- Tzeng, G.-H., Cheng, H.-J., Huang, T. (2007) Multi-objective optimal planning for designing relief delivery systems. *Transportation Research E* **43(6)**, 673–686.

- Sheu, J.-B. (2010) Dynamic relief-demand management for emergency logistics operations under large-scale disasters. *Transportation Research E* **46**, 1–17.

Additional Relevant Literature Chronology

- Nagurney, A., Yu, M., Qiang, Q. (2011) Supply chain network design for critical needs with outsourcing. *Papers in Regional Science* **90(1)**, 123–143.
- Ortuño, M.T., Tirado, G., Vitoriano, B. (2011) A lexicographical goal programming based decision support system for logistics of humanitarian aid. *TOP* **19(2)**, 464–479.
- Qiang, Q., Nagurney, A. (2012) A bi-criteria indicator to assess supply chain network performance for critical needs under capacity and demand disruptions. *Transportation Research A* **46(5)**, 801–812.
- Nagurney, A., Masoumi, A., Yu, M. (2012) Supply chain network operations management of a blood banking system with cost and risk minimization. *Computational Management Science* **9(2)**, 205–231.

Additional Relevant Literature Chronology

- Rottkemper, B., Fischer, K., Blecken, A. (2012) A transshipment model for distribution and inventory relocation under uncertainty in humanitarian operations. *Socio-Economic Planning Sciences* **46**, 98–109.

- Huang, M., Smilowitz, K., Balcik, B. (2012) Models for relief routing: Equity, efficiency and efficacy. *Transportation Research E* **48**, 2-18.

- Ortuño, M. T., Cristóbal, P., Ferrer, J. M., Martín-Campo, F. J., Muñoz, S., Tirado, G., Vitoriano, B. (2013) Decision aid models and systems for humanitarian logistics: A survey. In *Decision Aid Models for Disaster Management and Emergencies*, Atlantis Computational Intelligence Systems, vol. 7, Vitoriano, B., Montero, J., Ruan, D., Editors, Springer Business + Science Media, New York, pp. 17–44.

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Integrated Disaster Relief Supply Chain Network Model

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In addition, our model allows for the **pre-disaster and the post-disaster procurement of relief items**, and involves the time and the cost associated with each strategy or a combination of both.

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Our model, in contrast to much of the literature, does not consider targets for cost; instead, it minimizes the total operational costs of the activities in the supply chain network.

In addition, our model allows for the **pre-disaster and the post-disaster procurement of relief items**, and involves the time and the cost associated with each strategy or a combination of both.

Moreover, **we handle nonlinear costs, which capture congestion effects, a big issue in disaster relief**, and an aspect that has been missing from much of the literature on the topic (cf. Haghani and Oh (1996)).

Our Goal is to Assist in Disaster Relief (Pre and Post)

Integrated Disaster Relief Supply Chain Network Model

The disaster relief supply chain network model captures both the **preparedness phase** and the **response phase** of the disaster management cycle (with the other two phases being mitigation and recovery) (Tomasini and Van Wassenhove (2009)).

• We take into account the pre-disaster preparations including the procurement, the pre-positioning, and the storage of disaster relief items given the estimated demand in disaster-prone areas.

• We also take into consideration the relevant issues surrounding the transportation and the ultimate distribution of the relief goods to the demand points once a potential disaster takes place.

• Furthermore, the case where an organization procures the humanitarian aid items after the occurrence of a disaster is also integrated into our model.

We propose an **integrated** supply chain network model for disaster relief.

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Our mathematical framework is of **system-optimization** type where the organization aims to satisfy the uncertain demands subject to the minimization of total operational costs while the sequences of activities leading to the ultimate delivery of the relief good are **targeted to be completed within a certain time.**

Integrated Disaster Relief Supply Chain Network Model

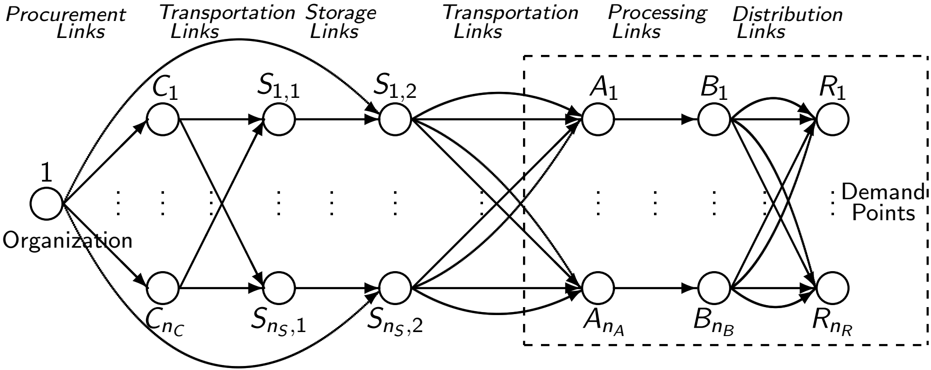

Figure: Network Topology of the Integrated Disaster Relief Supply Chain

$G = [N, L]$: supply chain network graph,
N: set of nodes, L: set of links (arcs).

Notation

$\mathcal{P}_k$: the set of paths connecting the origin (node 1) to demand point k,
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n_p: the total number of paths in the supply chain.

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Total Operational Cost Functions

$$\hat{c}_a(f_a) = f_a \times c_a(f_a), \qquad \forall a \in L, \tag{1}$$

where:
f_a: the flow of the disaster relief product on link a, and
$c_a(f_a)$: the unit operational cost function on link a.

Probability Distributions of Demand at Demand Points

$$P_k(D_k) = P_k(d_k \leq D_k) = \int_0^{D_k} \mathcal{F}_k(t)dt, \qquad k = 1, \ldots, n_R, \quad (2)$$

d_k: the actual value of demand at point k,
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Demand Shortages and Surpluses

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \qquad k = 1, \ldots, n_R, \qquad (3a)$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \qquad k = 1, \ldots, n_R, \qquad (3b)$$

v_k: the "projected demand" for the disaster relief item at point k.

The Expected Values of Shortages and Surpluses

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k)\mathcal{F}_k(t)dt, \qquad k = 1, \ldots, n_R, \qquad (4a)$$

$$E(\Delta_k^+) = \int_{0}^{v_k} (v_k - t)\mathcal{F}_k(t)dt, \qquad k = 1, \ldots, n_R. \qquad (4b)$$

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The Expected Penalties due to Shortages and Surpluses

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \qquad k = 1, \ldots, n_R, \qquad (5)$$

λ_k^-: the unit penalty associated with the shortage of the relief item at point k,
λ_k^+: the unit penalty associated with the surplus of the relief item at point k.

Path Flows

$$x_p \geq 0, \qquad \forall p \in \mathcal{P}. \tag{6}$$

x_p: the flow of the disaster relief goods on path p joining node 1 with a demand node.

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Relationship between Path Flows and Projected Demand

$$v_k \equiv \sum_{p \in \mathcal{P}_k} x_p, \qquad k = 1, \ldots, n_R. \tag{7}$$

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Relationship between Link Flows and Path Flows

$$f_a = \sum_{p \in \mathcal{P}} x_p \, \delta_{ap}, \qquad \forall a \in L. \tag{8}$$

Completion Time on Links

$$\tau_a(f_a) = g_a f_a + h_a, \qquad \forall a \in L, \tag{9}$$

where:
τ_a: the completion time of the activity on link a. Note: $h_a \geq 0$, and $g_a \geq 0$.

Completion Time on Paths

$$\tau_p = \sum_{a \in L} \tau_a(f_a)\delta_{ap} = \sum_{a \in L}(g_a f_a + h_a)\delta_{ap}, \qquad \forall p \in \mathcal{P}. \qquad (10)$$

τ_p: the completion time of the sequence of activities on path p.

$$h_p = \sum_{a \in L} h_a \delta_{ap}, \qquad \forall p \in \mathcal{P}. \qquad (11)$$

Hence,

$$\tau_p = h_p + \sum_{a \in L} g_a f_a \delta_{ap}, \qquad \forall p \in \mathcal{P}. \qquad (12)$$

h_p: the sum of the uncongested terms h_as on path p.

Time Targets

$$\tau_p \leq T_k, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R, \tag{13}$$

T_k: the target for the completion time of the activities on paths corresponding to demand point k determined by the organization.

$$\sum_{a \in L} g_a f_a \delta_{ap} \leq T_k - h_p, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{14}$$

$$T_{kp} = T_k - h_p, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{15}$$

T_{kp}: the target time for demand point k with respect to path p.

$$\sum_{a \in L} g_a f_a \delta_{ap} \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{16}$$

Late Delivery with Respect to Target Times

$$\sum_{a \in L} g_a f_a \delta_{ap} - z_p \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{17}$$

z_p: the amount of deviation with respect to target time T_{kp} corresponding to the "late" delivery of product to point k on path p.

Nonnegativity Assumption

$$z_p \geq 0, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{18}$$

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Time Constraints

$$\sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q \delta_{aq} \delta_{ap} - z_p \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{19}$$

Capturing Time

Nonnegativity Assumption

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Time Constraints

$$\sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q \delta_{aq} \delta_{ap} - z_p \leq T_{kp}, \qquad \forall p \in \mathcal{P}_k; \quad k = 1, \ldots, n_R. \tag{19}$$

$$\hat{C}_p(x) = x_p \times C_p(x) = x_p \times \sum_{a \in L} c_a(f_a) \delta_{ap}, \qquad \forall p \in \mathcal{P}, \tag{20}$$

$\hat{C}_p(x)$: the total operational cost function on path p.
C_p: the unit operational cost on path p.

The Optimization Formulation

$$\text{Minimize} \sum_{p \in \mathcal{P}} \hat{C}_p(x) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z), \tag{21}$$

subject to: constraints (6), (18), and (19), where $\gamma_k(z)$ **is the tardiness penalty function** corresponding to demand point k.

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$$\text{Minimize} \sum_{p \in \mathcal{P}} \hat{C}_p(x) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) + \sum_{k=1}^{n_R} \gamma_k(z), \quad (21)$$

subject to: constraints (6), (18), and (19), where $\gamma_k(z)$ **is the tardiness penalty function** corresponding to demand point k.

The Feasible Set

$$K = \{(x, z, \omega) | x \in R_+^{n_p}, z \in R_+^{n_p}, \text{ and } \omega \in R_+^{n_p}\}, \quad (22)$$

where x is the vector of path flows, z is the vector of time deviations on paths, and ω is the vector of Lagrange multipliers corresponding to (19).

Theorem: The Variational Inequality Formulation

The optimization problem (21), subject to its constraints (6), (18), and (19), is equivalent to the variational inequality problem: determine the vectors of optimal path flows, optimal path time deviations, and optimal Lagrange multipliers $(x^*, z^*, \omega^*) \in K$*, such that:*

$$\sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \lambda_k^+ P_k(\sum_{q \in \mathcal{P}_k} x_q^*) - \lambda_k^- (1 - P_k(\sum_{q \in \mathcal{P}_k} x_q^*)) \right.$$

$$\left. + \sum_{q \in \mathcal{P}} \sum_{a \in L} \omega_q^* g_a \delta_{aq} \delta_{ap} \right] \times [x_p - x_p^*] + \sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[\frac{\partial \gamma_k(z^*)}{\partial z_p} - \omega_p^* \right] \times [z_p - z_p^*]$$

$$+ \sum_{k=1}^{n_R} \sum_{p \in \mathcal{P}_k} \left[T_{kp} + z_p^* - \sum_{q \in \mathcal{P}} \sum_{a \in L} g_a x_q^* \delta_{aq} \delta_{ap} \right] \times [\omega_p - \omega_p^*] \geq 0, \; \forall (x, z, \omega) \in K, \quad (23)$$

where

$$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}, \quad \forall p \in \mathcal{P}_k; \; k = 1, \ldots, n_R. \quad (24)$$

An Illustrative Example

Consider the simple disaster relief supply chain network topology in Figure 2. There are 2 modes of transportation with links d and e representing ground and air transportation, respectively.

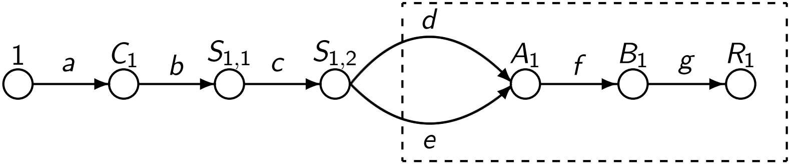

Figure: Supply Chain Network Topology for an Illustrative Numerical Example

An Illustrative Example

The total operational cost functions on the links are:

$$\hat{c}_a(f_a) = 3f_a^2 + 2f_a, \quad \hat{c}_b(f_b) = f_b^2 + 3f_b, \quad \hat{c}_c(f_c) = 2f_c^2 + f_c,$$

$$\hat{c}_d(f_d) = 4f_d^2 + 3f_d, \quad \hat{c}_e(f_e) = 7f_e^2 + 5f_e,$$

$$\hat{c}_f(f_f) = f_f^2 + 4f_f, \quad \hat{c}_g(f_g) = 3f_g^2 + 2f_g.$$

There are two paths: $p_1 \equiv (a, b, c, d, f, g)$, $p_2 \equiv (a, b, c, e, f, g)$.

An Illustrative Example

The demand for the relief item at the demand point follows a uniform distribution on the interval [5,10]; therefore, the probability distribution function of demand at the demand point is:

$$P_{R_1}(v_{R_1}) = \frac{v_{R_1} - 5}{10 - 5} = \frac{x_{p_1} + x_{p_2} - 5}{5}.$$

The unit shortage and surplus penalties are: $\lambda^-_{R_1} = 5000$ and $\lambda^+_{R_1} = 100$.

An Illustrative Example

The organization is interested in the pre-positioning strategy; i.e., it wishes to determine the amount of the relief item that should be stored beforehand. Hence, the completion time on links a, b, and c (procurement, transportation, and storage) is set to zero:

$$\tau_a(f_a) = \tau_b(f_b) = \tau_c(f_c) = 0.$$

The completion time functions on the rest of the links are:

$$\tau_d(f_d) = 9f_d + 6, \quad \tau_e(f_e) = 2f_e + 2,$$

$$\tau_f(f_f) = 1.5f_f + 2, \quad \tau_g(f_g) = 5f_g + 4.$$

The target time at demand point R_1 is 72 hours:

$$T_{R_1} = 72, \quad \forall p \in \mathcal{P}_{R_1}.$$

An Illustrative Example

The decision-maker assigns a higher tardiness penalty to p_2 in that the expectation of on-time delivery from the path with the air transportation link was higher, so the tardiness penalty function at the demand point was:

$$\gamma_{R_1}(z) = 3.5z_{p_1}^2 + 8z_{p_2}^2.$$

Solution to the Illustrative Example

The solution to this problem can be obtained by solving a system of equations as we demonstrate in our paper.

The optimal path flows are:

$$x^*_{p_1} = 1.04 \text{ and } x^*_{p_2} = 7.50$$

and the optimal values of link flows:

$$f^*_a = f^*_b = f^*_c = f^*_f = f^*_g = 8.54, f^*_d = 1.04, f^*_e = 7.50.$$

The value of the projected demand at R_1 is:

$$v^*_{R_1} = x^*_{p_1} + x^*_{p_2} = 8.54,$$

which is the amount that needs to be pre-positioned at the storage facility.

Solution to the Illustrative Example

The optimal time deviations on paths p_1 and p_2 with respect to the target of 72 hours are:

$$z^*_{p_1} = 4.85 \text{ and } z^*_{p_2} = 6.47.$$

Neither of the two transportation modes to the affected area would be able to satisfy the target time requirement. Interestingly, the time deviation is higher on the path that contains the air route, which is due to the majority of the load being allocated to this mode.

The optimal values of the Lagrange multipliers corresponding to the time goal constraints are:

$$\omega^*_{p_1} = 33.97 \text{ and } \omega^*_{p_2} = 103.55.$$

A Variant of the Illustrative Example

This example addresses the post-disaster procurement strategy – as opposed to pre-positioning of the supplies. We assumed that the organization did not store disaster items beforehand. In other words, the organization would procure relief items only once a disaster struck.

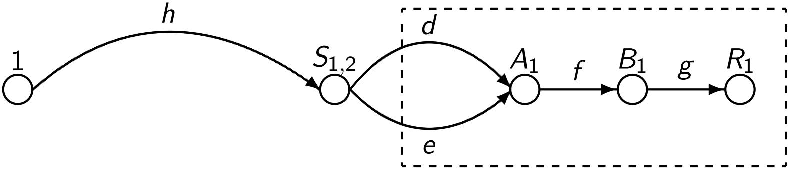

Figure: Supply Chain Network Topology for a Variabt of the Illustrative Numerical Example

A Variant of the Illustrative Example

The total cost functions and the time completion functions on links d, e, f, and g remain the same as in the illustrative example. As for link h, we have:

$$\hat{c}_h(f_h) = 5f_h^2 + 3f_h \quad \text{and} \quad \tau_h(f_h) = 3f_h + 3.$$

In this example, the total operational cost on the post-disaster procurement link, h, is higher than that on the pre-disaster procurement link a in the original example. The set of paths in this problem is $\mathcal{P} = \{p_3, p_4\}$ where $p_3 \equiv (h, d, f, g)$ and $p_4 \equiv (h, e, f, g)$.

The demand distribution, the shortage and surplus penalties, as well as the target time are identical to those in the previous problem.

A Variant of the Illustrative Example

The tardiness penalty function at the demand point is the same, except that now it is a function of the time deviations on new paths:

$$\gamma_{R_1}(z) = 3.5z_{p_3}^2 + 8z_{p_4}^2.$$

Solution of the Variant

This example can also be solved using a system of equations as in the paper. The optimal path flows are:

$$x^*_{p_3} = 0.33, \quad x^*_{p_4} = 6.26,$$

which yields the following optimal link flows:

$$f^*_h = f^*_f = f^*_g = 6.59, f^*_d = 0.33, f^*_e = 6.26.$$

Note that, under a post-disaster procurement strategy, the optimal flow on the ground transportation link is significantly lower than that in the pre-disaster procurement strategy. In addition, the optimal flow on the air transportation link has experienced a 17% decrease which is a consequence of increased operational costs.

Solution of the Variant

The value of the projected demand is:

$$v_{R_1}^* = x_{p_3}^* + x_{p_4}^* = 6.59,$$

which is 23% lower than that for the original example.

The optimal time deviations on paths p_3 and p_4 are:

$$z_{p_3}^* = 8.54, \quad z_{p_4}^* = 14.09,$$

which are higher than their respective values in the first example. The incurred tardiness penalty value, $\gamma_{R_1}(z^*)$, is equal to $1,844.16$, i.e., a 321% increase from that in the first example.

The optimal Lagrange multipliers on paths p_3 and p_4 are:

$$\omega_{p_3}^* = 59.77, \quad \omega_{p_4}^* = 225.49.$$

Sensitivity Analysis for the Variant Example

Table 1 displays the optimal values of the path flows, $x^*_{p_3}$ and $x^*_{p_4}$, the path time deviations, $z^*_{p_3}$ and $z^*_{p_4}$, and the Lagrange multipliers, $\omega^*_{p_3}$ and $\omega^*_{p_4}$, as the unit shortage penalty, $\lambda^-_{R_1}$, is increased from $2,500$ to $12,500$ in the variant.

Table: Sensitivity Analysis of the Optimal Solution to the Unit Shortage Penalty at the Demand Point for the Variant Example

$\lambda^-_{R_1}$	$x^*_{p_3}$	$x^*_{p_4}$	$z^*_{p_3}$	$z^*_{p_4}$	$\omega^*_{p_3}$	$\omega^*_{p_4}$	Value of OF (21)
2,500	0.50	5.56	5.09	7.66	35.66	122.58	5,081.96
5,000	0.33	6.26	8.54	14.09	59.77	225.49	8,440.02
7,500	0.20	6.79	11.18	19.02	78.25	304.39	11,021.81
10,000	0.09	7.22	13.26	22.91	92.80	366.49	13,035.31
12,500	0.01	7.56	14.94	26.05	104.57	416.72	14,655.25

Sensitivity Analysis for the Variant Example

As seen in Table 1, as the shortage penalty increases, the organization will be fulfilling a higher projected demand, $x^*_{p_3} + x^*_{p_4}$, by assigning higher quantities to the path that uses air transportation to the affected region.

At $\lambda^-_{R_1} = 12,500$, the optimal path flow on the ground transportation path is almost zero, which means that the organization relies on the air transport mode. Handling larger volumes of goods increases the congestion on paths which, in turn, worsens the lateness of deliveries to the region.

Explicit Formulae for the Euler Method (Dupuis and Nagurney (1993)) Applied to Variational Inequality (23)

At iteration $\tau + 1$, compute the path flows, the path time deviations, and the Lagrange multipliers, according to:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^-(1 - P_k(\sum_{q\in\mathcal{P}_k} x_q^\tau)) - \lambda_k^+ P_k(\sum_{q\in\mathcal{P}_k} x_q^\tau) - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p}$$

$$- \sum_{q\in\mathcal{P}} \sum_{a\in L} \omega_q^\tau g_a \delta_{aq}\delta_{ap})\}, \quad \forall p \in \mathcal{P}_k, \forall k, \tag{25}$$

$$z_p^{\tau+1} = \max\{0, z_p^\tau + a_\tau(\omega_p^\tau - \frac{\partial \gamma_k(z^\tau)}{\partial z_p})\}, \quad \forall p \in \mathcal{P}_k, \forall k, \tag{26}$$

$$\omega_p^{\tau+1} = \max\{0, \omega_p^\tau + a_\tau(\sum_{q\in\mathcal{P}} \sum_{a\in L} g_a x_q^\tau \delta_{aq}\delta_{ap} - T_{kp} - z_p^\tau\}, \forall p \in \mathcal{P}_k, \forall k. \tag{27}$$

Background for the Larger Numerical Examples

The scenario for the first larger numerical example is built on the possibility of another earthquake striking Haiti. We then construct a variant.

The next figure displays the disaster relief supply chain network topology corresponding to the case of a Haiti earthquake.

Node 1 represents the American Red Cross. We assume that the Red Cross can utilize two of its disaster aid zones in the US, one in Maryland, representing the Northeast and the East, and the other one in Florida, the closest state to the Caribbean. Each of the two zones is assumed to possess a single procurement facility, a single storage facility, and a single departure portal.

The Supply Chain Network Topology

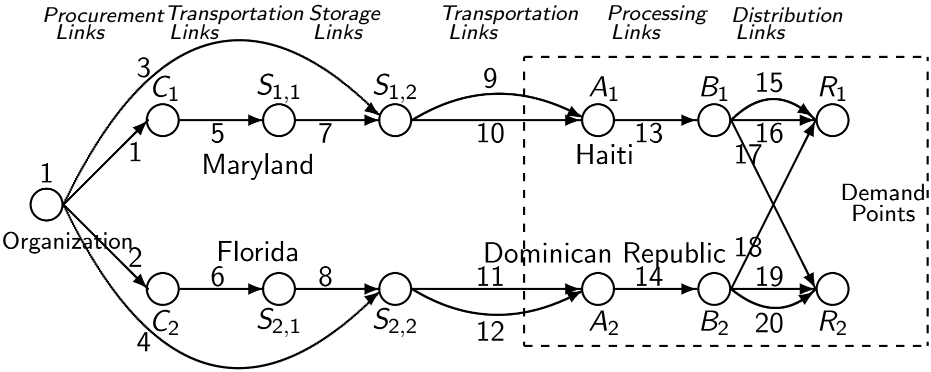

Figure: Network Topology of the Disaster Relief Supply Chain Numerical Examples

Background for the Larger Numerical Examples

Both locations have the ability to start procuring the relief goods after an earthquake strikes when and if the need arises (links 3 and 4). These procured/purchased goods at the zones are directly sent to their respective departure portals bypassing the storage phase. Next, from these departure facilities, the collected items whether pre-positioned beforehand or just procured are sent via air or sea to the affected region.

We assume that the facility $S_{1,2}$, Maryland, would cover the arrival facility A_1 in Haiti, and the facility $S_{2,2}$ would serve the arrival port A_2 in the Dominican Republic.

Links 9 and 12 represent air transportation whereas links 10 and 11 correspond to marine transportation. After the arrived cargo is sorted and processed, relief items are distributed to the points of demand, R_1 and R_2, both located in Haiti.

Background for the Larger Numerical Examples

Links 15 and 20 correspond to the distribution of goods by helicopter, whereas links 16 through 19 represent ground distribution.

We allow for each of the two processing facilities located in Haiti and the Dominican Republic, i.e., B_1 and B_2, to ship to both of the two demand points in Haiti.

Background for the Larger Numerical Examples

The Euler method (cf.(25)–(27)) for the solution of variational inequality (23) was implemented in FORTRAN on a Linux system at the University of Massachusetts Amherst. We set the sequence as $\{a^\tau\} = .1(1, \frac{1}{2}, \frac{1}{2}, \ldots)$, and the convergence tolerance was 10^{-6}.

The unit shortage and surplus penalties at demand points R_1 and R_2:

$$\lambda^-_{R_1} = 10,000, \ \lambda^+_{R_1} = 100,$$

$$\lambda^-_{R_2} = 7,500, \ \lambda^+_{R_2} = 150.$$

The target times of delivery at demand points:

$$T_{R_1} = 72, \ T_{R_2} = 70.$$

The tardiness penalty functions at demand points:

$$\gamma_{R_1}(z) = 3(\sum_{p \in \mathcal{P}_{R_1}} z_p^2), \quad \gamma_{R_2}(z) = 3(\sum_{p \in \mathcal{P}_{R_2}} z_p^2).$$

Table 2: Link Total Cost and Time Functions and Optimal **Link** Flows

Link	$\hat{c}_a(f_a)$	$\tau_a(f_a)$	f_a^*
1	$3f_1^2 + 2f_1$	0	19.22
2	$2f_2^2 + 2.5f_2$	0	20.02
3	$5f_3^2 + 4f_3$	$3f_3 + 3$	0.00
4	$4.5f_4^2 + 3f_4$	$4f_4 + 2$	0.00
5	$f_5^2 + 2f_5$	0	19.22
6	$f_6^2 + .5f_6$	0	20.02
7	$2.5f_7^2 + 3f_7$	0	19.22
8	$3.5f_8^2 + 2f_8$	0	20.02
9	$7f_9^2 + 5f_9$	$2f_9 + 2$	19.22
10	$4f_{10}^2 + 6f_{10}$	$10f_{10} + 6$	0.00
11	$2.5f_{11}^2 + 4f_{11}$	$7.5f_{11} + 5$	0.23
12	$4.5f_{12}^2 + 5f_{12}$	$1.5f_{12} + 1.5$	19.79
13	$2f_{13}^2 + 4f_{13}$	$2f_{13} + 2$	19.22
14	$f_{14}^2 + 3f_{14}$	$1.5f_{14} + 1$	20.02
15	$4f_{15}^2 + 5f_{15}$	$3f_{15} + 3$	13.95
16	$2.5f_{16}^2 + 2f_{16}$	$5f_{16} + 4$	5.28
17	$3f_{17}^2 + 4f_{17}$	$6.5f_{17} + 3$	0.00
18	$4f_{18}^2 + 4f_{18}$	$7f_{18} + 5$	6.85
19	$3f_{19}^2 + 3f_{19}$	$4f_{19} + 5$	5.68
20	$3.5f_{20}^2 + 5f_{20}$	$3.5f_{20} + 4$	7.49

As seen in Table 2, the optimal flows on links 3 and 4, i.e., the post-disaster procurement links, are zero. Hence, given the demand information and the cost and time functions on the supply chain network links, the organization would be better off by adopting the pre-positioning strategy.

In addition, links 10 and 11, corresponding to marine transportation of goods from the US to the affected region have zero or very small flows. Such an outcome is due to the importance of timely deliveries, and, thus, the organization needs to ship via air to minimize the lateness on the demand end.

Discussion

Similarly, among the distribution links, the ones representing shipments by helicopter (links 15 and 20) are assigned relatively higher loads whereas link 17 corresponding to one of the ground distribution links will not be utilized.

Also, the optimal flows are almost equal on links 1 and 2 suggests an even split of pre-positioning of the load between the two US aid regions.

Table 3: Path Definitions, Target Times, **Optimal Path Flows**, Time Deviations, and Lagrange Multipliers

	Path Definition	T_{kp}	x_p^*	z_p^*	ω_p^*
$\mathcal{P}_{R_1}$: Set of Paths Corresponding to Demand Point R_1	$p_1 = (1,5,7,9,13,15)$	65	13.95	53.66	321.99
	$p_2 = (1,5,7,9,13,16)$	64	5.28	39.23	235.39
	$p_3 = (1,5,7,10,13,15)$	61	0.00	19.32	115.90
	$p_4 = (1,5,7,10,13,16)$	60	0.00	4.83	28.99
	$p_5 = (2,6,8,11,14,18)$	61	0.06	18.67	112.03
	$p_6 = (2,6,8,12,14,18)$	64.5	6.79	43.12	258.75
	$p_7 = (3,9,13,15)$	62	0.00	56.66	339.99
	$p_8 = (3,9,13,16)$	61	0.00	42.23	253.39
	$p_9 = (3,10,13,15)$	58	0.00	22.34	134.05
	$p_{10} = (3,10,13,16)$	57	0.00	7.84	47.03
	$p_{11} = (4,11,14,18)$	59	0.00	20.71	124.24
	$p_{12} = (4,12,14,18)$	62.5	0.00	45.24	271.46
$\mathcal{P}_{R_2}$: Set of Paths Corresponding to Demand Point R_2	$p_{13} = (1,5,7,9,13,17)$	63	0.00	13.87	83.25
	$p_{14} = (1,5,7,10,13,17)$	59	0.00	0.00	0.00
	$p_{15} = (2,6,8,11,14,19)$	59	0.13	0.00	0.00
	$p_{16} = (2,6,8,11,14,20)$	60	0.04	0.00	0.00
	$p_{17} = (2,6,8,12,14,19)$	62.5	5.55	19.91	119.44
	$p_{18} = (2,6,8,12,14,20)$	63.5	7.45	22.40	134.43
	$p_{19} = (3,9,13,17)$	60	0.00	16.90	101.41
	$p_{20} = (3,10,13,17)$	56	0.00	0.00	0.00
	$p_{21} = (4,11,14,19)$	57	0.00	0.00	0.00
	$p_{22} = (4,11,14,20)$	58	0.00	0.00	0.00
	$p_{23} = (4,12,14,19)$	60.5	0.00	21.96	131.77
	$p_{24} = (4,12,14,20)$	61.5	0.00	24.48	146.85

Discussion

From Table 3 we see that, that among the 24 paths in the supply chain network, fewer than one-third have considerable positive flows since the others involve links that are either costlier or more time-consuming.

From the optimal values of time deviations on paths, one can observe that significant deviations from the target times have occurred on several paths in the network. This seems to be more of an issue in the paths connecting the origin to demand point R_1, i.e., the hypothetically more vulnerable location.

Such an outcome may mandate additional investments on critical transportation/distribution channels to R_1 which can be done in accordance with the optimal values of respective Lagrange multipliers.

Discussion

The higher the value of the Lagrange multiplier on a path, the more improvement in time can be attained by enhancing that path which, in turn, leads to a more efficient disaster response system.

A Variant of the Numerical Example

We assumed that the organization will now procure the items **locally** and, hence, the time functions associated with the direct procurement links 3 and 4 are now greatly reduced. The remainder of the input data remains the same as in the previous example.

Table 4: Numerical Example **Variant** - Optimal Link Flows

Link	$\hat{c}_a(f_a)$	$\tau_a(f_a)$	f_a^*
1	$3f_1^2 + 2f_1$	0	12.02
2	$2f_2^2 + 2.5f_2$	0	11.21
3	$5f_3^2 + 4f_3$	$.1f_3 + 1$	7.35
4	$4.5f_4^2 + 3f_4$	$.1f_4 + 1$	8.88
5	$f_5^2 + 2f_5$	0	12.02
6	$f_6^2 + .5f_6$	0	11.21
7	$2.5f_7^2 + 3f_7$	0	12.02
8	$3.5f_8^2 + 2f_8$	0	11.21
9	$7f_9^2 + 5f_9$	$2f_9 + 2$	19.37
10	$4f_{10}^2 + 6f_{10}$	$10f_{10} + 6$	0.00
11	$2.5f_{11}^2 + 4f_{11}$	$7.5f_{11} + 5$	0.24
12	$4.5f_{12}^2 + 5f_{12}$	$1.5f_{12} + 1.5$	19.86
13	$2f_{13}^2 + 4f_{13}$	$2f_{13} + 2$	19.37
14	$f_{14}^2 + 3f_{14}$	$1.5f_{14} + 1$	20.10
15	$4f_{15}^2 + 5f_{15}$	$3f_{15} + 3$	14.04
16	$2.5f_{16}^2 + 2f_{16}$	$5f_{16} + 4$	5.33
17	$3f_{17}^2 + 4f_{17}$	$6.5f_{17} + 3$	0.00
18	$4f_{18}^2 + 4f_{18}$	$7f_{18} + 5$	6.84
19	$3f_{19}^2 + 3f_{19}$	$4f_{19} + 5$	5.72
20	$3.5f_{20}^2 + 5f_{20}$	$3.5f_{20} + 4$	7.53

Discussion

As reported in Table 4, now both the storage links for pre-positioning (links 7 and 8) and for post-disaster procurement (links 3 and 4) have positive flows.

Hence, with the new data (and decision to procure locally) the organization should engage in both strategies.

The optimal solution suggests to the organization how much of the relief good should be stored and in which location and how much should also be procured (and from where) once the disaster strikes.

Summary and Conclusions

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- enables **prioritizing the demand points** based on the population, geographic location, etc., by assigning different time targets.