Lecture 7
Basic Sensitivity Analysis and the Braess Paradox

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Professor Braess's visit to UMass, Spring 2006

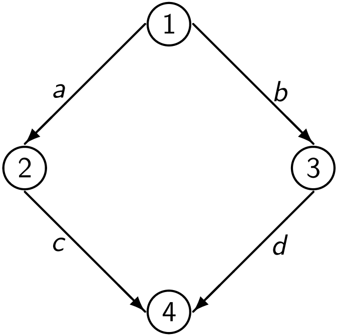
http://supernet.isenberg.umass.edu/cfoto/braess-visit/braessvisit.html

The Braess Paradox Illustrates Why Capturing the Behavior of Users on Networks is Essential

The Braess (1968) Paradox

Assume a network with a single O/D pair $w_1 = (1,4)$. There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand d_{w_1} of **6**, the U-O / equilibrium path flows are: $F^*_{p_1} = F^*_{p_2} = 3$ and **the U-O / equilibrium path travel costs are:** $C_{p_1} = C_{p_2} = 83.$

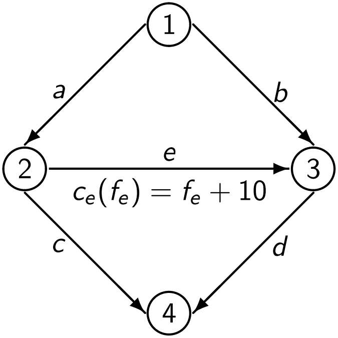


$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

Adding a Link Increases Travel Cost for All!

Adding a new link e creates a new path $p_3 = (a, e, d)$. The user link cost on e is: $c_e(f_e) = f_e + 10$ and d_{w_1} remains at 6.
The original flow distribution pattern is no longer a U-O pattern, since, at that level of flow, the cost on path p_3, $C_{p_3} = 70$.



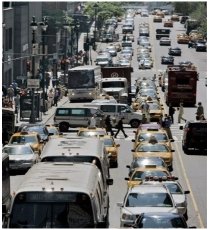
The new U-O flow pattern is $F^*_{p_1} = F^*_{p_2} = F^*_{p_3} = 2$.
The U-O path travel costs are now: $C_{p_1} = C_{p_2} = C_{p_3} = 92$.

The travel cost has increased for all from 83 to 92!

Under S-O behavior, the total cost in the network is minimized, and the new route p_3, under the same demand of 6, would not be used.

The Braess paradox never occurs in S-O networks and only in U-O networks!

The Braess Paradox Around the World



1969 - **Stuttgart, Germany** - The traffic worsened until a newly built road was closed.

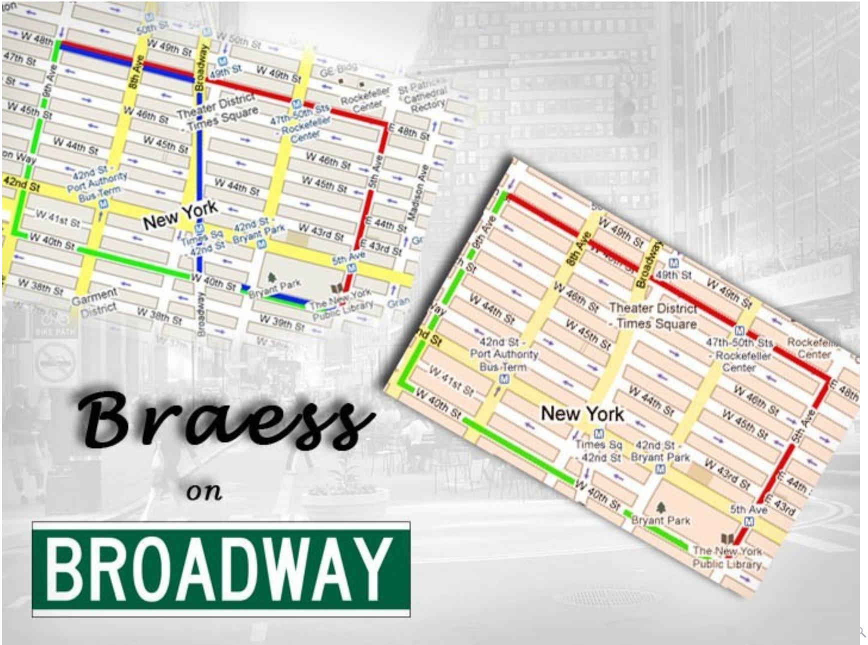
1990 - **Earth Day - New York City** - 42nd Street was closed and traffic flow improved.

2002 - **Seoul, Korea** - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.

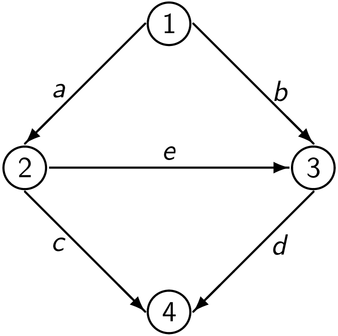
The Closing of Broadway in NYC to Traffic from 42nd to 47th Streets in 2009 Until Now

In May 2009, Mayor Bloomberg's administration implemented the closing of Broadway from 42nd Street (Times Square) to 47th Street to traffic and the creation of pedestrian plazas. This closure generated much discussion and was the subject of, among others, the World Science Festival *Traffic* panel in NYC in June 2009.

World Science Festival *Traffic* panel in NYC in June 2009


Braess
on
BROADWAY
New York
Theater District - Times Square
Bryant Park
The New York Public Library

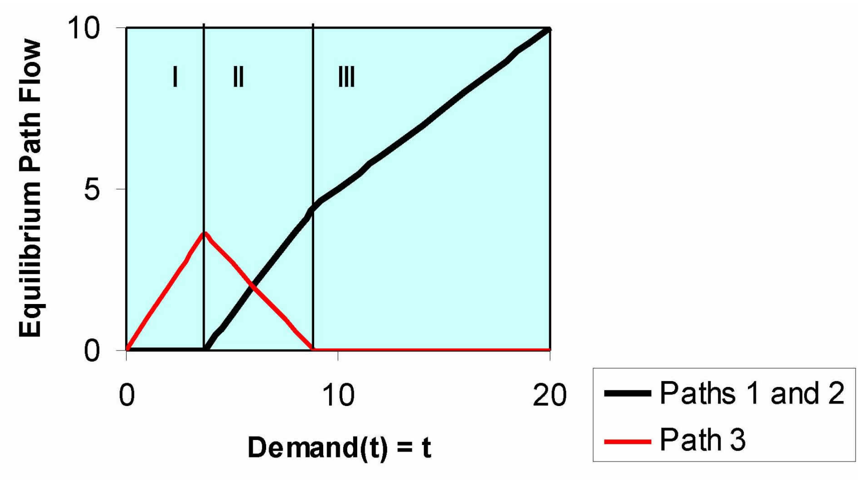
Interview on Broadway for *America Revealed* on March 15, 2011



Recall the Braess network with the added link e.

What happens as the demand increases?

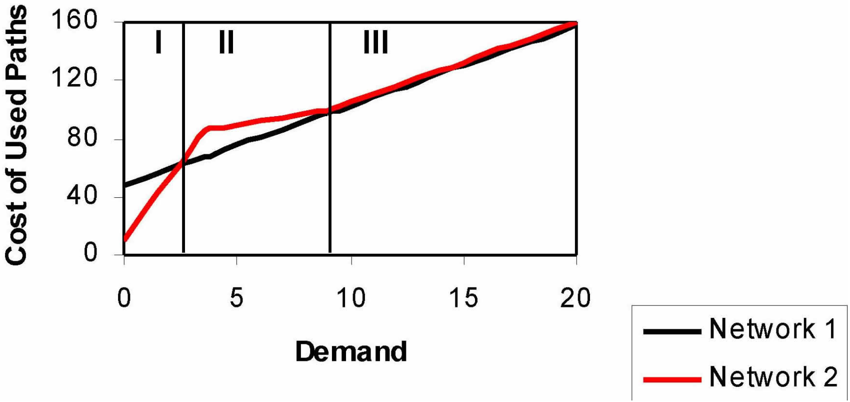
The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands Solved as an *Evolutionary Variational Inequality* (Nagurney, Daniele, and Parkes (2007)).



In Demand Regime I, **Only the New Path is Used**.

In Demand Regime II, the travel demand lies in the range [2.58, 8.89], and **the Addition of a New Link (Path) Makes Everyone Worse Off**!

In Demand Regime III, when the travel demand exceeds 8.89, **Only the Original Paths are Used!**



The new path is never used, under U-O behavior, when the demand exceeds 8.89, even out to infinity!

Note:

The addition of a new path on a network may: increase, decrease, or leave unchanged the equilibrium (U-O) travel path costs.

In the case of S-O solution, the addition of a new path can never increase the total system cost in the network.

Hence, from the system point of view, the network is "improved" or at least not worsened.

Question:

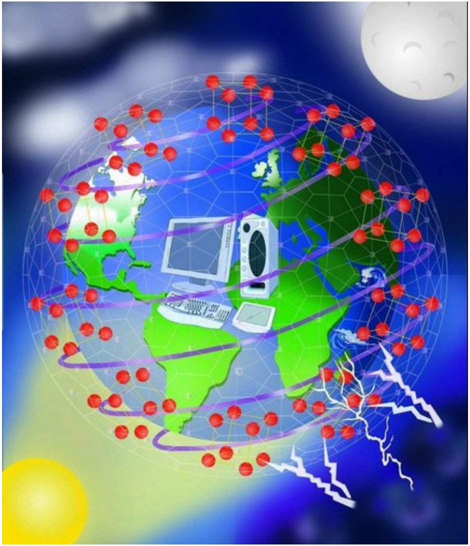
Can you design a new path connecting O/D pair w_1 in the original Braess paradox network so that the travelers can never be worse off, from a U-O perspective?

Basic Sensitivity Analysis

What can we say about the effect on users', that is, travelers', costs with respect to:

- ◇ an increase in travel demand?
- ◇ a decrease in travel demand?
- ◇ an increase in the link cost function?
- ◇ a decrease in the link cost function?

The Braess paradox occurs not only in transportation networks but also in other networks where there is "selfish" type of behavior, such as the Internet.



Internet Traffic

www.dailygalaxy.com

The Price of Anarchy

The *price of anarchy*, $\mathcal{P}$, is defined as

$$\mathcal{P} = \frac{S(f^*_{U-O})}{S(f_{S-O})},$$

where S is the total cost in the network and f^*_{U-O} denotes the U-O link flow solution for a given network and f_{S-O} denotes the S-O link flow solution for the same network.

This metric captures the relative difference in total cost to society under the two main types of traffic flow. Bounds for the value of the price of anarchy for networks ranging from transportation to the Internet have been determined for networks characterized by certain user link cost functions.

References

⇒ D. Braess, A. Nagurney and T. Wakolbinger (2005), On a Paradox of Traffic Planning, *Transportation Science*, Vol. 39, No. 4, pp. 446-450

http://homepage.rub.de/Dietrich.Braess/Paradox-BNW.pdf

∗ Original article in German: D. Braess (1968), ber ein Paradoxon aus der Verkehrsplanung, *Unternehmensforschung*, 12, pp. 258-268

http://homepage.ruhr-uni-bochum.de/Dietrich.Braess/paradox.pdf

∗ Additional information on Professor Braess's visit to UMass (Spring 2006) in the Supernetworks website:

http://supernet.isenberg.umass.edu/cfoto/braess-visit/braessvisit.html

References (continued)

⇒ A. Nagurney (1999), *Network Economics: A Variational Inequality Approach, Revised Second Edition*, Kluwer Academic Publishers.

http://supernet.isenberg.umass.edu/bookser/netbook.htm

⇒ A. Nagurney and J. Dong (2002), *Supernetworks: Decision-Making for the Information Age*, Edward Elgar Publishing, Cheltenham, England.

http://supernet.isenberg.umass.edu/bookser/supbook.html

⇒ A. Nagurney, D. Parkes and P. Daniele (2007), The Internet, Evolutionary Variational Inequalities, and the Time-Dependent Braess Paradox, *Computational Management Science* 4, pp 355-375.

⇒ T. Roughgarden (2005), *Selfish Routing and the Price of Anarchy*, MIT Press, Cambridge, Massachusetts.

Additional Reading

For more advanced formulations and associated theory, see Professor Nagurney's Fulbright Network Economics lectures.

http://supernet.isenberg.umass.edu/austria_lectures/fulmain.html