Lecture 5: Network Performance, Robustness, and Resiliency

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Disasters and Critical Infrastructure

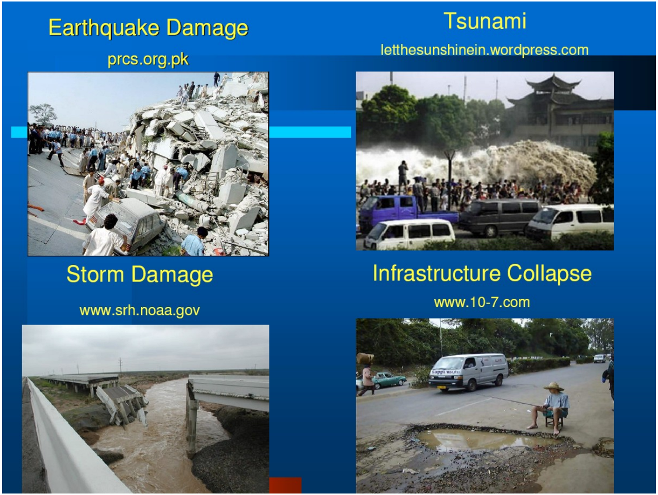


Figure: Disaster preparedness and response depends crucially on critical infrastructure, especially that of transportation networks

Disasters and Critical Infrastructure

Hence, it is essential to identify which transportation and logistical network components are the most important since their deterioration, ad even destruction, will have the biggest impacts.

Moreover, without adequate transportation infrastructure relief products cannot be delivered in a timely manner.

Some of the Recent Literature on Network Vulnerability

- Latora and Marchiori (2001, 2002, 2004)
- Holme, Kim, Yoon and Han (2002)
- Taylor and Deste (2004)
- Murray-Tuite and Mahmassani (2004)
- Chassin and Posse (2005)
- Barrat, Barthlemy and Vespignani (2005)
- Sheffi (2005)
- DallAsta, Barrat, Barthlemy and Vespignani (2006)
- Jenelius, Petersen and Mattson (2006)
- Taylor and DEste (2007)
- Nagurney and Qiang (2007, 2008, 2009)

Network Centrality Measures

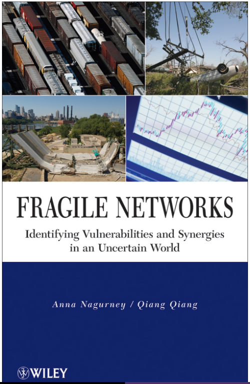
- Barrat et al. (2004, pp. 3748), The identification of the most central nodes in the system is a major issue in network characterization.
- **Centrality Measures for Non-Weighted Networks**
  - Degree, betweenness (node and edge), closeness (Freeman (1979), Girvan and Newman (2002))
  - Eigenvector centrality (Bonacich (1972))
  - Flow centrality (Freeman, Borgatti and White (1991))
  - Betweenness centrality using flow (Izquierdo and Hanneman (2006))
  - Random-work betweenness, Current-flow betweenness (Newman and Girvan (2004))
- **Centrality Measures for Weighted Networks (Very Few)**
  - Weighted betweenness centrality (Dall'Asta et al. (2006))
  - Network efficiency measure (Latora-Marchiori (2001))

Which Nodes and Links Really Matter?

Recall the U-O model of the previous lecture.

We first focus on U-O and then on S-O and construct relevant network performance measures and importance indicators as well as robustness measures.

These measures were developed by A. Nagurney and Q. Qiang in a series of publications. A reference is their book, which contains a complete set of citations.



The Nagurney and Qiang (N-Q) Network Efficiency / Performance Measure

Definition: A Unified Network Performance Measure
The network performance/efficiency measure, $\mathcal{E}(G, d)$*, for a given network topology* G *and the equilibrium (or fixed) demand vector* d*, is:*

$$\mathcal{E} = \mathcal{E}(G, d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where recall that n_W *is the number of O/D pairs in the network, and* d_w *and* λ_w *denote, for simplicity, the equilibrium (or fixed) demand and the equilibrium disutility for O/D pair* w*, respectively.*

Definition: Importance of a Network Component

The importance of a network component $g \in G$, $I(g)$, is measured by the relative network efficiency drop after g is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G,d) - \mathcal{E}(G-g,d)}{\mathcal{E}(G,d)}$$

where $G - g$ is the resulting network after component g is removed from network G.

The Approach to Identifying the Importance of Network Components

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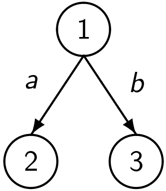
The N-Q measure is well-defined even in the case of disconnected networks.

A Numerical Example

Consider the network below in which there are two O/D pairs: $w_1 = (1, 2)$ and $w_2 = (1, 3)$ with demands given, respectively, by $d_{w_1} = 100$ and $d_{w_2} = 20$. We have that path $p_1 = a$ and path $p_2 = b$. Assume that the link cost functions are given by: $c_a(f_a) = .01f_a + 19$ and $c_b(f_b) = .05f_b + 19$. Clearly, we must have that $x^*_{p_1} = 100$ and $x^*_{p_2} = 20$ so that $\lambda_{w_1} = \lambda_{w_2} = 20$. The network efficiency measure $\mathcal{E}=\frac{1}{2}(\frac{100}{20} + \frac{20}{20}) = 3.0000$.



The importance values and the rankings of the links and the nodes for this Example are given, respectively, in the following Tables.

Table: Importance Values and Ranking of Links in the Example

Link	Importance Value from $\mathcal{E}$	Importance Ranking from $\mathcal{E}$
a	0.8333	1
b	0.1667	2

Table: Importance Values and Ranking of Nodes in the Example

Node	Importance Value from $\mathcal{E}$	Importance Ranking from $\mathcal{E}$
1	1.0000	1
2	0.8333	2
3	0.1667	3

$\mathcal{E}$, which captures flow information, is a precise measure, since, in the case of a disruption, the destruction of link a, with which was associated a flow 5 times the flow on link b, would result in a greater loss of efficiency. The same holds for the destruction of node 2 vs. node 3.

An Application to the Braess Paradox

In order to further reinforce the above concepts, we now recall the well-known Braess (1968) paradox; see also Braess, Nagurney, and Wakolbinger (2005). This paradox is as relevant to transportation networks as it is to telecommunication networks, and, in particular, to the Internet, since such networks are subject to flows operating in a decentralized decision-making manner.

Assume a network as the first network depicted in the next Figure in which there are four nodes: $1, 2, 3, 4$; four links: a, b, c, d; and a single O/D pair $w = (1, 4)$. There are, hence, two paths available for this O/D pair: $p_1 = (a, c)$ and $p_2 = (b, d)$.

An Application to the Braess Paradox

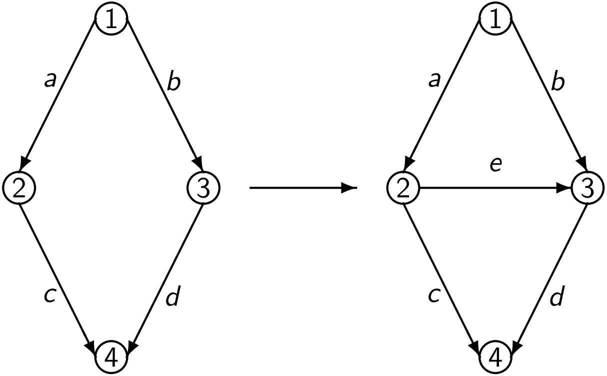


Figure: The Braess Network Example

The individual/user link cost functions are:

$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50, \quad c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

Assume a fixed demand $d_w = 6$.

It is easy to verify that the equilibrium path flows are: $x^*_{p_1} = 3$, $x^*_{p_2} = 3$, the equilibrium link flows are: $f^*_a = 3, \quad f^*_b = 3$, $f^*_c = 3, \quad f^*_d = 3$, with associated equilibrium path costs: $C_{p_1} = c_a + c_c = 83$, $C_{p_2} = c_b + c_d = 83$.

An Application to the Braess Paradox

Assume now that, as depicted in the Figure, a new link "e", joining node 2 to node 3 is added to the original network, with user link cost function $c_e(f_e) = f_e + 10$. The addition of this link creates a new path $p_3 = (a, e, d)$ that is available. Assume that the demand d_w remains at 6 units of flow. Note that the original flow distribution pattern $x_{p_1} = 3$ and $x_{p_2} = 3$ is no longer an equilibrium pattern, since at this level of flow the individual cost on path p_3, $C_{p_3} = c_a + c_e + c_d = 70$, so users of the network would switch paths.

The equilibrium flow pattern on the new network is: $x^*_{p_1} = 2$, $x^*_{p_2} = 2$, $x^*_{p_3} = 2$, with equilibrium link flows: $f^*_a = 4$, $f^*_b = 2$, $f^*_c = 2$, $f^*_e = 2$, $f^*_d = 4$, and with associated equilibrium user path travel costs: $C_{p_1} = C_{p_2} = C_{p_3} = 92$. Note that the cost increased for every user of the network from 83 to 92 without a change in the demand!

An Application to the Braess Paradox

We now apply the unified network efficiency measure $\mathcal{E}$ to the Braess network with the link e to identify the importance and ranking of nodes and links. The results are reported in the Tables.

Table: Link Results for the Braess Network

Link	$\mathcal{E}$ Measure Importance Value	$\mathcal{E}$ Measure Importance Ranking
a	.2069	1
b	.1794	2
c	.1794	2
d	.2069	1
e	-.1084	3

An Application to the Braess Paradox

Table: Nodal Results for the Braess Network

Node	$\mathcal{E}$ Measure Importance Value	$\mathcal{E}$ Measure Importance Ranking
1	1.0000	1
2	.2069	2
3	.2069	2
4	1.0000	1

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- It is **applicable to dynamic networks**, **including the Internet** (Nagurney and Qiang, *Netnomics* (2008)).

Some Applications of the N-Q Measure

The Sioux Falls Network

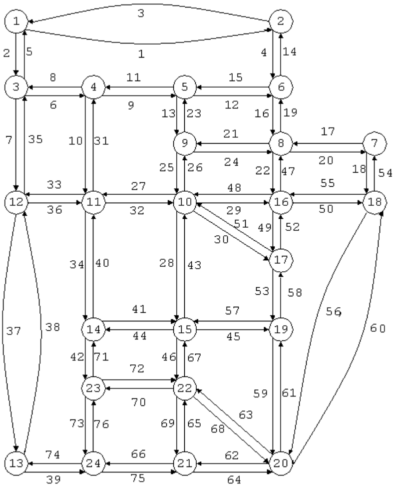

Figure: The Sioux Falls network with 24 nodes, 76 links, and 528 O/D pairs of nodes.

The computed network efficiency measure $\mathcal{E}$ for the Sioux Falls network is $\mathcal{E} = 47.6092$. Links 56, 60, 36, and 37 are the most important links, and hence special attention should be paid to protect these links accordingly, while the removal of links 10, 31, 4, and 14 would cause the least efficiency loss.

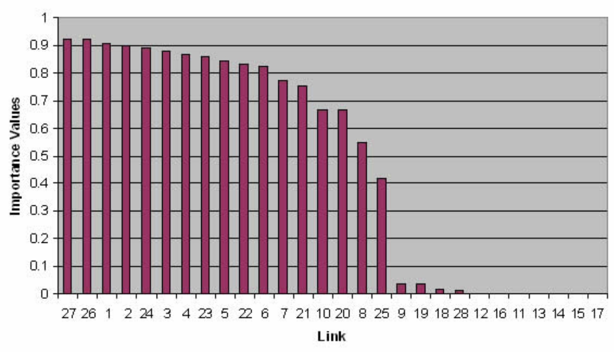

Figure: The Sioux Falls network link importance rankings

According to the European Environment Agency (2004), **since 1990, the annual number of extreme weather and climate-related events has doubled, in comparison to the previous decade**. These events account for approximately 80% of all economic losses caused by catastrophic events. In the course of climate change, catastrophic events are projected to occur more frequently (see Schulz (2007)).

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Schulz (2007) applied the **N-Q network efficiency measure to a German highway system in order to identify the critical road elements** and found that this measure provided more reasonable results than the measure of Taylor and DEste (2007).

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The N-Q measure can also be used to assess which links should be added to improve efficiency. **This measure was used for the evaluation of the proposed North Dublin (Ireland) Metro system** (October 2009 Issue of *ERCIM News*).

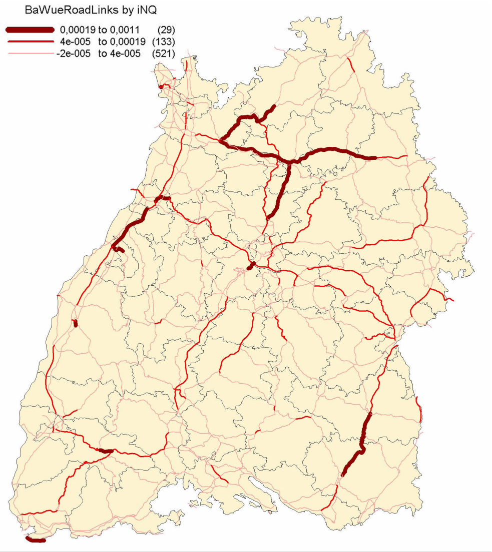

Figure: Comparative Importance of the links for the Baden - Wurttemberg Network – Modelling and analysis of transportation networks in earthquake prone areas via the N-Q measure, Tyagunov et al.

What About Transportation Network Robustness?

Network Robustness

The concept of *system robustness* has been studied in engineering and computer science. IEEE (1990) defined robustness as **"the degree to which a system or component can function correctly in the presence of invalid inputs or stressful environmental conditions."**

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Ali et al. (2003) considered an allocation mapping to be robust if it **"guarantees the maintenance of certain desired system characteristics despite fluctuations in the behavior of its component parts or its environment."**

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Holmgren (2007) stated: **"Robustness signifies that the system will retain its system structure (function) intact (remain unchanged or nearly unchanged) when exposed to perturbations."**

Network Robustness Measure Under User-Optimizing Decision-Making Behavior

Definition: Network Robustness Measure Under User-Optimizing Decision-Making Behavior

The robustness measure $\mathcal{R}^\gamma$ for a network G with the vector of user link cost functions c, the vector of link capacities u, the vector of demands d (either fixed or elastic) is defined as the relative performance retained under a given uniform capacity retention ratio γ with $\gamma \in (0,1]$ so that the new capacities are given by γu. Its mathematical definition is

$$\mathcal{R}^\gamma = \mathcal{R}(G, c, \gamma, u) = \frac{\mathcal{E}^\gamma}{\mathcal{E}} \times 100\%$$

where $\mathcal{E}$ and $\mathcal{E}^\gamma$ are the network performance measures with the original capacities and the remaining capacities, respectively.

Network Robustness Measure Under User-Optimizing Decision-Making Behavior

For example, if $\gamma = .8$, this means that the user link cost functions now have the link capacities given by $.8u_a$ for all links $a \in \mathcal{L}$; if $\gamma = .4$, then the link capacities become $.4u_a$ for all links $a \in \mathcal{L}$, and so on.

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We can also study network robustness from **the perspective of network capacity enhancement**.

Such an analysis **provides insights into link investments**. In this case $\gamma \geq 1$ and, for definiteness (and as suggested in Nagurney and Qiang (2009)), we refer to the network robustness measure in this context as the **"capacity increment ration."**

Each link of the Anaheim network has a link travel cost functional form of the BPR form. There are 461 nodes, 914 links, and 1,406 O/D pairs in the Anaheim network.

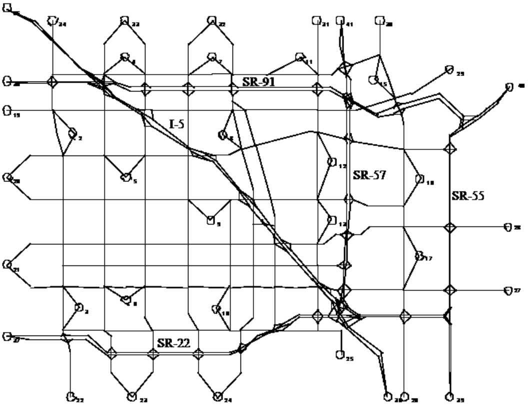

Figure: The Anaheim network

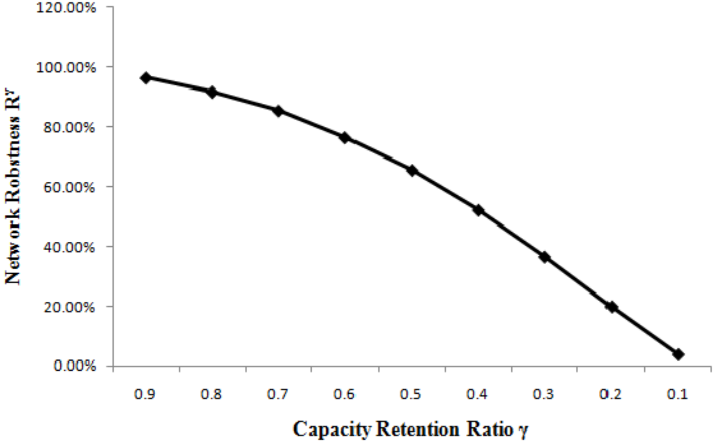

Figure: Robustness vs. Capacity Retention Ratio for the Anaheim Network

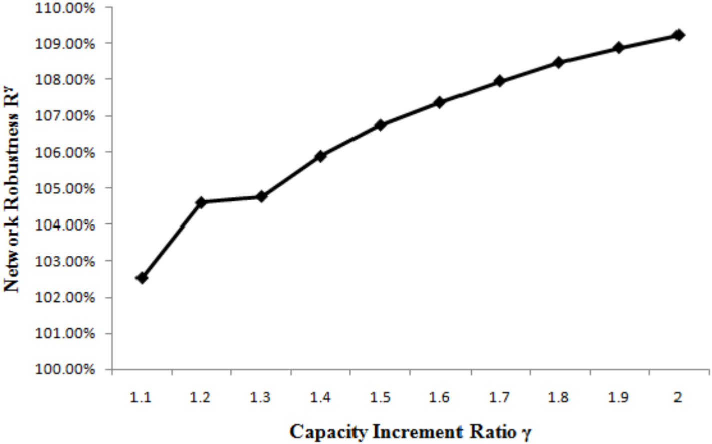

Figure: Robustness vs. Capacity Increment Ratio for the Anaheim Network

Different Perspectives on Transportation Network Robustness

Relative Total Cost Indices

The definition of the index under the user-optimizing flow pattern, denoted by $\mathcal{I}^{\gamma}_{U-O}$:

$$\mathcal{I}^{\gamma}_{U-O} = \mathcal{I}_{U-O}(G, c, d, \gamma, u) = \frac{TC^{\gamma}_{U-O} - TC_{U-O}}{TC_{U-O}} \times 100\%,$$

where TC_{U-O} and TC^{γ}_{U-O} are the total network costs evaluated under the U-O flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

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where TC_{U-O} and TC^{γ}_{U-O} are the total network costs evaluated under the U-O flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

The definition of the index under the system-optimizing flow pattern is:

$$\mathcal{I}^{\gamma}_{S-O} = \mathcal{I}_{S-O}(G, c, d, \gamma, u) = \frac{TC^{\gamma}_{S-O} - TC_{S-O}}{TC_{S-O}} \times 100\%,$$

where TC_{S-O} and TC^{γ}_{S-O} are the total network costs evaluated at the S-O flow pattern with the capacities as above.

Relative Total Cost Indices

From these definitions, a network, under a given capacity retention/deterioration ratio γ (and either S-O or U-O behavior) is **considered to be robust if the index $\mathcal{I}^{\gamma}$ is low.**

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From these definitions, a network, under a given capacity retention/deterioration ratio γ (and either S-O or U-O behavior) is **considered to be robust if the index $\mathcal{I}^{\gamma}$ is low.**

This means that the relative total cost does not change much; hence the network may be viewed as being more robust than if the relative total cost were large.

Relative Total Improvement Indices

We can also study the relative total cost improvement after capacity enhancement. In that case, because the relative total cost savings need to be computed, we reverse the order of subtraction in the previous expressions with $\gamma \geq 1$. Furthermore, γ is defined as the "capacity increment ratio."

Therefore, the larger the relative total cost index is, the greater the expected total cost savings for a capacity enhancement plan for a specific γ.

The **price of anarchy**, $\mathcal{P}$, defined as

$$\mathcal{P} = \frac{TC_{U-O}}{TC_{S-O}},$$

captures the relationship between total costs *across* distinct behavioral principles, whereas the above indices are focused on the degradation of network performance *within* U-O or S-O behavior.

The relationship between the ratio of the two indices and the price of anarchy

$$\frac{I^{\gamma}_{S-O}}{I^{\gamma}_{U-O}} = \frac{[TC^{\gamma}_{S-O} - TC_{S-O}]}{[TC^{\gamma}_{U-O} - TC_{U-O}]} \times \mathcal{P}.$$

The term preceding the price of anarchy may be less than 1, greater than 1, or equal to 1, depending on the network and data.

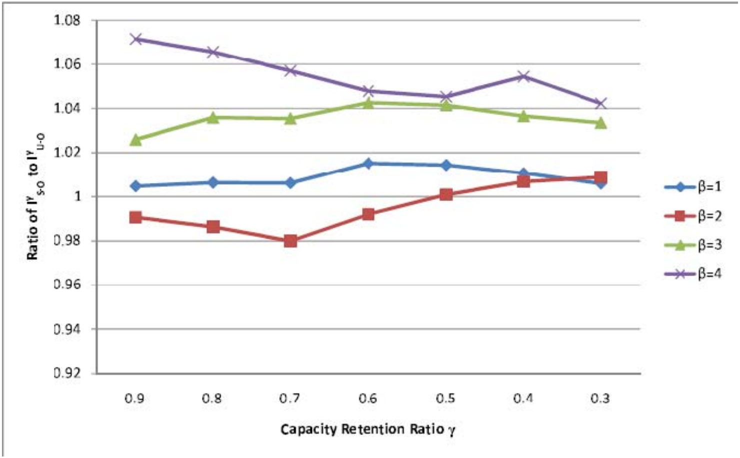


Figure: Example: The Sioux Falls network

This network is always more robust under U-O behavior except when β is equal to 2 (where β is the power to which the link flow is raised to into the BPR function) and $\gamma \in [0.5, 0.9]$.

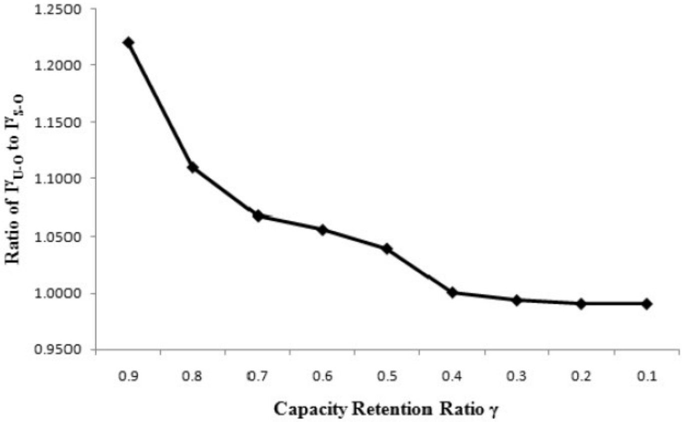
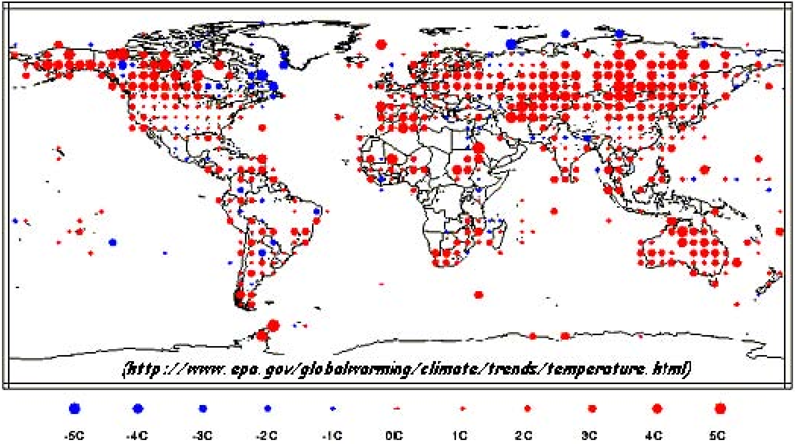


Figure: Example: The Anaheim network

This network is more robust under the S-O solution when the capacity retention ratio γ is above .3.

Which Nodes and Links Matter Environmentally?



Source: Global Historical Climate Network,
National Oceanic and Atmospheric Administration

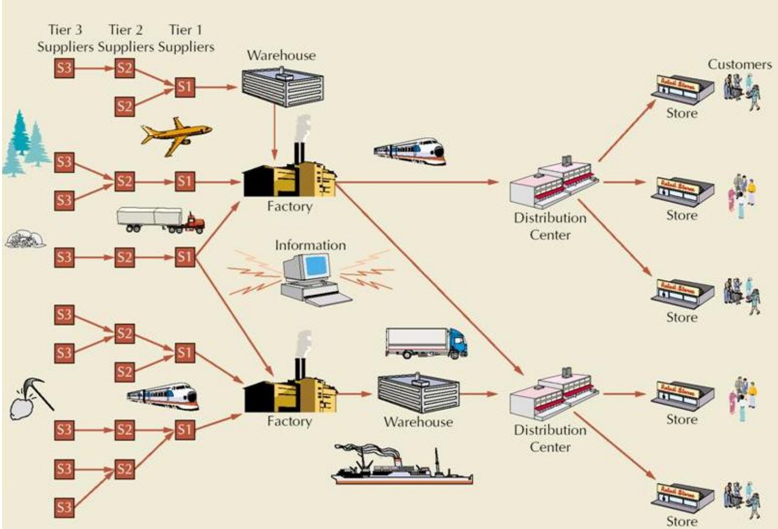
Figure: Global Annual Mean Temperature Trend 1950–1999

Figure: Impacts of climate change on transportation infrastructure

We have also extended our measures to construct environmental impact assessment indices and environmental link importance identifiers under either U-O or S-O behaviors.

What About Transportation's Role in Disaster Relief?

A General Supply Chain



Haiti Earthquake in 2010

Delivering the humanitarian relief supplies (water, food, medicines, etc.) to the victims was a major logistical challenge.

Humanitarian Relief

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The period between 2000-2004 experienced an average annual number of disasters that was 55% higher than the period of 1995-1999 with 33% more people affected in the more recent period (Balcik and Beamon (2008)).

According to ISDR (2006) 157 million people required immediate assistance due to disasters in 2005 with approximately 150 million requiring asistance the year prior (Balcik and Beamon (2008)).

The supply chain is a critical component not only of corporations but also of humanitarian organizations and their logistical operations.

At least 50 cents of each dollars worth of food aid is spent on transport, storage and administrative costs (Dugger (2005)).

Vulnerability of Humanitarian Supply Chains

Extremely poor logistic infrastructures: Modes of transportation include trucks, barges, donkeys in Afghanistan, and elephants in Cambodia (Shister (2004)).

To ship the humanitarian goods to the affected area in the first 72 hours after disasters is crucial. The successful execution is not just a question of money but a difference between life and death (Van Wassenhove (2006)).

Corporations expertise with logistics could help public response efforts for nonprofit organizations (Sheffi (2002), Samii et al.(2002)).

In the humanitarian sector, organizations are 15 to 20 years behind, as compared to the commercial arena, regarding supply chain network development (Van Wassenhove (2006)).

It is clear that better-designed supply chain networks in which transportation plays a pivotal role would have facilitated and enhanced various emergency preparedness and relief efforts and would have resulted in less suffering and lives lost.

Critical Needs Products

Critical needs products are those that are **essential to the survival of the population**, and can include, for example, vaccines, medicine, food, water, etc., depending upon the particular application.

The demand for the product should be met as nearly as possible since otherwise there may be additional loss of life.

In times of crises, a **system-optimization** approach is mandated since the demands for critical supplies should be met (as nearly as possible) at minimal total cost.

Critical Needs Products

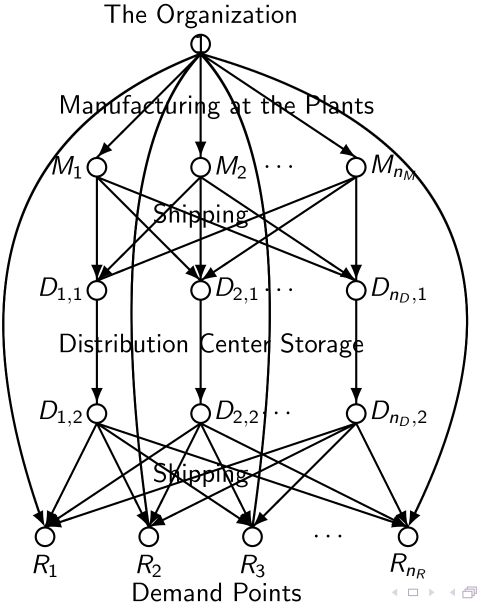
We have now developed a framework for the optimal design of critical needs product supply chains:

Supply chain network design for critical needs with outsourcing,

A. Nagurney, M. Yu, and Q. Qiang, 2011. *Papers in Regional Science* **90**, 123-142,

where additional background as well as references can be found.

Supply Chain Network Topology with Outsourcing



References

Many of the references noted in this lecture can be found in the papers below, the first paper of which contains a condensation of some of the major findings reported in the book: A. Nagurney and Q. Qiang, 2009. *Fragile Networks: Identifying Vulnerabilities and Synergies and an Uncertain World*, John Wiley & Sons, Hoboken, New Jersey.

⇒ A. Nagurney and Q. Qiang, 2012. Fragile networks: Identifying vulnerabilities and synergies in an uncertain age, *International Transactions in Operational Research* **19**, 123-160.

⇒ A. Nagurney, M. Yu, and Q. Qiang, 2011. *Papers in Regional Science* **90**, 123-142.