OIM 413 Logistics and Transportation
Lecture 8: Tolls

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Congestion is Not a New Phenomenon

The study of the efficient operation of transportation networks dates to ancient Rome with a classical example being the publicly provided Roman road network and the time of day chariot policy, whereby chariots were banned from the ancient city of Rome at particular times of day.

Tolls

Tolls are a policy that, when constructed and applied appropriately, can guarantee that the S-O flow pattern is, at the same time, U-O. Hence, through the use of tolls, travelers' decision-making behavior can be altered so that total costs to society associated with travel are minimized.

Let the S-O link flow pattern be f.

Let the U-O flow link pattern be f'.

What do we know?

The total cost on the network with the flow pattern f is **always**

$$\leq$$

the total cost on the network with the flow pattern f'.

Remedy

Modify the travel costs as perceived by users by charging them a toll. This procedure does not change the travel cost spent as perceived by society (tolls aren't lost).

Then try to determine the **toll pattern** in such a way that the S-O becomes at the same time U-O.

Conventional Toll Collection

Conventional Toll Collection

borgawker.com

Electronic Toll Collection

blog.pennlive.com

Electronic Tolls on the Massachusetts Turnpike

In late October 2016, the Massachusetts Turnpike switched to an all-electronic tolling (AET) system and the old toll booths have now been demolished as part of a months-long process.

Tolls Around the Globe

Toll roads are found in many countries. The way that they are funded and operated may differ from country to country and even from state to state or region to region. Some toll roads are privately owned and operated. Others are owned by the government. Some of the government-owned toll roads are privately operated.

Under BOT (Build-Operate-Transfer) systems, private companies build the roads and are given a limited franchise. Ownership is transferred to the government when the franchise expires.

Tolls Around the Globe

Around the globe, this type of arrangement is prevalent in Australia, South Korea, Japan, Philippines, and Canada.

The (BOT) system is a fairly new concept that is gaining ground in the United States, with Arkansas, California, Delaware, Florida, Illinois, Indiana, Mississippi, Texas, and Virginia already building and operating toll roads under this scheme.

Pennsylvania, Massachusetts, New Jersey, and Tennessee are also considering the BOT methodology for future highway projects.

Tolls and Congestion Charges

Tolls are also commonly referred to as congestion charges.

Congestion pricing is also back in the news for New York City with New York State Governor Mario Cuomo saying that this may be a way in which to obtain funds for investments in much needed transportation (including subway) infrastructure.

Congestion charges are relevant to other network systems from electric power generation and distribution networks to the Internet.

Two Types of Toll Policies

Link Toll Collection Policy - a toll r_a is associated with each link a of the network.

Then, we set $R_L = r_a; \ a \in L$ – a link toll policy.

Path Toll Collection Policy - a toll r_p is associated with each path p of the network.

Then we set $R_p = r_p; \ p \in P$ – a path toll policy.

Tolls

Before the imposition of tolls

Travel cost incurred by users as perceived by society and by users themselves is the same.

1. on a link $a -$ $c_a(f_a)$,
2. on a path $p -$ $C_p = \sum_{a \in L} c_a(f_a)\delta_{ap}$.

Tolls

After the imposition of tolls

Travel costs incurred by users as perceived by society **still** given by 1 and 2. The tolls aren't lost.

However, the travel cost as perceived by **users** changes as follows:

1. Link Toll Policy

$$(1.1)\ C_p^r = C_p^r(f) = \sum_{a\in L}[c_a(f_a) + r_a] \times \delta_{ap} = C_p(f) + \sum_{a\in L} r_a\delta_{ap}$$

2. Path Toll Policy

$$(2.1)\ C_p^r = C_p^r(f) = C_p(f) + r_p.$$

Tolls

After the imposition of tolls, the corresponding S-O pattern is the same as before the imposition of tolls.

However, the U-O pattern changes.

1. Link toll collection problem

Determine a link toll pattern $R_L = r_a; \ a \in L$ so that the S-O pattern for a given network is **at the same time** U-O.

2. Path toll collection problem

Determine a path toll pattern $R_p = r_p; \ p \in P$ so that the S-O pattern for a given network is **at the same time** U-O.

Tolls

We know that f is an S-O pattern, if and only if, it satisfies, for every O/D pair $w \in W$:

$$\hat{C}'_{p_1}(f) = \dots = \hat{C}'_{p_{s'}}(f) = \mu_w \leq \hat{C}'_{p_{s'+1}}(f) \leq \dots \leq \hat{C}'_{p_{n_w}}(f)$$

$F_{p_r} > 0; \ r = 1, \dots, s';$ **(II)**

$F_{p_r} = 0; \ r = s' + 1, \dots, n_w,$

where

$$\hat{C}'_p(f) = \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}.$$

Tolls

After the imposition of tolls, the above S-O pattern is at the same time U-O, if and only if, it satisfies the equilibrium conditions (U-O), that is,

$$C_{p_1}(f) = ... = C_{p_s}(f) = \lambda_w \leq C_{p_{s+1}}(f) \leq ... \leq C_{p_{n_w}}(f)$$

$F_{p_r} > 0; \;\; r = 1, ..., s;$ **(I)**

$F_{p_r} = 0; \;\; r = s+1, ..., n_w,$

where $C_p(f)$ is given by (1.1) for a link toll policy and by (2.1) for a path toll policy.

$*$ When do (I) and (II) have the same solution?

Tolls

If for an f that satisfies II:

$\hat{C}'_p(f) = C^r_p(f)$ for all $p \in P,\ w \in W$ **(1)**

then (I) and (II) coincide; hence, we will have identical S-O and U-O solutions.

From (1) we have:

(3.1) $\hat{C}'_p = C_p(f) + r_p$

(the case of a path toll policy)

(3.2) $\hat{C}'_p = \sum_{a \in L} (c_a(f_a) + r_a)\delta_{ap} = C_p(f) + \sum_{a \in L} r_a \delta_{ap}.$

(the case of a link toll policy)

(3.1) suggests the following **path toll policy:**

$r_p = \hat{C}'_p - C_p(f)$, for all paths p, where f = S-O pattern.

Hence, we have a path toll policy:

$R_p = r_p;\ \ p \in P.$

Link toll policy

We can replace (3.2) by:

$$\hat{C}'_p = C_p(f) + \sum_{a \in L} r_a \delta_{ap} = \sum_{a \in L} (c_a(f_a) + r_a)\delta_{ap}$$

or

$$\sum_{a \in L} \hat{c}'_a(f_a)\delta_{ap} = \sum_{a \in L} (c_a(f_a) + r_a)\delta_{ap}$$

or

$$\hat{c}'_a(f_a) = c_a(f_a) + r_a, \quad \forall a \in L,$$

or

$$r_a = \hat{c}'_a(f_a) - c_a(f_a), \quad \forall a \in L.$$

Note that the link toll policy according to the above formula guarantees that the link tolls $\{r_a\}$ are always nonnegative!

Hence, the procedure to determine link tolls is as follows:

Step 1: Solve for the S-O flow pattern for the network. Denote this S-O link flow solution by f.

Step 2: Use the formula for determining the link tolls:

$$r_a = \hat{c}_a'(f_a) - c_a(f_a), \quad \forall a \in L.$$

Remark

For each link toll policy $R_L = r_a;\ a \in L$, one can construct a corresponding path toll policy $\{R_p\}$ which is induced if we set:

$$r_p = \sum_{a \in L} r_a \delta_{ap}, \quad \forall p \in P.$$

But, the converse is **not** true.

Given a path toll policy $R_p = r_p;\ p \in P$ we cannot usually find a link toll policy R_L that satisfies the above equation for each path.

Tolls

We have a system of equations with the r_as as unknowns.

# of equations = # of paths
# of unknowns = # of links

Usually # of paths > # of links.

Hence, the system does not always have a solution.

However, we have shown that the system has always one solution given by:

$$r_p = \hat{C}'_p(f) - C_p(f).$$

The General Case

A toll policy renders a S-O flow pattern user-optimized, if and only if, for every O/D pair w:

$$\left.\begin{array}{l} C^r_{p_1}(f) = C_{p_1}(f) + r_{p_1} = \lambda^r_w \\ C^r_{p_2}(f) = C_{p_2}(f) + r_{p_2} = \lambda^r_w \\ \vdots \\ C^r_{p_s}(f) = C_{p_s}(f) + r_{p_s} = \lambda^r_w \end{array}\right\} F_{p_i} > 0; \qquad i = 1, \ldots, s;$$

$$\left.\begin{array}{l} C^r_{p_{s+1}}(f) = C_{p_{s+1}}(f) + r_{p_{s+1}} = \lambda^r_w \\ \vdots \\ C^r_{p_m}(f) = C_{p_m}(f) + r_{p_{n_w}} = \lambda^r_w \end{array}\right\} F_{p_i} = 0; \qquad i = s+1, \ldots, n_w.$$

Tolls

or, equivalently,

$$\begin{aligned}
&r_{p_1} = \lambda^r_w - C_{p_1}(f) \\
&\vdots \\
&r_{p_s} = \lambda^r_w - C_{p_s}(f) \\
&r_{p_{s+1}} \geq \lambda^r_w - C_{p_{s+1}}(f) \\
&\vdots \\
&r_{p_{n_w}} \geq \lambda^r_w - C_{p_{n_w}}(f).
\end{aligned}$$

In theory, the decision-maker (policy-maker) is free to choose λ^r_w at any value.

Solution of the Path Toll Problem

Clear from the previous pages that we can construct an infinite # of solutions $R_p = r_p; \; p \in P$ to our path toll problem.

Method: We select **a priori** for each O/D pair the level of the user's travel cost λ_w^r. Then determine: $r_{p_1}, ..., r_{p_s}$, that satisfy preceding relationships.

$*$ Freedom in choosing λ_w^r - User's Cost.

Hence, possibility of imposing additional requirements.

Solution of Link Toll Problem

We need: $C_p^r(f) = \hat{C}_p^r$

$$C_p^r(f) = C_p(f) + \sum r_a \delta_{ap} = \sum_{a \in L} \hat{c}_a'(f_a)\delta_{ap} =$$

$$\sum_{a \in L} [c_a(f_a) + r_a]\delta_{ap} = \sum_{a \in L} \hat{c}_a'(f_a)\delta_{ap}$$

and $r_a = \hat{c}_a'(f_a) - c_a(f_a)$, for all links $a \in L$.

This is, in general, the **only** solution to the link toll policy problem that is nonnegative.

Summary of Link Toll and Path Toll Policies

Step 1: First determine the S-O flow pattern f for a given network. Before proceeding to Step 3, distinguish between Link Toll Policy (LTP) and the Path Toll Policy (PTP).

Link Toll Policy

Step 2: For each link a in the network, compute the link toll $r_a, \; a \in L : r_a = \hat{c}'_a(f_a) - c_a(f_a)$ using f_as calculated in Step 1. The link toll policy R_L is a solution to the link toll problem.

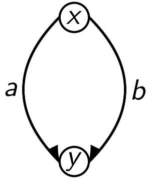
Path Toll Policy

Step 2: For each path $p \in P$ in the network, compute the user's travel cost $C_p(f) = \sum_{a \in L} c_a(f_a)\delta_{ap}$ using the f_as calculated in Step 1

Step 3: Select the level of user's cost λ_w^r after the imposition of tolls for every O/D pair w so that a certain objective is met.

Step 4: Compute now, for each w, the path tolls
$r_p = \lambda_w^r - C_p^r(f), \quad \forall p \in P.$

A Numerical Link Toll Example



Consider the above transportation network.
There is a single O/D pair $w_1 = (x, y)$ with demand $d_{w_1} = 10$. Let path $p_1 = a$ and path $p_2 = b$.
The user link cost functions are:

$$c_a(f_a) = 2f_a + 5, \quad c_b(f_b) = f_b + 10.$$

In the absence of any tolls, users would select the U-O flow pattern: $f_a^* = F_{p_1}^*$, $f_b^* = F_{p_2}^* = 5$, with user path costs: $C_{p_1} = c_a = 15, \quad C_{p_2} = c_b = 15$. This U-O flow pattern (before tolls) would result in a total network cost of $c_a \times f_a + c_b \times f_b = 75 + 75 = 150$.
The S-O link and path flow patterns are: $f_a = F_{p_1} = 4\frac{1}{6}$, $f_b = F_{p_2} = 5\frac{5}{6}$, with the marginal total path costs:

$$\hat{C}'_{p_1} = \hat{c}'_a = 21\frac{2}{3}, \quad \hat{C}'_{p_2} = \hat{c}'_b = 21\frac{2}{3},$$

with a total network cost of: $131\frac{7}{18}$.
Applying the link toll formula, we have that:

$$r_a = \hat{c}'_a(f_a) - c_a(f_a) = 21\frac{2}{3} - 13\frac{1}{3} = 8\frac{1}{3},$$

$$r_b = \hat{c}'_b(f_b) - c_b(f_b) = 21\frac{2}{3} - 15\frac{5}{6} = 5\frac{5}{6}.$$

Verification

Let's verify that if we impose the previous link tolls that the U-O solution will then coincide with the S-O solution.

Note that the user costs, after the imposition of these tolls, will be:

$$C^r_{p_1} = c^r_a = c_a + r_a = 13\frac{1}{3} + 8\frac{1}{3} = 21\frac{2}{3},$$

$$C^r_{p_2} = c^r_b = c_b + r_b = 15\frac{5}{6} + 5\frac{5}{6} + 21\frac{2}{3}.$$

Hence, the S-O flow pattern, after the imposition of the link tolls, is also U-O!

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