1 Financial and Economic Networks: An Overview

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1.1 Background

Networks throughout history have provided the physical means by which humans conduct their economic activities. Transportation and logistical networks have allowed for the movement of individuals, goods, and services, whereas communication networks have enabled the exchange of messages and information. Energy networks have provided the fuel for the transactions to take place.

The emergence and evolution of myriad physical networks over space and time and the effects of human decision-making on such networks through their utilization and management thereof has given rise, in turn, to the development of rich theories and scientific methodologies that are network-based. Networks, as a science, have impacted disciplines ranging from engineering, computer science, applied mathematics, and even biology to finance and economics. The novelty of networks is that they are pervasive, providing the fabric of connectivity for our societies and economies, while, methodologically, network theory has developed into a powerful and dynamic medium for abstracting complex problems, which, at first glance, may not even appear to be networks, with associated nodes, links, and flows.

The topic of networks as a subject of scientific inquiry originates in the paper by Euler (1736), which is credited with being the earliest paper on *graph* theory. By a graph in this context is meant, mathematically, a means of abstractly representing a system by its depiction in terms of vertices (or nodes) and edges (or arcs, equivalently, links) connecting various pairs of vertices. Euler was interested in determining whether it was possible to stroll around Königsberg (later called Kaliningrad) by crossing the seven bridges over the River Pregel exactly once. The problem was represented as a graph (cf. Figure 1.1) in which the vertices corresponded to land masses and the edges to bridges.

Interestingly, not long thereafter, Quesnay (1758), in his *Tableau Economique*, conceptualized the circular flow of financial funds in an economy as a network and this work can be identified as the first paper on the topic of

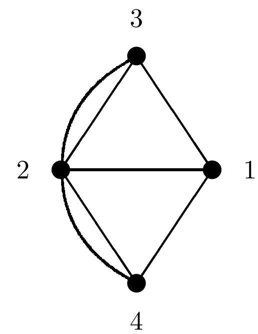

Fig. 1.1. The Euler graph representation of the seven bridge Königsberg problem

financial networks. Quesnay's basic idea has been utilized in the construction of financial flow of funds accounts, which are a statistical description of the flows of money and credit in an economy (see Cohen (1987)).

The concept of a network in economics was implicit as early as the classical work of Cournot (1838), who not only seems to have first explicitly stated that a competitive price is determined by the intersection of supply and demand curves, but had done so in the context of two spatially separated markets in which the cost associated with transporting the goods was also included. Pigou, subsequently, in 1920 studied a network system in the form of a transportation network consisting of two routes and noted that the decision-making behavior of the the users of such a system would lead to different flow patterns. Hence, the network of concern therein consists of the graph, which is now directed, with the edges or links represented by arrows, as well as the resulting flows on the links.

Monge, who had worked under Napoleon Bonaparte in providing infrastructure support for his army, published in 1781 what is probably the first paper on the transportation model (see, e.g., Buckard, Klinz, and Rudolf (1996)). In particular, he was interested in minimizing the cost associated with backfilling n places from m other places with surplus brash with cost c_{ij} being proportional to the distance between origin i and destination j. Much later, and following the first book on graph theory by König in 1936, the economists Kantorovich (1939), Hitchcock (1941), and Koopmans (1947) considered the network flow problem associated with this classical minimum cost transportation problem, provided insights into the special network structure of such problems, and yielded network-based algorithmic approaches. Hence, the study of network flows precedes that of optimization techniques, in general, with seminal work done by Dantzig in 1948 in linear programming with the simplex method and, subsequently, adapted for the classical transportation problem in 1951.

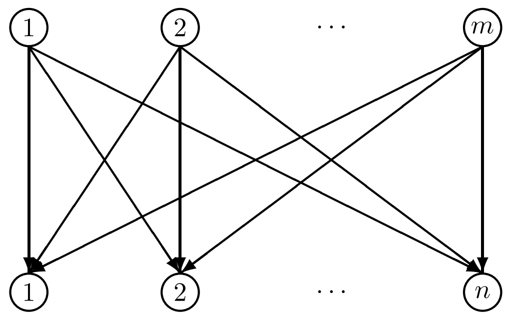

Fig. 1.2. A bipartite network with directed links

In 1952, Copeland in his book recognized the conceptualization of the interrelationships among financial funds as a network and raised the question, "Does money flow like water or electricity?" Moreover, he provided a "wiring diagram for the main money circuit." Note that Kirchhoff is credited with pioneering the field of electrical engineering by being the first to have systematically analyzed electrical circuits and with providing the foundations for the principal ideas of network flow theory. Interestingly, Enke in 1951 had proposed electronic circuits as a means of solving spatial price equilibrium problems, in which goods are produced, consumed, and traded, in the presence of transportation costs. Such analog computational devices, were soon to be superseded by digital computers along with advances in computational methodologies, notably, algorithms, based on mathematical programming, and including not only the aforementioned linear programming techniques but other optimization techniques, as well.

Samuelson (1952) provided a rigorous mathematical formulation of the spatial price equilibrium problem and explicitly recognized and utilized the network structure, which was bipartite (the same structure as in the classical transportation problems), that is, consisting of two sets of nodes (cf. Figure 1.2). Interestingly, he depicted the changing network of trade as the excess demand for the commodity at a particular market increased. In spatial price equilibrium problems, unlike classical transportation problems, the supplies and the demands are variables, rather than fixed quantities. The work was subsequently extended by Takayama and Judge (1964, 1971) and others (cf. Asmuth, Eaves, and Peterson (1979), Florian and Los (1982), Nagurney (1999), and the references therein) to include, respectively, multiple commodities, and asymmetric supply price and demand functions, as well as other extensions, made possible by such advances as quadratic programming techniques, complementarity theory, as well as variational inequality theory (which allowed for the formulation and solution of equilibrium problems for which no optimization reformulation of the governing equilibrium conditions was available).

Beckmann, McGuire, and Winsten (1956), in turn, in their book, provided a rigorous treatment of congested transportation networks, and formulated their solution as mathematical programming problems. Their contributions added significantly to the topic of network equilibrium problems, which was later advanced by the contributions of Dafermos and Sparrow (1969), who coined the terms *user-optimization* versus *system-optimization* and provided computational methods for the determination of the resulting flows on such networks. Subsequent notable contributions were made by Smith (1979) and Dafermos (1980) who allowed for asymmetric interactions associated with the link travel costs (resulting in no equivalent optimization reformulation of the equilibrium conditions) and established the methodology of variational inequalities as a primary tool for both the qualitative analysis and the solution of such and other related problems. For additional background, see the book by Nagurney (1999).

1.2 Financial Networks

We now further elaborate upon historical breakthroughs in the use of networks for the formulation, analysis, and solution of financial problems. Such a perspective allows one to trace the methodological developments as well as the applications of financial networks and provides a platform upon which further innovations can be constructed and evaluated. We begin with a discussion of financial optimization problems within a network context and then turn to financial network equilibrium problems.

1.2.1 Optimization Problems

Network models have been proposed for a spectrum of financial problems characterized by a single objective function to be optimized such as in portfolio optimization and asset allocation problems, currency translation, and risk management problems, among others. We now briefly highlight this literature recognizing that it was, of course, the innovative work of Markowitz (1952, 1959) that started a new era in financial economics and became the basis for many financial optimization models that exist today.

Interestingly, although many financial optimization problems (including the work by Markowitz) had an underlying network structure, and the advantages of network programming were becoming increasingly evident (cf. Charnes and Cooper (1958)), not many financial network optimization models were developed until some time later, with the exception of several early models due to Charnes and Miller (1957) and Charnes and Cooper (1961). Indeed, it was not until the last years of the 1960s and the first years of the 1970s that the network setting started to be extensively used for financial applications.

Among the first financial network optimization models that appear in the literature were a series of currency translating models. Rutenberg (1970) suggested that the translation among different currencies could be performed through the use of arc multipliers. The network model developed by Rutenberg was a multiperiod one with linear costs on the arcs (a characteristic common to the earlier financial networks models). The nodes of such generalized networks represented a particular currency in a specific period and the flow on the arcs the amount of cash moving from one period and/or currency to another. Related financial network models were subsequently introduced by Christofides, Hewins, and Salkin (1979) and Shapiro and Rutenberg (1976), among others. In most of these models, the currency prices were determined according to the amount of capital (network flow) that was moving from one currency (node) to the other.

Networks were also used to formulate a series of cash management problems (cf. Barr (1972) and Srinivasan (1974)), with a major contribution being the introduction (cf. Crum (1976)) of a generalized linear network model for the cash management of a multinational firm. The links in the network represented possible cash flow patterns and the multipliers incorporated costs, fees, liquidity changes, and exchange rates. A series of related cash management problems were modeled as network problems in subsequent years (see, e.g., Crum and Nye (1981), Crum, Klingman, and Tavis (1983)), thereby, further extending the applicability of network programming in financial applications. The focus therein was on linear network flow problems in which the cost on an arc was a linear function of the flow. Crum, Klingman, and Tavis (1979), in turn, showed how contemporary financial capital allocation problems could be modeled as an integer generalized network problem, in which the flows on particular arcs were forced to be integers.

We emphasize that in many financial network optimization problems the objective function must be nonlinear due to the modeling of the risk function and, hence, typically, such financial problems lie in the domain of nonlinear, rather than linear, network flow problems. Mulvey (1987) presented a collection of nonlinear financial network models that were based on previous cash flow and portfolio models in which the original authors (e.g., Rudd and Rosenberg (1979) and Soenen (1979)) did not realize (nor exploit) the underlying network structure. He also emphasized that the Markowitz (1952, 1959) mean-variance minimization problem was, in fact, a network optimization problem with a nonlinear objective function. Here, for completeness, we recall the classical Markowitz models and cast them into the framework of network optimization problems. See Figure 1.3 for the network structure of such problems.

Markowitz's model was based on mean-variance portfolio selection, where the average and the variability of portfolio returns were determined in terms of the mean and covariance of the corresponding investments. The mean is a measure of an average return and the variance is a measure of the distribution

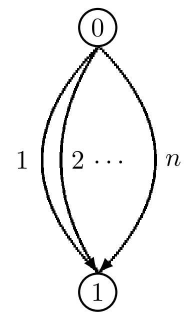

Fig. 1.3. Network structure of classical portfolio optimization

of the returns around the mean return. Markowitz formulated the portfolio optimization problem as associated with risk minimization with the objective function given by:

$$\text{Minimize} \quad V = X^T Q X$$

subject to constraints, representing, respectively, the attainment of a specific return, a budget constraint, and that no short sales were allowed, and, mathematically, given by:

$$R = \sum_{i=1}^{n} X_i r_i$$

$$\sum_{i=1}^{n} X_i = 1$$

$$X_i \geq 0, \quad i = 1, \ldots, n.$$

Here n denotes the total number of securities available in the economy, X_i represents the relative amount of capital invested in security i, with the securities being grouped into the column vector X, Q denotes the $n \times n$ variance-covariance matrix on the return of the portfolio, r_i denotes the expected value of the return of security i, and R denotes the expected rate of return on the portfolio. Within a network context (cf. Figure 1.3), the links correspond to the securities, with their relative amounts $X_1, \ldots, X_n$ corresponding to the flows on the respective links: $1, \ldots, n$. The budget constraint and the nonnegativity assumption on the flows are network conservation equations. Since the objective function here is risk minimization, it can be interpreted as the sum of the costs on the n links in the network. Obsrve that the network representation is abstract and does not correspond (as in the case of transportation) to physical locations and links.

In his work, Markowitz suggested that, for a fixed set of expected values r_i and covariances of the returns of all assets i and j, every investor can find an (R, V) combination that better fits his taste, solely limited by the constraints of the specific problem. Hence, according to the original work of Markowitz (1952), the efficient frontier had to be identified and then every investor had to select a portfolio through a mean-variance analysis that fitted his preferences.

A related mathematical optimization model (Markowitz (1959)) to the one above, which can be interpreted as the investor seeking to maximize his returns while minimizing his risk can be expressed by the quadratic programming problem:

$$\text{Maximize} \quad \alpha R - (1-\alpha)\, V$$

subject to:

$$\sum_{i=1}^{n} X_i = 1$$

$$X_i \geq 0, \quad i = 1, \ldots, n,$$

where α denotes an indicator of how risk-averse a specific investor is. Of course, this model is also a network optimization problem with the network as depicted in Figure 1.3.

Many versions and extensions of Markowitz's model have appeared in the literature, a collection of which can be found in Francis and Archer (1979), with $\alpha = 1/2$ being a frequently accepted value. A recent interpretation of the model as a multicriteria decision-making model along with theoretical extensions to multiple sectors can be found in Dong and Nagurney (2001).

Lastly, a part of the optimization literature on financial networks focused on variables that were stochastic and had to be treated as random variables in the optimization procedure. Clearly, since most financial optimization problems are of large size, the incorporation of stochastic variables made the problems more complicated and difficult to model and compute. Mulvey (1987) and Mulvey and Vladimirou (1989, 1991), among others, studied stochastic financial networks, utilizing a series of different theories and techniques (e.g., purchase power priority, arbitrage theory, scenario aggregation) that were then utilized for the estimation of the stochastic elements in the network in order to be able to represent them as a series of deterministic equivalents. The large size and the computational complexity of stochastic networks, at times, limited their usage to specially structured problems where general computational techniques and algorithms could be applied. See Rudd and Rosenberg (1979), Wallace (1986), and Rockafellar and Wets (1991) for a more detailed discussion on aspects of realistic portfolio optimization and implementation issues related to stochastic financial networks.

1.2.2 Equilibrium Problems

In 1969, Thore introduced networks, along with the mathematics, for the study of systems of linked portfolios. His work benefited from that of Charnes and Cooper (1967) which showed that systems of linked accounts could be represented as a network, where the nodes depict the balance sheets and the links depict the credit and debit entries. Thore considered credit networks, with the explicit goal of providing a tool for use in the study of the propagation of money and credit streams in an economy, based on a theory of the behavior of banks and other financial institutions. The credit network recognized that these sectors interact and its solution made use of linear programming. Thore (1970)) then extended the basic network model to handle holdings of financial reserves in the case of uncertainty. The approach utilized two-stage linear programs under uncertainty introduced by Ferguson and Dantzig (1956) and Dantzig and Madansky (1961). See Fei (1960) for a graph theoretic approach to the credit system.

Storoy, Thore, and Boyer (1975), in turn, developed a network representation of the interconnection of capital markets and demonstrated how decomposition theory of mathematical programming could be exploited for the computation of equilibrium. The utility functions facing a sector were no longer restricted to being linear functions. Thore (1980) further investigated network models of linked portfolios, financial intermediation, and decentralization/decomposition theory. However, the computational techniques at that time were not sufficiently well-developed to handle such problems in practice.

Thore (1984) proposed an international financial network for the Euro dollar market and viewed it as a logistical system, exploiting the above-mentioned ideas of Samuelson (1952) and Takayama and Judge (1971) for spatial price equilibrium problems. In this paper, as in Thore's preceding papers on financial networks, the micro-behavioral unit consisted of the individual bank, savings and loan, or other financial intermediary and the portfolio choices were described in some optimizing framework, with the portfolios being linked together into a network with a separate portfolio visualized as a node and assets and liabilities as directed links.

Notably, the above-mentioned contributions focused on the use and application of networks for the study of financial systems consisting of multiple economic decision-makers. In such systems, equilibrium was a central concept, along with the role of prices in the equilibrating mechanism. Rigorous approaches that characterized the formulation of equilibrium and the corresponding price determination were greatly influenced by the Arrow-Debreu economic model (cf. Arrow (1951), Debreu (1951)). In addition, the importance of the inclusion of dynamics in the study of such systems was explicitly emphasized (see, also, Thore and Kydland (1972)).

The first use of finite-dimensional variational inequality theory for the computation of multi-sector, multi-instrument financial equilibria is due to Nagurney, Dong, and Hughes (1992), who recognized the network structure underlying the subproblems encountered in their proposed decomposition scheme. Hughes and Nagurney (1992) and Nagurney and Hughes (1992) had, in turn, proposed the formulation and solution of estimation of financial flow of funds accounts as network optimization problems. Their proposed optimization scheme fully exploited the special network structure of these problems. Nagurney and Siokos (1997) then developed an international financial equilibrium model utilizing finite-dimensional variational inequality theory for the first time in that framework.

Finite-dimensional variational inequality theory is a powerful unifying methodology in that it contains, as special cases, such mathematical programming problems as: nonlinear equations, optimization problems, and complementarity problems. Moreover, with projected dynamical systems theory (see the book by Nagurney and Zhang (1996)) one can then also trace the dynamic behavior prior to an equilibrium state (formulated as a variational inequality). In contrast to classical dynamical systems, projected dynamical systems are characterized by a discontinuous right-hand side, with the discontinuity arising due to the constraint set underlying the application in question. Hence, this methodology allows one to model systems dynamically which are subject to limited resources, with a principal constraint in finance being budgetary restrictions.

Dong, Zhang, and Nagurney (1996) were the first to apply the methodology of projected dynamical systems to develop a dynamic multi-sector, multi-instrument financial model, whose set of stationary points coincided with the set of solutions to the variational inequality model developed in Nagurney (1994); and then to study it qualitatively, providing stability analysis results.

The book by Nagurney and Siokos (1997) presents the foundations of financial networks to that date as well as an overview of the basic methodologies for the formulation, analysis, and solution of such problems with a particular focus on equilibrium problems. Additional background can be found in Nagurney (2001). Finally, Nagurney and Ke (2001, 2003) focus on financial networks with intermediation and utilize variational inequalities in that problem domain.

1.3 Economic Networks

As the preceding discussion has noted, the development of the study of economic networks has been based heavily on transportation networks as well as on spatial price equilibrium networks in the form of interregional commodity trade. Extensive references on the subject as well as a variety of models and applications can be found in Nagurney (1999). Here, we recall two economic equilibrium problems and provide their network equilibrium formulations. The first is a Walrasian price equilibrium problem, which is an example of general equilibrium, whereas, the second is a spatial price equilibrium problem, which is a partial equilibrium problem.

1.3.1 Equilibrium Problems

Zhao and Nagurney (1993) (see also Zhao and Dafermos (1991)) considered the general economic equilibrium problem known as the Walrasian price equilibrium problem as a network equilibrium problem with precisely the network structure given in Figure 1.3. However, the flows on the network representing this problem correspond to prices of different commodities. For completeness and easy reference we briefly review the pure exchange economic equilibrium model and also give its variational inequality formulation. We consider a pure exchange economy with n commodities, price vector $p = (p_1, p_2, \ldots, p_n)^T$ taking values in the positive orthant R^n_+, and the induced aggregate excess demand function $z(p)$, with components $z_1(p), \ldots, z_n(p)$. Under certain technical assumptions (such as requiring that $z(p)$ be homogeneous of degree zero in p), we may normalize the prices so that they take values in the unit simplex, that is,

$$S^n = p : p \in R^n_+, \sum_{i=1}^{n} p_i = 1.$$

As is standard in general economic equilibrium theory, the aggregate excess demand function must satisfy Walras's law:

$$\langle p^T, z(p) \rangle = 0, \quad \forall p \in S^n,$$

with $\langle \, , \rangle$ denoting the inner product with the aformentioned equation being equivalent to: $\sum_{i=1}^{n} p_i z_i(p) = 0$.

We now recall the definition of a Walrasian equilibrium.

Definition 1.1: Walrasian Price Equilibrium
A price vector $p^* \in S^n$ is a Walrasian equilibrium if the market is cleared for valuable commodities and is in excess supply for free commodities, that is,

$$z_i(p^*) = 0, \quad \textit{if} \quad p_i^* > 0$$

$$z_i(p^*) \leq 0, \quad \textit{if} \quad p_i^* = 0.$$

The following theorem, due to Dafermos (1990), shows the equivalence between Walrasian price equilibria and solutions of a variational inequality problem.

Theorem 1.1: Variational Inequality Formulation of Walrasian Price Equilibrium
A price vector $p^* \in S^n$ is a Walrasian price equilibrium if and only if it satisfies the variational inequality: determine $p \in S^n$ such that

$$\langle -z(p^*), p - p^* \rangle \geq 0, \quad \forall p \in S^n.$$

This example vividly illustrates that fundamental economic equilibrium problems can be cast into a network equilibrium framework. We now (due to its importance in the development of economic network models) recall the spatial price equilibrium problem and provide its variational inequality formulation (for further details, see Florian and Los (1982), Dafermos and Nagurney (1987), and Nagurney (1999)).

Consider the spatial price equilibrium problem in quantity variables with m supply markets and m demand markets involved in the production and consumption of a homogeneous commodity under perfect competition. Denote a typical supply market by i and a typical demand market by j. Let s_i denote the supply and π_i the supply price of the commodity at supply market i. Let d_j denote the demand and ρ_j the demand price at demand market j. Group the supplies and supply prices, respectively, into a column vector $s \in R^m$ and a row vector $\pi \in R^m$. Similarly, group the demands and demand prices, respectively, into a column vector $d \in R^n$ and a row vector $\rho \in R^n$. Let Q_{ij} denote the nonnegative commodity shipment between the supply and demand market pair (i,j), and let c_{ij} denote the unit transaction cost associated with trading the commodity between (i,j). The unit transaction costs are assumed to include the unit costs of transportation from supply markets to demand markets, and, depending upon the application, may also include a tax/tariff, duty, or subsidy incorporated into these costs. Group the commodity shipments into a column vector $Q \in R^{mn}$ and the transaction costs into a row vector $c \in R^{mn}$. The network structure of the problem is depicted in Figure 1.2.

Assume that the supply price at any supply market may, in general, depend upon the supply of the commodity at every supply market, that is, $\pi = \pi(s)$, where π is a known smooth function. Similarly, the demand price at any demand market may depend upon, in general, the demand of the commodity at every demand market, that is, $\rho = \rho(d)$, where ρ is a known smooth function. The unit transaction cost between a pair of supply and demand markets may depend upon the shipments of the commodity between every pair of markets, that is, $c = c(Q)$, where c is a known smooth function.

The supplies, demands, and shipments of the commodity, in turn, must satisfy the following feasibility conditions, which are also referred to as the *conservation of flow equations*:

$$s_i = \sum_{j=1}^{n} Q_{ij}, \quad i = 1, ..., m$$

$$d_j = \sum_{i=1}^{m} Q_{ij}, \quad j = 1, ..., n$$

$$Q_{ij} \geq 0, \quad i = 1, ..., m; j = 1, ..., n.$$

In other words, the supply at each supply market is equal to the commodity shipments out of that supply market to all the demand markets.

Similarly, the demand at each demand market is equal to the commodity shipments from all the supply markets into that demand market.

Definition 1.2: Spatial Price Equilibrium
Following Samuelson (1952) and Takayama and Judge (1971), the supply, demand, and commodity shipment pattern (s^*, Q^*, d^*) constitutes a spatial price equilibrium, if it is feasible, and for all pairs of supply and demand markets (i, j), it satisfies the conditions:

$$\pi_i(s^*) + c_{ij}(Q^*) \begin{cases} = \rho_j(d^*), & \text{if} \quad Q^*_{ij} > 0 \\ \geq \rho_j(d^*), & \text{if} \quad Q^*_{ij} = 0. \end{cases}$$

Hence, if the commodity shipment between a pair of supply and demand markets is positive at equilibrium, then the demand price at the demand market must be equal to the supply price at the originating supply market plus the unit transaction cost. If the commodity shipment is zero in equilibrium, then the supply price plus the unit transaction cost can exceed the demand price.

The spatial price equilibrium can be formulated as a variational inequality problem. Precisely, we have following Florian and Los (1982), Dafermos and Nagurney (1987):

Theorem 1.2: Variational Inequality Formulation of Spatial Price Equilibrium
A commodity supply, shipment, and demand pattern $(s^*, Q^*, d^*) \in K$ is a spatial price equilibrium if and only if it satisfies the following variational inequality problem:

$$\langle \pi(s^*), s - s^* \rangle + \langle c(Q^*), Q - Q^* \rangle + \langle -\rho(d^*), d - d^* \rangle \geq 0, \quad \forall (s, Q, d) \in K,$$

where $K \equiv \{(s, Q, d) : \text{feasibility conditions hold}\}$.

An Example

For illustrative purposes, we now present a small example. Consider the spatial price equilibrium problem consisting of two supply markets and two demand markets. Assume that the functions are as follows:

$$\pi_1(s) = 5s_1 + s_2 + 1, \quad \pi_2(s) = 4s_2 + s_1 + 2$$

$$c_{11}(Q) = 2Q_{11} + Q_{12} + 3, \; c_{12}(Q) = Q_{12} + 5,$$

$$c_{21}(Q) = 3Q_{21} + Q_{22} + 7, \; c_{22}(Q) = 3Q_{22} + 2Q_{21} + 9$$

$$\rho_1(d) = -d_1 - d_2 + 21, \quad \rho_2(d) = -5d_2 - 3d_1 + 29.$$

It is easy to verify that the spatial price equilibrium pattern is given by:

$$s^*_1 = 2, \; s^*_2 = 1, \; Q^*_{11} = 1, \; Q^*_{12} = 1, \; Q^*_{21} = 1, \; Q^*_{22} = 0, \; d^*_1 = 2, \; d^*_2 = 1.$$

In one of the simplest models, in which the Jacobians of the supply price functions, $\left[\frac{\partial \pi}{\partial s}\right]$, the transportation (or transaction) cost functions, $\left[\frac{\partial c}{\partial Q}\right]$, and minus the demand price functions, $-\left[\frac{\partial \rho}{\partial d}\right]$ are diagonal and positive definite, then the spatial price equilibrium pattern coincides with the Kuhn-Tucker conditions of the strictly convex optimization problem:

$$\text{Minimize}_{Q \in R_+^{MN}} \quad \sum_{i=1}^{M} \int_0^{\sum_{j=1}^{N} Q_{ij}} \pi_i(x)dx + \sum_{i=1}^{M} \sum_{j=1}^{N} \int_0^{Q_{ij}} c_{ij}(y)dy$$
$$- \sum_{j=1}^{N} \int_0^{\sum_{i=1}^{M} Q_{ij}} \rho_j(z)dz.$$

We note also that the classical Cournot (1838) (see also Nash (1950, 1951)) oligopoly problem which possesses a variational inequality formulation (cf. Gabay and Moulin (1980)) can also be cast into a network framework (cf. Nagurney (1999)) with the structure as given in Figure 1.3. Numerous spatial oligopoly problems including those having the network structure given in Figure 1.2) have also been developed and algorithms that exploit the network structure proposed (see Dafermos and Nagurney (1987) and Nagurney, Dupuis, and Zhang (1994), and the references therein).

Finally, it is worth mentioning that a variety of migration problems in economics have a network structure as do knowledge network problems (cf. Nagurney (1999)).

1.4 The Internet and New Directions

The advent of the Internet, along with associated communication methodologies, has further elevated the interest in networks and the importance thereof. It has brought greater focus to the study of financial and economic networks, the interaction among networks (as in the case of supernetworks (cf. Nagurney and Dong (2002)), and to the entire field of network economics, including network industries (cf. Shapiro and Varian (1999)). Entirely new subject areas such as electronic commerce (cf. Whinston, Stahl, and Choi (1997)) and electronic finance (cf. Claessens and Jansen (2000)) have been born whereas others such as supply chain networks (cf. Nagurney et al. (2002)) have evolved both in dimension and complexity. In such contexts the role of intermediaries becomes increasingly important as well as the dynamics and network structures.

Coupled with the growth of the Internet have come new and more powerful tools for the modeling, analysis, and solution of financial and economic network problems. Such tools, some of which are revealed in this volume, provide new methodologies, both analytical and conceptual, from which further advances can be expected. Above we have provided an overview of financial and

Table 1.1. Methodologies utilized in this volume

Methodology	Chapter
Agent-based simulation	11, 12
Fluid dynamics	10
Game theory	5, 7, 9, 11 – 13
Graph theory	2, 9
Network theory	4, 7, 9 – 12
Optimization	2 – 5, 7 – 11, 13
Stochastic programming	3, 4
Variational inequalities	5, 7, 9

economic networks from both optimization and equilibrium perspectives with a view towards historical developments. It is worth noting also the growth of simulation approaches, notably, those based on agent-based computational economics (ACE), which provide computational studies of economies modeled as evolving systems of autonomous interacting agents (cf. Tesfatsion (2002)) for the study of economic network phenomena.

1.5 Outline of the Volume

This volume presents a broad collection of recent innovations in the study of financial and economic networks by leading scholars in different parts of the world.

Part I of this book focuses on financial networks, ranging from the use of graph theory to provide a new perspective on the stock market, to the conceptualization, modeling, and solution of international financial networks with electronic transactions, and, finally, to the use of financial options to hedge transportation capacity in the rail industry. Part II of this book turns to economic networks, beginning with new frameworks for supply chain and distribution problems, to the use of agent-based computational economics for the study of trade with intermediaries (and varying amounts of information) and for the evolution of worker-employer networks. This part as well as the volume concludes with a market framework for trade in demand-side Web caching.

The innovations in this volume are broad, original, and timely and include conceptual, theoretical, methodological, empirical, as well as application-based contributions. Table 1.1 lists the principal methodologies utilized and in what chapters they may be found. Table 1.2, in turn, highlights some of the applications in this volume.

In addition, for the convenience of the reader, we provide a snapshot of

Table 1.2. Applications in this volume

Application	Chapter
Capacity provision networks	13
Competition	5, 6, 7, 9, 11, 13
Corporate planning	4
Distribution systems	9, 10
Electronic transactions	6, 7, 11, 13
Equilibrium analysis	5, 7, 9
Financial planning	3, 4, 6, 7, 8
Financial services	6, 7
Hedging instruments	8
Intermediation	6, 7, 9, 10, 11, 13
International finance	6, 7
Internet effects	6, 7, 11, 13
Labor markets	12
Network industries	6, 8
Pension planning	4
Policy analysis	6, 12
Portfolio optimization	3, 4, 5, 7
Pricing	7 – 11
Risk management	3, 4, 5, 7, 8
Stock market	2
Supply chains and logistics	9, 10
Trade networks	11, 13
Transportation networks	8, 10
Worker-employer networks	12

the contents of the volume in Table 1.3 in terms of the type of innovations (conceptual, theoretical, methodological, etc.).

Chapter 2 by Boginski, Butenko, and Pardalos presents a detailed study of the stock market graph, which yields a new tool for the analysis of the market structure through the classification of stocks into different groups. The authors first demonstrate how information generated by the stock market can be used to construct a market graph, consisting of vertices and edges, and based on the cross-correlations of price fluctuations. This graph, which may be massive in size, is then analyzed from the perspective of finding cliques and independent sets. Boginski, Butenko, and Pardalos then make evident, through experiments, that the distribution of the correlation coefficients between the stocks in the US stock market remains very stable over time. The authors also establish, for the first time in the field of finance, the

Table 1.3. Snapshot of innovations in this volume

Chapter	Conceptual	Theoretical	Methodological	Empirical	Applications
2	x	x	x	x	x
3	x		x	x	x
4	x		x	x	x
5	x	x	x		x
6	x				x
7	x	x	x		x
8	x			x	x
9	x	x	x		x
10	x	x	x		x
11	x		x	x	x
12	x		x	x	x
13	x	x			x

applicability of the power-law model, which describes massive graphs arising in telecommunications and the Internet.

Chapter 3 by Gülpinar, Rustem, and Settergren uses multistage stochastic programming in order to model the problem of financial portfolio management with transaction costs, given stochastic data provided in the form of a scenario tree. The mean or variance of the total wealth at the end of the planning horizon can be optimized in view of the transaction costs by solving either a linear or a quadratic stochastic program. Moreover, the incorporation of proportional transaction costs yields a model that reflects the effect of these costs on portfolio performance. Numerical experiments backtesting the optimization strategies at different levels of risk and transaction cost are reported, as well as tests that do not optimize over the affected transaction costs. The results show that the incorporation of transaction costs improves investment performance.

The fourth chapter, by Mulvey, Simsek, and Pauling, also utilizes stochastic programming for financial decision-making, and presents a multi-period stochastic network model for integrating corporate financial and pension planning. The model maximizes the combined company's value over a finite planning time horizon and has certain advantages over general nonlinear programs, especially in regards to the model's understandability. The empirical results demonstrate that the integration of pension planning is feasible and that it can improve a company's performance. Moreover, the authors, by analyzing historical data for some typical pension plans, demonstrate that the recent loss of surplus by many large US companies was largely preventable. The methodology proposed requires three elements: a stochastic scenario generator, a pension/corporate simulator, and a stochastic network optimizer. Solving such stochastic programming problems has become practical due to the large improvements in computer hardware and software.

Chapter 5 shifts from stochastic optimization to the investigation of financial equilibria in the case of multiple sectors and multiple financial instruments. Daniele in this chapter proposes a new framework for the modeling, analysis, and computation of financial equilibria through a novel evolutionary model. In contrast to earlier multi-sector, multi-instrument financial equilibrium models, the new model allows for variance-covariance matrices associated with risk perception to be time-dependent as well as the sector financial volumes. Daniele identifies the network structure of the sectors' optimization problems out of equilibrium and provides the network structure of the financial economy in equilibrium. The methodology utilized for the formulation, qualitative analysis, and solution of such problems is that of infinite-dimensional variational inequalities.

Chapter 6 focuses on financial services industries, which have been undergoing rapid change due to globalization and technological advances, including electronic finance, and examines the importance of networks in finance and its effects on competition. Claessens, Dobos, Klingebiel, and Laeven, in this chapter, argue that, as part of the changes, financial services are becoming less special, making policies to preserve the franchise value of financial institutions less necessary while competition policy becomes more feasible. They identify the main characteristics of networks, from an economic perspective, along with related public policy issues. They conclude that as financial services heavily and increasingly depend on networks for their production and distribution, that competition policy for financial services becomes more necessary and will need to resemble that used in other network industries, such as telecommunications. Hence, the institutional and functional approaches to competition in the financial sector need to be complemented with more production-based approaches to competition.

The seventh chapter, by Nagurney and Cruz, is complementary to Chapter 6 in that international finance with intermediaries and electronic transactions is of primary concern. However, rather than a focus on competition and appropriate policies with a view of the financial sector as a network industry, it considers the modeling, analysis, and computation of such international financial network problems when the optimizing behavior of those with sources of funds, that of the intermediaries, as well as the consumers is assumed known and given. The authors identify the international financial network with intermediaries in which transactions can take place either physically or electronically, model the behavior of the various decision-makers (which includes net revenue maximization as well as risk minimization), derive the equilibrium conditions, and establish the governing finite-dimensional variational inequality formulation. This formulation is then utilized to obtain both qualitative properties of the equilibrium price and financial flow pattern, as well as a computational procedure. Numerical examples are provided to illustrate both the model and the algorithm. The model is sufficiently general to include as many countries, financial sectors in each country, currencies, and intermediaries, as well as financial products, as needed.

In Chapter 8, Law, MacKay, and Nolan describe a potential derivatives, in particular, financial options, market for rail car capacity pricing in the case of a deregulated rail industry. The authors discuss the rail industry in the United States and Canada, identifying the similarities and differences. They delineate the features of the rail car capacity market that they expect to evolve if there is entry deregulation in this industry, noting the existing capacity allocation mechanisms that will be kept and discussing those that will be entirely new. They draw links between the possible future for the rail industry and the path that has been followed by other network industries, such as natural gas and electricity, that have deregulated in recent years. An empirical example is constructed to show how such a market might actually function, using coal as the commodity to be transported by rail.

Part II of this book, which focuses on economic networks, starts out with Chapter 9.

Chapter 9 develops a new framework for supply chain networks, using concepts from graph theory, optimization theory, and variational inequality theory. Co-authored by Zhang, Dong, and Nagurney, it develops a general network model of a supply chain economy to study supply chain competition. The proposed network framework considers the transformation and pricing of the material flows as they propagate through the network from origins, associated, for example, with raw material suppliers, to destinations – the consumer markets. The network includes both operation links and interface links, with an operation link representing a business operation such as manufacturing, storage, and/or transportation, while an interface link represents a business to business bridge. The model allows for the formulation and analysis of both intra-chain cooperation and inter-chain competition, and predicts the *winning* supply chains, which are those that carry positive chain flows. The model has the notable feature that as many links (and any topology as needed) can be utilized to describe the supply chain structures to capture both cooperation as well as competition. The network model is also analyzed qualitatively in terms of existence and uniqueness of solutions.

Chapter 10, in turn, by Kachani and Perakis, proposes a fluid model of dynamic pricing and inventory control in the context of supply chain management under make-to-stock regime. In particular, the supply chain system that the authors consider is a distribution system that consists of several wholesalers, intermediate distributors, and retailers. All entities are subsidiaries of the same company that is producing and selling multiple products. The model does not require the determination of how prices at each wholesaler, distributor, and retailer affect the corresponding demands. Instead, the model accounts for how price and level of inventory affect the sojourn time of the products at each wholesaler, distributor, and retailer in the distribution system. The model, hence, allows for joint pricing, production, and inventory decisions in a capacitated, multi-product, and dynamically evolving distribution system. Kachani and Perakis also analyze the properties of the model and establish conditions under which the model gives rise to reasonable policies. Finally, the authors discuss how this delay approach connects with more traditional demand models.

The following two chapters utilize agent-based computational modeling within a trade network game simulation framework to investigate economic questions in electronic commerce and in labor institutions, respectively. Chapter 11, by Alkemade, Poutré, and Amman, investigates whether intermediaries can still make a profit in an information economy. In particular, the authors study the influence of the network structure and the information level of the agents, in the form of producers, consumers, and intermediaries, on the level of intermediated trade. The main conclusion of the simulations is that intermediaries that have better knowledge of the market than the average consumer will continue to exist and make a profit if the market dynamics are sufficiently complex. For example, intermediaries that are experts at finding the best price quotes can survive in an electronic trade network where consumers can also form direct links to producers. However, ultimately, most consumers bypass the intermediary if direct trade is more profitable. Interestingly, the authors find that in the case of higher purchase prices, consumers compensate for the higher purchase price by maintaining fewer links, and this has a stabilizing effect on the architecture of the electronic trade network.

Chapter 12, by Pingle and Tesfatsion, applies agent-based computational modeling to analyze the impact of labor institutions. Determining the effects of labor institutions on macroeconomic performance is a central concern of economic policymakers. Specifically, the authors utilize an agent-based computational labor market model to conduct systematic experiments testing the sensitivity of macroeconomic performance to changes in the level of a non-employment payment. The computational experiments are implemented by means of the *Trade Network Game Laboratory*, which is an agent-based computational laboratory for studying the evolution of trade networks via real-time simulations, tables, and graphical displays. The experiments allow the authors to examine the effects of a non-employment payoff on network formation and work-site behaviors among workers and employers participating in a sequential employment game with incomplete contracts.

The final chapter in this volume, authored by Geng, Gopal, Ramesh, and Whinston, introduces Capacity Provision Networks (CPN) as a market framework for demand-side Web cache trading. According to the authors, the need for demand-side cache trading is supported by the fact that there exist positive network externalities across individual Internet Service Providers who provide caching services to their respective users. The need for a CPN market is further supported by the existence of convexity in capacity discounting and the consequential potential for intermediation in cache trading. This chapter develops three critical components in an implementation of Capacity Provision Networks: a technical framework, the economic foundations of such networks, and tactical models of real-time trading, and demonstrates the technical and economic viability of a CPN to effectively support demand-side distributed caching systems through the mechanisms of cache trading and deployment. Possible future research will include futures contract, options, and the development of indices as instruments for Internet Service Providers to coordinate capacity decisions through trading mechanisms.

1.6 Notes

In this chapter we have attempted to provide a preface to the subject matter at hand so that the innovations in this volume can be appreciated. The chapters in this volume contain additional source and background material as well as appropriate references.

In addition to the above-mentioned references and the other citations following each chapter in this book, we also mention several books that provide further interesting material and background on networks. For a history of graph theory and contributions over two centuries, see Biggs, Lloyd, and Wilson (1976). For numerous models, algorithms, and applications of network flows, see the book by Ahuja, Magnanti, and Orlin (1993). For a classical book on network flows, see Ford and Fulkerson (1962).

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