Game Theory Network Models for Disaster Relief

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Masterclass - Network Equilibrium
Lancaster University, England, Spring 2018

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- Methodology - The VI Problem
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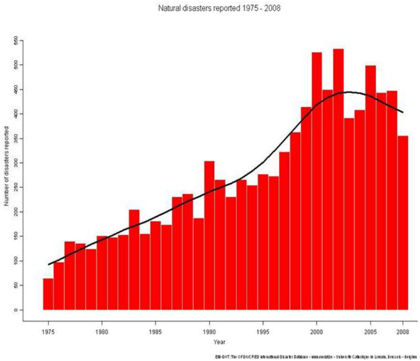
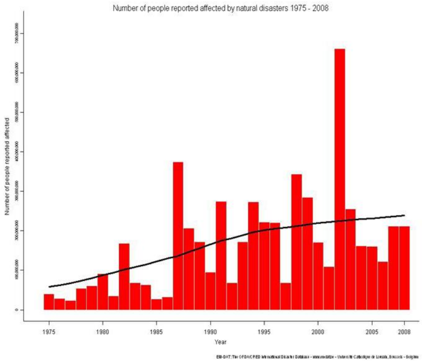
Background and Motivation

Network Models Are Also Very Useful in Disaster Relief

Also for Healthcare Supply Chains

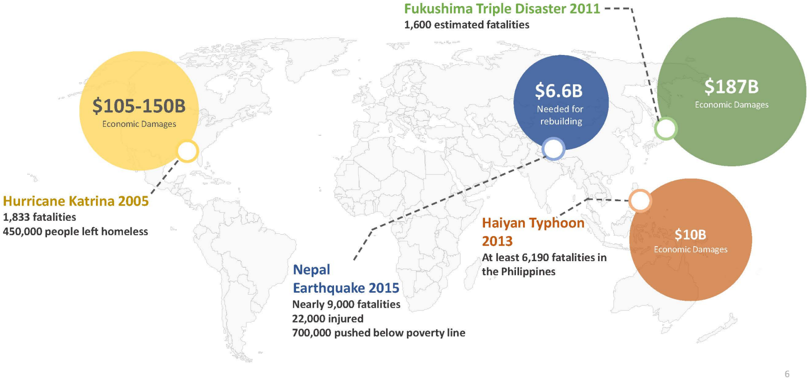
Natural Disasters (1975–2008)





Disasters have a catastrophic effect on human lives and a region's or even a nation's resources. A total of 2.3 billion people were affected by natural disasters from 1995-2015 (UN Office of Disaster Risk (2015)).

Some Recent Disasters



Hurricane Katrina in 2005

Hurricane Katrina has been called an **“American tragedy,”** in which essential services failed completely.

The Triple Disaster in Japan on March 11, 2011

Now the world is reeling from the aftereffects of the triple disaster

Superstorm Sandy and Power Outages

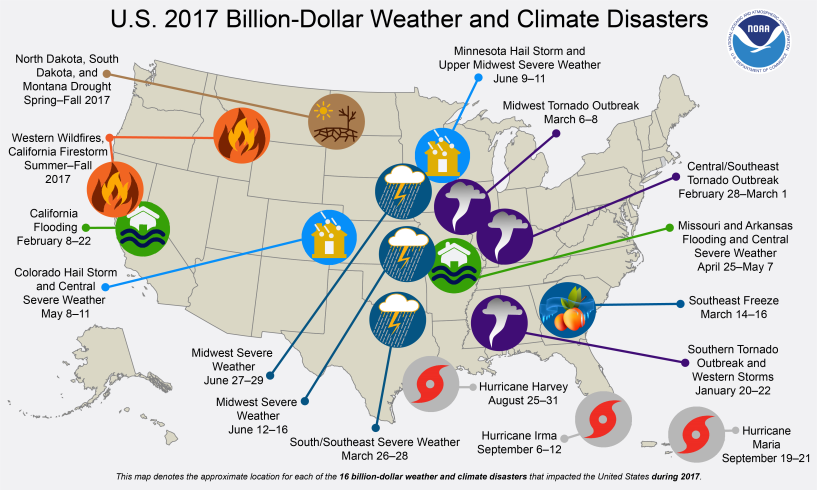
Manhattan without power October 30, 2012 as a result of the devastation wrought by Superstorm Sandy.

2017 Set a Record for Losses from Natural Disasters

Hurricane Harvey, which made landfall in Texas in August 2017, was the most costly disaster of 2017, causing losses of $85 billion. *The New York Times* reports that, together with Hurricanes Irma (hitting Florida) and Maria (devastating Puerto Rico), **the 2017 hurricane season caused the most damage ever, with losses reaching $215 billion**.

Plus, the **damage of wildfires in California drove insured losses to about $8 billion**.

Billion Dollar Disasters in the United States in 2017



Challenges Associated with Disaster Relief

• Timely delivery of relief items **is challenged by damaged and destroyed infrastructure (transportation, telecommunications, hospitals, etc.)**.

• Shipments of **the wrong supplies create congestion and materiel convergence** (sometimes referred to as the second disaster).

• • Within three weeks following the 2010 earthquake in Haiti, 1,000 NGOs were operating in Haiti. News media attention of insufficient water supplies resulted in immense donations to the Dominican Red Cross to assist its island neighbor. Port-au-Price was saturated with both cargo and gifts-in-kind.

• • After the Fukushima disaster, there were too many blankets and items of clothing shipped and even broken bicycles.

• • After Katrina, even tuxedos were delivered to victims.

Better coordination among NGOs is needed.

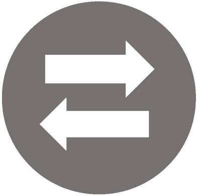
Challenges Associated with Disaster Relief - The NGO Balancing Act

There were 1.5 million registered NGOs in the US in 2012. $300 billion in donations given yearly to US charities.

Challenges Associated with Disaster Relief - Driving Forces

Disasters

Will pose an ever increasing risk to the most vulnerable people on the planet.



NGOs

Will need to adapt their delivery mechanisms to an era of uncertainty and increased competition.

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Therefore

there is a need to *develop appropriate analytical tools* that can assist NGOs, as well as governments in modeling the complex interactions in disaster relief to improve outcomes.

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Methodology - The VI Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized supply chain network problems.

Definition: The Variational Inequality Problem

The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$, *is to determine a vector* $X^* \in \mathcal{K}$, *such that:*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F *is a given continuous function from* $\mathcal{K}$ *to* R^N, $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* R^N.

The vector X consists of **the decision variables** – typically, the flows (products, prices, etc.).

$\mathcal{K}$ is the **feasible set representing how the decision variables are constrained** – for example, the flows may have to be nonnegative; budget constraints may have to be satisfied; similarly, quality and/or time constraints may have to be satisfied.

The function F that enters the variational inequality represents **functions that capture the behavior in the form of the functions such as costs, profits, risk, etc.**

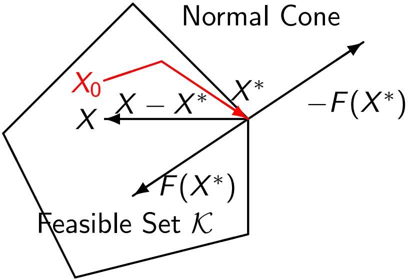
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- game theory problems, operating under Nash equilibrium,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of supply chain network problems from centralized to decentralized ones.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System (Dupuis and Nagurney, Nagurney and Zhang)

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides the natural underlying dynamics.

To model the **dynamic behavior of complex networks**, including supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in

evolutionary games (Sandholm (2005, 2011)),

ecological predator-prey networks (Nagurney and Nagurney (2011a, b)),

even **neuroscience** (Girard et al. (2008), and

dynamic spectrum model for cognitive radio networks (Setoodeh, Haykin, and Moghadam (2012)).

Game Theory Model for Post-Disaster Humanitarian Relief

Game Theory and Disaster Relief

We developed **the first Generalized Nash Equilibrium (GNE) model for post-disaster humanitarian relief, which contains both a financial component and a supply chain component**.

"A Generalized Nash Equilibrium Network Model for Post-Disaster Humanitarian Relief," Anna Nagurney, Emilio Alvarez Flores, and Ceren Soylu, *Transportation Research E* **95** (2016), pp 1-18.

The model is a Generalized Nash Equilibrium model since not only do the utilities of each NGO depend on its strategies and those of the other NGOS' but their feasible sets do, as well, since they have shared, that is, common, constraints.

Some Literature

Our disaster relief game theory framework entails competition for donors as well as media exposure plus supply chain aspects. We now highlight some of the related literature on these topics.

• Natsios (1995) contends that the **cheapest way for relief organizations to fundraise is to provide early relief in highly visible areas**.

• Balcik et al. (2010) note that **the media is a critical factor affecting relief operations with NGOs seeking visibility to attract more resources from donors**. They also review the challenges in coordinating humanitarian relief chains and describe the current and emerging coordination practices in disaster relief.

Some Literature

• Olsen and Carstensen (2003) confirmed the frequently repeated argument that media coverage is critical in relation to emergency relief allocation in a number of cases that they analyzed.

• Van Wassenhove (2006) also emphasizes the role of the media in humanitarian logistics and states that following appeals in the media, humanitarian organizations are often flooded with unsolicited donations that can create bottlenecks in the supply chain.

• Zhuang, Saxton, and Wu (2014) develop a model that reveals the amount of charitable contributions made by donors is positively dependent on the amount of disclosure by the NGOs. **They also emphasize that there is a dearth of existing game-theoretic research on nonprofit organizations**. Our model attempts to help to fill this void.

Game Theory and Disaster Relief

Although there have been quite a few optimization models developed for disaster relief there are very few game theory models.

Toyasaki and Wakolbinger (2014) constructed the **first models of financial flows that captured the strategic interaction between donors and humanitarian organizations using game theory and also included earmarked donations**.

Muggy and Stamm (2014), in turn, provide **an excellent review of game theory in humanitarian operations and emphasize that there are many untapped research opportunities for modeling in this area**.

Additional references to disaster relief and humanitarian logistics can be found in our paper.

The Network Structure of the Model

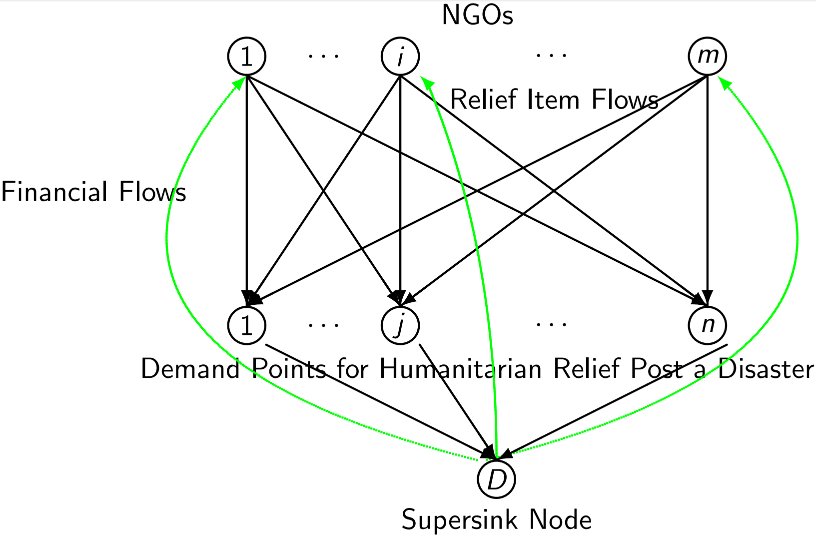


Figure: The Network Structure of the Game Theory Model

The Game Theory Model

We assume that each NGO i has, at its disposal, an amount s_i of the relief item that it can allocate post-disaster. Hence, we have the following conservation of flow equation, which must hold for each i; $i = 1, \ldots, m$:

$$\sum_{j=1}^{n} q_{ij} \leq s_i. \tag{1}$$

In addition, we know that the product flows for each i; $i = 1, \ldots, m$, must be nonnegative, that is:

$$q_{ij} \geq 0, \quad j = 1, \ldots, n. \tag{2}$$

Each NGO i encumbers a cost, c_{ij}, associated with shipping the relief items to location j, denoted by c_{ij}, where we assume that

$$c_{ij} = c_{ij}(q_{ij}), \quad j = 1, \ldots n, \tag{3}$$

with these cost functions being strictly convex and continuously differentiable.

Each NGO i; $i = 1, \ldots, m$, derives satisfaction or utility associated with providing the relief items to j; $j = 1, \ldots, n$, with its utility over all demand points given by $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$. Here γ_{ij} is a positive factor representing a measure of satisfaction/utility that NGO i acquires through its supply chain activities to demand point j.

Each NGO i; $i = 1, \ldots, m$, associates a positive weight ω_i with $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$, which provides a monetization of, in effect, this component of the objective function.

The Game Theory Model

Finally, each NGO i; $i = 1, \ldots, m$, based on the media attention and the visibility of NGOs at location j; $j = 1, \ldots, n$, acquires funds from donors given by the expression

$$\beta_i \sum_{j=1}^{n} P_j(q), \tag{4}$$

where $P_j(q)$ represents the financial funds in donation dollars due to visibility of all NGOs at location j. Hence, β_i is a parameter that reflects the proportion of total donations collected for the disaster at demand point j that is received by NGO i.

Expression (4), therefore, represents the financial flow on the link joining node D with node NGO i.

Each NGO i seeks to maximize its utility with the utility corresponding to the financial gains associated with the visibility through media of the relief item flow allocations, $\beta_i \sum_{j=1}^n P_j(q)$, plus the utility associated with the supply chain aspect of delivery of the relief items, $\omega_i \sum_{j=1}^n \gamma_{ij} q_{ij} - \sum_{j=1}^n c_{ij}(q_{ij})$.

The optimization problem faced by NGO i; $i = 1, \ldots, m$, is, hence,

$$\text{Maximize} \quad \beta_i \sum_{j=1}^n P_j(q) + \omega_i \sum_{j=1}^n \gamma_{ij} q_{ij} - \sum_{j=1}^n c_{ij}(q_{ij}) \tag{5}$$

subject to constraints (1) and (2).

The Game Theory Model

We also have that, at each demand point j; $j = 1, \ldots, n$:

$$\sum_{i=1}^{m} q_{ij} \geq \underline{d}_j, \tag{6}$$

and

$$\sum_{i=1}^{m} q_{ij} \leq \bar{d}_j, \tag{7}$$

where $\underline{d}_j$ denotes a lower bound for the amount of the relief items needed at demand point j and $\bar{d}_j$ denotes an upper bound on the amount of the relief items needed post the disaster at demand point j.

We assume that

$$\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} \underline{d}_j, \tag{8}$$

so that the supply resources of the NGOs are sufficient to meet the minimum financial resource needs.

Each NGO i; $i = 1, \ldots, m$, seeks to determine its optimal vector of relief items or strategies, q_i^*, that maximizes objective function (5), subject to constraints (1), (2), and (6), (7).

Because of the structure of the objective functions of the NGOs, based on a result of Li and Lin (2013), rather than having to formulate the GNE problem as a quasivariational inequality, we have an optimization reformulation.

Theorem: Optimization Formulation of the Generalized Nash Equilibrium Model of Financial Flow of Funds

The above Generalized Nash Equilibrium problem, with each NGO's objective function (5) rewritten as:

$$\textit{Minimize} \quad -\beta_i \sum_{j=1}^{n} P_j(q) - \omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} + \sum_{j=1}^{n} c_{ij}(q_{ij}) \qquad (9)$$

and subject to constraints (1) and (2), with common constraints (6) and (7), is equivalent to the solution of the following optimization problem:

$$\textit{Minimize} \quad -\sum_{j=1}^{n} P_j(q) - \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\omega_i \gamma_{ij}}{\beta_i} q_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{1}{\beta_i} c_{ij}(q_{ij}) \quad (10)$$

subject to constraints: (1), (2), (6), and (7).

We associate Lagrange multipliers λ_k^*, $\forall k$, with the supply constraints; Lagrange multipliers: $\lambda_l^{1^*}$, $\forall l$, for the lower bound demand constraints, and Lagrange multipliers: $\lambda_l^{2^*}$, $\forall k$, for the upper bound demand constraints, and we group these multipliers, respectively, into the vectors: $\lambda^* \in R_+^m$, $\lambda^{1^*} \in R_+^n$, and $\lambda^{2^*} \in R_+^n$.

Variational Inequality (VI) Formulation

The solution to the optimization problem: $(q^*, \lambda^*, \lambda^{1*}, \lambda^{2*}) \in R_+^{mn+m+2n}$ *coincides with the solution to the VI:*

$$\sum_{k=1}^{m}\sum_{l=1}^{n}\left[-\sum_{j=1}^{n}(\frac{\partial P_j(q^*)}{\partial q_{kl}}) - \frac{\omega_k \gamma_{kl}}{\beta_k} + \frac{1}{\beta_k}\frac{\partial c_{kl}(q^*_{kl})}{\partial q_{kl}} + \lambda^*_k - \lambda^{1*}_l + \lambda^{2*}_l\right]$$

$$\times [q_{kl} - q^*_{kl}]$$

$$+\sum_{k=1}^{m}(s_k - \sum_{l=1}^{n} q^*_{kl}) \times (\lambda_k - \lambda^*_k) + \sum_{l=1}^{n}(\sum_{k=1}^{n} q^*_{kl} - \underline{d}_l) \times (\lambda_l - \lambda^{1*}_l)$$

$$+\sum_{l=1}^{n}(\bar{d}_l - \sum_{k=1}^{m} q^*_{kl}) \times (\lambda^2_l - \lambda^{2*}_l) \geq 0, \ \forall (q, \lambda, \lambda^1, \lambda^2) \in R_+^{mn+m+2n}. \quad (11)$$

The Algorithm

Explicit Formulae for the Euler Method Applied to the Model

We have the following closed form expression for the product flows $k = 1, \ldots, m$; $l = 1, \ldots, n$, at each iteration:

$$q_{kl}^{\tau+1} = \max$$

$$\{0, \{q_{kl}^{\tau} + a_{\tau}(\sum_{j=1}^{n}(\frac{\partial P_j(q^{\tau})}{\partial q_{kl}}) + \frac{\omega_k \gamma_{kl}}{\beta_{kl}} - \frac{1}{\beta_k}\frac{\partial c_{kl}(q_{kl}^{\tau})}{\partial q_{kl}} - \lambda_k^{\tau} + \lambda_l^{1^{\tau}} - \lambda_l^{2^{\tau}})\}\},$$

the following closed form expressions for the Lagrange multipliers associated with the supply constraints, respectively, for $k = 1, \ldots, m$:

$$\lambda_k^{\tau+1} = \max\{0, \lambda_k^{\tau} + a_{\tau}(-s_k + \sum_{l=1}^{n} q_{kl}^{\tau})\}.$$

The Algorithm

The following closed form expressions are for the Lagrange multipliers associated with the lower bound demand constraints, respectively, for $l = 1, \ldots, n$:

$$\lambda_l^{1\,\tau+1} = \max\{0, \lambda_l^{1\,\tau} + a_\tau(-\sum_{k=1}^{n} q_{kl}^\tau + \underline{d}_l)\}.$$

The following closed form expressions are for the Lagrange multipliers associated with the upper bound demand constraints, respectively, for $l = 1, \ldots, n$:

$$\lambda_l^{2\,\tau+1} = \max\{0, \lambda_l^{2\,\tau} + a_\tau(-\bar{d}_l + \sum_{k=1}^{m} q_{kl}^\tau)\}.$$

Hurricane Katrina Case Study

Hurricane Katrina Case Study

Making landfall in August of 2005, Katrina caused extensive damage to property and infrastructure, **left 450,000 people homeless, and took 1,833 lives in Florida, Texas, Mississippi, Alabama, and Louisiana (Louisiana Geographic Information Center (2005))**.

Given the hurricane's trajectory, most of the damage was concentrated in Louisiana and Mississippi. In fact, 63% of all insurance claims were in Louisiana, a trend that is also reflected in FEMA's post-hurricane damage assessment of the region (FEMA (2006)).

The total damage estimates range from \$105 billion (Louisiana Geographic Information Center (2005)) to \$150 billion (White (2015)), making Hurricane Katrina not only a far-reaching and costly disaster, but also a very challenging environment for providing humanitarian assistance.

We consider 3 NGOs: the Red Cross, the Salvation Army, and Others and 10 Parishes in Louisiana.

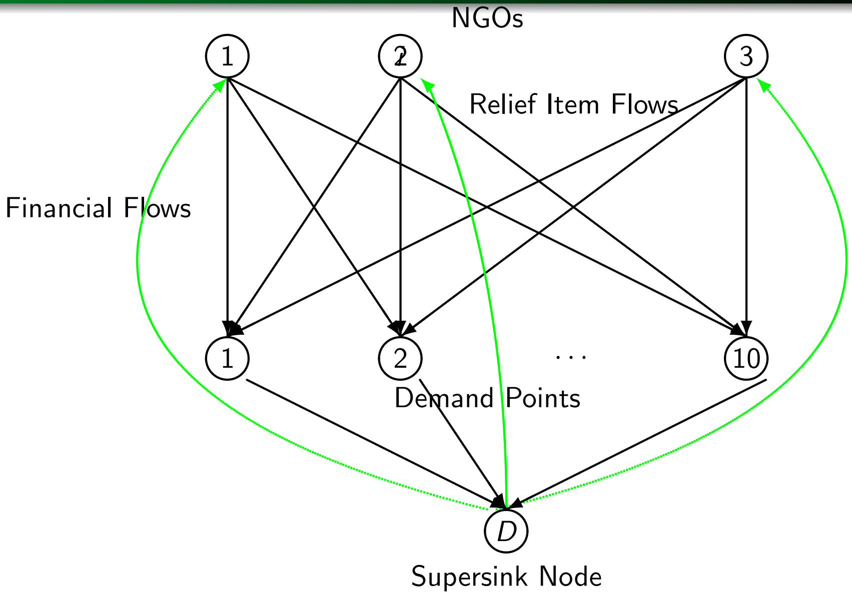


Figure: Hurricane Katrina Relief Network Structure

Hurricane Katrina Case Study

The structure of the P_j functions is as follows:

$$P_j(q) = k_j \sqrt{\sum_{i=1}^{m} q_{ij}}.$$

The weights are:

$$\omega_1 = \omega_2 = \omega_3 = 1,$$

with $\gamma_{ij} = 950$ for $i = 1, 2, 3$ and $j = 1, \ldots, 10$.

Hurricane Katrina Case Study

Hurricane Katrina Demand Point Parameters					
Parish	Node j	k_j	$\underline{d}_j$	$\bar{d}_j$	p_j (in %)
St. Charles	1	8	16.45	50.57	2.4
Terrebonne	2	16	752.26	883.82	6.7
Assumption	3	7	106.36	139.24	1.9
Jefferson	4	29	742.86	1,254.89	19.5
Lafourche	5	6	525.53	653.82	1.7
Orleans	6	42	1,303.99	1,906.80	55.9
Plaquemines	7	30	33.28	62.57	57.5
St. Barnard	8	42	133.61	212.43	78.4
St. James	9	9	127.53	166.39	1.2
St. John the Baptist	10	7	19.05	52.59	6.7

Table: Demand Point Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

Hurricane Katrina Case Study

We then estimated the cost of providing aid to the Parishes as a function of the total damage in the area and the supply chain efficiency of each NGO. We assume that these costs follow the structures observed by Van Wassenhove (2006) and randomly generate a number based on his research with a mean of $\hat{p} = .8$ and standard deviation of $s = \sqrt{\frac{.8(.2)}{3}}$.

We denote the corresponding coefficients by π_i. Thus, each NGO i; $i = 1, 2, 3$, incurs costs according the the following functional form:

$$c_{ij}(q_{ij}) = \left(\pi_i q_{ij} + \frac{1}{1 - p_j}\right)^2.$$

Hurricane Katrina Case Study

Data Parameters for NGOs Providing Aid					
NGO	i	π_i	γ_{ij}	β_i	s_i
Others	1	.82	950	.355	1,418
Red Cross	2	.83	950	.55	2,200
Salvation Army	3	.81	950	.095	382

Table: NGO Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

Hurricane Katrina Case Study

Generalized Nash Equilibrium Product Flows (in Millions of Aid Units)			
Demand Point	Others	Red Cross	Salvation Army
St. Charles	17.48	28.89	4.192
Terrebonne	267.023	411.67	73.57
Assumption	49.02	77.26	12.97
Jefferson	263.69	406.68	72.45
Lafourche	186.39	287.96	51.18
Orleans	463.33	713.56	127.1
Plaquemines	21.89	36.54	4.23
St. Barnard	72.31	115.39	16.22
St. James	58.67	92.06	15.66
St. John the Baptist	18.2	29.99	4.40

Table: Flows to Demand Points under Generalized Nash Equilibrium

The total utility obtained through the above flows for the Generalized Nash Equilibrium for Hurricane Katrina is $9,257,899$, with the Red Cross capturing 3,022,705, the Salvation Army 3,600,442.54, and Others 2,590,973.

In addition, we have that the Red Cross, the Salvation Army, and Others receive 2,200.24, 1418.01, and 382.31 million in donations, respectively.

The relief item flows meet at least the lower bound, even if doing so is very expensive due to the damages to the infrastructure in the region.

Furthermore, the above flow pattern behaves in a way that, after the minimum requirements are met, any additional supplies are allocated in the most efficient way. For example, only the minimum requirements are met in New Orleans Parish, while the upper bound is met for St. James Parish.

The Nash Equilibrium Solution

If we remove the shared constraints, we obtain a Nash Equilibrium solution, and we can compare the outcomes of the humanitarian relief efforts for Hurricane Katrina under the Generalized Nash Equilibrium concept and that under the Nash Equilibrium concept.

The Nash Equilibrium Solution

Nash Equilibrium Product Flows			
Demand Point	Others	Red Cross	Salvation Army
St. Charles	142.51	220.66	38.97
Terrebonne	142.50	220.68	38.93
Assumption	142.51	220.66	38.98
Jefferson	142.38	220.61	38.74
Lafourche	142.50	220.65	38.98
Orleans	141.21	219.59	37.498
Plaquemines	141.032	219.28	37.37
St. Barnard	138.34	216.66	34.59
St. James	142.51	220.65	38.58
St. John the Baptist	145.51	220.66	38.98

Table: Flows to Demand Points under Nash Equilibrium

The Nash Equilibrium Solution

Under the Nash Equilibrium, the NGOs obtain a higher utility than under the Generalized Nash Equilibrium. Specifically, of the total utility $10,346,005.44$, 2,804,650 units are received by the Red Cross, 5,198,685 by the Salvation Army, and 3,218,505 are captured by all other NGOs.

Under this product flow pattern, there are total donations of 3,760.73, of which 2,068.4 are donated to the Red Cross, 357.27 to the Salvation Army, and 1,355 to the other players.

It is clear that there is a large contrast between the flow patterns under the Generalized Nash and Nash Equilibria. For example, the Nash Equilibrium flow pattern results in about $500 million less in donations.

While this has strong implications about how collaboration between NGOs can be beneficial for their fundraising efforts, the differences in the general flow pattern highlights a much stronger point.

Additional Insights

Under the Nash Equilibrium, NGOs successfully maximize their utility. Overall, the Nash Equilibrium solution leads to an increase of utility of roughly 21% when compared to the flow patterns under the Generalized Nash Equilibrium.

But they do so at the expense of those in need. In the Nash Equilibrium, each NGO chooses to supply relief items such that costs can be minimized. On the surface, this might be a good thing, but recall that, given the nature of disasters, it is usually more expensive to provide aid to demand points with the greatest needs.

Additional Insights

With this in mind, one can expect oversupply to the demand points with lower demand levels, and undersupply to the most affected under a purely competitive scheme. This behavior can be seen explicitly in the results summarized in the Tables.

For example, St. Charles Parish receives roughly 795% of its upper demand, while Orleans Parish only receives about 30.5% of its minimum requirements. That means that much of the 21% in 'increased' utility is in the form of waste.

In contrast, the flows under the Generalized Nash Equilibrium guarantee that minimum requirements will be met and that there will be no waste; that is to say, *as long as there is a coordinating authority that can enforce the upper and lower bound constraints, the humanitarian relief flow patterns under this bounded competition will be significantly better than under untethered competition.*

An Extension of the Model

At the Dynamics of Disasters conference in Greece, July 5-9, 2017, we presented the paper: “A Variational Equilibrium Network Framework for Humanitarian Organizations in Disaster Relief: Effective Product Delivery Under Competition for Financial Funds,” A. Nagurney, P. Daniele, E. Alvarez Flores, and V. Caruso, to be published in a new *Dynamics of Disasters* Springer volume.

The extended model captures competition for logistic services, has more general cost functions as well as financial donation functions and uses general altruism benefit functions, where the costs associated with logistics are now given by:

$$c_{ij} = c_{ij}(q), \quad i = 1, \ldots, m; j = 1, \ldots n.$$

Each NGO i; $i = 1, \ldots, m$, based on the media attention and the visibility of NGOs at demand point j; $j = 1, \ldots, n$, receives financial funds from donors given by the expression

$$\sum_{j=1}^{n} P_{ij}(q),$$

where $P_{ij}(q)$ denotes the financial funds in donation dollars given to NGO i due to visibility of NGO i at location j. We introduce an altruism/benefit function B_i; $i = 1, \ldots, m$, such that

$$B_i = B_i(q).$$

Extension of the Model

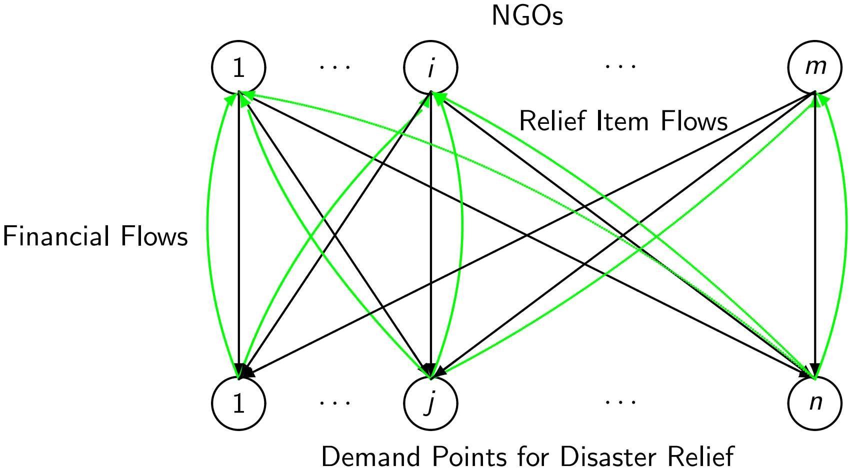


Figure: The Network Structure of the Extended Game Theory Model

The Extended Model

The utility function of NGO i; $i = 1, \dots, m$, is now:

$$\text{Maximize} \quad U_i(q) = \sum_{j=1}^{n} P_{ij}(q) + \omega_i B_i(q) - \sum_{j=1}^{n} c_{ij}(q)$$

with the same constraints imposed as the original Generalized Nash Equilibrium model for post-disaster relief.

In the new model, we can no longer reformulate the Generalized Nash Equilibrium as an optimization problem but do so as a Variational Equilibrium and, hence, we can apply variational inequality theory.

The Extended Model

We define the feasible set K_i for each NGO i as:

$$K_i \equiv \{q_i | \, (1) \, \text{and} \, (2) \, \text{hold}\}$$

and we let $K \equiv \prod_{i=1}^{m} K_i$.

In addition, we define the feasible set $\mathcal{S}$ consisting of the shared constraints as:

$$\mathcal{S} \equiv \{q | \, (6) \, \text{and} \, (7) \, \text{hold}\}.$$

Observe that now not only does the utility of each NGO depend on the strategies, that is, the relief item flows, of the other NGOs, but so does the feasible set because of the common constraints. Hence, the above game theory model, in which the NGOs compete noncooperatively is a Generalized Nash Equilibrium problem.

We make use of the following:

Definition: Disaster Relief Generalized Nash Equilibrium

A relief item flow pattern $q^* \in K = \prod_{i=1}^{m} K_i$, $q^* \in \mathcal{S}$, *constitutes a disaster relief Generalized Nash Equilibrium if for each NGO* i; $i = 1, \ldots, m$:

$$U_i(q_i^*, \hat{q}_i^*) \geq U_i(q_i, \hat{q}_i^*), \quad \forall q_i \in K_i, \forall q \in \mathcal{S},$$

where $\hat{q}_i^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_m^*)$.

An equilibrium is established if no NGO can unilaterally improve upon its utility by changing its relief item flows in the disaster relief network, given the relief item flow decisions of the other NGOs, and subject to the constraints. Both K and $\mathcal{S}$ are convex sets.

Nash Equilibrium

If there are no coupling, that is, shared, constraints in the above model, then q and q^* in the Definition need only lie in the set K, and, under the assumption of concavity of the utility functions and that they are continuously differentiable, we know that (cf. Gabay and Moulin (1980) and Nagurney (1999)) the solution to what would then be a Nash equilibrium problem (see Nash (1950, 1951)) would coincide with the solution of the following variational inequality problem: determine $q^* \in K$, such that

$$-\sum_{i=1}^{m} \langle \nabla_{q_i} U_i(q^*), q_i - q_i^* \rangle \geq 0, \quad \forall q \in K,$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{q_i} U_i(q)$ denotes the gradient of $U_i(q)$ with respect to q_i.

Variational Equilibrium

As emphasized in Nagurney, Yu, and Besik (2017), a refinement of the Generalized Nash Equilibrium is what is known as a variational equilibrium and it is a specific type of GNE (see Kulkarni and Shabhang (2012)).

Definition: Variational Equilibrium

A strategy vector q^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $q^* \in K, q^* \in \mathcal{S}$ is a solution of the variational inequality:

$$-\sum_{i=1}^{m} \langle \nabla_{q_i} U_i(q^*), q_i - q_i^* \rangle \geq 0, \quad \forall q \in K, \forall q \in \mathcal{S}.$$

The Extended Model

In a GNE defined by a variational equilibrium, the Lagrange multipliers associated with the common/shared/coupling constraints are all the same. **This feature provides a fairness interpretation and is reasonable from an economic standpoint.**

By utilizing a variational equilibrium, we can take advantage of the well-developed theory of variational inequalities, including algorithms (cf. Nagurney (1999) and the references therein), which are in a more advanced state of development and application than algorithms for quasivariational inequality problems.

The Extended Model

We now expand the terms in the variational inequality for the GNE. We have that the previous VI is equivalent to the variational inequality: determine $q^* \in K, q^* \in \mathcal{S}$, such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\sum_{k=1}^{n}\frac{\partial c_{ik}(q^*)}{\partial q_{ij}} - \sum_{k=1}^{n}\frac{\partial P_{ik}(q^*)}{\partial q_{ij}} - \omega_i \frac{\partial B_i(q^*)}{\partial q_{ij}}\right]$$

$$\times \left[q_{ij} - q_{ij}^*\right] \geq 0, \quad \forall q \in K, \forall q \in \mathcal{S}.$$

An Alternative Variational Inequality Formulation of the Generalized Nash Equilibrium for the Extended Model

Find $(q^*, \delta^*, \sigma^*, \varepsilon^*) \in R_+^{mn+m+2n}$:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\sum_{k=1}^{n}\frac{\partial c_{ik}(q^*)}{\partial q_{ij}} - \sum_{k=1}^{n}\frac{\partial P_{ik}(q^*)}{\partial q_{ij}} - \omega_i\frac{\partial B_i(q^*)}{\partial q_{ij}} + \delta_i^* - \sigma_j^* + \varepsilon_j^*\right]$$

$$\times(q_{ij} - q_{ij}^*) + \sum_{i=1}^{m}\left(s_i - \sum_{j=1}^{n} q_{ij}^*\right) \times (\delta_i - \delta_i^*)$$

$$+\sum_{j=1}^{n}\left(\sum_{i=1}^{m} q_{ij}^* - \underline{d}_j\right)\times(\sigma_j - \sigma_j^*)+\sum_{j=1}^{n}\left(\overline{d}_j - \sum_{i=1}^{m} q_{ij}^*\right)\times(\varepsilon_j - \varepsilon_j^*) \geq 0,$$

$$\forall q \in R_+^{mn}, \forall\delta \in R_+^{m}, \forall\sigma \in R_+^{n}, \forall\varepsilon \in R_+^{n}.$$

Our case study is inspired by a disaster consisting of a series of tornadoes that hit western Massachusetts on June 1, 2011. The largest tornado was measured at EF3. It was the worst tornado outbreak in the area in a century (see Flynn (2011)). A wide swath from western to central MA of about 39 miles was impacted.

The tornado killed 4 persons, injured more than 200 persons, damaged or destroyed 1,500 homes, left over 350 people homeless in Springfield's MassMutual Center arena, left 50,000 customers without power, and brought down thousands of trees.

FEMA estimated that 1,435 residences were impacted with the following breakdowns: 319 destroyed, 593 sustaining major damage, 273 sustaining minor damage, and 250 otherwise affected. FEMA estimated that the primary impact was damage to buildings and equipment with a cost estimate of $24,782,299.

Total damage estimates from the storm exceeded $140 million, the majority from the destruction of homes and businesses.

Especially impacted were the city of Springfield and the towns of Monson and Brimfield. It has been estimated that, in the aftermath, the Red Cross served about 11,800 meals and the Salvation Army about 20,000 meals (cf. Western Massachusetts Regional Homeland Security Advisory Council (2012)).

We consider the American Red Cross and the Salvation Army as the NGOs, who provide the meals, which are the flows. The demand points are: Springfield, Monson, and Brimfield.

We find in multiple examples comprising our case study of Massachusetts tornadoes that the NGOs garner greater financial funds through the Generalized Nash Equilibrium solution, rather than the Nash equilibrium one. Moreover, the needs of the victims are met under the Generalized Nash Equilibrium solution

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How disaster relief efforts could be improved with game theory

March 9, 2017 11.21pm EST

Buddhist monks and family members of victims of the Fukushima tsunami and earthquake face the sea to pray on March 11, 2016 while mourning the victims of the March 11, 2011 disaster. REUTERS/Kyodo

Email

The number of disasters has doubled globally since the 1980s, with the

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Disclosure statement

Anna Nagurney does not work for, consult, own shares in or receive funding from any company or organization that would benefit from this article, and has disclosed no relevant affiliations beyond the academic appointment above.

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Response to natural disasters like Harvey could be helped with game theory

By Anna Nagurney
Associated Press

AUGUST 29, 2017 | BOSTON, MASSACHUSETTS

(The Conversation is an independent and nonprofit source of news, analysis and commentary from academic experts.)

Anna Nagurney, University of Massachusetts Amherst

(THE CONVERSATION) The devastation by Hurricane Harvey continues, with the National Weather Service calling the event unprecedented, thus making the response even more complicated.

Nearly half a million people are expected to seek federal aid in the aftermath of the Category 4 hurricane, which already has dumped more than 30 inches on the Houston

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Disaster response

How disaster relief efforts could be improved with game theory

By Anna Nagurney

Published 13 March 2017

Share

The number of disasters has doubled globally since the 1980s, with the damage and losses estimated at an average

"A Multitiered Supply Chain Network Equilibrium Model for Disaster Relief with Capacitated Freight Service Provision," A. Nagurney (2017), to appear in new *Dynamics of Disasters* Springer volume.

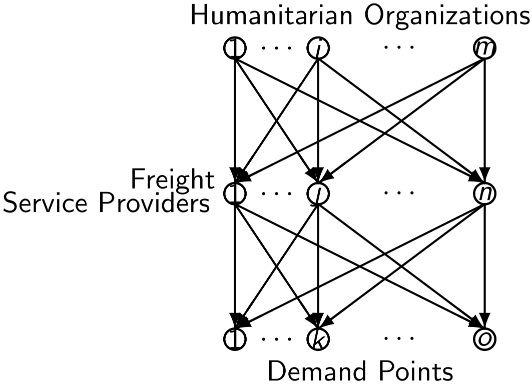


Figure: The Multitiered Disaster Relief Humanitarian Organization and Freight Service Provision Supply Chain Network

Game Theory and Blood Supply Chains

Blood Supply Chains

Nagurney, Masoumi, and Yu (2012) developed a supply chain network optimization model for the management of the procurement, testing and processing, and distribution of human blood.

Novel features of the model include:

- It captures **perishability of this life-saving product** through the use of arc multipliers;
- It contains **discarding costs** associated with waste/disposal;
- It handles **uncertainty** associated with demand points;
- It assesses **costs associated with shortages/surpluses at the demand points**, and
- It quantifies the **supply-side risk** associated with procurement.

In the paper, "Mergers and Acquisitions in Blood Banking Systems: A Supply Chain Network Approach," A.H. Masoumi, M. Yu, and A. Nagurney, *International Journal of Production Economics* **193** (2017), pp 406-421, we constructed network models to assess possible synergies associated with mergers and acquisitions among blood service organizations, taking into account capacities and frequencies of various supply chain network link activities.

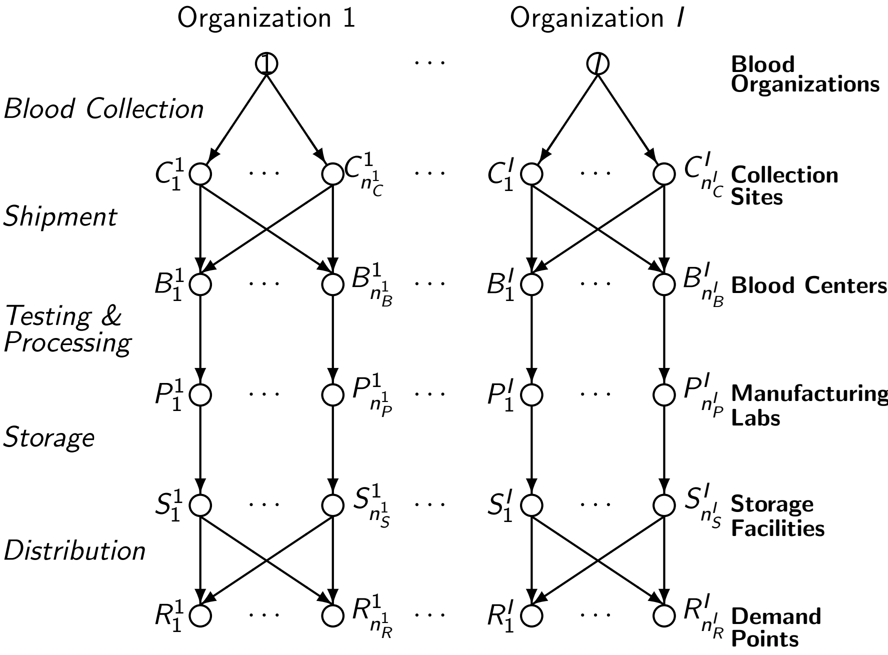


Figure: Supply Chain Network Topology Pre-Merger

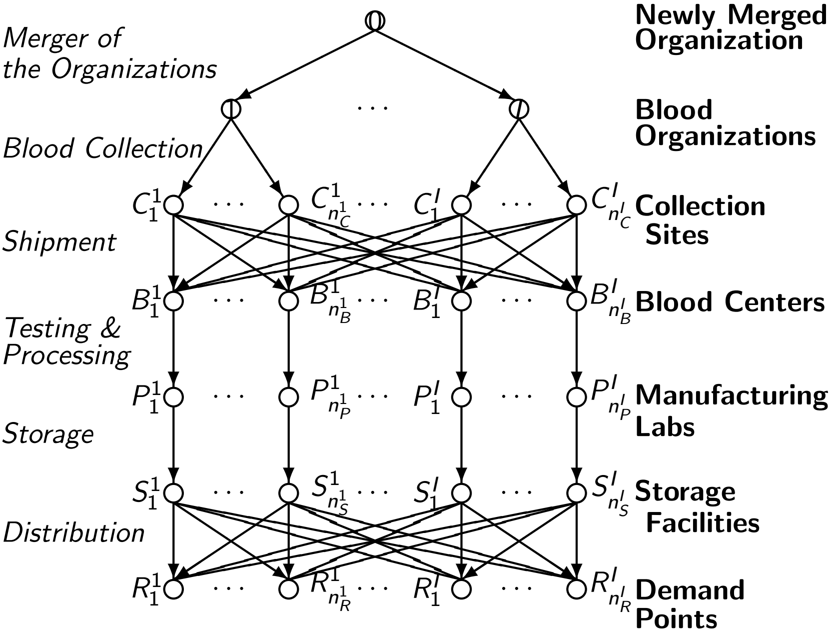
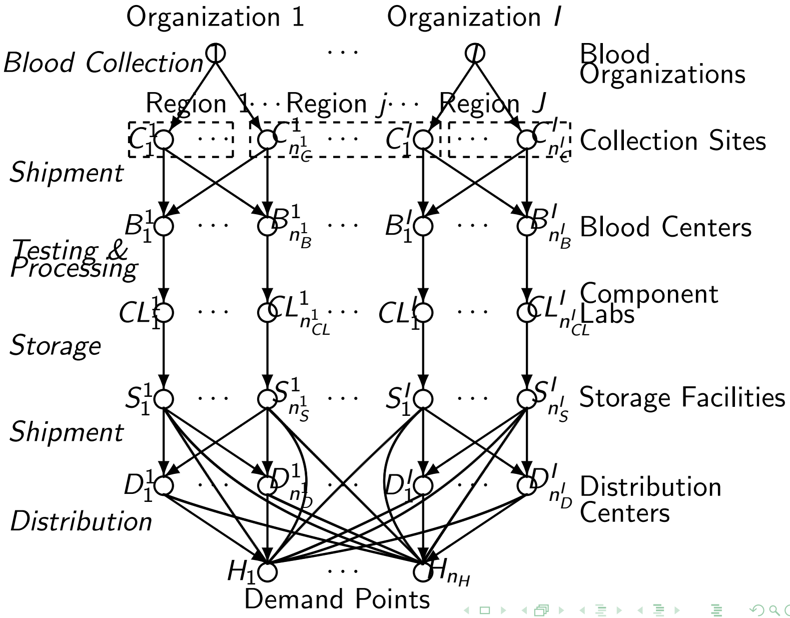


Figure: Supply Chain Network Topology Post-Merger

With a doctoral student, Pritha Dutta, we completed the paper, "Supply Chain Network Competition Among Blood Service Organizations: A Generalized Nash Equilibrium Framework," which we are presenting at 2018 NEDSI in Providence, Rhode Island in April.

This paper builds on our work, "Competition for Blood Donations: A Nash Equilibrium Network Framework."

Blood Supply Chain Competition



Summary and Conclusions

- In this lecture, a game theory network model for post-disaster relief was presented, **which integrates financial flows and logistical flows**, with NGOs competing for financial funds from donors while also seeking to deliver the needed supplies.

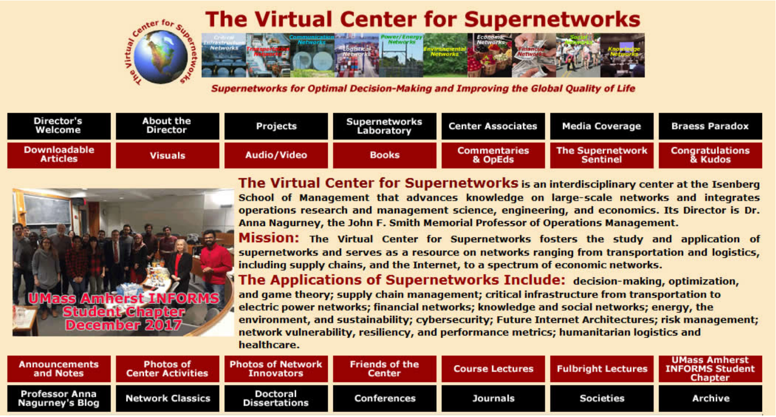
- The model, because of common constraints on the demand side, in order to ensure that the needed supplies are delivered in the correct amounts without an oversupply, is a **Generalized Nash Equilibrium (GNE) model**, which can be challenging to solve.

- Because of the structure of the functions comprising the objective functions of the NGOs, the governing GNE conditions can be reformulated as an optimization problem. We utilize then a VI construct for effective and efficient computational purposes when we consider **a case study on Hurricane Katrina**.

Summary and Conclusions

- An extension of the model is then given, which makes use of **the concept of a Variational Equilibrium** and results from a case study based on tornadoes in Massachusetts outline.
- The results show that, **by doing better for a victims' perspective, the NGOs can also gain financially**.
- Additional recent related game theory models in the nonprofit sector for both disaster relief and **blood supply chains** are also highlighted.

THANK YOU!



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