4 Supply Chain Networks and Electronic Commerce

This chapter begins the second part of this book, which is devoted to multitiered supernetworks, that is, supernetworks, which consist of distinct tiers of decision-makers, whose behavior, in turn, affects the variables on the networks in the form of flows as well as prices. Chapters 4 and 5 focus on supply chain networks. Chapter 6, subsequently, through the use of the multitiered (and, also, multilevel) supernetwork concept, explores another application area – that of financial networks with intermediation.

The study of supply chain networks here is in the context of the Information Age with the innovations brought about by electronic commerce. As is well-recognized, electronic commerce (e-commerce), with the advent of the Information Age, has had an enormous effect on the manner in which businesses as well as consumers order goods and have them transported. Electronic commerce is defined as a "trade" that takes place over the Internet usually through a buyer visiting a seller's website and making a transaction there. The major portion of e-commerce transactions is in the form of business-to-business (B2B) with estimates ranging from approximately 1 billion dollars to 1 trillion dollars in 1998 and with forecasts reaching as high as \$4.8 trillion dollars in 2003 in the United States (see Federal Highway Administration (2000), Southworth (2000)). The business-to-consumer (B2C) component, on the other hand, has seen tremendous growth in recent years but its impact on the US retail activity is still relatively small. Nevertheless, this segment should grow to \$80 billion per year (Southworth (2000)).

As noted by Handfield and Nichols (1999) and by the National Research Council (2000), the principal effect of B2B commerce, estimated to be 90% of all e-commerce by value and volume, is in the creation of new and more profitable *supply chain networks*. A supply chain is a chain of relationships which synthesizes and integrates the movement of goods between suppliers, manufacturers, distributors, retailers, and consumers.

The topic of supply chain analysis is multidisciplinary by nature since it involves aspects of manufacturing, transportation and logistics, retailing/marketing, as well as economics. It has been the subject of a growing body of literature with researchers focusing both on the conceptualization of the underlying problems (see, e.g., Andersson et al. (1993), Poirier (1996, 1999), Bovet (2000), Mentzer (2001)), due to the complexity of the problem and the numerous agents, such as manufacturers, retailers, or consumers involved in the transactions, as well as on the analytics (cf. Federgruen (1993), Graves, Rinooy Kan, and Zipkin (1993), Slats et al. (1995), Bramel and Simchi-Levi (1997), Stadtler and Kilger (2000), Miller (2001), Hensher, Button, and Brewer (2001), and the references therein).

The introduction of e-commerce has unveiled new opportunities in terms of research and practice in supply chain analysis and management (see, e.g., Kuglin and Rosenbaum (2001)). Indeed, the primary benefit of the Internet for business is its open access to potential suppliers and customers both within a particular country and past national boundaries. Consumers, on the other hand, may obtain goods, which they physically could not locate otherwise.

In this chapter, a supernetwork framework is constructed for the study of supply chains with electronic commerce in the form of B2C and B2B transactions. The framework is sufficiently general to allow for the modeling, analysis, and computation of solutions to such problems. Here the focus is on the network interactions of the underlying agents and on the underlying competitive processes. Moreover, the emphasis is placed on the equilibrium aspects of the problems rather than, simply, the optimization ones. Of course, it is assumed that the agents in the supply chain behave in some optimal fashion. An equilibrium approach is necessary and valuable since it provides a benchmark against which one can evaluate both prices and product flows. Moreover, it captures the independent behavior of the various decision-makers as well as the effect of their interactions. Finally, it provides for the development of dynamic models, with possible disequilibrium behavior, which is the topic of Chapter 5, to enable the study of the evolution of supply chains.

In this chapter, manufacturers are considered who are involved in the production of a homogeneous commodity, referred to also as the product, which can then be shipped to the retailers or to the consumers directly or to both. The manufacturers obtain a price for the product (which is endogenous) and seek to determine their optimal production and shipment quantities, given the production costs as well as the *transaction* costs associated with conducting business with the different retailers and demand markets. Here a transaction cost is considered to be sufficiently general, for example, to include the transportation/shipping cost. On the other hand, in the case of an e-commerce link, the transaction costs can include the cost associated with the use of such a link, the lack of productivity due to congestion, an associated risk, etc.

The retailers, in turn, must agree with the manufacturers as to the volume of shipments, either ordered physically or through the Internet, since they are faced with the handling cost associated with having the product in their retail outlet. In addition, they seek to maximize their profits with the price that the consumers are willing to pay for the product being endogenous.

Finally, in this supply chain, the consumers provide the "pull" in that, given the demand functions at the various demand markets, they determine their optimal consumption levels from the various retailers (transacted either physically or through the Internet) and from the manufacturers (transacted through the Internet), subject both to the prices charged for the product as well as the cost of conducting the transaction (which, of course, may include the cost of transportation associated with obtaining the product from the manufacturer or the retailer). Thus, the demand for the product is a central part of the supply chain framework.

In this chapter, it is shown that, in equilibrium, the structure of the supply chain network is that of a three-tiered network, with links connecting the top tier (the manufacturers) with the bottom tier (the demand markets) to represent e-commerce links and additional links from the top tier to the middle tier (the retailers) and from the middle tier to the bottom tier nodes to also represent the e-commerce links. The variational inequality formulation of the governing equilibrium conditions is then utilized in order to obtain both qualitative properties as well as an algorithm for the computation of the equilibrium flows and prices.

The chapter is organized as follows. In Section 4.1, the supply chain network model with electronic commerce is presented, the optimality conditions for each set of network agents or decision-makers derived, and the governing equilibrium conditions given. The finite-dimensional variational inequality formulation of the equilibrium conditions is also established. We then discuss two applications of the model to an online grocery and to an online bookseller, respectively. In Section 4.2, some qualitative properties of the equilibrium pattern are obtained as well as the necessary properties for proving convergence of a computational procedure. In Section 4.3, the computational procedure is described, which, in the context of the supply chain application, resolves the supernetwork problem into subproblems, each of which can be solved exactly and in closed form. In Section 4.4, the algorithm is applied to numerical examples to determine the equilibrium flows and prices.

4.1 The Supply Chain Network Model with Electronic Commerce

In this section, the supply chain network model is developed. It consists of manufacturers, retailers, and consumers. The manufacturers can sell directly to the consumers at the demand markets through the Internet and can also

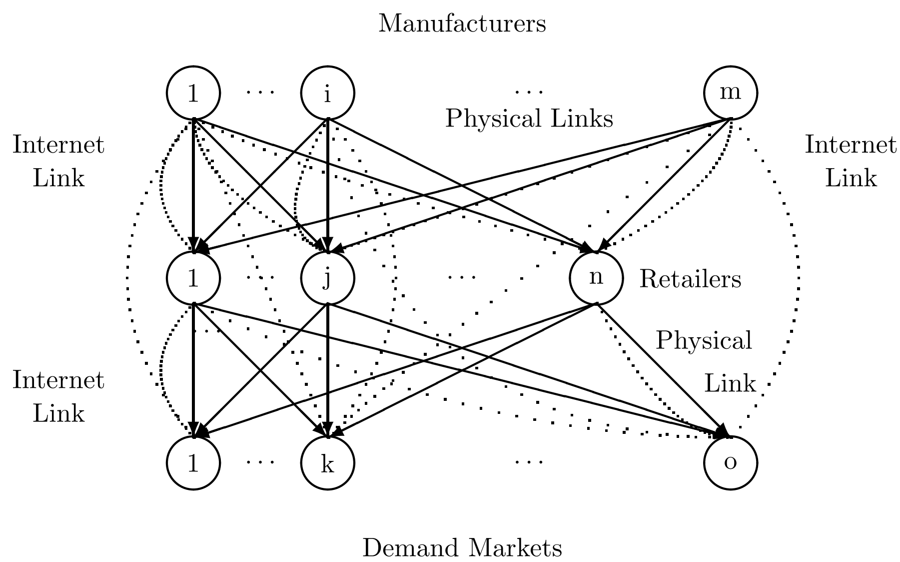

Fig. 4.1. The Multitiered Supernetwork Structure of the Supply Chain Network with E-Commerce at Equilibrium

conduct their business transactions with the retailers through the Internet. The consumers, in turn, can also purchase the product from the retailers either "physically" or electronically, that is, "on-line," through the Internet. Figure 4.1 depicts the multitiered supernetwork structure of the supply chain network, at equilibrium, which is established in this section.

Specifically, consider m manufacturers involved in the production of a homogeneous product which can then be purchased by n retailers and/or directly by the consumers located at the o demand markets. Denote a typical manufacturer by i, a typical retailer by j, and a typical demand market by k. Note that the manufacturers are located at the top tier of nodes of the network, the retailers at the middle tier, and the demand markets at the third or bottom tier of nodes.

The links in the supply chain supernetwork in Figure 4.1 include classical physical links as well as Internet links to allow for e-commerce. The introduction of e-commerce allows for "connections" that were, heretofore, not possible, such as enabling consumers, for example, to purchase a product directly from the manufacturers. In order to conceptualize this B2C type of transaction, a direct link has been constructed from each top tier node to each bottom tier node. In addition, since manufacturers can transact not only with the consumers directly but also with the retailers through the Internet, an additional link is added (to represent such a possible B2B transaction) between each top tier node and each middle tier node. Hence, a manufacturer may transact with a retailer through either a physical link or through an Internet link, or through both. Finally, consumers can transact with retailers either via a physical link, or through an Internet link, or through both.

The behavior of the various economic network agents represented by the three tiers of nodes in Figure 4.1 is now described. We first focus on the manufacturers. We then turn to the retailers and, subsequently, to the consumers.

The Behavior of the Manufacturers and Their Optimality Conditions

Let q_i denote the nonnegative production output of manufacturer i. Group the production outputs of all manufacturers into the column vector $q \in R^m_+$. Here it is assumed that each manufacturer i is faced with a production cost function f_i, which can depend, in general, on the entire vector of production outputs, that is,

$$f_i = f_i(q), \quad \forall i. \tag{4.1}$$

Hence, the production cost of a particular manufacturer can depend not only on his production output but also on those of the other manufacturers. This allows one to model competition.

Figure 4.2 depicts the allowable transactions of a typical manufacturer i with the consumers at the demand markets and with the retailers. Note that a manufacturer may transact with a retailer via a physical link, and/or via an Internet link.

The transaction cost associated with manufacturer i transacting with retailer j via link (also referred to as *mode*) l, where $l = 1$ denotes a physical link and $l = 2$ denotes an Internet link, is denoted by c_{ijl}. The product shipment associated with manufacturer i, retailer j, and mode of transaction l is denoted by q_{ijl}, and these product shipments into the column vector $Q^1 \in R^{2mn}_+$. In addition, a manufacturer i may transact directly with consumers located at a demand market k with this transaction cost associated with the Internet transaction denoted by c_{ik} and the associated product shipment from manufacturer i to demand market k by q_{ik}. Group these product shipments into the column vector $Q^2 \in R^{mo}_+$.

The transaction cost between a manufacturer and retail pair and the transaction cost between a manufacturer and consumers at a demand market may depend upon the volume of transactions between each such pair, and are given, respectively, by:

$$c_{ijl} = c_{ijl}(q_{ijl}), \quad \forall i, j, l, \tag{4.2a}$$

and

$$c_{ik} = c_{ik}(q_{ik}), \quad \forall i, k. \tag{4.2b}$$

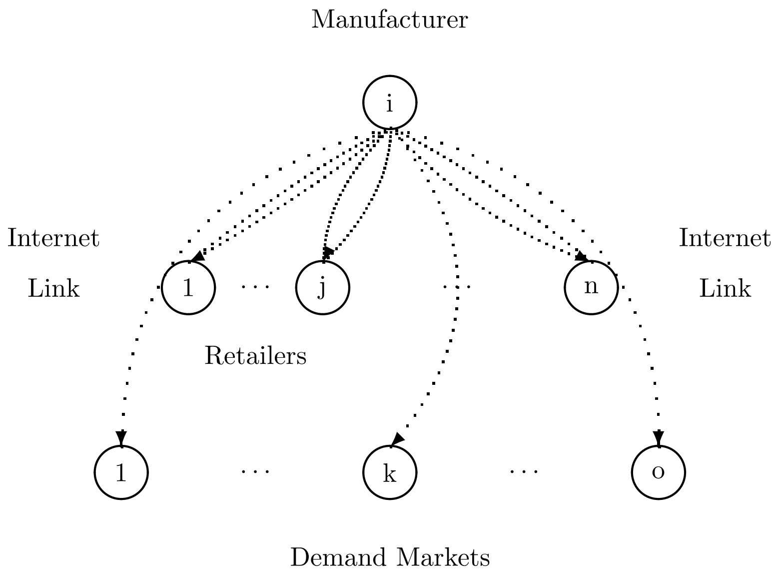

Fig. 4.2. Network Structure of Manufacturer i's Transactions

The quantity produced by manufacturer i must satisfy the following conservation of flow equation:

$$q_i = \sum_{j=1}^{n} \sum_{l=1}^{2} q_{ijl} + \sum_{k=1}^{o} q_{ik}, \tag{4.3}$$

which states that the quantity produced by manufacturer i is equal to the sum of the quantities shipped from the manufacturer to all retailers and to all demand markets.

The total costs incurred by a manufacturer i, thus, are equal to the sum of the manufacturer's production cost plus the total transaction costs. His revenue, in turn, is equal to the price that the manufacturer charges for the product (and the consumers are willing to pay) times the total quantity obtained/purchased of the product from the manufacturer by all the retail outlets and consumers at all demand markets. Let ρ^*_{1ijl} denote the price charged for the product by manufacturer i to retailer j who has transacted using mode l, and let ρ^*_{1ik} denote the price charged by manufacturer i for the product to consumers at demand market k. Hence, manufacturers can price according to their location, as to whether the product is sold to the retailers or to the consumers directly, and according to whether the transaction was conducted via the Internet or not. How these prices are arrived at is discussed later in this section.

Noting the conservation of flow equations (4.3) and the production cost functions (4.1), one can express the criterion of profit maximization for manufacturer i as:

$$\text{Maximize} \sum_{j=1}^{n}\sum_{l=1}^{2}\rho^*_{1ijl}q_{ijl} - f_i(Q^1,Q^2) - \sum_{j=1}^{n}\sum_{l=1}^{2}c_{ijl}(q_{ijl}) - \sum_{k=1}^{o}c_{ik}(q_{ik})$$

$$+\sum_{k=1}^{o}\rho^*_{1ik}q_{ik}, \tag{4.4}$$

subject to $q_{ijl} \geq 0$, for all j, l, and $q_{ik} \geq 0$, for all k.

The manufacturers are assumed to compete in a noncooperative fashion. Also, it is assumed that the production cost functions and the transaction cost functions for each manufacturer are continuous and convex (see Appendix A). The governing optimization/equilibrium concept underlying noncooperative behavior is that of Nash (1950, 1951), which states, in this context, that each manufacturer will determine his optimal production quantity and shipments, given the optimal ones of the competitors. Hence, the optimality conditions for all manufacturers *simultaneously* can be expressed as the following inequality (see also Gabay and Moulin (1980), Dafermos and Nagurney (1987), Bazarra, Sherali, and Shetty (1993), and Nagurney (1999)): Determine the solution (Q^{1^*}, Q^{2^*}), which satisfies:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(Q^{1^*},Q^{2^*})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}} - \rho^*_{1ijl}\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1^*},Q^{2^*})}{\partial q_{ik}} + \frac{\partial c_{ik}(q^*_{ik})}{\partial q_{ik}} - \rho^*_{1ik}\right] \times \left[q_{ik} - q^*_{ik}\right] \geq 0, \tag{4.5}$$

$$\forall Q^1 \in R^{2mn}_+, \forall Q^2 \in R^{mo}_+.$$

The inequality (4.5), which is a *variational inequality* (see also Appendix B) has a nice economic interpretation. In particular, from the first term one can infer that, if there is a positive shipment of the product transacted either in a classical manner or via the Internet from a manufacturer to a retailer, then the marginal cost of production plus the marginal cost of transacting must be equal to the price that the retailer is willing to pay for the product. If the marginal cost of production plus the marginal cost of transacting exceeds that price, then there will be zero volume of flow of the product on that link. The second term in (4.5) has a similar interpretation; in particular, there will be a positive volume of flow of the product from a manufacturer to a demand market if the marginal cost of production of the manufacturer plus the marginal cost of transacting with the consumers at a demand market via the Internet is equal to the price the consumers are willing to pay for the product at the demand market.

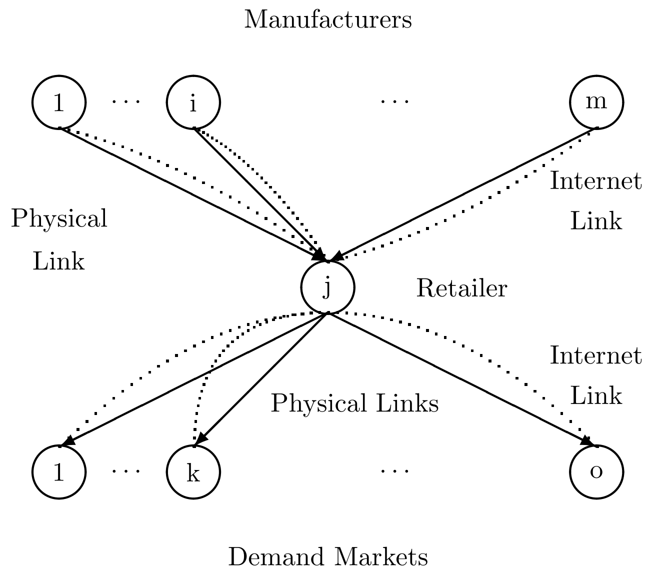

Fig. 4.3. Network Structure of Retailer j's Transactions

The Behavior of the Retailers and Their Optimality Conditions

The retailers, in turn, are involved in transactions both with the manufacturers since they wish to obtain the product for their retail outlets, as well as with the consumers, who are the ultimate purchasers of the product. Thus, a retailer conducts transactions both with the manufacturers as well as with the consumers. See Figure 4.3 for a graphical depiction.

A retailer j is faced with what is termed a *handling* cost, which may include, for example, the display and storage cost associated with the product. Denote this cost by c_j and, in the simplest case, one would have that c_j is a function of $\sum_{i=1}^m \sum_{l=1}^2 q_{ijl}$, that is, the holding cost of a retailer is a function of how much of the product he has obtained from the various manufacturers via the two different modes of transacting. However, for the sake of generality, and to enhance the modeling of competition, allow the function to, in general, depend also on the amounts of the product held by other retailers and, therefore, one may write:

$$c_j = c_j(Q^1), \quad \forall j. \tag{4.6}$$

The retailers, in turn, also have associated transaction costs in regards to transacting with the manufacturers via either modal alternative. Denote the transaction cost associated with retailer j transacting with manufacturer i using mode l by $\hat{c}_{ijl}$ and assume that the function can depend upon the product shipment q_{ijl}, that is,

$$\hat{c}_{ijl} = \hat{c}_{ijl}(q_{ijl}), \quad \forall i, j, l. \tag{4.7}$$

Let q_{jkl} denote the amount of the product purchased/consumed by consumers located at demand market k from retailer j, using transaction mode l, where, as in the case of the manufacturer/retailer transactions discussed above, $l = 1$ denotes a physical mode (and corresponding link) of transaction, whereas $l = 2$ denotes an electronic mode of transaction (through an Internet link). Group these consumption quantities into the column vector $Q^3 \in R^{2no}_+$.

The retailers associate a price with the product at their retail outlet, which is denoted by γ_j^*, for retailer j. This price, as will be shown, will also be endogenously determined in the model and will be, given a positive volume of flow between a retailer and any demand market, equal to a clearing-type price. Assuming, as mentioned earlier in this chapter, that the retailers are also profit-maximizers, the optimization problem of a retailer j is given by:

$$\text{Maximize} \quad \gamma_j^* \sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl} - c_j(Q^1) - \sum_{i=1}^{m} \sum_{l=1}^{2} \hat{c}_{ijl}(q_{ijl}) - \sum_{i=1}^{m} \sum_{l=1}^{2} \rho_{1ijl}^* q_{ijl} \tag{4.8}$$

subject to:

$$\sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl} \leq \sum_{i=1}^{m} \sum_{l=1}^{2} q_{ijl}, \tag{4.9}$$

and the nonnegativity constraints: $q_{ijl} \geq 0$, and $q_{jkl} \geq 0$, for all i, l, and k. Objective function (4.8) expresses that the difference between the revenues and the handling cost plus the transaction costs and the payout to the manufacturers should be maximized. Constraint (4.9) simply expresses that consumers cannot purchase more from a retailer than is held in stock.

The optimality conditions of the retailers are now obtained, assuming that each retailer is faced with the optimization problem (4.8), subject to (4.9), and the nonnegativity assumption on the variables. Here it is also assumed that the retailers compete in a noncooperative manner so that each maximizes his profits, given the actions of the other retailers. Note that, at this point, we consider that retailers seek to determine not only the optimal amounts purchased by the consumers from their specific retail outlet but, also, the amount that they wish to obtain from the manufacturers. In equilibrium, all the shipments between the tiers of network agents will have to coincide.

Assuming that the handling cost for each retailer is continuous and convex as are the transaction costs, the optimal $(Q^{1^*}, Q^{3^*}, \rho_2^*)$ satisfy the optimality conditions for all the retailers (cf. Appendix B) or, equivalently, the variational inequality:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{l=1}^{2} \left[\frac{\partial c_j(Q^{1^*})}{\partial q_{ijl}} + \rho_{1ijl}^* + \frac{\partial \hat{c}_{ijl}(q_{ijl}^*)}{\partial q_{ijl}} - \rho_{2j}^* \right] \times \left[q_{ijl} - q_{ijl}^* \right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[-\gamma_j^* + \rho_{2j}^*\right] \times \left[q_{jkl} - q_{jkl}^*\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}^* - \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl}^*\right] \times \left[\rho_{2j} - \rho_{2j}^*\right] \geq 0, \qquad (4.10)$$

$$\forall Q^1 \in R_+^{2mn}, \forall Q^3 \in R_+^{2no}, \forall \rho_2 \in R_+^n,$$

where ρ_{2j} is the Lagrange multiplier associated with constraint (4.9) for manufacturer j and ρ_2 is the column vector of all the manufacturers' multipliers. For further background on such a derivation, see Bertsekas and Tsitsiklis (1989) and Appendix B. In this derivation, as in the derivation of inequality (4.5), the prices charged were not variables. We let γ^* be the column vector of endogenous equilibrium retailer prices with components $(\gamma_1^*, \ldots, \gamma_n^*)$.

The economic interpretation of the retailers' optimality conditions is now highlighted. From the second term in inequality (4.10), one has that, if consumers at demand market k purchase the product from a particular retailer j using mode l, that is, if the q_{jkl}^* is positive, then the price charged by retailer j, γ_j^*, is precisely equal to ρ_{2j}^*, which, from the third term in the inequality, serves as the price to clear the market from retailer j. Also, note that, from the second term, one sees that if no product is sold by a particular retailer, then the price associated with holding the product can exceed the price charged to the consumers. Furthermore, from the first term in inequality (4.10), one can infer that, if a manufacturer transacts with a retailer via a particular mode resulting in a positive flow of the product between the two, then the price ρ_{2j}^* is precisely equal to the retailer j's payment to the manufacturer, ρ_{1ijl}^*, plus his marginal cost of handling the product plus the retailer's marginal cost of transaction associated with transacting with the particular manufacturer.

The Consumers at the Demand Markets and the Equilibrium Conditions

We now describe the consumers located at the demand markets. The consumers take into account in making their consumption decisions not only the price charged for the product by the retailers and the manufacturers but also their transaction costs associated with obtaining the product. The consumers at the demand markets can transact either directly with the producing manufacturers through the Internet or physically with the retailers. In Figure 4.4, a depiction of consumers transacting at a typical demand market k is given.

Let $\hat{c}_{jkl}$ denote the unit transaction cost associated with obtaining the product by consumers at demand market k from retailer j via mode l and recall that q_{jkl} is the amount of the product puchased (or flowing) between retailer j and consumers at demand market k on the connecting link l. Assume that the transaction cost is continuous and can, in general, depend

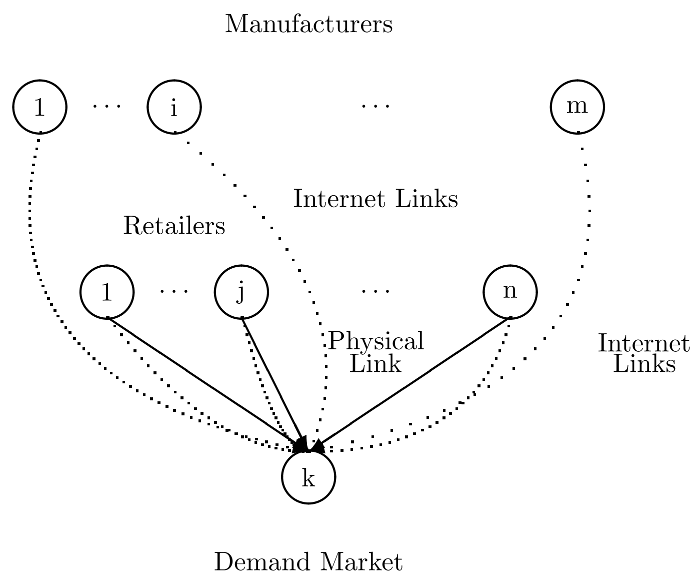

Fig. 4.4. Network Structure of Consumers' Transactions at Demand Market k

upon all the product shipments to all the demand markets, that is:

$$\hat{c}_{jkl} = \hat{c}_{jkl}(Q^2, Q^3), \quad \forall j, k, l. \tag{4.11}$$

Also, let $\hat{c}_{ik}$ denote the unit transaction cost, from the perspective of the consumers at demand market k, associated with manufacturer i. Assume that

$$\hat{c}_{ik} = \hat{c}_{ik}(Q^2, Q^3), \quad \forall i, k. \tag{4.12}$$

Hence, the cost of conducting a transaction with a manufacturer via the Internet can depend, in general, upon the volumes of the product transacted via all the Internet links as well as those transacted from all the retailers.

Let now ρ_{3k} denote the demand price of the product at demand market k. Further, denote the demand for the product at demand market k by d_k and assume, as given, the continuous demand functions:

$$d_k = d_k(\rho_3), \quad \forall k, \tag{4.13}$$

where ρ_3 is the o-dimensional column vector of demand market prices. Thus, according to (4.13), the demand of consumers for the product at a demand market depends, in general, not only on the price of the product at that demand market but also on the prices of the product at the other demand markets. Consequently, consumers at a demand market, in a sense, also compete with consumers at other demand markets.

The consumers take the price charged by the retailers for the product, which, recall was denoted by γ_j^* for retailer j, plus the transaction cost associated with obtaining the product, in making their consumption decisions. In addition, they take the price charged by a producer, ρ_{1ik}^*, plus that associated transaction cost into consideration.

The equilibrium conditions for consumers at demand market k, thus, take the form: for all retailers: $j; j = 1, \ldots, n$, and for all transaction modes l; $l = 1, 2$:

$$\gamma_j^* + \hat{c}_{jkl}(Q^{2^*}, Q^{3^*}) \begin{cases} = \rho_{3k}^*, & \text{if} \quad q_{jkl}^* > 0 \\ \geq \rho_{3k}^*, & \text{if} \quad q_{jkl}^* = 0, \end{cases} \tag{4.14}$$

for all manufacturers i; $i = 1, \ldots, m$:

$$\rho_{1ik}^* + \hat{c}_{ik}(Q^{2^*}, Q^{3^*}) \begin{cases} = \rho_{3k}^*, & \text{if} \quad q_{ik}^* > 0 \\ \geq \rho_{3k}^*, & \text{if} \quad q_{ik}^* = 0, \end{cases} \tag{4.15}$$

and for demand market k:

$$d_k(\rho_3^*) \begin{cases} = \sum_{j=1}^{n} \sum_{l=1}^{2} q_{jkl}^* + \sum_{i=1}^{m} q_{ik}^*, & \text{if} \quad \rho_{3k}^* > 0 \\ \leq \sum_{j=1}^{n} \sum_{l=1}^{2} q_{jkl}^* + \sum_{i=1}^{m} q_{ik}^*, & \text{if} \quad \rho_{3k}^* = 0. \end{cases} \tag{4.16}$$

Conditions (4.14) state that consumers at demand market k will purchase the product from retailer j, transacted via mode l, if the price charged by the retailer for the product plus the transaction cost (from the perspective of the consumers) does not exceed the price that the consumers are willing to pay for the product. Conditions (4.15), in turn, state the analogue for the manufacturers and demand market. Conditions (4.16), on the other hand, reflect that, if the price the consumers are willing to pay for the product at a demand market is positive, then the quantity consumed by the consumers at the demand market is precisely equal to the demand. These conditions correspond to the well-known spatial price equilibrium conditions (cf. Samuelson (1952), Takayama and Judge (1971), and Nagurney (1999) and the references therein).

In equilibrium, conditions (4.14), (4.15), and (4.16) will have to hold for all demand markets k, and these, in turn, can also be expressed as a variational inequality problem akin to (4.5) and (4.10) and given by: Determine $(Q^{2^*}, Q^{3^*}, \rho_3^*) \in R^{mo+2no+n}$, such that

$$\sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{2} \left[\gamma_j^* + \hat{c}_{jkl}(Q^{2^*}, Q^{3^*}) - \rho_{3k}^*\right] \times \left[q_{jkl} - q_{jkl}^*\right]$$

$$+ \sum_{i=1}^{m} \sum_{k=1}^{n} \left[\rho_{1ik}^* + \hat{c}_{ik}(Q^{2^*}, Q^{3^*}) - \rho_{3k}^*\right] \times \left[q_{ik} - q_{ik}^*\right]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2}q^*_{jkl}+\sum_{i=1}^{m}q^*_{ik}-d_k(\rho_3^*)\right]\times[\rho_{3k}-\rho^*_{3k}]\geq 0, \qquad (4.17)$$

$$\forall (Q^2,Q^3,\rho_3)\in R_+^{mo+2no+n}.$$

Note that, in the context of the consumption decisions, demand functions, rather than utility functions, have been utilized, in contrast to the manufacturers and the retailers, who were assumed to be faced with profit functions, which correspond to utility functions. Of course, demand functions can be derived from utility functions (cf. Arrow and Intrilligator (1982)). One can expect the number of consumers to be much greater than that of the manufacturers and retailers and, hence, the above formulation is the more natural and tractable one.

The Equilibrium Conditions of the Supply Chain

In equilibrium, the shipments of the product that the manufacturers ship to the retailers must be equal to the shipments that the retailers accept from the manufacturers. In addition, the amounts of the product purchased by the consumers must be equal to the amounts sold by the retailers and directly to the consumers by the manufacturers. Furthermore, the equilibrium shipment and price pattern in the supernetwork must satisfy the sum of the inequalities (4.5), (4.10), and (4.17) in order to formalize the agreements between the tiers. This is now stated formally in the following definition.

Definition 4.1: Supply Chain Network Equilibrium
The equilibrium state of the supernetwork consisting of the supply chain with electronic commerce is one where the flows between the tiers of the supernetworks coincide and the product shipments and prices satisfy the sum of the optimality conditions (4.5), (4.10), and the conditions (4.17).

The variational inequality formulation of the governing equilibrium conditions is now derived. It is used as the basis for obtaining qualitative properties of the equilibrium shipment and price pattern as well as an algorithm for its computation. We also utilize it to provide an alternative but equivalent interpretation of the equilibrium state as defined above.

Theorem 4.1: Variational Inequality Formulation
A product shipment and price pattern $(Q^{1^*},Q^{2^*},Q^{3^*},\rho_2^*,\rho_3^*)\in\mathcal{K}$ *is an equilibrium pattern of the supply chain model with electronic commerce according to Definition 4.1 if and only if it satisfies the variational inequality problem:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(Q^{1^*},Q^{2^*})}{\partial q_{ijl}}+\frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}}+\frac{\partial c_j(Q^{1^*})}{\partial q_{ijl}}+\frac{\partial \hat{c}_{ijl}(q^*_{ijl})}{\partial q_{ijl}}-\rho^*_{2j}\right]$$

$$\times\left[q_{ijl}-q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ik}}+\frac{\partial c_{ik}(q^*_{ik})}{\partial q_{ik}}+\hat{c}_{ik}(Q^{2*},Q^{3*})-\rho^*_{3k}\right]\times[q_{ik}-q^*_{ik}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\rho^*_{2j}+\hat{c}_{jkl}(Q^{2*},Q^{3*})-\rho^*_{3k}\right]\times\left[q_{jkl}-q^*_{jkl}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2}q^*_{ijl}-\sum_{k=1}^{o}\sum_{l=1}^{2}q^*_{jkl}\right]\times[\rho_{2j}-\rho^*_{2j}]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2}q^*_{jkl}+\sum_{i=1}^{m}q^*_{ik}-d_k(\rho^*_3)\right]\times[\rho_{3k}-\rho^*_{3k}]\geq 0, \tag{4.18}$$

$$\forall(Q^1,Q^2,Q^3,\rho_2,\rho_3)\in\mathcal{K},$$

where $\mathcal{K}\equiv\{(Q^1,Q^2,Q^3,\rho_2,\rho_3)|(Q^1,Q^2,Q^3,\rho_2,\rho_3)\in R_+^{2mn+mo+2no+n+o}\}$.

Proof: It is first established that the equilibrium conditions imply variational inequality (4.18). Summing up inequalities (4.5), (4.10), and (4.17) yields, after algebraic simplification variational inequality (4.18).

We now establish the converse, that is, that a solution to variational inequality (4.18) satisfies the sum of inequalities (4.5), (4.10), and (4.17). and is, hence, an equilibrium according to Definition 4.1.

To inequality (4.18), add the term: $-\rho^*_{1ijl}+\rho^*_{1ijl}$ to the term in the first set of brackets preceding the multiplication sign. Similarly, add the term: $-\rho^*_{1ik}+\rho^*_{1ik}$ to the term preceding the second multiplication sign, and, finally, add the term: $-\gamma^*_j+\gamma^*_j$ to the term preceding the third multiplication sign. Such "terms" do not change the value of the inequality since they are identically equal to zero, with the resulting inequality of the form:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ijl}}+\frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}}+\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}}+\frac{\partial \hat{c}_{ijl}(q^*_{ijl})}{\partial q_{ijl}}-\rho^*_{2j}\right.$$

$$\left.-\rho^*_{1ijl}+\rho^*_{1ijl}\right]\times\left[q_{ijl}-q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ik}}+\frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}}+\hat{c}_{ik}(Q^{2*},Q^{3*})-\rho^*_{3k}-\rho^*_{1ik}+\rho^*_{1ik}\right]$$

$$\times[q_{ik}-q^*_{ik}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\rho^*_{2j}+\hat{c}_{jk}(Q^{2*},Q^{3*})-\rho^*_{3k}-\gamma^*_j+\gamma^*_j\right]\times\left[q_{jkl}-q^*_{jkl}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2}q^*_{ijl}-\sum_{k=1}^{o}\sum_{l=1}^{2}q^*_{jkl}\right]\times[\rho_{2j}-\rho^*_{2j}]$$

$$\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2} q^*_{jkl} + \sum_{i=1}^{m} q^*_{ik} - d_k(\rho^*_3)\right] \times [\rho_{3k} - \rho^*_{3k}] \geq 0, \tag{4.19}$$

$$\forall (Q^1, Q^2, Q^3, \rho_2, \rho_3) \in \mathcal{K},$$

which, in turn, can be rewritten as:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(Q^{1^*}, Q^{2^*})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}} - \rho^*_{1ijl}\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1^*}, Q^{2^*})}{\partial q_{ik}} + \frac{\partial c_{ik}(q^*_{ik})}{\partial q_{ik}} - \rho^*_{1ik}\right] \times [q_{ik} - q^*_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_j(Q^{1^*})}{\partial q_{ijl}} + \rho^*_{1ijl} + \frac{\partial \hat{c}_{ijl}(q^*_{ijl})}{\partial q_{ijl}} - \rho^*_{2j}\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[-\gamma^*_j + \rho^*_{2j}\right] \times \left[q_{jkl} - q^*_{jkl}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2} q^*_{ijl} - \sum_{k=1}^{o}\sum_{l=1}^{2} q^*_{jkl}\right] \times \left[\rho_{2j} - \rho^*_{2j}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\gamma^*_j + \hat{c}_{jkl}(Q^{2^*}, Q^{3^*}) - \rho^*_{3k}\right] \times \left[q_{jkl} - q^*_{jkl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\rho^*_{1ik} + \hat{c}_{ik}(Q^{2^*}, Q^{3^*}) - \rho^*_{3k}\right] \times [q_{ik} - q^*_{ik}]$$

$$\sum_{k=1}^{o}\left[\sum_{j=1}^{n}\sum_{l=1}^{2} q^*_{jkl} + \sum_{i=1}^{m} q^*_{ik} - d_k(\rho^*_3)\right] \times [\rho_{3k} - \rho^*_{3k}] \geq 0, \tag{4.20}$$

$$\forall (Q^1, Q^2, Q^3, \rho_2, \rho_3) \in \mathcal{K}.$$

But inequality (4.20) is equivalent to the price and shipment pattern satisfying the sum of conditions (4.5), (4.10), and (4.17). The proof is complete. □

We now utilize variational inequality (4.18) to derive alternative but equivalent equilibrium conditions to those in Definition 4.1 which highlight the consistency of the supernetwork framework and the integration of the preceding conditions for the distinct networks tiers.

In particular, we note that if we let $(Q^2, Q^3, \rho_2, \rho_3) = (Q^{2^*}, Q^{3^*}, \rho_2^*, \rho_3^*)$ and substitute into (4.18), the resulting inequality is equivalent to the statement that: For all i, j, l, in equilibrium, we must have that:

$$\frac{\partial f_i(Q^{1^*}, Q^{2^*})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl}^*)}{\partial q_{ijl}} + \frac{\partial c_j(Q^{1^*})}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl}^*)}{\partial q_{ijl}} \begin{cases} = \rho_{2j}^*, & \text{if} \quad q_{ijl}^* > 0 \\ \geq \rho_{2j}^*, & \text{if} \quad q_{ijl}^* = 0. \end{cases} \tag{4.21}$$

Similarly, if we set $(Q^1, Q^3, \rho_2, \rho_3) = (Q^{1^*}, Q^{3^*}, \rho_2^*, \rho_3^*)$ and substitute the resultant into inequality (4.18), we obtain an inequality which is equivalent to the statement that, in equilibrium, we must have that, for all i, k:

$$\frac{\partial f_i(Q^{1^*}, Q^{2^*})}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^*)}{\partial q_{ik}} + \hat{c}_{ik}(Q^{2^*}, Q^{3^*}) \begin{cases} = \rho_{3k}^*, & \text{if} \quad q_{ik}^* > 0 \\ \geq \rho_{3k}^*, & \text{if} \quad q_{ik}^* = 0. \end{cases} \tag{4.22}$$

If we now make the substitution $(Q^1, Q^2, \rho_2, \rho_3) = (Q^{1^*}, Q^{2^*}, \rho_2^*, \rho_3^*)$ in (4.18) we obtain the inequality which is equivalent to the statement that, for all j, k, l:

$$\rho_{2j}^* + \hat{c}_{jkl}(Q^{2^*}, Q^{3^*}) \begin{cases} = \rho_{3k}^*, & \text{if} \quad q_{jkl}^* > 0 \\ \geq \rho_{3k}^*, & \text{if} \quad q_{jkl}^* = 0. \end{cases} \tag{4.23}$$

Similarly, if we now let $(Q^1, Q^2, Q^3, \rho_3) = (Q^{1^*}, Q^{2^*}, Q^{3^*}, \rho_3^*)$ and substitute into (4.18), the resulting inequality is equivalent to the statement that, for all j, we must have that:

$$\sum_{j=1}^{n} \left[\sum_{i=1}^{m} \sum_{l=1}^{2} q_{ijl}^* - \sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl}^* \right] \begin{cases} = 0, & \text{if} \quad \rho_{2j}^* > 0 \\ \geq 0, & \text{if} \quad \rho_{2j}^* = 0. \end{cases} \tag{4.24}$$

Analogously, if we let $(Q^1, Q^2, Q^2, \rho_2) = (Q^{1^*}, Q^{2^*}, Q^{3^*}, \rho_2^*)$ and make this substitution into (4.18), we obtain precisely the inequality representing conditions (4.16).

We now provide an economic interpretation for the equilibrium conditions (4.21) , (4.22), and (4.24). The economic interpretation of (4.16) was given earlier, whereas that for (4.23) coincides with that for (4.14).

Specifically, we have that, according to (4.21), if the product shipment transacted via a mode is positive between a manufacturer and retailer, then the marginal production cost plus the marginal transaction costs and marginal handling cost is equal to the price of the product associated at the retailer. If the sum of all those marginal costs exceeds the price, then there will be a zero amount of the product transacted via that mode and between that manufacturer and retailer pair.

Similarly, according to (4.22), we have that if there is a positive amount of the product shipped between a manufacturer and demand market in equilibrium, then the marginal production cost of the manufacturer plus the associated marginal transaction cost plus the unit transaction cost from the consumers' perspective must be equal to the demand price at the demand market. The product shipment will be zero between the manufacturer and demand market pair if the sum of the above described marginal and unit costs exceeds the demand market price.

According to (4.24), in turn, we have that if the price associated with a retailer is positive in equilibrium, then the product shipments into that retailer must be equal to the product shipments out (that is, to the consumers). If the product shipments to a retailer exceed the product shipments out, then the associated price of the product at the retailer will be zero in equilibrium.

Standard Variational Inequality Formulation

For easy reference in the subsequent sections, variational inequality problem (4.18) can be rewritten in standard variational inequality form (cf. Definition 3.1 and Appendix B) as follows:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{4.25}$$

where $X \equiv (Q^1, Q^2, Q^3, \rho_2, \rho_3)$, $F(X) \equiv (F_{ijl}, F_{ik}, F_{jkl}, F_j, F_k)$ for $\{i = 1, ..., m; j = 1, ..., n; l = 1, 2; k = 1, ..., o\}$, with the specific components of F given by the functional terms preceding the multiplication signs in (4.18), respectively. The feasible set $\mathcal{K}$ was defined previously following (4.18). The term $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space.

We now discuss how to recover the prices ρ^*_{1ijl}, for all i, j, l, and γ^*_j, for all j, from the solution of variational inequality (4.18). (In Section 4.3 we describe an algorithm for computing the solution.) Recall that, in the preceding discussions, it was noted that if $q^*_{jkl} > 0$, for some k, j, and l, then γ^*_j is precisely equal to ρ^*_{2j}, which can be obtained from the solution of (4.18). The prices ρ^*_{1ijl}, in turn (cf. also (4.20)), can be obtained by finding a $q^*_{ijl} > 0$, and then setting $\rho^*_{1ijl} = \left[\frac{\partial f(Q^{1*}, Q^{2*})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}}\right]$, or, equivalently, to $\left[\rho^*_{2j} - \frac{\partial c_j(Q^{1*})}{\partial q_{ijl}} - \frac{\partial \hat{c}_{ijl}(q^*_{ijl})}{\partial q_{ijl}}\right]$, for all such i, j, l. The prices ρ^*_{1ik} can be obtained by finding a $q^*_{ik} > 0$ and setting $\rho^*_{1ik} = \left[\frac{\partial f_i(Q^{1*}, Q^{2*})}{\partial q_{ik}} + \frac{\partial c_{ik}(q^*_{ik})}{\partial q_{ik}}\right]$, or, equivalently, to $\left[\rho^*_{3k} - \hat{c}_{ik}(Q^{2*}, Q^{3*})\right]$, for all such i, k.

The supernetwork representing the supply chain network in equilibrium (cf. Figure 4.1) is now constructed, using, as building blocks, the previously drawn networks in Figures 4.2 through 4.4 corresponding, respectively, to the transactions of the manufacturers, the retailers, and the consumers. First, however, one needs to establish the result that, in equilibrium, the sum of the product shipments to each retailer is equal to the sum of the product shipments out. Hence, the corresponding ρ^*_{2j}s will all be positive. This means that each retailer, assuming profit-maximization, only purchases from the manufacturers the amount of the product that is actually consumed by the consumers. In order to establish this result, variational inequality (4.18) is utilized. Clearly, one knows that, if $\rho^*_{2j} > 0$, then the "market clears" for that retailer, that is, $\sum_{i=1}^m \sum_{l=1}^2 q^*_{ijl} = \sum_{k=1}^o \sum_{l=1}^2 q^*_{jkl}$. Let us now consider the case where $\rho_{2j}^* = 0$ for some retailer j. From the first term in inequality (4.18), since the production cost functions, and the transaction cost functions and handling cost functions have been assumed to be convex, and, assuming further, which is not unreasonable, that either the marginal cost of production or the marginal transaction costs or the marginal holding cost for each manufacturer/mode/retailer combination is strictly positive at equilibrium, then one knows that $\frac{\partial f_i(Q^{1^*},Q^{2^*})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl}^*)}{\partial q_{ijl}} + \frac{\partial c_j(Q^{1^*})}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl}^*)}{\partial q_{ijl}} > 0$, which implies that $q_{ijl}^* = 0$, and this holds for all i, l. It follows then from the fourth term in (4.18), that $\sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl}^* = 0$, and, hence, the market clears also in this case since the flow into a retailer is equal to the flow out and equal to zero. The following result has, thus, been established:

Corollary 4.1: The Market Clears for the Retailers
The market for the product clears for each retailer in the supply chain network with e-commerce at equilibrium.

In Figure 4.1, the supernetwork structure of the supply chain network in equilibrium, is depicted. It consists of all the manufacturers, all the retailers, and all the demand markets. In order to construct the supernetwork in Figure 4.1, Figure 4.2 has been replicated for all the manufacturers; Figure 4.3 has been replicated for all the retailers, and Figure 4.4 for all the demand markets. The supernetwork represents the possible transactions of all the economic agents. In addition, since there must be agreement between/among the transactors at equilibrium, the analogous links (and equilibrium flows on them) must coincide, yielding the network structure given in Figure 4.1. The vectors of prices ρ_1^*, γ^* and ρ_2^*, and ρ_3^* are associated, respectively, with the top tier, the middle tier, and the bottom tier of nodes in the Figure 4.1 network. The components of the vector of equilibrium product shipments Q^{1^*} correspond to the flows on the links joining the manufacturing (top tier) nodes with the retailer (middle tier) nodes. The components of the vector of equilibrium product shipments Q^{2^*} correspond to the flows on the Internet links joining the manufacturer nodes with the demand market nodes, whereas the components of the vector of equilibrium product shipments Q^{3^*} correspond to the flow on the links (physical or Internet) joining the retailer nodes with the demand market nodes.

Clearly, the special cases of our model in which there is only B2B commerce or only B2C commerce can be studied in this framework, as well, with a suitable reduction of the links and associated transaction costs and product shipments (and prices). In particular, in the absence of electronic commerce the supernetwork depicted in Figure 4.1 reduces to the network drawn in Figure 4.5.

Note that in the network in Figure 4.5 there are no Internet links and, thus, there are no Q^2 variables. Moreover, since there is only 1 mode of transacting between the manufacturers and the retailers, and 1 mode between the retailers and the demand markets, one can drop the subscript l

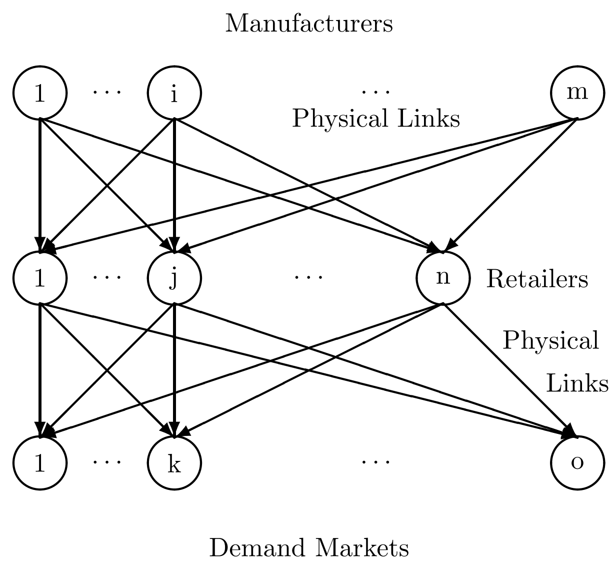

Fig. 4.5. The Multitiered Supernetwork Structure of the Supply Chain Network without E-Commerce

from those cost and flow terms and one has immediately the variational inequality formulation of the governing equilibrium conditions for this special case model:

Corollary 4.2: Variational Inequality Formulation of Supply Chain Network Equilibrium without E-Commerce
The variational inequality formulation of the supply chain network without Internet links is given by: Determine $(Q^{1^*}, Q^{3^*}, \rho_2^*, \rho_3^*) \in \mathcal{K}^1$ *satisfying:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i(Q^{1^*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}} + \frac{\partial c_j(Q^{1^*})}{\partial q_{ij}} + \frac{\partial \hat{c}_{ij}(q_{ij}^*)}{\partial q_{ij}} - \rho_{2j}^*\right] \times \left[q_{ij} - q_{ij}^*\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\rho_{2j}^* + \hat{c}_{jk}(Q^{3^*}) - \rho_{3k}^*\right] \times \left[q_{jk} - q_{jk}^*\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m} q_{ij}^* - \sum_{k=1}^{o} q_{jk}^*\right] \times \left[\rho_{2j} - \rho_{2j}^*\right]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n} q_{jk}^* - d_k(\rho_3^*)\right] \times [\rho_{3k} - \rho_{3k}^*] \geq 0, \quad \forall (Q^1, Q^3, \rho_2, \rho_3) \in \mathcal{K}^1, \quad (4.26)$$

where $\mathcal{K}^1 \equiv \{(Q^1, Q^3, \rho_2, \rho_3) | (Q^1, Q^3, \rho_2, \rho_3) \in R_+^{mn+no+n+o}\}$.

4.1.1 Applications

We now consider two applications of the above supernetwork framework for supply chain analysis. In Section 4.1.1.1, we describe an online grocer, whereas in Section 4.1.1.2, we discuss an online book retailer. Both of these businesses are examples of B2C electronic commerce (in particular, between a retailer and consumers at the demand markets) with the online grocer also consisting of brick and mortar supermarkets.

4.1.1.1 An Online Grocer – Tesco

Tesco, a supermarket located in the United Kingdom, had revenues in 2000 of \$30 billion. It uses its hundreds of supermarkets as bases for deliveries of products purchased from it online. According to Kapner (2001), Tesco invested only \$56 milllion in its online grocery initiative, a business that presently has \$422 million in sales annually, and is the largest of its type of business in the world. Last year, according to analysts (cf. Kapner (2001)), the company earned in the range of \$7 million from its online service. Hence, as reflected in Figure 4.1, Tesco is a retailer (in the form of an online grocer) who sells to consumers at a variety of demand markets either physically or electronically. Of course, it also competes with other supermarkets and grocery outlets and can obtain its products from different producers/manufacturers. Interestingly, for its online service it has opted not to use any warehouses, but has its outlets serve, in effect, as distribution centers. Moreover, according to Kapner (2001), Tesco charges a fee of \$7 per online transaction, which can also be handled by the above model through the concept of transaction cost. Fascinatingly, the company, well-aware of congestion and its effect on timely deliveries, has also devised, in a sense, "optimal" routes for product acquisition within its outlets, as well as planned routes of delivery in London, which should not exceed 25 minutes for delivery. Clearly, this enterprise reflects a strategic integration of logistics, transportation, and telecommunication networks.

Of course, in this as in any application, one may have to modify the supply chain network to reflect the particularities of the specific scenario. For example, in the case of multiple retail outlets controlled by the same firm, one may wish to optimize across the retail outlets, being aware, that these, in turn, have to compete with other retailers providing the same or similar products. In addition, if certain links are absent (or if additional ones are present) one would have to revise the supernetwork in Figure 4.1 accordingly. Nevertheless, the above framework is valuable for both conceptualization and theoretical purposes.

4.1.1.2 An Online Book Retailer – Amazon.com

No discussion of electronic commerce and supply chain issues would be complete without the inclusion of information about Amazon.com, a leader of the "dot.comers." Established in July, 1995 by Jeff Bezos, with a goal of changing book buying into the most convenient and easiest shopping experience, its web site (cf. Amazon.com (2001)) states that "29 million people in more than 160 countries have made us the leading online shopping site," with its selection of products vastly expanded from its original business of online book selling. Although in business six years, however, it has yet to make a pro forma profit (see *The Economist* (2001b)). Nevertheless, its size is huge with nearly $3 billion of sales a year and it is believed to be close to realizing profits.

Amazon.com obtains books from manufacturers/publishers and, in the case of a particular book, the supply chain (cf. Figure 4.1) would correspond to a single node in the top tier, with links connecting it to the second tier nodes, that is, the book retailers, who could be exclusively online, as is Amazon.com, or be both physical and online, or only physical. Amazon.com, hence, competes for consumers with other book retailers. Interestingly, as the above model reflects, each retailer prices the product identically, irregardless of whether the transaction is online or physical with the consumers. Indeed, as reported in *The Economist* (2001b), Amazon.com discovered that customers strongly dislike the idea of price discrimination, which the company attempted to achieve by charging different customers different prices for the same book.

In addition, the above model explicitly incorporates handling costs and, as reported in *The Economist* (2001b), Amazon.com has relatively fixed handling costs and this is viewed as one of its advantages, especially in regards to brick and mortar-type of competitors. However, as also reported therein, Amazon's business model depends on its supply chain efficiency, which is "held hostage by the pace at which its partners, from suppliers to transportation companies upgrade their own systems to complement Amazon's." Our supply chain framework explicitly allows for decentralized decision-making through a variety of network agents and, thus, allows one to capture and study such complex behavior and relationships.

4.1.2 An Extension

The above discussions focused on a 3-tiered network. It is now shown that the multitiered network concept can also be used to handle even more complex supply chain situations. In particular, suppliers are now explicitly considered. For example, in Figure 4.6, a 4-tiered supply chain is depicted where now suppliers represent the top tier of network agents and the remainder of the supernetwork is as drawn in Figure 4.1. There are two possible types of B2B e-commerce transactions: the B2B transactions via the Internet between manufacturers and the retailers and those between the suppliers and the manufacturers. Depending on the product, the suppliers may supply the same or different inputs to the manufacturers' production processes.

We emphasize that physical links of the supply chain models described

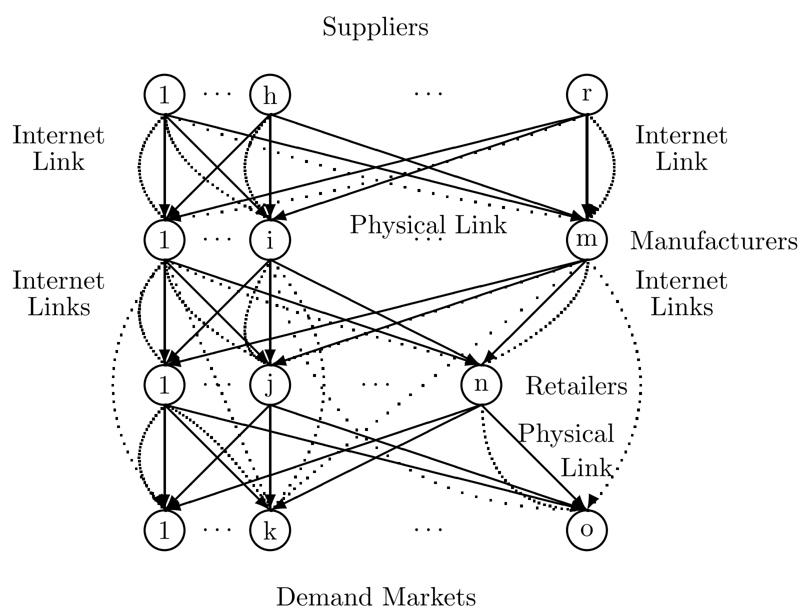

Fig. 4.6. The 4-Tiered Supernetwork Structure of the Supply Chain Network with Suppliers, Manufacturers, Retailers, and Demand Markets

in this and the next chapter are, in actuality, aggregations of transportation networks with associated origins, destinations, and routes. On these transportation networks, additional agents, in the form of shippers/carriers, interact in moving the freight (see Friesz, Gottfried, and Morlok (1986)). The freight consists of product inputs from suppliers to the manufacturers, and of products from manufacturers to retailers and/or to consumers. On the other hand, consumers in obtaining the product from the retailers must also cross physical distance to obtain the product and, hence, they interact on a transportation network as well. They, as the carriers of the freight, need to determine their optimal routes of transportation but in support of their retail activities. We discuss decision-making on transportation networks in a variety of contexts in Part III of this book.

Furthermore, the e-commerce links joining the manufacturers to the retailers and to the consumers; the suppliers to manufacturers, and the retailers to the consumers actually consist of telecommunication networks over which such transactions take place. E-commerce transactions, hence, trigger flows over the telecommunication networks, which, in turn, can trigger flows over transportation networks.

The above discussion points to further interrelationships among a variety of networks in regards to supply chains. We discuss such multilevel supernetworks in Chapter 5.

4.2 Qualitative Properties

In this section, some qualitative properties of the solution to variational inequality (4.18) are provided, notably, existence and uniqueness results. Properties of the function F (cf. (4.25)) that enters the variational inequality are also investigated. Existence of a solution is important to determine because without knowing that a solution exists, a model is vacuous.

Since the feasible set is not compact (that is, closed and bounded) one cannot derive existence of a solution, according to basic variational inequality theory, simply from the assumption of continuity of the functions (see also Appendix B). Nevertheless, one can impose a rather weak condition to guarantee existence of a solution pattern. Let

$$\mathcal{K}_b = \{(Q^1, Q^2, Q^3, \rho_2, \rho_3)|$$

$$0 \leq Q^1 \leq b_1;\ 0 \leq Q^2 \leq b_2; 0 \leq Q^3 \leq b_3;\ 0 \leq \rho_2 \leq b_4;\ 0 \leq \rho_3 \leq b_5\}, \quad (4.27)$$

where $b = (b_1, b_2, b_3, b_4, b_5) \geq 0$ and $Q^1 \leq b_1; Q^2 \leq b_2; Q^3 \leq b_3; \rho_2 \leq b_4; \rho_3 \leq b_5$ means that $q_{ijl} \leq b_1$; $q_{ik} \leq b_2$; $q_{jkl} \leq b_3$; $\rho_{2j} \leq b_4$; and $\rho_{3k} \leq b_5$ for all i, j, l, k. Then $\mathcal{K}_b$ is a bounded closed convex subset of $R^{2mn+mo+2no+n+o}$. Thus, the following variational inequality

$$\langle F(X^b), X - X^b \rangle \geq 0, \quad \forall X^b \in \mathcal{K}_b, \quad (4.28)$$

admits at least one solution $X^b \in \mathcal{K}_b$, from the standard theory of variational inequalities, since $\mathcal{K}_b$ is compact and F is continuous. Following Kinderlehrer and Stampacchia (1980) (see also Theorem B.3 in Appendix B), one then has:

Theorem 4.2
Variational inequality (4.18) admits a solution if and only if there exists a $b > 0$, such that variational inequality (4.28) admits a solution in $\mathcal{K}_b$ with

$$Q^{1b} < b_1, \quad Q^{2b} < b_2, \quad Q^{3b} < b_3, \quad \rho_2^b < b_4, \quad \rho_3^b < b_5. \tag{4.29}$$

Theorem 4.3: Existence
Suppose that there exist positive constants M, N, R with $R > 0$, such that:

$$\frac{\partial f_i(Q^1,Q^2)}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl})}{\partial q_{ijl}} + \frac{\partial c_j(Q^1)}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl})}{\partial q_{ijl}} \geq M, \quad \forall Q^1 \textit{ with } q_{ijl} \geq N,$$
$$\forall i,j,l, \tag{4.30}$$

$$\frac{\partial f_i(Q^1,Q^2)}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}} + \hat{c}_{ik}(Q^2,Q^3) \geq M, \quad \forall Q^2 \textit{ with } q_{ik} \geq N, \quad \forall i,k,$$
$$\hat{c}_{jkl}(Q^2,Q^3) \geq M, \quad \forall Q^3 \textit{ with } q_{jkl} \geq N, \quad \forall j,k,l,$$
$$d_k(\rho_3^*) \leq N, \quad \forall \rho \textit{ with } \rho_{3k} > R, \quad \forall k. \tag{4.31}$$

Then variational inequality (4.18); equivalently, variational inequality (4.25), admits at least one solution.

Proof: Under the conditions (4.30) and (4.31) it is possible to construct a $b > 0$ such that (4.29) holds and existence, hence, follows. See also Nagurney and Zhao (1993) and Nagurney, Dong, and Zhang (2000). □

Assumptions (4.30) and (4.31) are reasonable from an economics perspective, since when the product shipment between a manufacturer and demand market pair or a manufacturer and retailer is large, one can expect the corresponding marginal cost of production and/or the marginal cost of transaction from either the manufacturer's or the retailer's/consumers' perspectives to be bounded from below by positive constants. Moreover, in the case where the demand price of the product as perceived by a demand market is high, one can expect that the demand for the product will be low at that market.

We now recall the definition of an additive production cost introduced in Zhang and Nagurney (1996) for establishing some qualitative properties in dynamic network oligopoly problems, which will be utilized as an assumption for establishing additional qualitative properties of the supply chain network model.

Definition 4.2: Additive Production Cost
Suppose that for each manufacturer i, the production cost f_i is additive, that is,

$$f_i(q) = f_i^1(q_i) + f_i^2(\bar{q}_i), \tag{4.32}$$

where $f_i^1(q_i)$ is the internal production cost that depends solely on the manufacturer's own output level q_i, which may include the cost of production operation and facility maintenance, etc., and $f_i^2(\bar{q}_i)$ is the interdependent part of the production cost that is a function of all the other manufacturers' output levels $\bar{q}_i = (q_1, \cdots, q_{i-1}, q_{i+1}, \cdots, q_m)$ and reflects the impact of the other manufacturers' production patterns on manufacturer i's cost. This interdependent part of the production cost may describe the competition for the resources, consumption of the homogeneous raw materials, etc.

The properties of monotonicity and Lipschitz continuity of F (see also Appendix B) are now established and these properties will be utilized in the subsequent section for proving convergence of the algorithmic scheme. Furthermore, these properties will be utilized to obtain a variety of results for the dynamic version of this model in Chapter 5. Hence, proofs are provided here for completeness and easy reference.

Theorem 4.4: Monotonicity
Suppose that the production cost functions $f_i; i = 1, ..., m$, are additive, as defined in Definition 4.2, and $f_i^1; i = 1, ..., m$, are convex functions. If the c_{ijl}, c_j, and $\hat{c}_{ijl}$, and c_{ik} functions are convex; the $\hat{c}_{jkl}$ and the $\hat{c}_{ik}$ functions are monotone increasing, and the d_k functions are monotone decreasing functions of the demand prices, for all i, l, j, k, then the vector function F that enters the variational inequality (4.25) is monotone, that is,

$$\langle F(X') - F(X''), X' - X'' \rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}. \tag{4.33}$$

Proof: Let $X' = (Q^{1\prime}, Q^{2\prime}, Q^{3\prime}, \rho_2', \rho_3')$, $X'' = (Q^{1\prime\prime}, Q^{2\prime\prime}, Q^{3\prime\prime}, \rho_2'', \rho_3'')$. Then,

$$\langle F(X') - F(X''), X' - X'' \rangle$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{l=1}^{2} \left[\frac{\partial f_i(Q^{1\prime}, Q^{2\prime})}{\partial q_{ijl}} - \frac{\partial f_i(Q^{1\prime\prime}, Q^{2\prime\prime})}{\partial q_{ijl}} \right] \times [q_{ijl}' - q_{ijl}'']$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{l=1}^{2} \left[\frac{\partial c_{ijl}(q_{ijl}')}{\partial q_{ijl}} - \frac{\partial c_{ijl}(q_{ijl}'')}{\partial q_{ijl}} \right] \times [q_{ijl}' - q_{ijl}'']$$

$$+ \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{l=1}^{2} \left[\frac{\partial c_j(Q^{1\prime})}{\partial q_{ijl}} - \frac{\partial c_j(Q^{1\prime\prime})}{\partial q_{ijl}} \right] \times [q_{ijl}' - q_{ijl}'']$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial \hat{c}_{ijl}(q'_{ijl})}{\partial q_{ijl}}-\frac{\partial \hat{c}_{ijl}(q''_{ijl})}{\partial q_{ijl}}\right]\times[q'_{ijl}-q''_{ijl}]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1\prime},Q^{2\prime})}{\partial q_{ik}}-\frac{\partial f_i(Q^{1\prime\prime},Q^{2\prime\prime})}{\partial q_{ik}}\right]\times[q'_{ik}-q''_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial c_{ik}(q'_{ik})}{\partial q_{ik}}-\frac{\partial c_{ik}(q''_{ik})}{\partial q_{ik}}\right]\times[q'_{ik}-q''_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\hat{c}_{ik}(Q^{2\prime},Q^{3\prime})-\hat{c}_{ik}(Q^{2\prime\prime},Q^{3\prime\prime})\right]\times[q'_{ik}-q''_{ik}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\hat{c}_{jkl}(Q^{2\prime},Q^{3\prime})-\hat{c}_{jkl}(Q^{2\prime\prime},Q^{3\prime\prime})\right]\times\left[q'_{jkl}-q''_{jkl}\right]$$

$$+\sum_{k=1}^{o}[-d_k(\rho'_3)+d_k(\rho''_3)]\times[\rho'_{3k}-\rho''_{3k}]$$

$$=(I)+(II)+(III)+(IV)+(V)+(VI)+(VII)+(VIII)+(IX). \quad (4.34)$$

Since the f_i; $i=1,...,m$, are assumed to be additive, and the f_i^1; $i=1,...,m$, are convex functions, one has

$$(I)+(V)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i^1(Q^{1\prime},Q^{2\prime})}{\partial q_{ijl}}-\frac{\partial f_i^1(Q^{1\prime\prime},Q^{2\prime\prime})}{\partial q_{ijl}}\right]\times\left[q'_{ijl}-q''_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i^1(Q^{1\prime},Q^{2\prime})}{\partial q_{ik}}-\frac{\partial f_i^1(Q^{1\prime\prime},Q^{2\prime\prime})}{\partial q_{ik}}\right]\times[q'_{ik}-q''_{ik}]\geq 0. \quad (4.35)$$

The convexity of the transaction and handling cost functions: c_{ijl}, c_j, $\hat{c}_{ijl}$, and of c_{ik}, $\forall i,j,k,l$, gives, respectively:

$$(II)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_{ijl}(q'_{ijl})}{\partial q_{ijl}}-\frac{\partial c_{ijl}(q''_{ijl})}{\partial q_{ijl}}\right]\times\left[q'_{ijl}-q''_{ijl}\right]\geq 0, \quad (4.36)$$

$$(III)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_j(Q^{1\prime})}{\partial q_{ijl}}-\frac{\partial c_j(Q^{1\prime\prime})}{\partial q_{ijl}}\right]\times\left[q'_{ijl}-q''_{ijl}\right]\geq 0, \quad (4.37)$$

$$(IV)=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial \hat{c}_{ijl}(q'_{ijl})}{\partial q_{ijl}}-\frac{\partial \hat{c}_{ijl}(q''_{ijl})}{\partial q_{ijl}}\right]\times[q'_{ijl}-q''_{ijl}]\geq 0, \quad (4.38)$$

$$(VI)=\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial c_{ik}(q'_{ik})}{\partial q_{ik}}-\frac{\partial c_{ik}(q''_{ik})}{\partial q_{ik}}\right]\times[q'_{ik}-q''_{ik}]\geq 0. \quad (4.39)$$

Since the transaction cost functions $\hat{c}_{ik}$, $\forall i, k$, and $\hat{c}_{jkl}$, $\forall j, k, l$, in turn, are assumed to be monotone increasing, and the demand functions d_k, $\forall k$, are assumed to be monotone decreasing, one has

$$(VII) = \sum_{i=1}^{m} \sum_{k=1}^{o} \left[\hat{c}_{ik}(Q^{2\prime}, Q^{3\prime}) - \hat{c}_{ik}(Q^{2\prime\prime}, Q^{3\prime\prime})\right] \times [q'_{ik} - q''_{ik}] \geq 0, \quad (4.40)$$

$$(VIII) = \sum_{j=1}^{n} \sum_{k=1}^{o} \sum_{l=1}^{2} \left[\hat{c}_{jkl}(Q^{2\prime}, Q^{3\prime}) - \hat{c}_{jkl}(Q^{2\prime\prime}, Q^{3\prime\prime})\right] \times \left[q'_{jkl} - q''_{jkl}\right] \geq 0, \quad (4.41)$$

and

$$(IX) = \sum_{k=1}^{o} [-d_k(\rho'_3) + d_k(\rho''_3)] \times [\rho'_{3k} - \rho''_{3k}] \geq 0. \quad (4.42)$$

Bringing (4.35)–(4.42) into the right-hand side of (4.34), the conclusion follows. □

Theorem 4.5: Strict Monotonicity
Assume all the conditions of Theorem 4.4. In addition, suppose that one of the five families of convex functions $f_i^1; i = 1, ..., m$; $c_{ijl}; i = 1, ..., m$; $j = 1, ..., n$; $l = 1, 2$; c_j; $j = 1, ..., n$; $\hat{c}_{ijl}$; $i = 1, \ldots, m$; $j = 1, \ldots, n$; $l = 1, 2$; *and* c_{ik}; $i = 1, \ldots, m$; $k = 1, \ldots, o$, *is a family of strictly convex functions. Suppose that* $\hat{c}_{ik}$; $i = 1, \ldots, m$; $k = 1, \ldots, o$; $\hat{c}_{jkl}; j = 1, ..., n; k = 1, ..., o; l = 1, 2$, *and* -$d_k$; $k = 1, ..., o$, *are strictly monotone. Then, the vector function* F *that enters the variational inequality (4.25) is strictly monotone, with respect to* (Q^1, Q^2, Q^3, ρ_3), *that is, for any two* X', X'' *with* $(Q^{1'}, Q^{2'}, Q^{3'}, \rho'_3) \neq (Q^{1''}, Q^{2''}, Q^{3''}, \rho_3{}'')$

$$\langle F(X') - F(X''), X' - X'' \rangle > 0. \quad (4.43)$$

Proof: For any two distinct $(Q^{1\prime}, Q^{2\prime}, Q^{3\prime}\rho'_3)$, $(Q^{1\prime\prime}, Q^{2\prime\prime}, Q^{3\prime\prime}, \rho''_3)$, one must have at least one of the following four cases:
(i). $Q^{1\prime} \neq Q^{1\prime\prime}$,
(ii). $Q^{2\prime} \neq Q^{2\prime\prime}$,
(iii). $Q^{3\prime} \neq Q^{3\prime\prime}$,
(iv). $\rho'_3 \neq \rho''_3$.

Under the condition of the theorem, if (i) holds true, then, on the right-hand side of (4.34), at least one of (I), (II), (III), or (IV) is positive. If (ii) is true, then (V), (VI), or (VII) is positive. In the case of (iii), (VIII) is positive. In the case of (iv), (IX) is positive. Hence, one can conclude that the right-hand side of (4.34) is greater than zero. The proof is complete. □

Theorem 4.5 has an important implication for the uniqueness of the manufacturer shipments, Q^1, the retailer shipments, Q^2, and the prices at the demand markets, ρ_3, at the equilibrium as is now established. Note also that no guarantee of a unique ρ_{2j}; $j = 1, ..., n$, can generally be expected at the equilibrium.

Theorem 4.6: Uniqueness
Under the conditions of Theorem 4.5, there must be a unique shipment pattern (Q^{1*}, Q^{2*}, Q^{3*})*, and a unique demand price vector* ρ_3^* *satisfying the equilibrium conditions of the supply chain. In other words, if the variational inequality (4.25) admits a solution, then that is the only solution in* (Q^1, Q^2, Q^3, ρ_3)*.*

Proof: Under the strict monotonicity result of Theorem 4.5, uniqueness follows from the standard variational inequality theory (cf. Theorem B.4 in Appendix B). □

Theorem 4.7: Lipschitz Continuity
The function that enters the variational inequality problem (4.25) is Lipschitz continuous, that is,

$$\|F(X') - F(X'')\| \leq L\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}, \tag{4.44}$$

under the following conditions:
(i). Each f_i*;* $i = 1, ..., m$*, is additive and has a bounded second-order derivative;*
(ii). c_{ijl}*,* c_j*,* $\hat{c}_{ijl}$ *and* c_{ik} *have bounded second-order derivatives, for all* i, j, l, k*;*
(iii). $\hat{c}_{ik}$*,* $\hat{c}_{jkl}$*, and* d_k *have bounded first-order derivatives for all* i, j, k, l*.*

Proof: The result is direct by applying a mid-value theorem from calculus to the vector function F that enters the variational inequality problem (4.25). □

4.3 The Algorithm

In this section, we consider the computation of solutions to variational inequality (4.18). The algorithm that will be used is the modified projection method of Korpelevich (1977), which is guaranteed to solve any variational inequality problem in standard form (see (4.25)) provided that the function F that enters the variational inequality is monotone and Lipschitz continuous (and that a solution exists). The statement of the algorithm in its general form is given in Appendix C. Below is its realization for the solution of variational inequality problem (4.18) representing the equilibrium for the supply chain network model with electronic commerce. In the next section, the algorithm is applied to several numerical examples. An iteration counter is denoted by $\mathcal{T}$.

Modified Projection Method for the Solution of Variational Inequality (4.18)

Step 0: Initialization

Set $(Q^{1^0}, Q^{2^0}, Q^{3^0}, \rho_2^0, \rho_3^0) \in \mathcal{K}$. Let $\mathcal{T} = 1$ and set α such that $0 < \alpha \leq \frac{1}{L}$, where L is the Lipschitz constant for the problem (cf. (4.44)).

Step 1: Computation

Compute $(\bar{Q}^{1^{\mathcal{T}}}, \bar{Q}^{2^{\mathcal{T}}}, \bar{Q}^{3^{\mathcal{T}}}, \bar{\rho}_2^{\mathcal{T}}, \bar{\rho}_3^{\mathcal{T}}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\bar{q}_{ijl}^{\mathcal{T}} + \alpha\left(\frac{\partial f_i(Q^{1^{\mathcal{T}-1}}, Q^{2^{\mathcal{T}-1}})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl}^{\mathcal{T}-1})}{\partial q_{ijl}} + \frac{\partial c_j(Q^{1^{\mathcal{T}-1}})}{\partial q_{ijl}}\right.\right.$$
$$\left.\left. + \frac{\partial \hat{c}_{ijl}(q_{ijl}^{\mathcal{T}-1})}{\partial q_{ijl}} - \rho_{2j}^{\mathcal{T}-1}\right) - q_{ijl}^{\mathcal{T}-1}\right] \times \left[q_{ijl} - \bar{q}_{ijl}^{\mathcal{T}}\right]$$
$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\bar{q}_{ik}^{\mathcal{T}} + \alpha\left(\frac{\partial f_i(Q^{1^{\mathcal{T}-1}}, Q^{2^{\mathcal{T}-1}})}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^{\mathcal{T}-1})}{\partial q_{ik}}\right.\right.$$
$$\left.\left. + \hat{c}_{ik}(Q^{2^{\mathcal{T}-1}}, Q^{3^{\mathcal{T}-1}}) - \rho_{3k}^{\mathcal{T}-1}\right) - q_{ik}^{\mathcal{T}-1}\right] \times \left[q_{ik} - \bar{q}_{ik}^{\mathcal{T}}\right]$$
$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\bar{q}_{jkl}^{\mathcal{T}} + \alpha\left(\rho_{2j}^{\mathcal{T}-1} + \hat{c}_{jkl}(Q^{2^{\mathcal{T}-1}}, Q^{3^{\mathcal{T}-1}}) - \rho_{3k}^{\mathcal{T}-1}\right) - q_{jkl}^{\mathcal{T}-1}\right]$$
$$\times \left[q_{jkl} - \bar{q}_{jkl}^{\mathcal{T}}\right]$$
$$+\sum_{j=1}^{n}\left[\bar{\rho}_{2j}^{\mathcal{T}} + \alpha\left(\sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}^{\mathcal{T}-1} - \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jk}^{\mathcal{T}-1}\right) - \rho_{2j}^{\mathcal{T}-1}\right] \times \left[\rho_{2j} - \bar{\rho}_{2j}^{\mathcal{T}}\right]$$
$$+\sum_{k=1}^{o}\left[\bar{\rho}_{3k}^{\mathcal{T}} + \alpha\left(\sum_{j=1}^{n}\sum_{l=1}^{2} q_{jkl}^{\mathcal{T}-1} + \sum_{i=1}^{m} q_{ik}^{\mathcal{T}-1} - d_k(\rho_3^{\mathcal{T}-1})\right) - \rho_{3k}^{\mathcal{T}-1}\right]$$
$$\times \left[\rho_{3k} - \bar{\rho}_{3k}^{\mathcal{T}}\right] \geq 0, \qquad \forall (Q^1, Q^2, Q^3, \rho_2, \rho_3) \in \mathcal{K}. \tag{4.45}$$

Step 2: Adaptation

Compute $(Q^{1^{\mathcal{T}}}, Q^{2^{\mathcal{T}}}, Q^{3^{\mathcal{T}}}, \rho_2^{\mathcal{T}}, \rho_3^{\mathcal{T}}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[q_{ijl}^{\mathcal{T}} + \alpha\left(\frac{\partial f_i(\bar{Q}^{1\mathcal{T}}, \bar{Q}^{2\mathcal{T}})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(\bar{q}_{ijl}^{\mathcal{T}})}{\partial q_{ijl}} + \frac{\partial c_j(\bar{Q}^{1\mathcal{T}})}{\partial q_{ijl}}\right.\right.$$
$$\left.\left. + \frac{\partial \hat{c}_{ijl}(\bar{q}_{ijl}^{\mathcal{T}})}{\partial q_{ijl}} - \bar{\rho}_{2j}^{\mathcal{T}}\right) - q_{ijl}^{\mathcal{T}-1}\right] \times \left[q_{ijl} - q_{ijl}^{\mathcal{T}}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[q_{ik}^{\mathcal{T}}+\alpha(\frac{\partial f_i(\bar{Q}^{1\mathcal{T}},\bar{Q}^{2\mathcal{T}})}{\partial q_{ik}}+\frac{\partial c_{ik}(\bar{q}_{ijk}^{\mathcal{T}})}{\partial q_{ik}}+\hat{c}_{ik}(\bar{Q}^{2\mathcal{T}},\bar{Q}^{3\mathcal{T}})\right.$$
$$\left.-\bar{\rho}_{3k}^{\mathcal{T}})-q_{ik}^{\mathcal{T}-1}\right]\times\left[q_{ik}-q_{ik}^{\mathcal{T}}\right]$$
$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[q_{jkl}^{\mathcal{T}}+\alpha(\bar{\rho}_{2j}^{\mathcal{T}}+\hat{c}_{jkl}(\bar{Q}^{2\mathcal{T}},\bar{Q}^{3\mathcal{T}})-\bar{\rho}_{3k}^{\mathcal{T}})-q_{jkl}^{\mathcal{T}-1}\right]\times\left[q_{jkl}-q_{jkl}^{\mathcal{T}}\right]$$
$$+\sum_{j=1}^{n}\left[\rho_{2j}^{\mathcal{T}}+\alpha(\sum_{i=1}^{m}\sum_{l=1}^{2}\bar{q}_{ijl}^{\mathcal{T}}-\sum_{k=1}^{o}\sum_{l=1}^{2}\bar{q}_{jkl}^{\mathcal{T}})-\rho_{2j}^{\mathcal{T}-1}\right]\times\left[\rho_{2j}-\rho_{2j}^{\mathcal{T}}\right]$$
$$+\sum_{k=1}^{o}\left[\rho_{3k}^{\mathcal{T}}+\alpha(\sum_{j=1}^{n}\sum_{l=1}^{2}\bar{q}_{jkl}^{\mathcal{T}}+\sum_{i=1}^{m}\bar{q}_{ik}^{\mathcal{T}}-d_k(\bar{\rho}_3^{\mathcal{T}}))-\rho_{3k}^{\mathcal{T}-1}\right]\times\left[\rho_{3k}-\rho_{3k}^{\mathcal{T}}\right]\geq 0,$$
$$\forall(Q^1,Q^2,Q^3,\rho_2,\rho_3)\in\mathcal{K}. \tag{4.46}$$

Step 3: Convergence Verification

If $|q_{ijl}^{\mathcal{T}}-q_{ijl}^{\mathcal{T}-1}|\leq\epsilon$, $|q_{ik}^{\mathcal{T}}-q_{ik}^{\mathcal{T}-1}|\leq\epsilon$, $|q_{jkl}^{\mathcal{T}}-q_{jkl}^{\mathcal{T}-1}|\leq\epsilon$, $|\rho_{2j}^{\mathcal{T}}-\rho_{2j}^{\mathcal{T}-1}|\leq\epsilon$, $|\rho_{3k}^{\mathcal{T}}-\rho_{3k}^{\mathcal{T}-1}|\leq\epsilon$, for all $i=1,\cdots,m$; $j=1,\cdots,n$; $l=1,2$; $k=1,\cdots,o$, with $\epsilon>0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T}:=\mathcal{T}+1$, and go to Step 1.

Although the solution of Steps 1 and 2 as stated by (4.45) and (4.46), at first glance, may appear ominous, the variational inequality subproblems (4.45) and (4.46) can be solved explicitly and in closed form since the feasible set is that of the nonnegative orthant. Indeed, they yield subproblems in the q_{ijl}, q_{ik}, q_{jkl}, ρ_{2j}, and ρ_{3k} variables $\forall i,j,l,k$. Hence, the computation of solutions to subproblems (4.45) and (4.46) is actually remarkably simple.

Specifically, subproblem (4.45) can be solved *exactly* and in *closed form* as follows:

Computation of the Product Shipments:

At iteration $\mathcal{T}$, compute the $\bar{q}_{ijl}^{\mathcal{T}}$s, $\forall i,j,l$, according to:

$$\bar{q}_{ijl}^{\mathcal{T}}=\max\left\{0,q_{ijl}^{\mathcal{T}-1}-\alpha(\frac{\partial f_i(Q^{1\mathcal{T}-1},Q^{2\mathcal{T}-1})}{\partial q_{ijl}}+\frac{\partial c_{ijl}(q_{ijl}^{\mathcal{T}-1})}{\partial q_{ijl}}+\frac{\partial c_j(Q^{1\mathcal{T}-1})}{\partial q_{ijl}}\right.$$
$$\left.+\frac{\partial\hat{c}_{ijl}(q_{ijl}^{\mathcal{T}-1})}{\partial q_{ijl}}-\rho_{2j}^{\mathcal{T}-1})\right\}. \tag{4.47}$$

In addition, at iteration $\mathcal{T}$, compute the $\bar{q}_{ik}^{\mathcal{T}}$s, $\forall i,k$, according to:

$$\bar{q}_{ik}^{\mathcal{T}}=\max\left\{0,q_{ik}^{\mathcal{T}-1}-\alpha(\frac{\partial f_i(Q^{1\mathcal{T}-1},Q^{2\mathcal{T}-1})}{\partial q_{ik}}+\frac{\partial c_{ik}(q_{ik}^{\mathcal{T}-1})}{\partial q_{ik}}\right.$$

$$+\hat{c}_{ik}(Q^{2^{\mathcal{T}-1}}, Q^{3^{\mathcal{T}-1}}) - \rho_{3k}^{\mathcal{T}-1})\Big\}. \quad (4.48)$$

Also, at iteration $\mathcal{T}$ compute the $\bar{q}_{jkl}^{\mathcal{T}}$s according to:

$$\bar{q}_{jkl}^{\mathcal{T}} = \max\{0, q_{jkl}^{\mathcal{T}-1} - \alpha(\rho_{2j}^{\mathcal{T}-1} + \hat{c}_{jkl}(Q^{2^{\mathcal{T}-1}}, Q^{3^{\mathcal{T}-1}}) - \rho_{3k}^{\mathcal{T}-1})\}, \quad \forall j, k, l. \quad (4.49)$$

Computation of the Prices:

The prices, $\bar{\rho}_{2j}^{\mathcal{T}}$, in turn, are computed at iteration $\mathcal{T}$ explicitly according to:

$$\bar{\rho}_{2j}^{\mathcal{T}} = \max\{0, \rho_{2j}^{\mathcal{T}-1} - \alpha(\sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl}^{\mathcal{T}-1} - \sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl}^{\mathcal{T}-1})\}, \quad \forall j, \quad (4.50)$$

whereas the prices, $\bar{\rho}_{3k}^{\mathcal{T}}$, are computed according to:

$$\bar{\rho}_{3k}^{\mathcal{T}} = \max\{0, \rho_{3k}^{\mathcal{T}-1} - \alpha(\sum_{j=1}^{n}\sum_{l=1}^{2} q_{jkl}^{\mathcal{T}-1} + \sum_{i=1}^{m} q_{ik}^{\mathcal{T}-1} - d_k(\rho_3^{\mathcal{T}-1}))\}, \quad \forall k. \quad (4.51)$$

The solution to subproblem (4.46) can be obtained in analogous fashion. Hence, although the solution of (4.45) and (4.46) may, at first, appear challenging, the resulting computations can be accomplished exactly and in closed form as described above.

An Adjustment Process Interpretation

The solution of (4.45) and (4.46), with the former being solved according to (4.47) through (4.51), has an elegant interpretation as an adjustment process. Note that, according to (4.47), the product shipments between the manufacturers and the retailers are determined from iteration to iteration (which may also be interpreted as a time period) independently and simultaneously using only the shipments from the manufacturers from the preceding iteration and the retailers' prices. The shipments from the manufacturers to the demand markets, in turn, are determined at a given iteration according to (4.48) using only the demand market prices from the preceding iteration as well as the shipments. Finally, the shipments from the retailers to the demand markets at a particular iteration are computed according to (4.49) using the retailers' prices and the demand market prices from the preceding iteration and the product shipments to the demand markets.

The computation of the retail prices according to (4.50) at an iteration requires the price of the particular retailer at the preceding iteration as well as the product shipments to and from the retailer. Finally, the demand market prices, according to (4.51), can be computed using the prices at the demand markets at the preceding iteration and the product shipments to the particular demand market

The solution of subproblem (4.46), which is an adaptation step, has a similar interpretation as given above, but now the information in terms of the product flows and prices computed as the solution to (4.45) are used as inputs along with the original flows and prices at the beginning of the iteration. Hence, the nomenclature of "Adaptation" for this step.

The convergence result for the modified projection method for this model is now given.

Theorem 4.8: Convergence
Assume that the function that enters the variational inequality (4.18) (or (4.25)) satisfies the conditions in Theorems 4.3, 4.4, and in Theorem 4.7. Then the modified projection method described above converges to the solution of the variational inequality (4.18) or (4.25).

Proof: According to Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem of the form (4.25), provided that the function F that enters the variational inequality is monotone and Lipschitz continuous and that a solution exists. Existence of a solution follows from Theorem 4.3. Monotonicity follows Theorem 4.4. Lipschitz continuity, in turn, follows from Theorem 4.7. □

It is worth noting that the algorithm may, nevertheless, converge even if the above conditions are not satisfied and, if it converges, it converges to a solution of the variational inequality problem; equivalently, it determines an equilibrium flow and price pattern.

4.4 Numerical Examples

In this section, the modified projection method is applied to several numerical examples. The algorithm was implemented in FORTRAN and the computer system used was a DEC Alpha system located at the University of Massachusetts at Amherst. The convergence criterion utilized was that the absolute value of the product flows and prices between two successive iterations differed by no more than 10^{-4}. For the examples, α was set to .01 in the algorithm. The numerical examples had the network structure depicted in Figure 4.7 and consisted of two manufacturers, two retailers, and two demand markets, with both B2B and B2C transactions permitted, with the B2C transactions being between the manufacturers and the demand markets, for simplicity.

Example 4.1

The data for the first example were constructed for easy interpretation purposes. The production cost functions for the manufacturers were given by:

$$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1, \quad f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2.$$

The transaction cost functions faced by the manufacturers and associated with transacting with the retailers using the physical link, that is, mode 1,

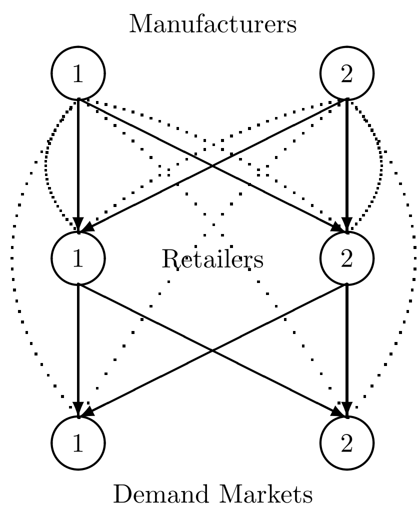

Fig. 4.7. Supply Chain Network Structure for the Numerical Examples

were given by:

$$c_{111}(q_{111}) = .5q_{111}^2 + 3.5q_{111}, \quad c_{121}(q_{121}) = .5q_{121}^2 + 3.5q_{121},$$

$$c_{211}(q_{211}) = .5q_{211}^2 + 3.5q_{211}, \quad c_{221}(q_{221}) = .5q_{221}^2 + 3.5q_{221},$$

whereas the analogous transaction costs, but for mode 2, were given by:

$$c_{112}(q_{112}) = 1.5q_{112}^2 + 3q_{112}, \quad c_{122}(q_{122}) = 1.5q_{122}^2 + 3q_{122},$$

$$c_{212}(q_{212}) = 1.5q_{212}^2 + 3q_{212}, \quad c_{222}(q_{222}) = 1.5q_{222}^2 + 3q_{222},$$

The transaction costs of the manufacturers associated with dealing with the consumers at the demand markets via the Internet were given by:

$$c_{11}(q_{11}) = q_{11}^2 + 2q_{11}, \quad c_{12}(q_{12}) = q_{12}^2 + 2q_{12},$$

$$c_{21}(q_{21}) = q_{21}^2 + 2q_{21}, \quad c_{22}(q_{22}) = q_{22}^2 + 2q_{22}.$$

The handling costs of the retailers, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i1l})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i2l})^2.$$

The transaction costs of the retailers associated with transacting with the manufacturers via mode 1 and mode 2 were, respectively, given by:

$$\hat{c}_{111}(q_{111}) = 1.5q_{111}^2 + 3q_{111}, \quad \hat{c}_{121}(q_{121}) = 1.5q_{121}^2 + 3q_{121},$$

$$\hat{c}_{211}(q_{211}) = 1.5q_{211}^2 + 3q_{211}, \quad \hat{c}_{221}(q_{221}) = 1.5q_{221}^2 + 3q_{221},$$
$$\hat{c}_{112}(q_{112}) = 1.5q_{112}^2 + 3q_{112}, \quad \hat{c}_{122}(q_{122}) = 1.5q_{122}^2 + 3q_{122},$$
$$\hat{c}_{212}(q_{212}) = 1.5q_{212}^2 + 3q_{212}, \quad \hat{c}_{222}(q_{222}) = 1.5q_{222}^2 + 3q_{222}.$$

The demand functions at the demand markets were:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000, \quad d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000,$$

and the transaction costs between the retailers and the consumers at the demand markets (denoted for a typical pair by $\hat{c}_{jkl}$ with the associated shipment by q_{jkl} with $l = 1$) were given by:

$$\hat{c}_{111}(Q^2, Q^3) = q_{111} + 5, \quad \hat{c}_{121}(Q^2, Q^3) = q_{121} + 5,$$
$$\hat{c}_{211}(Q^2, Q^3) = q_{211} + 5, \quad \hat{c}_{221}(Q^2, Q^3) = q_{221} + 5,$$

whereas the transaction costs associated with transacting with the manufacturers via the Internet for the consumers at the demand markets (denoted for a typical such pair by $\hat{c}_{ik}$ with the associated shipment of q_{ik}) were given by:

$$\hat{c}_{11}(Q^2, Q^3) = q_{11} + 1, \quad \hat{c}_{12}(Q^2, Q^3) = q_{12} + 1,$$
$$\hat{c}_{21}(Q^2, Q^3) = q_{21} + 1, \quad \hat{c}_{22}(Q^2, Q^3) = q_{22} + 1.$$

The modified projection method converged in 1055 iterations and yielded the following equilibrium pattern: the product shipments between the two manufacturers and the two retailers associated with the physical links, and with the Internet links, respectively, that is, with transacting via mode 1 and mode 2 were:

$$Q^{1^*} : q_{111}^* = q_{121}^* = q_{211}^* = q_{221}^* = 3.4611,$$
$$q_{112}^* = q_{122}^* = q_{212}^* = q_{222}^* = 2.3907.$$

The product shipments between the two manufacturers and the two demand markets with transactions conducted through the Internet were:

$$Q^{2^*} : q_{11}^* = q_{12}^* = q_{21}^* = q_{22}^* = 13.3033.$$

The product shipments (consumption volumes) between the two retailers and the two demand markets were:

$$Q^{3^*} : q_{111}^* = q_{121}^* = q_{211}^* = q_{221}^* = 5.8513.$$

The vector ρ_2^*, which was equal to the prices charged by the retailers γ^*, had components:

$$\rho_{21}^* = \rho_{22}^* = 263.9088,$$

and the demand prices at the demand markets were:

$$\rho_{31}^* = \rho_{32}^* = 274.7701.$$

It is easy to verify that the optimality/equilibrium conditions were satisfied with good accuracy.

The prices charged by the manufacturers were as follows and were recovered according to the discussion following variational inequality (4.25). The ρ^*_{1ijl}s were for $l = 1$ and for $l = 2$, respectively: All ρ^*_{1ij1}s= 238.8218 and all ρ^*_{1ij2}s= 242.0329. All the ρ^*_{1ik}s were equal to 260.4673. These values were obtained in both ways as discussed following (4.25) and either manner yielded the same value for the corresponding price. Note that the price charged by the manufacturers to the consumers at the demand markets, approximately 260, was higher than the price charged to the retailers, regardless of the mode of transacting. The price charged to the retailers for the product transacted via the Internet, in turn, exceeded that charged using the classical physical manner.

Example 4.2

Example 4.1 was then modified as follows: The production cost function for manufacturer 1 was now given by:

$$f_1(q) = 2.5q_1^2 + q_1q_2 + 12q_1,$$

whereas the transaction costs for manufacturer 1 were now given by:

$$c_{11}(Q^1) = q_{11}^2 + 3.5q_{11}, \quad c_{12}(Q^1) = q_{12}^2 + 3.5q_{12}.$$

The remainder of the data was as in Example 4.1. Hence, both the production costs and the transaction costs increased for manufacturer 1.

The modified projection method converged in 1056 iterations and yielded the following equilibrium pattern: the product shipments between the two manufacturers and the two retailers associated with the physical links, and with the Internet links, respectively, that is, with transacting via mode 1 and mode 2 were:

$$Q^{1^*}: q^*_{111} = q^*_{121} = 3.3265, \quad q^*_{211} = q^*_{221} = 3.5408,$$

$$q^*_{112} = q^*_{122} = 2.3010, \quad q^*_{212} = q^*_{222} = 2.4438.$$

The product shipments between the two manufacturers and the two demand markets with transactions conducted through the Internet were:

$$Q^{2^*}: q^*_{11} = q^*_{12} = 12.5781, \quad q^*_{21} = q^*_{22} = 13.3638.$$

The product shipments (consumption volumes) between the two retailers and the two demand markets were:

$$Q^{3^*}: q^*_{11} = q^*_{12} = q^*_{21} = q^*_{22} = 5.8056.$$

The vector ρ^*_2 had components:

$$\rho^*_{21} = \rho^*_{22} = 264.1706,$$

and the demand prices at the demand markets were:

$$\rho_{31}^* = \rho_{32}^* = 274.9861.$$

The optimality/equilibrium conditions were, again, satisfied at the desired accuracy.

The ρ_{1ijl}^*s were as follows for $l = 1$ and for $l = 2$, respectively: The ρ_{11j1}^*s= 239.5789 for both j and the ρ_{11j2}^*s= 242.6553 for both j. For manufacturer 2, on the other hand, $\rho_{12j1}^* = 238.9360$ for both j, whereas $\rho_{12j2}^* = 242.268$ for both j. The ρ_{11k}^*s were equal to 261.4085, for both k, whereas the ρ_{12k}^*s were equal to 260.6223 for both k. Note that these values were obtained in both ways as discussed following (4.25) and either manner yielded the same value for the corresponding price. Note that, again, the prices charged by the manufacturers to the consumers at the demand markets were higher than the prices charged to the retailers. Of course, the demand price was, nevertheless, equal for all consumers at a given demand market. In fact, both in this and in the preceding example the equilibrium demand prices were the same for each demand market.

Hence, manufacturer 1 now produced less than it did in Example 4.1, whereas manufacturer 2 increased its production output. The prices charged by the retailers to the consumers increased, as did the prices at the demand markets, with a decrease in the incurred demand.

Example 4.3

Example 4.3 was constructed by changing Example 4.2 as follows: The data were identical to that in Example 4.2 except that the demand function for demand market 1 was now:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 2000.$$

The modified projection method converged in 1466 iterations and yielded the following equilibrium pattern: the product shipments between the two manufacturers and the two retailers associated with the physical links, and with the Internet links, respectively, that is, with transacting via mode 1 and mode 2 were:

$$Q^{1^*} : q_{111}^* = q_{121}^* = 16.1444, \quad q_{211}^* = q_{221}^* = 16.4974,$$

$$q_{112}^* = q_{122}^* = 10.8463, \quad q_{212}^* = q_{222}^* = 11.0816.$$

The product shipments between the two manufacturers and the two demand markets with transactions conducted through the Internet were:

$$Q^{2^*} : q_{11}^* = 60.2397, \quad q_{12}^* = 0.0000, \quad q_{21}^* = 61.2103, \quad q_{22}^* = 0.0000.$$

The product shipments (consumption volumes) between the two retailers and the two demand markets were:

$$Q^{3^*} : q_{111}^* = 54.5788, \quad q_{121}^* = 0.0000, \quad q_{211}^* = 54.5788, \quad q_{221}^* = 0.0000,$$

the vector ρ_2^*, which was equal to the prices charged by the retailers γ^*, had components:

$$\rho_{21}^* = \rho_{22}^* = 825.1216,$$

and the demand prices at the demand markets were:

$$\rho_{31}^* = 884.694, \quad \rho_{32}^* = 0.0000.$$

It is easy to verify that the optimality/equilibrium conditions were satisfied with good accuracy.

The prices charged by the manufacturers were as follows and were, again, recovered according to the discussion following variational inequality (4.25). The ρ_{1ijl}^*s were as follows for $l = 1$ and for $l = 2$, respectively: $\rho_{1111}^* = 719.1185 = \rho_{1121}^*$; $\rho_{1211}^* = 718.0597 = \rho_{1221}^*$; $\rho_{1112}^* = \rho_{1122}^* = 735.019$, and $\rho_{1212}^* = \rho_{1222}^* = 734.3071$. The ρ_{111}^*s was equal to 823.4536, whereas the ρ_{121}^* was equal to 822.4830. In this example, only the consumers at demand market 1 consume a positive amount. Indeed, there is no consumption of the product by consumers located at demand market 2.

4.5 Sources and Notes

Nagurney, Dong, and Zhang (2001) introduced the supply chain network model without electronic commerce and formulated it as a variational inequality problem akin to the one given in (4.26). Later, Nagurney, Loo, Dong, and Zhang (2001) extended that framework which focused on decentralized decision-making in supply chains to include electronic commerce. This chapter extends the model in the latter paper to include B2C electronic commerce between retailers and consumers, as well. This chapter provides complete proofs of the results, and includes new ideas, topical applications, and extensions. In addition, the work is now framed in a supernetwork context. The numerical examples in Section 4.4 are from Nagurney, Loo, Dong, and Zhang (2001).

This chapter has provided the foundations for supply chain networks with decentralized decision-makers in a multitiered context. It considered both models with electronic commerce as well as one without. Given the complexity of interactions among the network agents, the formalism provided here is both conceptual in nature, since it abstracts the problem as a supernetwork, as well as theoretically rigorous. Moreover, the theoretical foundations serve as the basis for the development and analysis of other supply chain network structures. For background reading on the economics of electronic commerce, see the book by Whinston, Dahl, and Choi (1997).