A Retrospective on Beckmann, McGuire, and Winsten's *Studies in the Economics of Transportation*

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It is an outgrowth of a Special Panel on the Impacts of BMW at the 50th North American Meeting of RSAI on November 22, 2003 in Philadelphia, PA.

Transportation-Based Innovations Stemming from BMW

- **Algorithms and Computations for the Standard Models of BMW**
- **Toll Policies**
- **Extended Traffic Network Models Including Models of Urban Location**
- **Variational Inequality Formulations and Algorithms**
- **Multicriteria Decision-Making**
- **Stochastic Route Choice Modeling**
- **Dynamic Transportation Networks**

Novel Network Equilibrium Applications

We now highlight *some* of the many applications whose further development has benefited from *Studies in the Economics of Transportation.*

Applications

- **Spatial Price Equilibrium Networks**
- **General Economic Equilibrium**
- **Oligopolistic Market Equilibrium and Game Theory**
- **Supply Chain Networks**
- **Financial Networks with Intermediation**
- **Electric Power Supply Chain Networks**
- **Knowledge Networks**
- **Computer Scientists Discover Beckmann, McGuire, and Winsten**

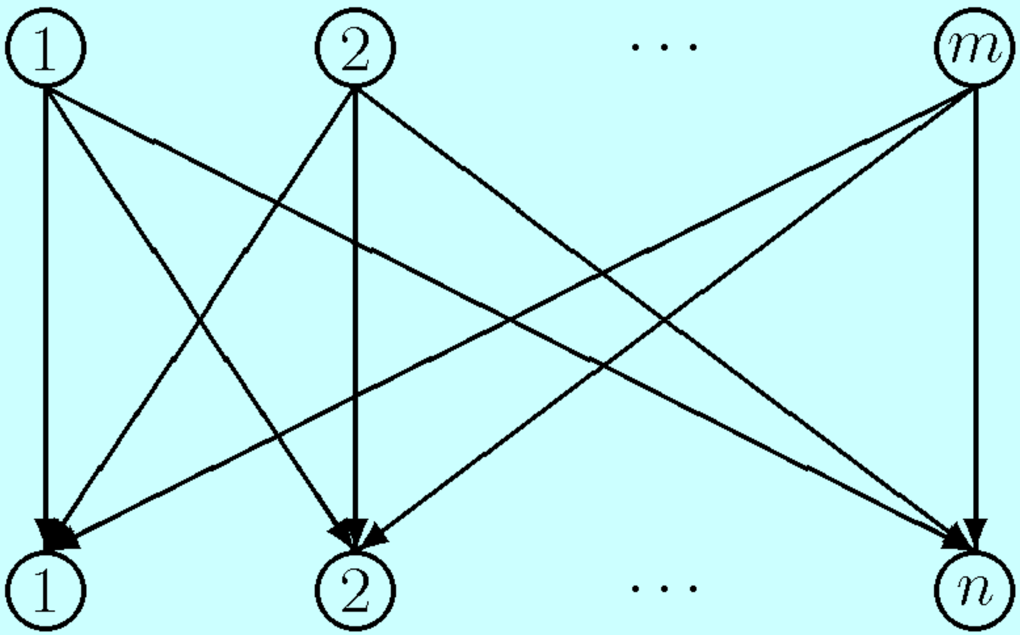
Spatial Price Equilibrium Networks

Koopmans, in his introduction to BMW, in discussing the railroad transportation contributions in the BMW book, noted the work of Enke (1951) and Samuelson (1952) in the development of frameworks (the former using analogues to electronic circuits and the latter to a linear programming problem) for the determination of interregional commodity flows and prices in the case of separated markets.

BMW stated on page 105 that *The principal difference between passenger transportation as discussed in this study and commodity transportation lies in the substitutability of commodity shipments from different origins to different destinations.*

Takayama and Judge (1964) demonstrated how, in the case of linear regional supply and demand functions and fixed interregional transportation costs, the governing spatial price equilibrium conditions could be reformulated as the Kuhn-Tucker conditions of a quadratic programming problem. The authors thank Martin Beckmann for helpful comments.

The Structure of Classical Spatial Price Networks



Florian and Los (1982) provided a synthesis of the Samuelson (1952) model and the BMW network equilibrium model with elastic/variable demand to construct a spatial price equilibrium model on a general network. They also considered multicommodity models and demonstrated that the governing equilibrium conditions satisfy a variational inequality problem akin to those arising in traffic network equilibrium models.

Others had also been developing and extending the basic spatial price equilibrium models of Samuelson (1952) and Takayama and Judge (1964, 1971) (see Nagurney (1999)).

Nagurney and Dafermos (1985) established an isomorphism between spatial price and traffic network equilibrium problems which was further elaborated upon by Dafermos (1986) in the context of multicommodity/multiclass networks.

Friesz et al. (1983, 1984), citing BMW, provided additional contributions to the modeling, analysis, and solution of spatial price network equilibrium problems and forged the topic of freight network equilibrium.

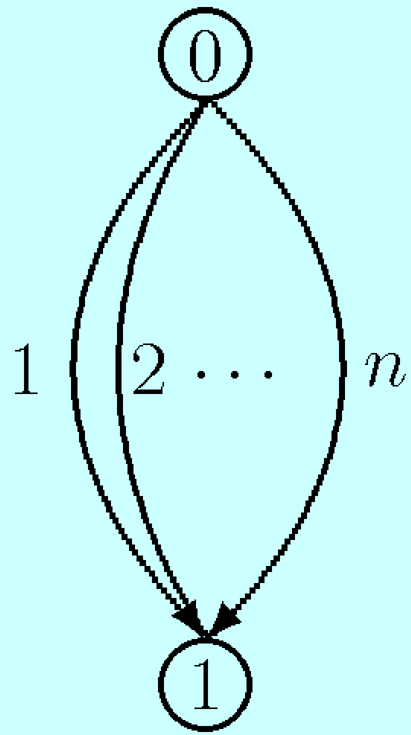
It was researchers in transportation science that truly exploited the connections between the two subjects which had actually been identified as early as the seminal book.

General Economic Equilibrium

Spatial price equilibrium models, in contrast to general economic equilibrium models, are necessarily partial equilibrium models. The network structure of spatial price equilibrium problems considered today often corresponds to the physical transportation network.

The general economic equilibrium problem due to Walras (1874) has also been extensively studied (see, e.g., Border (1985)) both from qualitative as well as quantitative perspectives (cf. Dafermos (1990)). The Walrasian price equilibrium problem can also be cast into a network equilibrium form as shown in Zhao and Nagurney (1993).

Network Structure of Walrasian Price Equilibrium



Oligopolistic Market Equilibrium and Game Theory

Game theory, although not explicitly recognized in the sense of Nash (1951) in the work of BMW, but noted in the Dafermos and Sparrow (1969) paper and cited by Charnes and Cooper (1958, 1961), has had an enormous impact not only on economics but lately also in computer science.

Such problems date to Cournot (1838) and Nash equilibria in the context of oligopoly problems have been shown to satisfy variational inequalities by Gabay and Moulin (1982) and solved thus by Harker (1984) and by Nagurney (1988) (see also Murphy, Sherali, and Soyster (1982)).

Nagurney (1993) demonstrated that the classical aspatial Cournot oligopoly market equilibrium problem could also be cast into a network equilibrium framework on an abstract network of the same structure as that underlying the Walrasian price equilibrium problem but with elastic demand.

In the network setting, the links correspond to the firms and the flows on the links are the production outputs.

Dafermos and Nagurney (1987) established the connection between spatial oligopolies operating in a Nash-Cournot sense and spatial price equilibrium problems.

Devarajan (1981), motivated by the Dafermos and Sparrow (1969) paper, established that a continuous flow, user-optimized network is a pure-strategy Nash equilibrium in a game with a continuum of pure strategies.

Haurie and Marcotte (1985) further tightened the connection between Nash-Cournot equilibria and Wardrop equilibria.

Supply Chain Networks

Beckmann, McGuire, and Winsten explicitly anticipated supply chain networks.

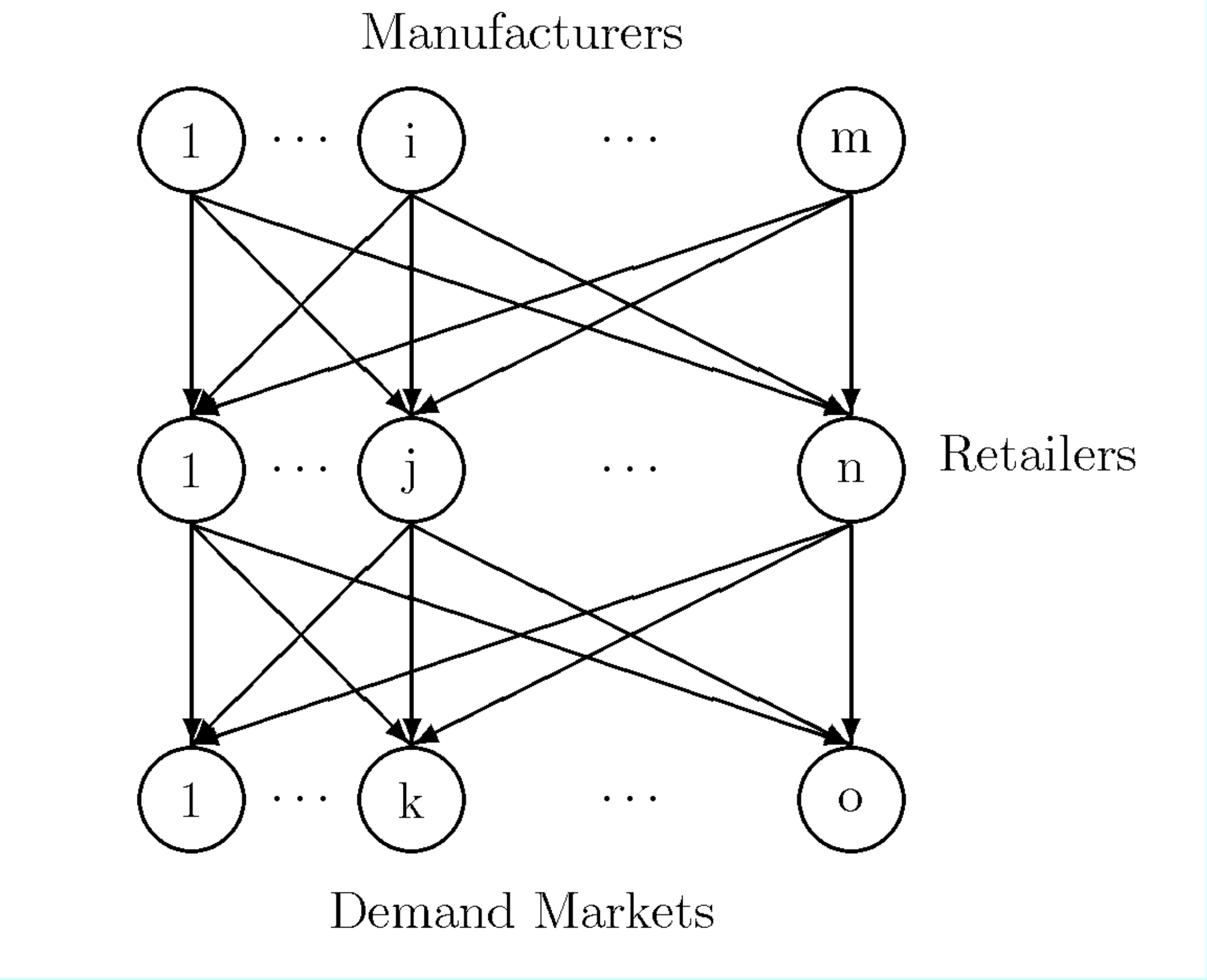
BMW on page 88 stated, ***consider a chemical or metallurgical material which is capable of various stages or modifications, and a firm which undertakes to transform it from certain stages to other stages … Here the stages of the material correspond to the locations, the transitions correspond to roads, and sequences of transformation processes … of the material -- that is, the production methods -- correspond to the routes.***

The first work on utilizing network equilibrium concepts in the context of supply chain applications is due to Nagurney, Dong, and Zhang (2002).

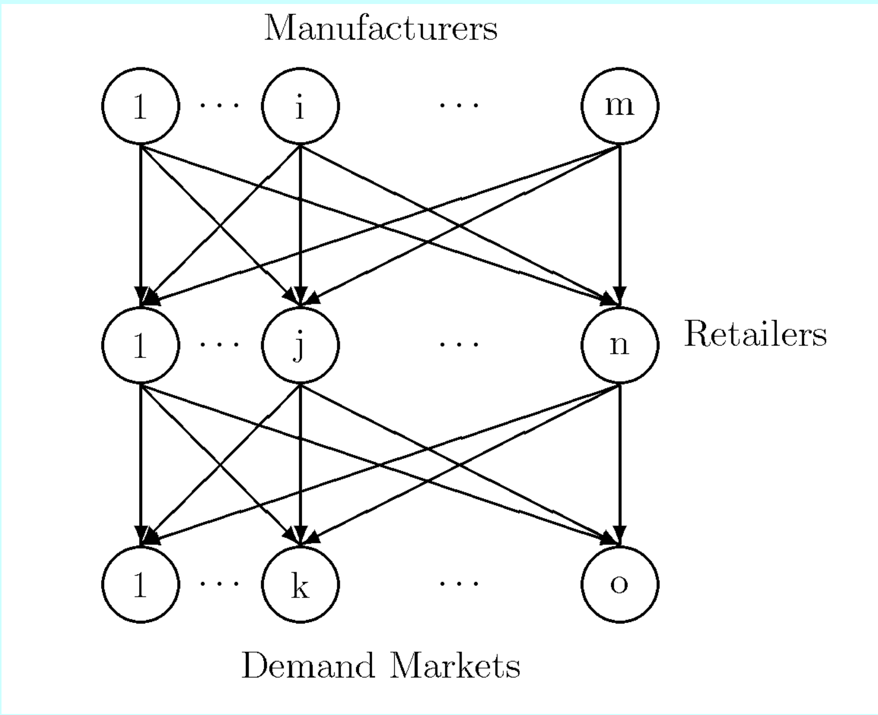
The decision-makers, now located at the nodes of the network, are faced with their individual objective functions, which can include profit-maximization, and one seeks to determine not only the optimal/equilibrium flows between tiers of nodes but also the prices of the product at the various tiers.

Nagurney (2005) has shown that supply chain network equilibria can be reformulated as transportation network equilibria.

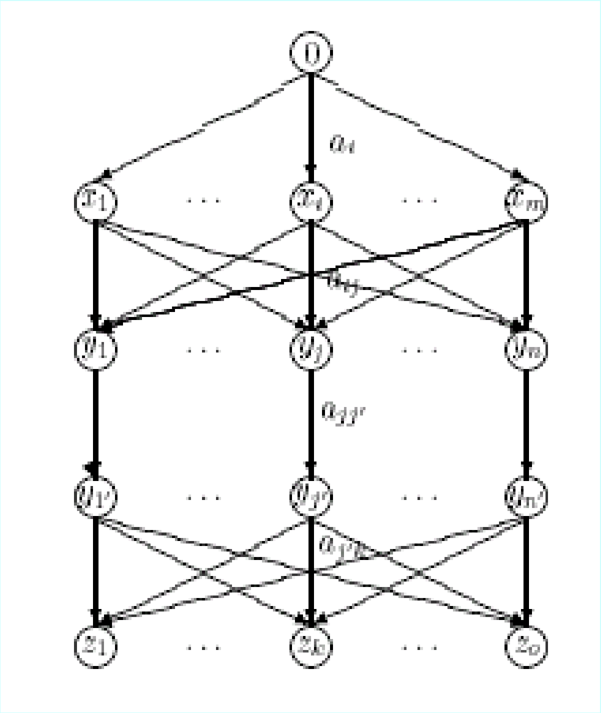
A Supply Chain Network



The Transportation Network Equilibrium Reformulation



Supply Chain Network



Transportation Network

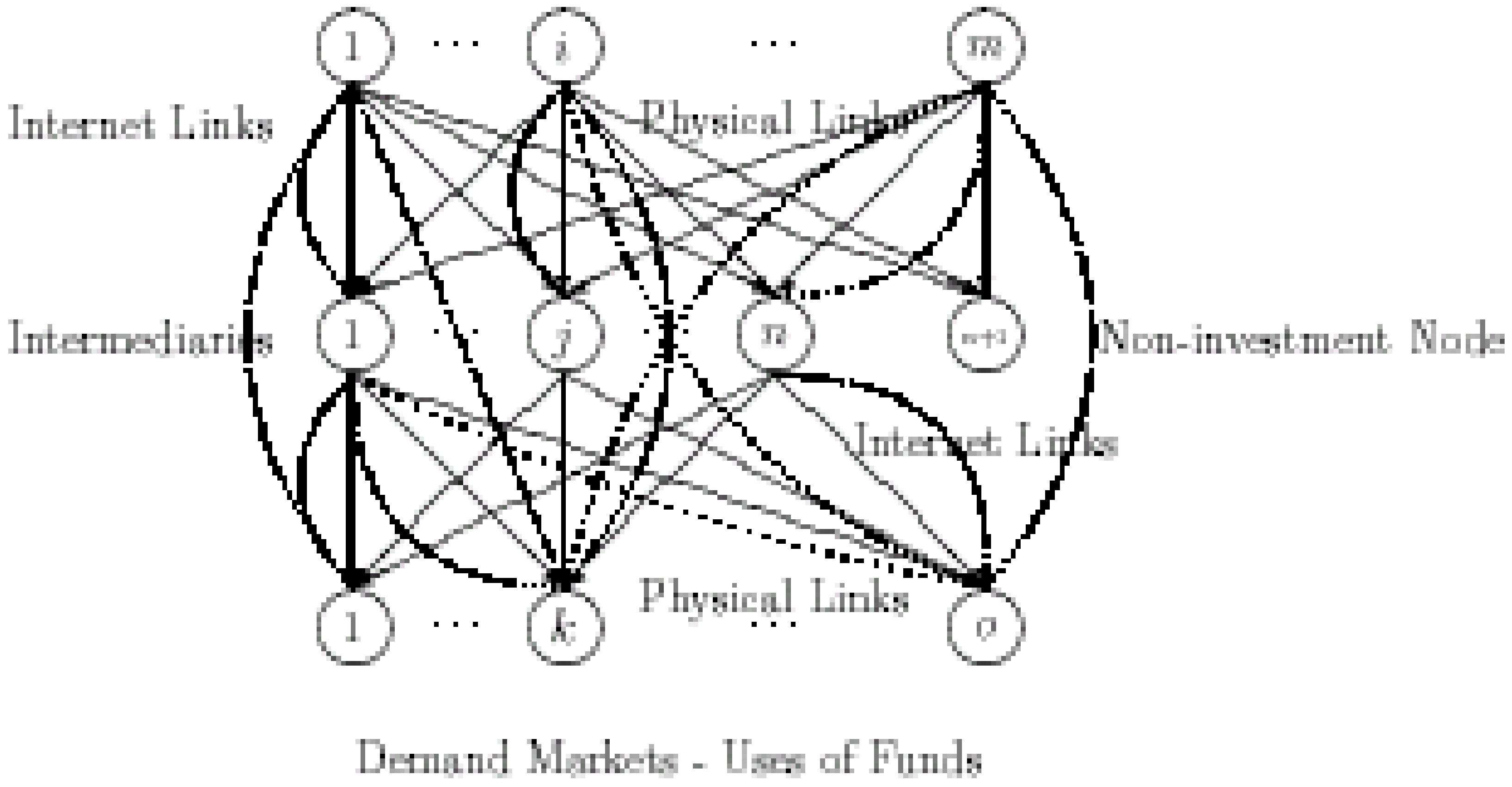
Financial Networks

Markowitz's classical portfolio optimization model (1952, 1959) can be transformed into a system-optimized transportation network problem of special structure and fixed demand.

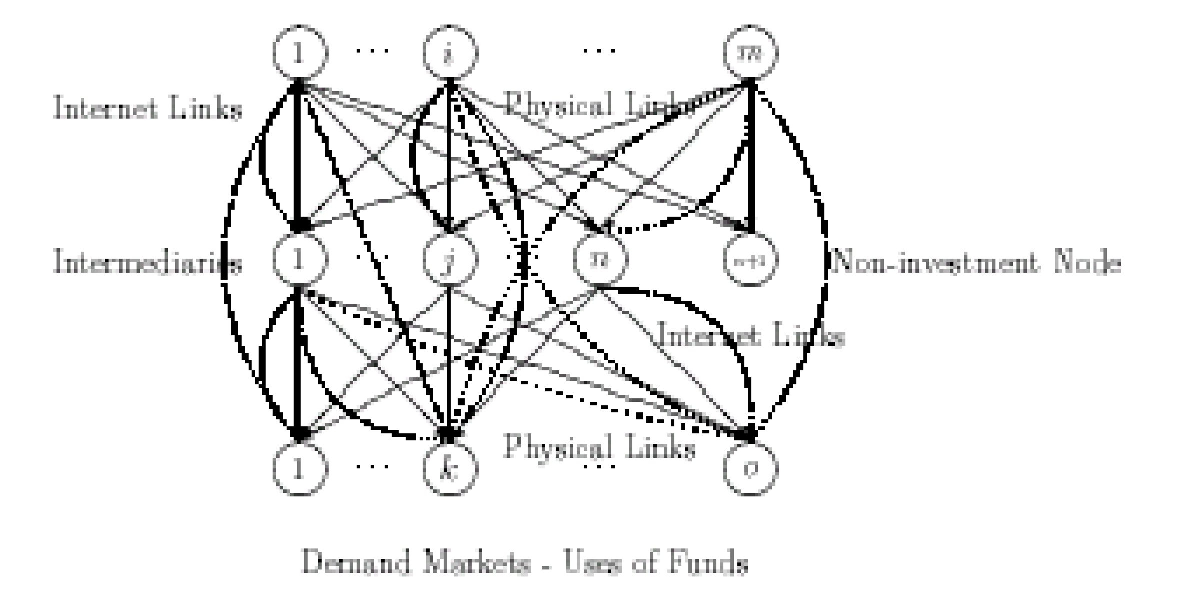
The book by Nagurney and Siokos (1997) surveys financial networks.

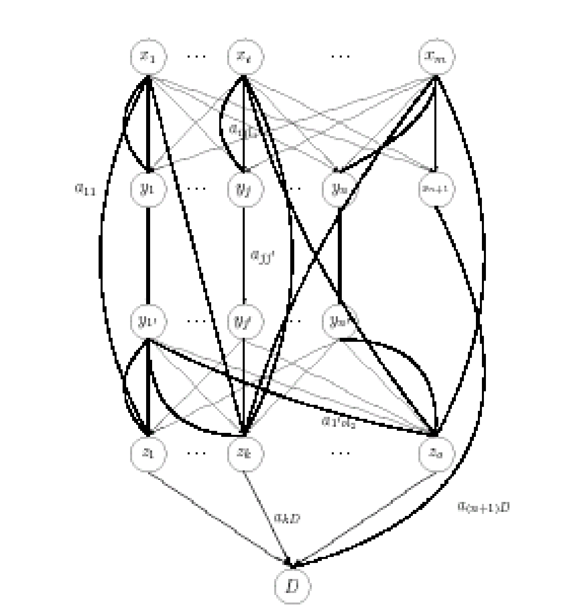
Liu and Nagurney (2005) have shown that financial networks with intermediation can be reformulated as transportation network equilibrium problems.

A Financial Network with Intermediation



The Transportation Network Equilibrium Reformulation





Financial Network with Intermediation

Transportation Network

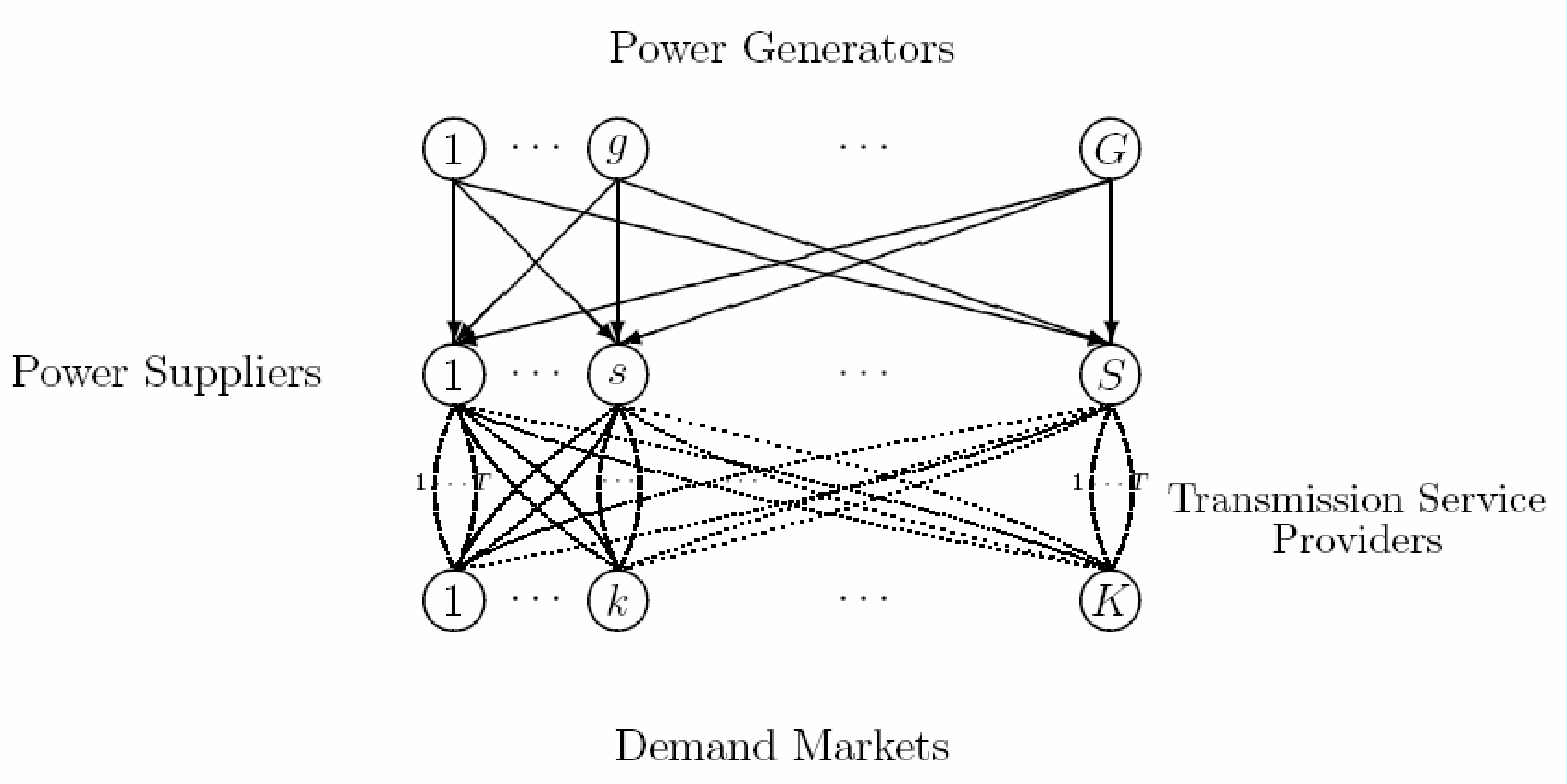
Electric Power Supply Chain Networks

The fifth chapter of BMW described some *unsolved problems* including a single commodity network equilibrium problems that the authors intuited could be generalized to capture electric power networks.

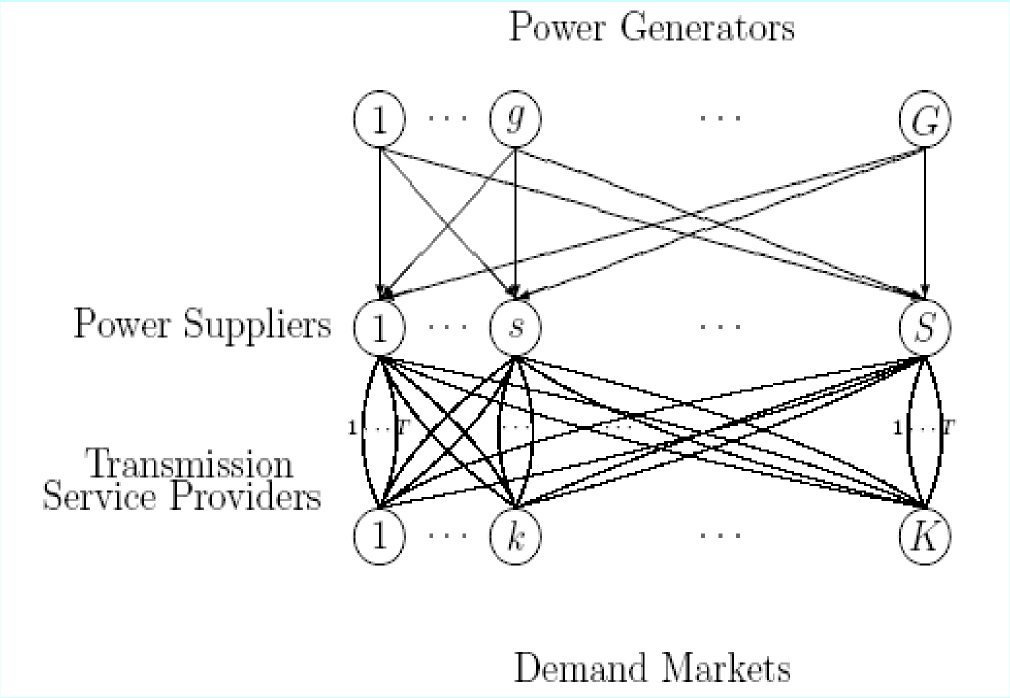
The unsolved problems concern the application of the model to particular cases. In particular, the problem of generation and distribution of electric energy in a network comes to mind.

Nagurney and Liu (2005) have shown that electric power supply chains can be reformulated as transportation network equilibrium problems.

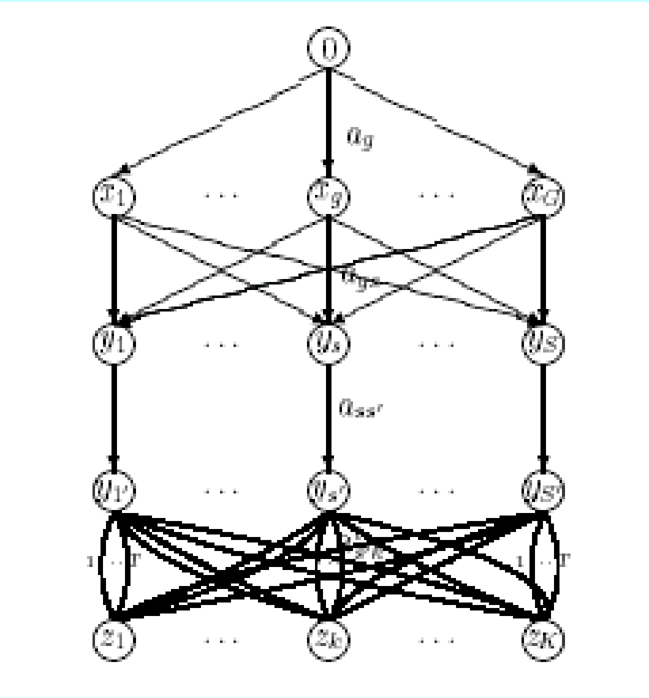
The Electric Power Supply Chain Network



The Transportation Network Equilibrium Reformulation



Electric Power Supply Chain Network



Transportation Network

Knowledge Networks

There has been much research conducted in the modeling of knowledge networks from an economic perspective and, notably, by researchers in transportation (Karlqvist and Lundqvist (1972), Batten, Kobayashi, and Andersson (1989), Kobayashi (1995), Nagurney (1999)) and even Beckmann (1993, 1994) and the volume edited by Beckmann et al. (1998).

Beckmann (1994) noted BMW but in the sense that the topic of transportation networks had been the study of operations researchers, applied mathematicians, and economic theorists while that of knowledge networks had not.

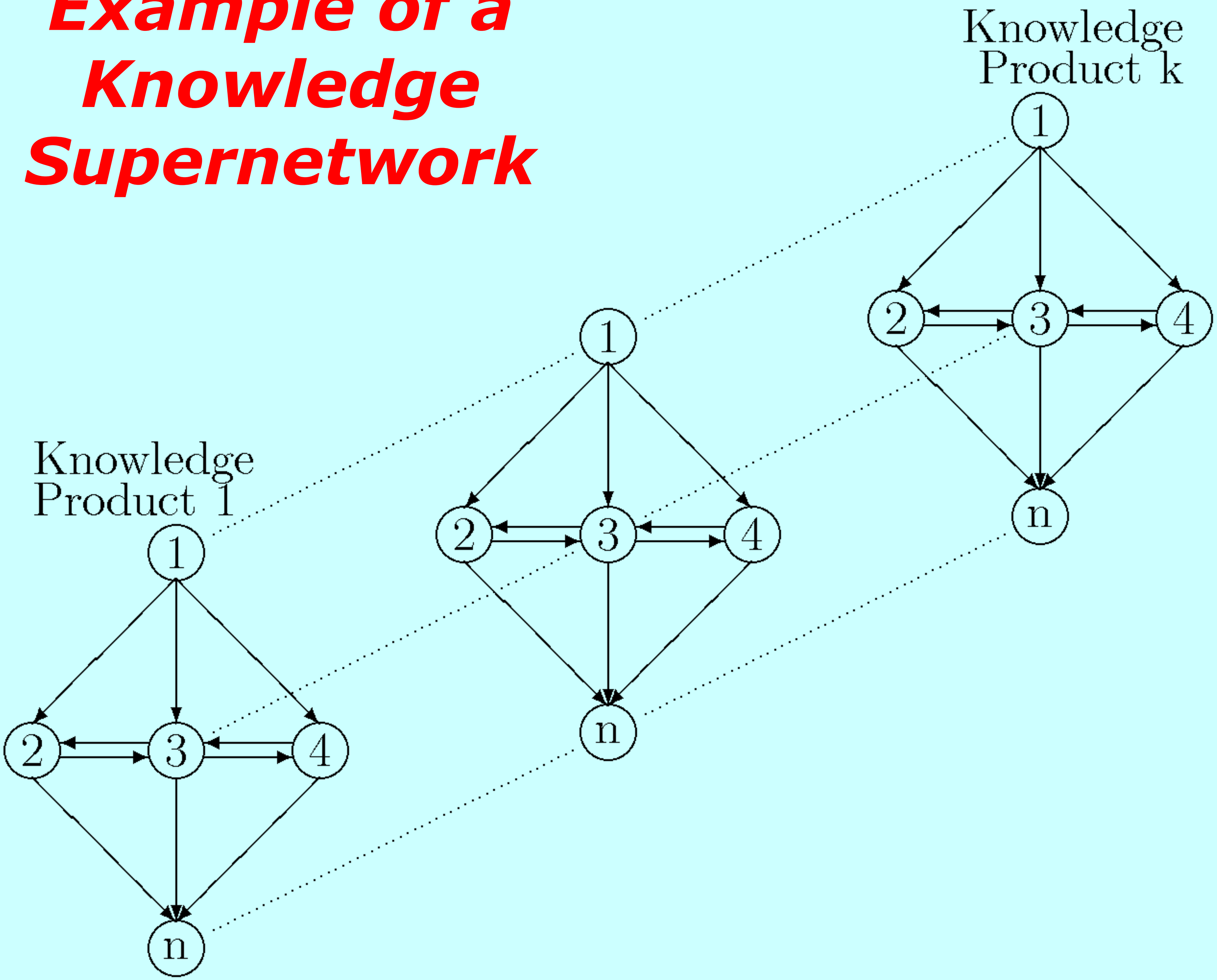
Nagurney and Dong (2005) proposed a supernetwork framework for the modeling and analysis of knowledge intensive organizations:

- **news organizations**
- **intelligence agencies**
- **global financial institutions.**

They identified the knowledge products, the O/D pairs, the paths and their meanings, along with the links and flows in a variety of knowledge organization contexts.

The need for research on such topics as postulated by Beckmann (1994) was now becoming a reality.

Example of a Knowledge Supernetwork



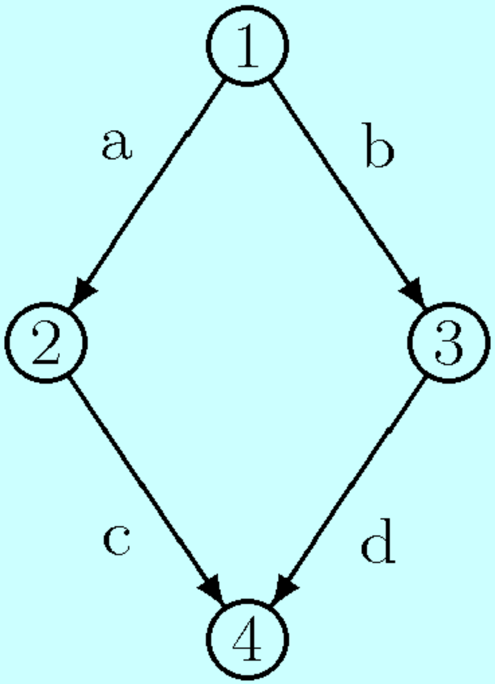
Sensitivity Analysis and Stability Analysis

The importance of stability analysis was recognized in Beckmann, McGuire, and Winsten (1956). Dafermos and Sparrow (1969), subsequently, obtained stability analysis results in the context of user-optimized models in the static setting.

Braess (1968), whose well-known paradox motivated much of the subsequent research in sensitivity analysis and networks, cited neither Wardrop (1952) nor BMW.

That paper was followed by the contributions of Murchland (1970), who elaborated upon the Braess paradox and reflected upon it in the context of BMW and Beckmann (1967).

Braess Paradox



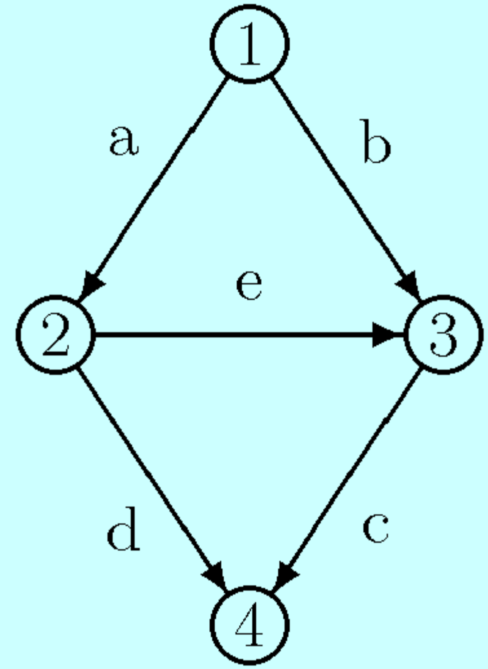
$c_a(f_a)=10f_a$

$c_b(f_b)=f_b+50$

$c_c(f_c)=f_c+50$

$c_d(f_d)=10f_d$

$c_c(f_e)=f_e+10$



O/D w=(1,4) travel demand is $d_w=6$.
Paths p1=(a,c) p2=(b,d).
The user-optimized solution in path flows: $x_{p1}^* = x_{p2}^* = 3$
The induced link flow pattern:
The link travel costs:
$c_a=30$ $c_b=53$ $c_c=53$ $c_d=30$,
and the user path travel costs:
$C_{p1}=c_a+c_c=83$ $C_{p2}=c_b+c_d=83$.

Consider the addition of a new road/link e to the network
A new path p3=(a,e,c) is available.
The new user-optimized solution in path flows: $x_{p1}^* = x_{p2}^* = x_{p3}^* = 2$
The induced the link flow pattern:
$f_a^*=4$ $f_b^*=2$ $f_c^*=2$ $f_d^*=4$ $f_e^*=2$
and associated user path travel costs:
$C_{p1} = C_{p2} = C_{p3} = 92$.

The 1968 Braess article has been translated from German to English by Braess, Nagurney, Wakolbinger and appears as

On a Paradox of Traffic Planning

in the November 2005 issue of

Transportation Science

with a preface by Boyce and Nagurney.

Computer Scientists Discover Beckmann, McGuire, and Winsten

Beckmann in 1967 noted the relevance of network equilibrium concepts to communication networks.

Bertsekas and Gallager (1987) realized the similarities between communication and transportation networks as well and were familiar with the algorithms of Dafermos and Sparrow (1969).

It was, however, the Braess paradox which, subsequently, provided one of the main linkages between transportation science and computer science.

Korilis, Lazar, and Orda (1999), in turn, developed methods to show how resources could be added efficiently to a noncooperative network, including the Internet, so that the Braess paradox would not occur and cited the work of Dafermos and Nagurney (1984).

Today, paradoxes on networks, due to alternative behaviors of decision-makers, are garnering increasing attention in other scientific communities, including that of computer science.

Roughgarden in his 2002 thesis, further elaborated upon the Braess paradox and focused on the quantification of the worst possible loss in network performance arising from noncooperative behavior.

He recognized the importance of the work of Koutsoupias and Papadimitrou (1999), who are computer scientists, and who proposed the idea of bounding of the inefficiency of Nash equilibria, and that of BMW and Dafermos and Sparrow (1969).

Almost 50 years after its publication, *Studies in the Economics of Transportation* is finding applications in disciplines that did not even exist when the book was published!

We expect that there will be continuing cross-fertilization between many fields in which networks play a prominent role, with BMW serving as one of the fundamental references.

Anna Nagurney's Personal Reflections and Comments

I was privileged to have had Martin Beckmann on my doctoral dissertation committee at Brown University with the chair of the committee being Stella Dafermos, who passed away in 1990.

Although I could not locate a copy of BMW for purchase, Stella had given me copies of parts of it for use in my research and it became a reference that has served me well and that I have carried with me on many travels and while living abroad and doing research.

Amazingly, Brown University had been home to such luminaries in transportation as:

- **William Prager, who in 1954 published a paper, which discussed the importance of extended type of traffic network models in which the cost on a link could depend not only on its own flow,**
- **Gordon Newell,**
- **Martin Beckmann,**
- **Stella Dafermos.**

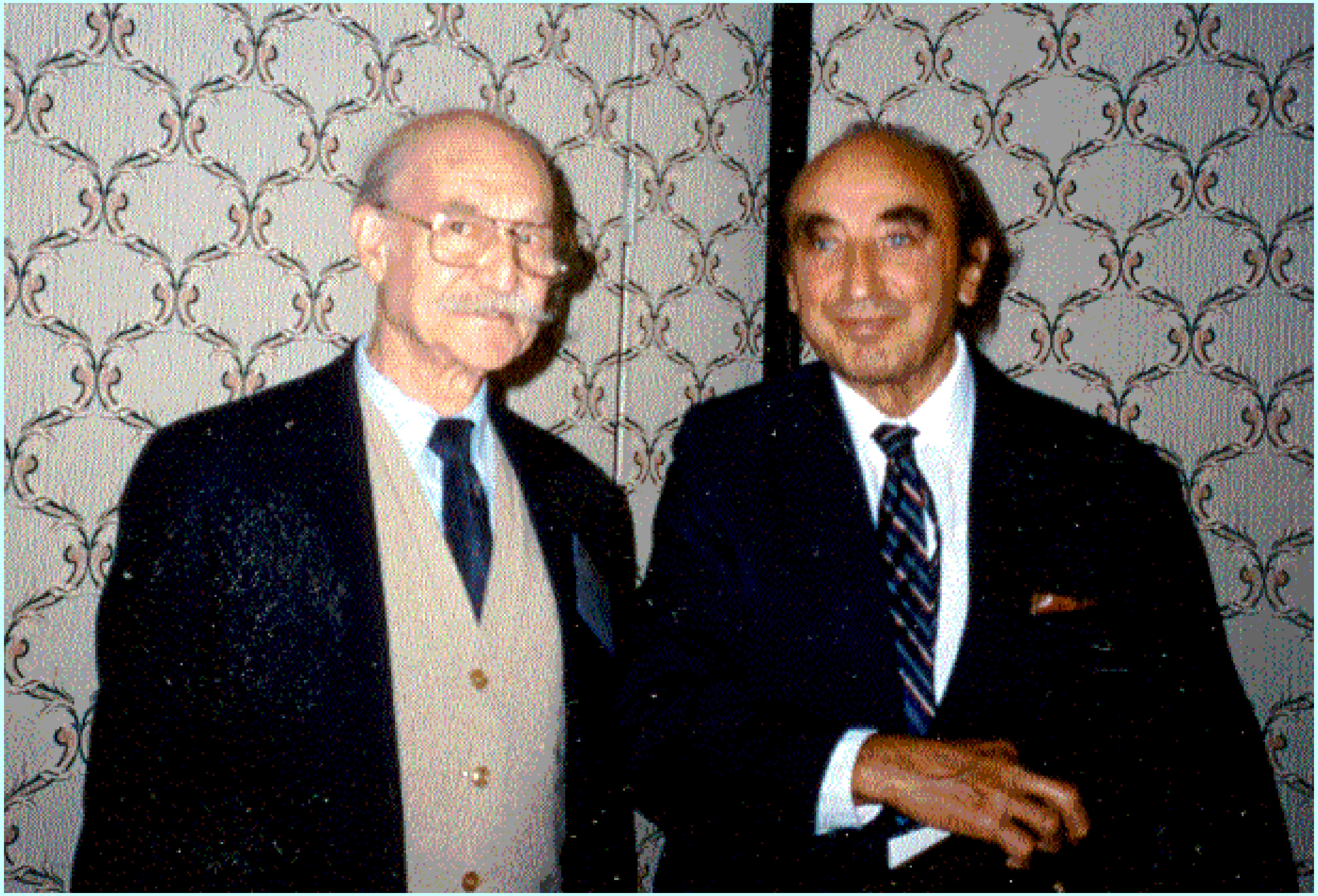
The intellectual journey that Beckmann and Dafermos started me on and influenced numerous others has been fascinating and never dull.

It has taken me to many countries, including Canada, Sweden, Russia, Japan, and Australia, and the intellectual inquiries and excitement continue to be fueled by interactions with students, collaborators, and many international colleagues.

Through the Robert Herman Lifetime Achievement Award sponsored by the Transportation and Logistics Section of INFORMS (and named after its first recipient, Robert Herman), the achievements and sustained contributions of innovators in transportation science have been recognized.

Past winners: Robert Herman, Martin Beckmann, Michael Florian, Denos Gazis, Amedeo Odoni, and, most recently, David E. Boyce.

Martin Beckmann with Robert Herman in 1994 upon his receipt of the Lifetime Achievement Award in Transportation

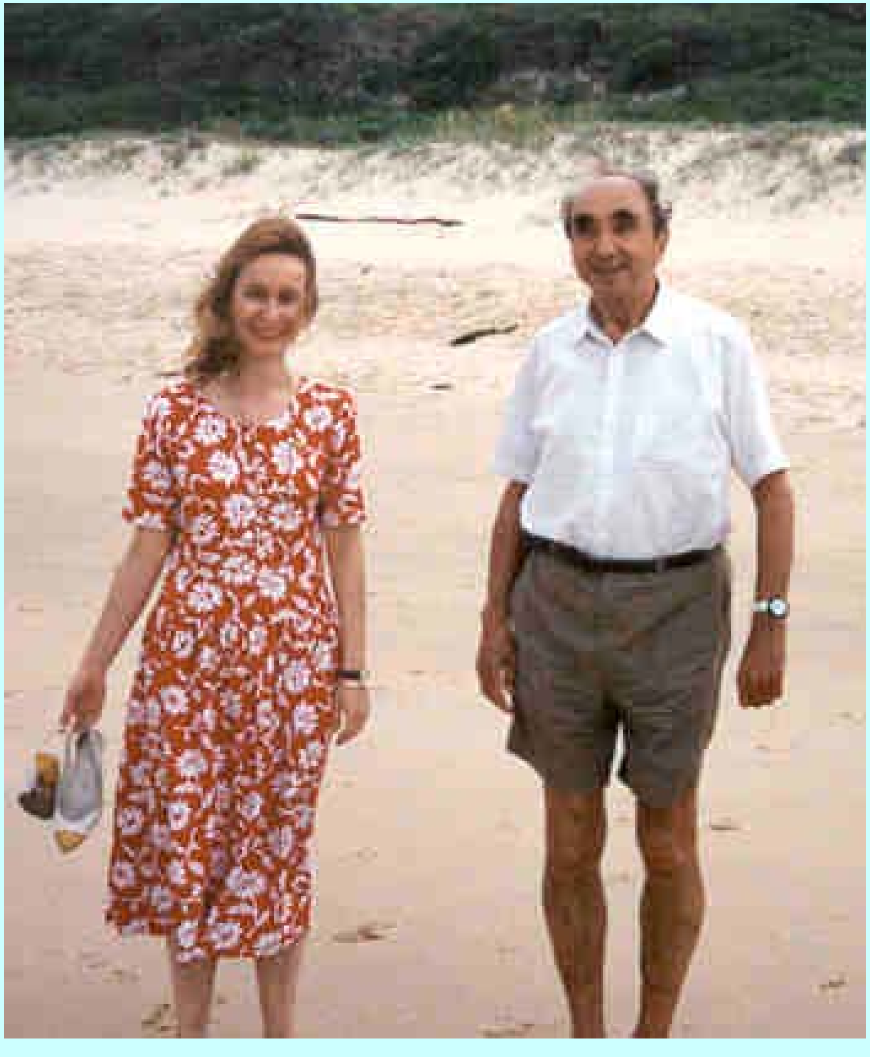


Beckmann being Congratulated by Michael Florian

Regional Science Conference Mallacoota Victoria Australia - 1992

Mallacoota, December 1992

On the Beach
Mallacoota, Australia - 1992



Anna Nagurney with her PhD Committee, Professors Majda, Beckmann, and Dafermos at the Post-Defense Party at Brown.

Thank you!

A copy of the paper accompanying this talk can be found at:

http://supernet.som.umass.edu