OIM 413 Logistics and Transportation Lecture 11: The Multimodal Model

Professor Anna Nagurney

John F. Smith Memorial Professor
and
Director – Virtual Center for Supernetworks
Department of Operations & Information Management

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Multimodal Transportation in the Real World

Multimodal Transportation

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Different Modes of Transportation

Minnesota Department of Transportation

Transport and Traffic Panel - New York Times Energy for Tomorrow Conference

New York Times Energy for Tomorrow Conference, April 25, 2013, Transport and Traffic Panel

The Multimodal Model

There are k different modes of transportation using the network, denoted by 1, 2, ..., k.

Assumptions

(1). Each mode (or class of user) has its own cost function.

(2). Each mode (or class of user) contributes to its own and other modes' costs in an individual way.

The Multimodal Model

Topological characteristics of networks remain the same as in the single-modal models.

We use a superscript i on the various parameters and variables to denote mode i.

The O/D travel demands, path flows, link flows, and path costs now become k-dimensional vectors, where k denotes the number of different modes of transportation on the network.

The Path and Link Flows

We denote the multimodal flow on paths and links as follows:

F_p^i is the flow on path p by mode i; $i = 1, \ldots, k$;

f_a^i is the flow on link a of mode i; $i = 1, \ldots, k$.

The User Link Travel Costs and the Total Costs

The user link cost of mode i on link a:

$$c_a^i = c_a^i(f_a^1, \cdots, f_a^k)$$

- an extension of the standard model.

Hence, the user cost on a link associated with a mode of transportation may, in general, depend upon the flow of all the different modes of transportation using that link.

The total link cost of mode i on link a:

$$\hat{c}_a^i = c_a^i(f_a^1, \cdots, f_a^k) \times f_a^i.$$

The User Path Travel Costs

The user travel cost on path p for mode i:

$$C_p^i = C_p^i(f) = \sum_{a \in L} c_a^i(f_a^1, \ldots, f_a^k)\delta_{ap},$$

that is, the cost on a path of using a mode is equal to the sum of the costs on the links of that mode making up that path.

This model is also relevant to multiclass network models since different classes of users perceive travel cost in a different way (business travelers versus students versus stay at home workers, for example).

An Example of Multimodal User Link Cost Functions

Linear User Travel Cost Functions For a mode i;

$i = 1, \dots, k$:

$$c_a^i(f_a^1, ..., f_a^k) = \sum_j g_a^{ij} f_a^j + h_a^i.$$

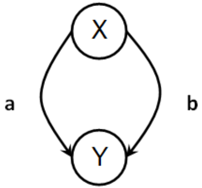
For mode 1:

$$c_a^1 = g_a^{11} f_a^1 + g_a^{12} f_a^2 + \dots + h_a^1.$$

The Multimodal Travel Demands

With every O/D pair w, we associate a vector of travel demands $(d_w^1, d_w^2, ..., d_w^k)$, where d_w^i is the travel demand associated with O/D pair w and mode i.

An Example: 2 Modes



$$c_a^1 = 10f_a^1 + 5f_a^2 + 10, \quad c_a^2 = 5f_a^2 + 4f_a^1 + 5,$$

$$c_b^1 = 6f_b^1 + 4f_b^2 + 10, \quad c_b^2 = 3f_b^2 + 2f_b^1 + 15,$$

$$d_{xy}^1 = 10 \qquad d_{xy}^2 = 20.$$

The Conservation of Flow Equations

$d_w^i = \sum_{p \in P_w} F_p^i$, for all O/D pairs w and all modes i.

A multimodal path flow pattern is feasible if $F_p^i \geq 0$, for all i and p, and the conservation of flow equations above holds.

The link flows are related to the path flows by:

$$f_a^i = \sum_{p \in P} F_p^i \delta_{ap}, \quad \forall i, a,$$

that is, the flow on a link of a mode is equal to the sum of the flows of the mode on paths that use that link.

A link flow pattern is feasible when the above equations are satisfied by a feasible path flow pattern.

The Multimodal User-Optimized or Equilibrium Conditions

For each mode i, and every O/D pair w, F^* is an equilibrium path flow pattern, if and only if:

$$C^i_{p_1}(f^*) = \cdots = C^i_{p_s}(f^*) = \lambda^i_w \leq C^i_{p_{s+1}}(f^*) \leq \cdots \leq C^i_{p_{n_w}}(f^*)$$

$F^{i\ *}_{p_r} > 0; r = 1, \cdots, s,$

$F^{i\ *}_{p_r} = 0; r = s+1, \cdots, n_w.$

where $C^i_p(f) = \sum_{a \in L} c^i_a(f^1_a, \cdots, f^k_a)\delta_{ap}$.

Hence, all used paths by each mode have equal and minimal travel costs.

The Multimodal System-Optimized Problem

$$\text{Minimize} \quad S(f) = \sum_{i} \sum_{a \in L} \hat{c}_a^i(f_a^1, \cdots, f_a^k)$$

subject to:

$$d_w^i = \sum_{p \in P_w} F_p^i, \quad \forall i, \forall w,$$

$$f_a^i = \sum_{p \in P_w} F_p^i \delta_{ap}, \quad \forall i, \forall a,$$

$$F_p^i \geq 0, \quad \forall i, \forall p.$$

The Optimality Conditions for the Multimodal S-O Problem

For each mode i and O/D pair w, f is S-O, if and only if:

$$\hat{C}^{i'}_{p_1}(f) = \ldots = \hat{C}^{i'}_{p_{s'}}(f) = \mu^i_w \leq \hat{C}^{i'}_{p_{s'+1}}(f) \leq \ldots \leq \hat{C}^{i'}_{p_{n_w}}(f)$$

$$F^i_{p_r} > 0; r = 1, \ldots, s';$$

$$F^i_{p_r} = 0; r = s' + 1, \ldots, n_w,$$

where $\hat{C}^{i'}_p(f) = \frac{\partial S}{\partial F^i_p} =$ by the chain rule $\sum_{b \in L} \sum_j \frac{\partial \hat{c}^j_b}{\partial f^i_b} \delta_{bp}$, where $\hat{C}^{i'}_p(f)$ is the marginal of the total cost with respect to i and p.

Transformation of Multimodal Networks to Single Mode Networks

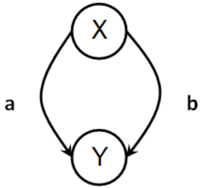
Suppose that we have a multimodal network. We can construct an equivalent single-modal (but extended cost) network as follows. The network will have a new topology, new travel demands, etc.

We do this by making multiple copies of the network, one copy for each mode of transportation.

Also, we have to then define the flows, costs, and demands accordingly on the network copies.

The *superscripts* denoting the specific modes, then become *subscripts* to represent the flows, demands, and costs, as well as the links and the paths, appropriately, on the network copies.

A Numerical Example of A Multimodal Network

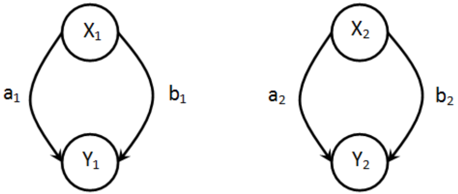


$$d_{xy}^1 = 5, \quad d_{xy}^2 = 10,$$

$$c_a^1 = 10f_a^1 + 5f_a^2 + 10, \qquad c_a^2 = 5f_a^2 + 3f_a^1 + 6,$$

$$c_b^1 = 5f_b^1 + 4f_b^2 + 3, \qquad c_b^2 = 6f_b^2 + 3f_b^1 + 10.$$

Transformation into a single-modal (extended cost) problem



O/D pair $w_1 = (x_1, y_1)$, O/D pair $w_2 = (x_2, y_2)$

$$c_{a_1} = 10f_{a_1} + 5f_{a_2} + 1, \quad c_{a_2} = 5f_{a_2} + 3f_{a_1} + 6,$$

$$c_{b_1} = 5f_{b_1} + 4f_{b_2} + 2, \quad c_{b_2} = 6f_{b_2} + 3f_{b_1} + 10,$$

$$d_{x_1 y_1} = 5, \quad d_{x_2 y_2} = 10.$$

Reminder

We always denote the U-O, equivalently, the transportation network equilibrium solution, in either link or path flows with a "*".

Applications of Multimodal Network Models

As we have emphasized in this course, the models and analytical tools covered are applicable not only to congested transportation networks, but also to other network systems.

For example, the user-optimized (U-O) multimodal model has been applied to study **the Internet**, where different modes correspond to different class of messages.

Also, the multimodal system-optimized (S-O) model can be applied in **humanitarian logistics operations** as well as **healthcare supply chains** in which the goal is to deliver multiple needed goods (including medicines) to the populations in need in a total cost minimizing manner.

Remember, here we have been treating cost as a *generalized* **cost which can also capture time.**

References

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