Recent Results on Congestion Toll Pricing of Traffic Networks

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Traffic Congestion

Electronic Toll Collection Facilities

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Introduction

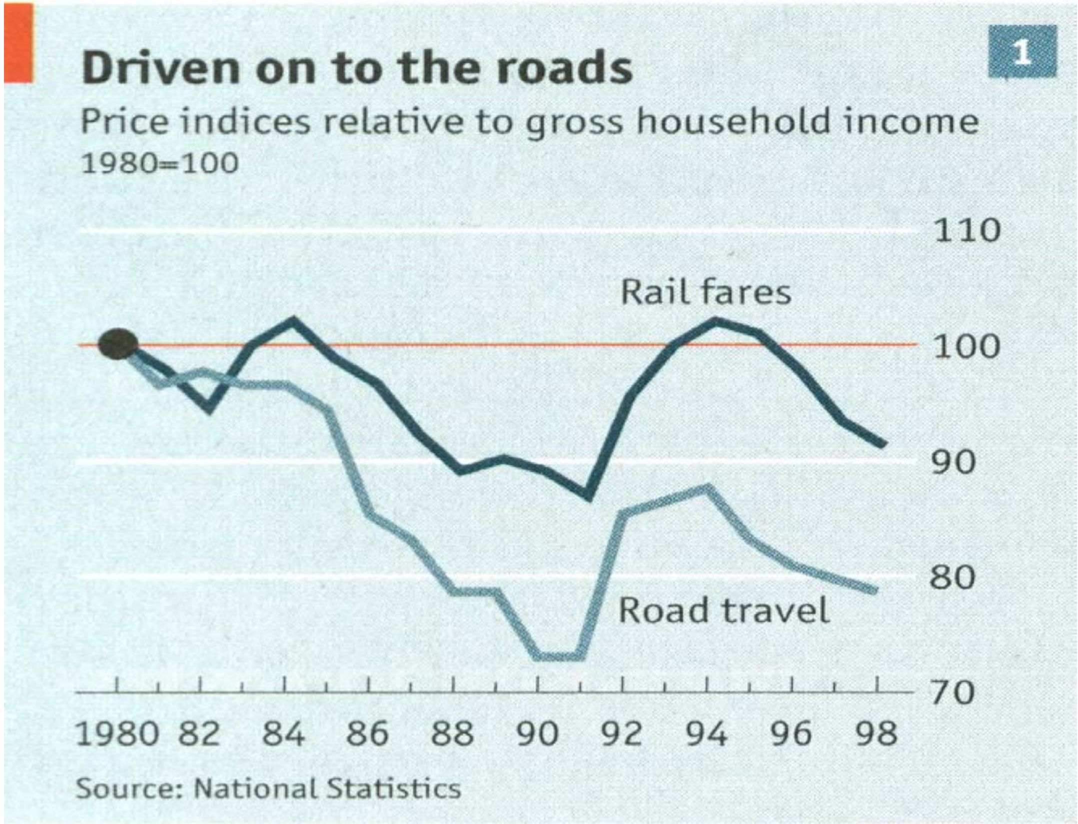
- Traffic congestion has become part of everyday life in major metropolitan areas.
  - An article in *The Economist*, April 27, 2002, discusses the congestion in Britain.

The Economist

Tolled you so

Introduction

- "The real costs of motoring (in Britain) have been falling for decades."



Introduction

- "Nearly all the recent road studies the government has commissioned have supported the use of road tolls."
- "A big road-building programme without pricing is as ludicrous as giving a heroine addict a last fix."
  [David Begg, Chairman, Commission for Integrated Transport.]
- "The capital's mayor, Ken Livingstone, is committed to introducing a £5 daily fee on cars entering the city centre from next January. London is the first big city in the world to try this, . . ."
- Toll is now £8

Congestion Charging in London – Feb. 17, 2003

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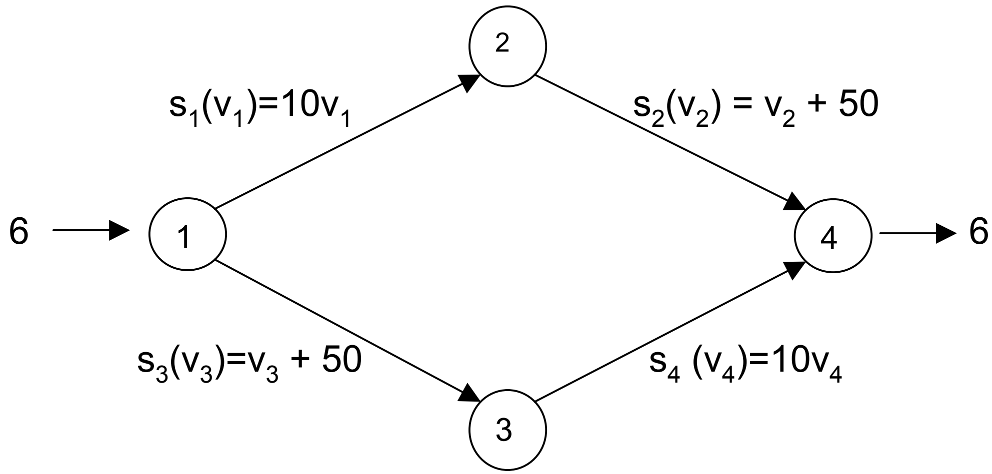
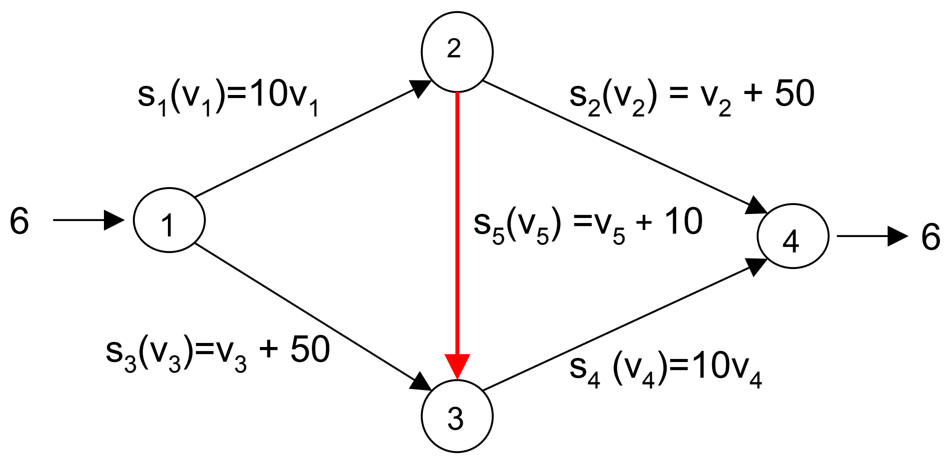
Introduction

- Two types of models in traffic assignment
  - User equilibrium (or optimum) models assume that at equilibrium, no traveler has any incentive to change his or her route.
    - An example of Nash equilibrium.
  - System optimum models choose routes that minimize total system cost when demand is fixed, or maximize Net User Benefit (*NUB*) when demand is elastic.
    - Implicitly assume that it is possible to control travelers' behavior.
- These models generally distribute travel demands to routes in the network differently.

Braess' Paradox



User Cost = 83



User Cost = 92

In either case, System Cost = 6 x 83

Traffic Assignment Models: Fixed Demand

- $V^{FD} = \{v\colon v = \sum_k x^k,\ Ax^k = b_k,\ x^k \geq 0,\ \forall\ k \in K\}$
- User Equilibrium: Find $v^U \in V^{FD}$ such that

$$s(v^U)^T(v - v^U) \geq 0, \ \ \forall\ v \in V^{FD}$$

- Tolled User Equilibrium: Find $v[\beta] \in V^{FD}$ such that

$$(s(v[\beta]) + \beta)^T(v - v[\beta]) \geq 0, \ \ \forall\ v \in V^{FD}$$

where β is a given toll vector.

- System Optimum:

$$v^S = \operatorname{argmin}\{s(v)^T v\colon v \in V^{FD}\}$$

Marginal Social Cost Pricing

- Toll Pricing Problem: Find β so that $v[\beta] = v^S$.

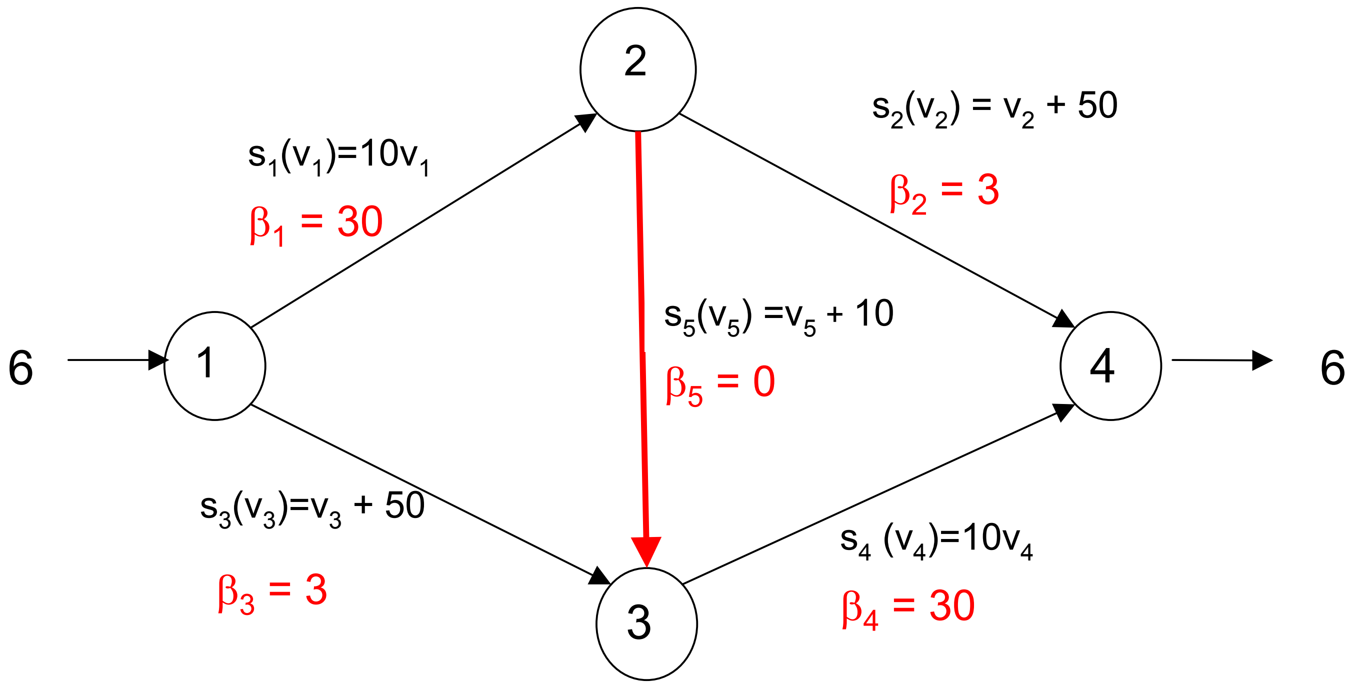
- An optimality condition for the system problem:

$$(s(v^S) + \nabla s(v^S)^T v^S)^T (u - v^S) \geq 0, \forall u \in V^{FD}$$

- Marginal Social Cost Pricing tolls

$$\beta = \nabla s(v^S)^T v^S$$

Braess' Paradox with MSCP Tolls



Note:

- User Cost = 83 + 33 = 116
- Four out of five arcs are tolled.

First-Best Toll Pricing

- Stockholm Network – Fixed Demand
  - Morning Rush 278,873 trips
  - Nodes/Links/Centroids = 417/963/46
- Results per Vehicle
  - Travel Time 42.96 minutes
  - MSCP tolls 128.53 minutes (88.86 Kr), 914 toll booths
  - MINREV tolls 9.4 minutes (8.125 Kr), 192 toll booths

First-Best Toll Set

- The system flow, v^S, is in a tolled user equilibrium with β being the toll vector if and only if there is ρ such that (β,ρ) satisfies the following:

$$s(v^S) + \beta \geq A^T \rho^k \qquad \forall k$$

$$(s(v^S) + \beta)^T v^S = \sum_{k \in K} b_k^T \rho^k$$

Bergendorff, P., D. W. Hearn, and M. V. Ramana, "Congestion Toll Pricing of Traffic Networks," *Network Optimization*, P. M. Pardalos, D. W. Hearn and W. W. Hager (Eds.), Springer-Verlag Series, Lecture Notes in Economics and Mathematical Systems, 1997, pp. 51-71.

First-Best Toll Pricing Framework

- Solve $v^S = \text{argmin}\{s(v)^T v: v \in V^{FD}\}$
- Solve a toll selection problem :

$$
\begin{aligned}
\min \quad & f(\beta) \\
s.t. \quad & s(v^S) + \beta \geq A^T \rho^k \qquad \forall k \\
& (s(v^S) + \beta)^T v^S = \sum_{k \in K} b_k^T \rho^k \\
& \beta \geq 0
\end{aligned}
$$

where, e.g.,

$$f(\beta) = s(v^S)^T \beta, \qquad \text{(MINREV)}$$

$$f(\beta) = \sum_{a:\beta_a > 0} 1, \text{ or} \qquad \text{(MINTB)}$$

$$f(\beta) = \max\{\beta_a : a \notin Y\} \qquad \text{(MINMAX)}$$

Hearn, D. W. and Ramana, M. V., "Solving Congestion Toll Pricing Models," in *Equilibrium and Advanced Transportation Modeling,* P. Marcotte and S. Nguyen (Eds.), Kluwer Academic Publishers, 1998, pp. 109-124.

Numerical Results for MINREV

- Customized DWD versus CPLEX

				Dantzig-Wolfe			CPLEX 7.0		
Network	**Nodes**	**Arcs**	**OD**	**Iter**	**Sec.**	**Toll Rev.**	**Iter**	**Sec.**	**Toll Rev.**
Sioux Falls	24	76	528	7	0.49	20.67	77	0.12	20.67
Hull	501	798	142	15	5.98	3462.82	2298	0.97	3464.67
Stockholm	416	962	1623	40	116.02	1.851K	8090	25.72	1.860K
Winnipeg	1052	2836	4345	68	9491.41	85186.7	Out of memory		

Bai, L., Hearn, D.W., and Lawphongpanich, S., "Decomposition Techniques for the Minimum Toll Revenue Problem," *Networks,* Vol. 44, No. 2, 142 - 150, 2004.

Numerical Results for MINTB

- Dynamic Slope Scaling Procedure (Modified)

Test Set	Nodes (Ave.)	Arcs (Ave.)	OD Pairs (Ave.)	Original DSSP			Modified DSSP		
				Iter	Sec.	# Booths	Iter	Sec.	# Booths
1	100	366	25	7	2.65	29	5	2.03	29
2	200	827	30	7	10.70	42	5	7.77	42
3	300	2155	35	8	27.85	45	3	12.71	45
4	400	3067	45	10	81.83	58	7	56.97	58
5	500	4860	50	10	117.33	69	6	71.53	69
Ave. Improvement							29%	30%	0%

Hearn, D. W., Yildirim, M. B., Ramana, M. V. and Bai, L. H., "Computational Methods for Congestion Toll Pricing Models," *Proceedings of The 4th International IEEE Conference on Intelligent Transportation Systems*, 2001.

Numerical Results for MINTB (cont.)

- Real Networks

Network	Nodes	Arcs	OD Pairs	Original DSSP Iter	Original DSSP Sec.	Original DSSP # Booths	Modified DSSP Iter	Modified DSSP Sec.	Modified DSSP # Booths
Sioux Falls	24	76	528	8	1.68	38	8	1.57	39
					Improvement		0%	7%	-3%
Hull	501	798	142	12	26.87	48	5	11.82	49
					Improvement		58%	56%	-2%
Stockholm	416	962	1623	26	660.01	127	11	301.32	127
					Improvement		58%	54%	0%

Bai, L., Hearn, D.W., and Lawphongpanich, S., “A Heuristic Method for the Minimum Toll Booth Problem,” submitted to TRB, January, 2006.

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Traffic Assignment Models: Elastic Demand

- $V^{ED} = \{v: v = \sum_k x^k, Ax^k = t_k E_k, x^k \geq 0, t_k \geq 0, \forall\ k \in K\}$

- User Equil.: Find $(v^U, t^U) \in V^{ED}$ such that

$$s(v^U)^T(v - v^U) - w(t^U)^T(d - t^U) \geq 0, \quad \forall\ (u,d) \in V^{ED}$$

  – where $w(t)$ = inverse demand function.

- Tolled User Equil.: Find $(v[\beta], t[\beta]) \in V^{ED}$ such that

$$(s(v[\beta]) + \beta)^T(u - v[\beta]) - w(t[\beta])^T(d - t[\beta]) \geq 0, \forall (u, d) \in V^{ED}$$

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Traffic Assignment Models: Elastic Demand (cont)

- System Optimum (Maximize NUB):

$$(v^S, t^S) = \arg\max \left\{ \sum_k \int_0^{t_k} w_k(z)dz - s(v)^t v : (v,t) \in V^{ED} \right\}$$

- Toll Set:

$$s(v^S) + \beta \geq A^T \rho^k \qquad \forall k$$

$$w(t^S)^T t^S \leq E_k^T \rho^k \qquad \forall k$$

$$(s(v^S) + \beta)^T v^S - w(t^S)^T t^S = 0$$

Hearn, D. W. and M. B. Yildirim, "A Toll Pricing Framework for Traffic Assignment Problems with Elastic Demand," *Current Trends in Transportation and Network Analysis: Papers in honor of Michael Florian*, M. Gendreau and P. Marcotte (Eds.), Kluwer Academic Publishers, 135-145, 2002.

Yildirim, M. B. and Hearn, D. W., “A First Best Toll Pricing Framework for Variable Demand Traffic Assignment Problems,”*Transportation Research,* 2004.

Numerical Issue: Approximate System Solution

- Existing algorithms for TA (e.g., Frank-Wolfe, PARTAN, RSD) only provide an approximate SOPT solution. This resulted in empty (nonegative) toll sets for Hull, Winnipeg and Stockholm even with 10^{-6} optimality gap.

- In general, feasible flows (even near optimal) may not have nonegative toll sets. See Bai et al., "Relaxed Toll Sets for Congestion Pricing Problems," in ***Mathematical and Computational Models for Congestion Pricing***, *S.* Lawphongpanich, D. W. Hearn and M. J. Smith (eds.), forthcoming, Springer-Verlag, 2005/06.

Relaxed Toll Sets

- For a given feasible flow vector $\hat{u} \in V^{FD}$ and $\varepsilon > 0$, the relaxed toll set at $\hat{u}$, $T^{+}(\hat{u}, \varepsilon)$, is the set of all β for which these exists a corresponding ρ satisfying the following conditions:

$$(s(\hat{u}) + \beta) \geq A^{T} \rho^{k} \qquad \forall k \in K$$

$$(s(\hat{u}) + \beta)^{T} \hat{u} \leq \sum_{k \in K} b_{k}^{T} \rho^{k} + \varepsilon$$

$$\beta \geq 0$$

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Relaxed Toll Sets – Primary result

- Let
  $$\varepsilon_{mscp} = -\min\{(s(\hat{u}) + (\nabla s(\hat{u})\hat{u})^T (v - \hat{u}) : v \in V^{FD}\}$$
- Then,
  - $\varepsilon_{mscp} > 0$
  - If $\nabla s(\hat{u}) > 0$, then $T^{+}(\hat{u}, \varepsilon_{mscp})$ is nonempty.
- Theorem: Let $s(.)$ be strongly monotone with modulus $\alpha > 0$. For any $\eta > 0$, there exists a $\delta > 0$ such that $\|v^{\beta} - v^{s}\| \leq \eta$ whenever $\beta \in T^{+}(\hat{u}, \varepsilon_{mscp})$ and $\|\hat{u} - v^{s}\| \leq \delta$.
  - A toll vector from a "good" relaxed toll set induces a user equilibrium that is approximately system optimal.

First-Best Toll Pricing - Results

- Characterized toll sets as polyhedra
- Toll Pricing Framework allows secondary objectives:
  - MINREV, MINTB, MINMAX, and ROBINHOOD
- Decomposition techniques for MINREV
  - Cutting Plane Algorithm
  - Dantzig-Wolfe Decomposition
- Modified DSSP algorithm for MINTB
- Extended results to all variable demand models
  - Elastic Demand
  - Combined Distribution-Assignment
- Relaxed toll sets

Second-Best Toll Pricing

- For political reasons or otherwise, there are some roads that are not tollable.
  - The second-best problem belongs to a harder class of problems − Mathematical Programs with Equilibrium Constraints (MPECs).
  - Problems of current interest such as pricing of cordon, HOT (High Occupancy Toll) and FAIR (Fast and Intertwined Regular) Lanes.

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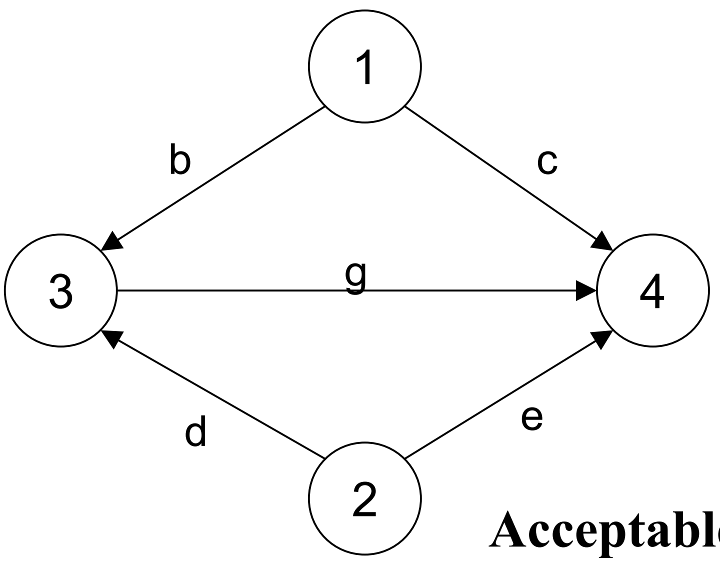
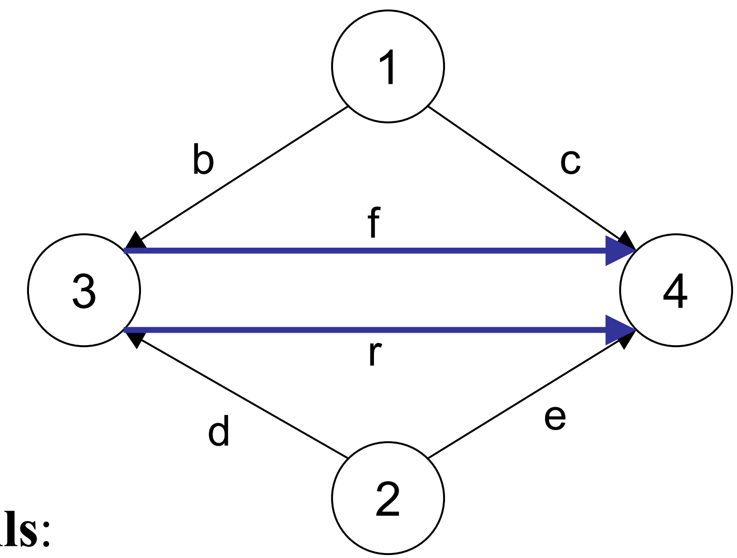
FAIR Lanes

- In the FAIR LANE concept, lanes in a designated highway are separated into two sections, fast and regular lanes.
  - Fast lanes would be electronically tolled and users of the regular lanes would receive credits that can be used as toll payments on days when they choose to use the fast lanes.





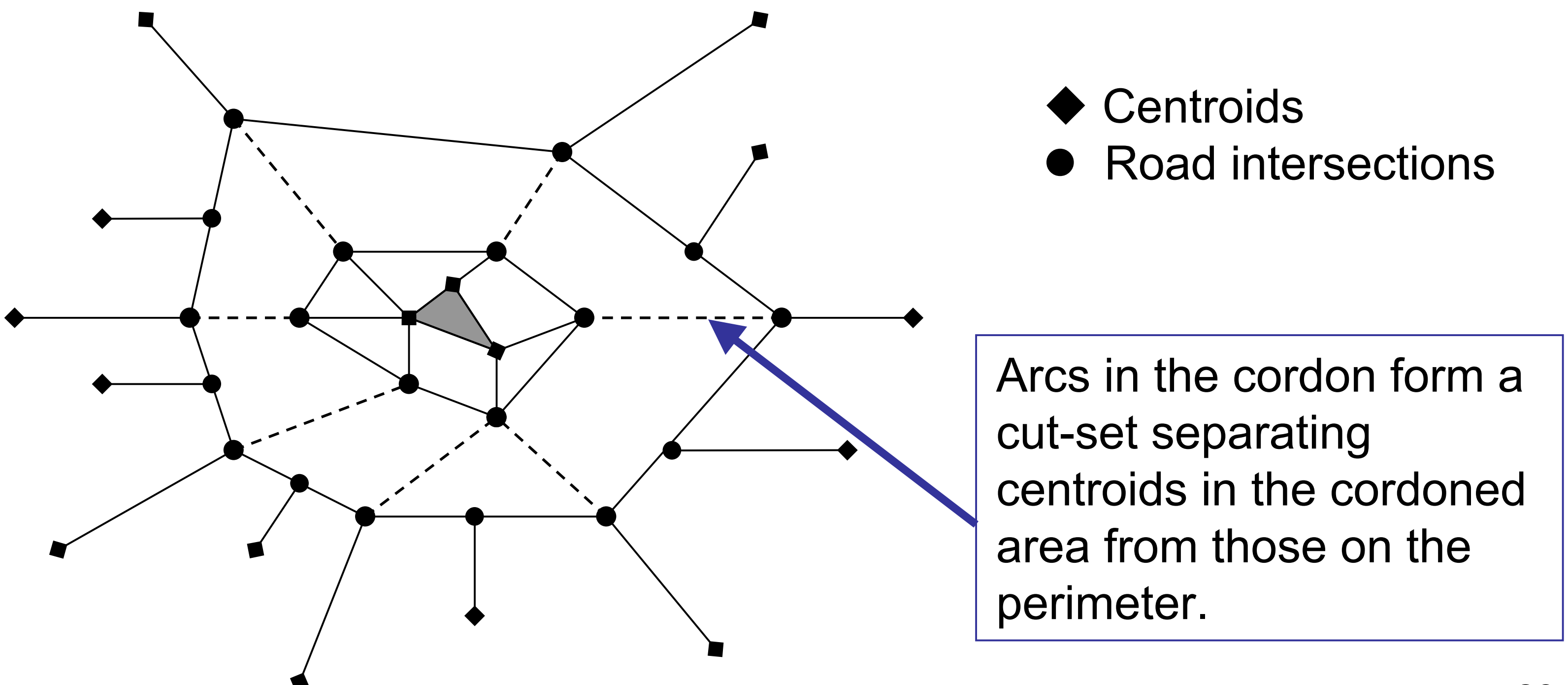
Acceptable tolls:

$\beta_b, \beta_c, \beta_d, \beta_e = 0, \beta_f \geq 0, \beta_r \leq 0$

$v_f\beta_f + v_r\beta_r \geq \mathbf{0}$

Cordon Pricing

- Cordon Pricing is a system that collects tolls from vehicles that passes through certain roads or points in a traffic network. Typically, these points form a loop around a defined area, e.g., a city center or a historical area, where traffic needs to be restricted.



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2nd Best Toll Pricing Problem – ED-VI

$$\max \quad \sum_k \int_0^{t_k} w_k(z)\,dz - s(v)^T v$$

$$\text{s.t.} \quad (v,t) \in V^{ED}$$

$$(s(v)+\beta)^T (u-v) - w(t)^T (d-t) \geq 0 \qquad \forall (u,d) \in V^{ED}$$

$$\beta \geq 0 \qquad \forall a \notin Y$$

$$\beta_a = 0 \qquad \forall a \in Y$$

Equivalent Formulation – ED-KKT

- The sequentially bounded constraint qualification (SBCQ) holds for ED-VI.

$$
\begin{aligned}
\max \quad & \sum_k \int_0^{t_k} w_k(z)dz - s(v)^T v \\
\text{s.t.} \quad & (v,t) \in V^{ED} \\
& s(v) + \beta \geq A^T \rho^k & \forall k \\
& w(t)^T t \leq E_k^T \rho^k & \forall k \\
& (s(v)+\beta)^T v = w(t)^T t \\
& \beta_a \geq 0 & \forall a \notin Y \\
& \beta_a = 0 & \forall a \in Y
\end{aligned}
$$

Properties of 2nd Best Tolls – NUB bounds

- **Theorem**: Assume that the user and system problems have solutions (v^U, t^U) and (v^S, t^S). Further, assume that $s(v)$ and $-w(t)$ are monotonic and continuous and $(\nabla s(v), \nabla w(t))$ exists and is continuous. Then, ED-VI has a global optimal solution with objective value in the interval $[NUB(v^U, t^U), NUB(v^S, t^S)]$.

Properties of 2nd Best Tolls – Constant Toll Revenue*

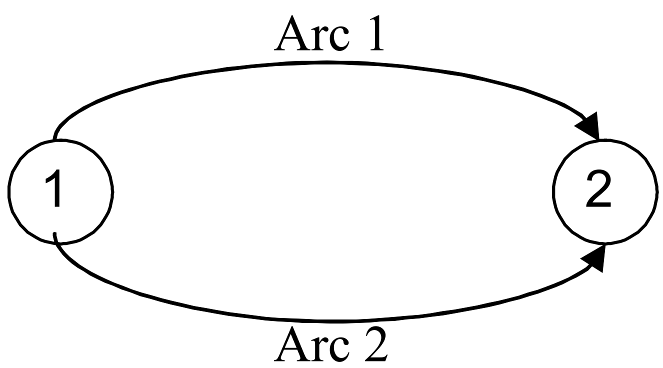
- Let $(\bar{v}, \bar{t}, \bar{\beta})$ be an optimal solution to ED - VI.

- Any β such that (β, ρ) satisfies the following system of equations is a valid toll.

$$
\begin{aligned}
s(\bar{v}) + \beta &\geq A^T \rho^k, \quad \forall k \\
w_k(\bar{t}_k) &\leq E_k^T \rho^k, \quad \forall k \\
(s(\bar{v}) + \beta)^T \bar{v} &= w(\bar{t})^T \bar{t} \qquad (*) \\
\beta_a &\geq 0 \quad \forall a \notin Y \\
\beta_a &= 0 \quad \forall a \in Y
\end{aligned}
$$

Properties of 2nd Best Tolls – Formulas?

- To motivate another property, consider the following two-arc problem where Arc 1 is tollable and Arc 2 is not.



where $s_1(v_1) = v_1$, $s_2(v_2) = v_2 + 2$, and $w(t) = 9 - t/2$

Properties of 2nd Best Tolls – Formulas?

- In the literature (see, e.g., McDonald, 1995, and Verhoef, 2000), the optimal toll for Arc 1 is

$$\bar{\beta}_1 = s_1'(\bar{v}_1)\bar{v}_1 + \frac{w'(\bar{t})}{s_2'(\bar{v}_2) - w'(\bar{t})} s_2'(\bar{v}_2)\bar{v}_2$$

$$= 3.3636 - \frac{0.5}{(1+0.5)} 3.5455 = 2.1818$$

- In this expression, the optimal toll includes a portion of MSCP from the non-tollable arc.
  - Are there similar formulas for general networks?

Properties of 2nd Best Tolls – Formulas?

- Results related to the previous question:
  - When the KKT multipliers exist, the second-best tolls can always be written as an expression involving marginal social cost pricing (MSCP) terms.
  - The KKT conditions associated with ED-KKT yields the following expression of an optimal toll vector.

$$\bar{\beta} = \frac{1}{\theta}\left[\delta^k - (1+\theta)[s(\bar{v}) + \nabla s(\bar{v})^T \bar{v}] - A^T \lambda^k + \nabla s(\bar{v})^T \sum_{k \in K} \psi^k \right]$$

  - An interpretation:
    - An optimal 2nd best toll on a link involves its own MSCP as well as those from non-tollable arcs via the KKT multipliers.

Properties of 2nd Best Tolls – Formulas?

- ED-KKT for the two arc example

			Multiplier
max	$v_1^2 + v_2^2 + 2v_2 - 9t + (t^2/4)$		
$s.t.$	$v_1 + v_2 - t$	$= 0$	λ
	$\rho - v_1 - \beta_1$	≤ 0	ψ_1
	$\rho - v_2 - 2$	≤ 0	ψ_2
	$-\rho + (9 - \frac{t}{2})$	≤ 0	ξ
	$(v_1 + \beta_1)v_1 + (v_2 + 2)v_2 - (9 - \frac{t}{2})t$	$= 0$	θ
	v_1, v_2, t, β_1	≥ 0	

Properties of 2nd Best Tolls – Formulas?

- An optimal solution

The net user benefit $=19.2727$

$(\bar{v}_1, \bar{v}_2, \bar{t}) = (3.3636, 3.5455, 6.9091), (\bar{\beta}_1, \bar{\rho}) = (2.1818, 5.5455)$

KKT Mult.

$(\lambda, \psi_1, \psi_2, \xi, \theta) = (-12.2727, 3.3636, 5.9091, 9.2727, 1.0)$

- Using the expression,

$$\begin{bmatrix} \bar{\beta}_1 \\ \bar{\beta}_2 \end{bmatrix} = \frac{1}{\theta}\left\{ \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} - (1+\theta)\begin{bmatrix} s_1(\bar{v}_1) + s_1'(\bar{v}_1)^T \bar{v}_1 \\ s_2(\bar{v}_2) + s_2'(\bar{v}_2)^T \bar{v}_2 \end{bmatrix} - \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} s_1'(\bar{v}_1)\psi_1 \\ s_2'(\bar{v}_2)\psi_2 \end{bmatrix} \right\}$$

$$= \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix} - 2\begin{bmatrix} 6.7272 \\ 9.0909 \end{bmatrix} + 12.2727\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3.3636 \\ 5.9091 \end{bmatrix} \right] = \begin{bmatrix} 2.1819 \\ 0 \end{bmatrix}.$$

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Properties of 2^{nd} Best Tolls – Formulas?

- The above result assumes that the multipliers exist.
  - Scheel and Scholtes [2000] show that MFCQ is violated at every feasible solution of ED-KKT.
  - However, the multipliers exist when the strong stationarity conditions hold at an optimal solution to ED-KKT
  - A similar expression for the tolls can be obtained using the ‘tightened’ NLP associated with ED-KKT.
    - The multipliers for this problem exist, e.g., when $s(v)$ and $w(t)$ are linear.

Equivalent Formulation: ED-EX

- The set V^{ED} can be expressed as a convex combination of its extreme points, (u^i, d^i), $i = 1,\ldots, n$.

$$
\begin{array}{lll}
\max\limits_{(v,t,\beta)} & \sum\limits_{k\in K} \int_0^{t_k} w_k(z)dz - s(v)^T v & \\
\text{s.t} & (v,t) \in V^{ED} & \\
& \beta_a \geq 0 & \forall a \notin Y \\
& \beta_a = 0 & \forall a \in Y \\
& (s(v)+\beta)^T (u^i - v) - w(t)^T (d^i - t) \geq 0 & \forall i = 1,\cdots,n
\end{array}
$$

Cutting Constraint Algorithm for ED-EX

- Let (u^1, d^1) be a system optimal solution. Set $r = 1$.
- Solve the following master problem:

$$
\begin{aligned}
(v^r, t^r, \beta^r) = \quad & \arg\max_{(v,t,\beta)} && \sum_{k \in K} \int_0^{t_k} w_k(z)dz - s(v)^T v \\
& \text{s.t} && (v,t) \in V^{ED}; \\
& && (s(v) + \beta)^T (u^i - v) - w(t)^T (d^i - t) \geq 0 \quad i = 1, \cdots, r \\
& && \beta_a \geq 0, \forall a \notin Y; \beta_a = 0, \forall a \in Y
\end{aligned}
$$

- Solve the subproblem:

$$
(u^{r+1}, d^{r+1}) = \arg\min \left\{ (s(v^r + \beta^r)^T u - w(t^r)^T d : (u,d) \in V^{ED} \right\}.
$$

If $(s(v^r + \beta^r)^T (u^{r+1} - u^r) - w(t^r)^T (d^{r+1} - d^r) \geq 0$, stop.

Otherwise, set $r = r + 1$ and go to 1.

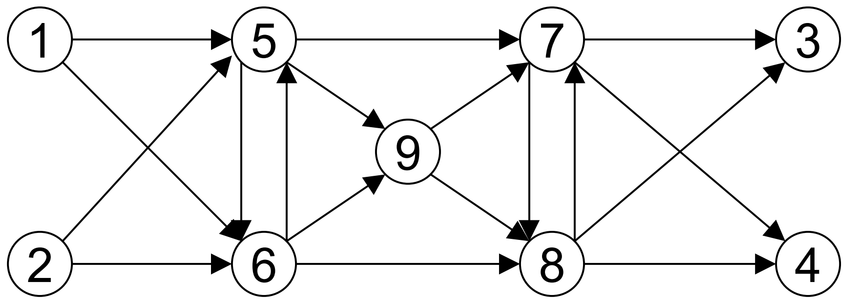
Cutting constraint algorithm for ED-EX

- The solutions to the problem in Step 3 are distinct.
  - Because the number of extreme points of V^{ED} is finite, the algorithm must stop after a finite number of iterations.
- In Step 2, the master problem is generally nonconvex and may not satisfy MFCQ.
  - Unless we obtain global solutions, the sequence of objective values for the master problem may not decrease monotonically.
  - In our implementation, MINOS is able to solve the master problem when the cutting constraints are relaxed, i.e.,

$$(s(v)+\beta)^T(u^i - v) - w(t)^T(d^i - t) \geq -\varepsilon, \quad i = 1,\cdots,r$$

Example: Elastic Demand

Network



Inverse Demand Function: $w_k(t) = a_k + b_k t$

OD pair	a_k	b_k
(1, 3)	20	-2
(1, 4)	40	-2
(2, 3)	60	-2
(2, 4)	80	-2

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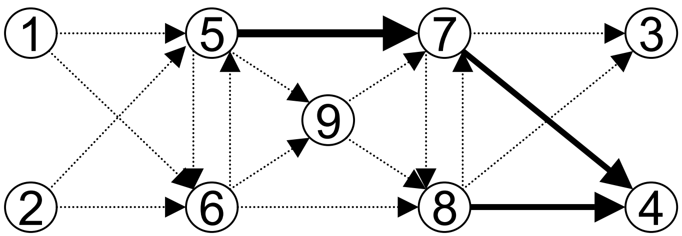
Example: Elastic Demand

Travel Cost function: $s_a(v) = T_a(1+0.15(v_a/C_a))$

Arcs	T_a	C_a	Arcs	T_a	C_a
(1, 5)	6	11	(6, 9)	7	29
(1, 6)	7	9	(7, 3)	2	34
(2, 5)	2	2	(7, 4)	7	9
(2, 6)	8	35	(7, 8)	1	49
(5, 6)	5	20	(8, 3)	4	13
(5, 7)	2	5	(8, 4)	4	5
(5, 9)	3	44	(8, 7)	2	47
(6, 5)	10	6	(9, 7)	5	42
(6, 8)	9	48	(9, 8)	5	5

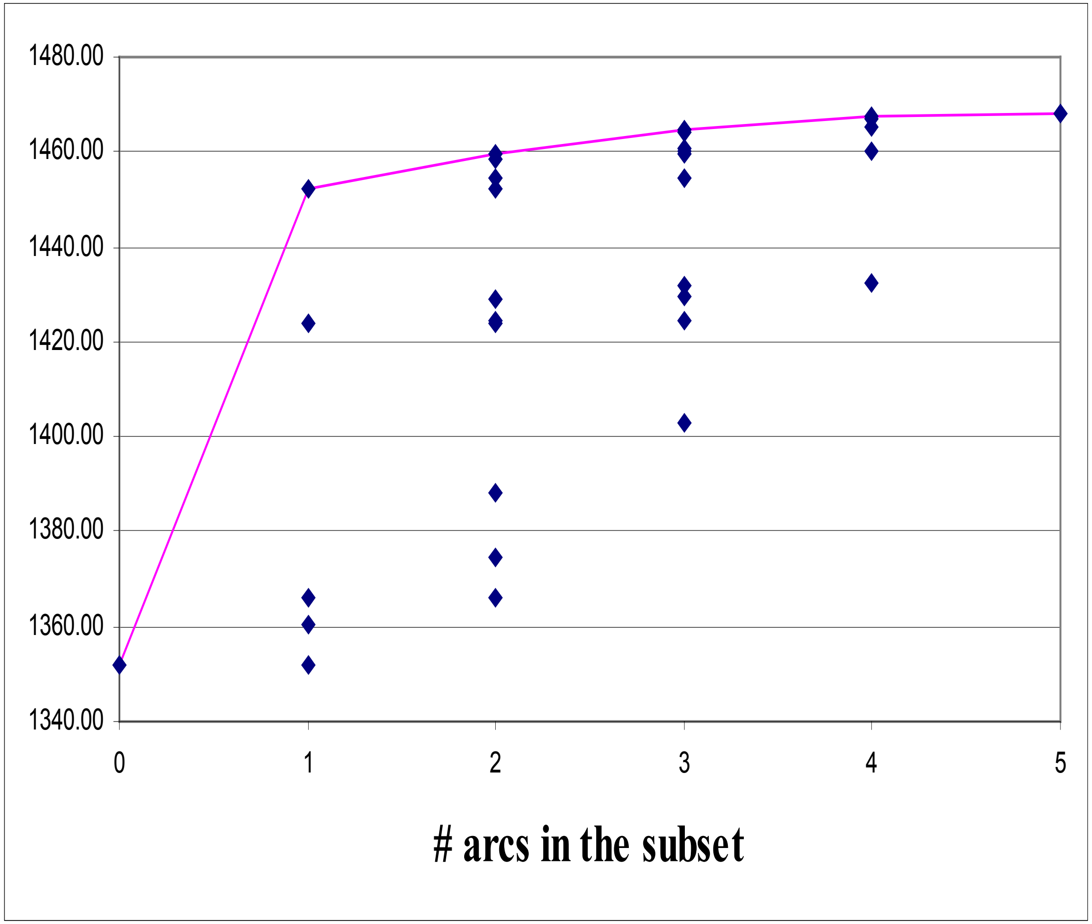
Example: Elastic Demand

It.	Objective Value: Master	Objective Value: Subprob	% Equil. Gap
1	1468.11	-740.15	27.7850
2	1466.31	-289.68	19.7560
3	1446.59	-91.22	1.8326
4	1441.52	-35.64	2.4725
5	1431.88	0.00	-4.7638E-14

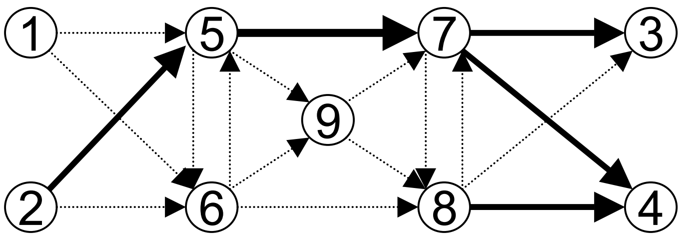


Tollable arcs: (5,7), (7,4), and (8,4).

Example: NUB versus Arcs Tolled



Tollable arcs:



Numerical Results: Fixed Demand

- Use GAMS
  - CPLEX to solve the subproblem in Step 3.
  - MINOS to solve the master problem in Step 2.
- Two networks from the literature
  - Sioux Falls: 76 links, 24 nodes, 528 OD pairs.
  - Hull: 798 links, 501 nodes, 158 OD pairs.
- Tollable arc selection
  - An arc is tollable if its user equilibrium flow exceeds its system optimum flow by a given percentage (‘excess’ percentage).

Numerical Results: Fixed Demand

- Sioux Falls:

Excess %	# of Tollable Arcs	Total Delay	Relative Gap (%)	Iterations Required	Master Problem (sec)	Sub-problem (sec)
5%	18	72.1036	0.9354	49	1731.61	11.77
10%	12	72.1861	0.9024	36	879.26	8.3
15%	4	73.0681	0.7764	14	182.13	2.65
25%	2	73.4916	0.4992	10	107.73	1.9

- Total delay at SOPT = 71.9426
- Total delay at UOPT = 74.8023

Numerical Results: Fixed Demand

- Hull

Excess %	# of Tollable Arcs	Total Delay	Relative Gap (%)	Iterations Required	Master Problem (sec)	Sub-problem (sec)
5%	179	179117	< 0.0001	16	6251.09	133.23
10%	135	179420	< 0.0001	8	1081.88	59.76
15%	93	179988	< 0.0001	7	1671.44	55.06
25%	58	180629	< 0.0001	7	2007.45	53.14
50%	21	181092	< 0.0001	10	3947.43	66.82
75%	12	181315	< 0.0001	8	3293.28	46.84
100%	10	181326	< 0.0001	11	5270.45	63.39
200%	8	181329	< 0.0001	11	5620.71	61.49

- Total delay at SOPT = 179063
- Total delay at UOPT = 186720

Toll Pricing Framework.

1. Solve ED - VI or one of the equivalent problems to obtain $(\bar{v}, \bar{t}, \bar{\beta})$.

- Solve the following toll selection problem:

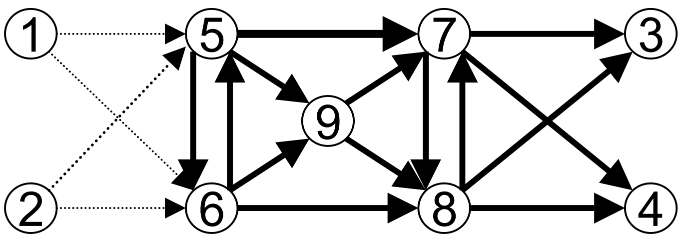
$$
\begin{array}{lll}
\min & f(\beta) & \\
s.t. & s(\bar{v}) + \beta \geq A^T \rho^k & \forall k \\
 & w_k(\bar{t}_k) \leq E_k^T \rho^k & \forall k \\
 & (s(\bar{v}) + \beta)^T \bar{v} = w(\bar{t})^T \bar{t} & \\
 & \beta_a \geq 0 & \forall a \notin Y \\
 & \beta_a = 0 & \forall a \in Y
\end{array}
$$

where, e.g., $f(\beta) = \sum_{a:\beta_a > 0} 1$ or $\max\{\beta_a : a \notin Y\}$.

Numerical Example

Arcs	ED-VI Flow	ED-VI Toll	Min Max Toll	MINTB Toll
(1,5)	5.12			
(1,6)	2.82			
(2,5)	36.35			
(2,6)	10.56			
(5,7)	41.42	3.56	3.23	4.78
(5,9)	0.05	0.05	0.73	2.28
(6,8)	13.37	2.69	1.69	
(6,9)		0.61		
(7,3)	21.24	1.59	1.92	0.37
(7,4)	12.91	2.9	3.23	1.69
(7,8)	7.28	3.9	3.23	
(8,4)	20.7		1	2.69
(9,8)	0.05	4.92	3.23	
Tot Rev.		283.46	283.46	283.46
Max Toll		4.92	3.23	4.78
# booths		8	8	5
NUB		1453.27	1453.27	1453.27

Tollable arcs



Note: Nonessential arcs are not listed.

2^{nd} Best Tolls - Conclusions

- Three equivalent formulations (ED-VI, ED-KKT, and ED-EX) for the 2^{nd} best toll pricing problem
- Properties of the 2^{nd} best tolls
  - Via the KKT multipliers, optimal 2^{nd} best tolls involves MSCP tolls on individual arcs as well as those from non-tollable arcs.
  - Toll revenue is constant.
- Cutting constraint algorithm for ED-EX
  - Converges finitely
  - Relaxed version can be implemented using existing software for LP and NLP
  - Can potentially solve large problems
- Toll pricing framework
  - Find a 2^{nd} best toll vector that optimizes a (secondary) objective.

Lawphongpanich, S. and Hearn, D. W., "An MPEC Approach to Second Best Toll Pricing," *Mathematical Programming*, 33-55, 7 July 2004.

Current Research

- Dynamic tolls to vary with time and traffic conditions – Ph. D. research of Artyom Nahapetyan
  - Period DTA – tomorrow TSL session on DTA (I) at 3:30
- Toll pricing for systems with multiple modes of transportation, e.g., tolls on the roads and fares on the transit network. Partial support from Volvo Research Foundation. PIs are Toi Lawphongpanich (UF), Younes Hammdouch (UAE), Agachai Sumalee (U. of Leeds)

Traffic Assignment Problem with Multiple Modes

- Consider two travel options:
  - Automobile only (auto-only)
  - Mixed modes are

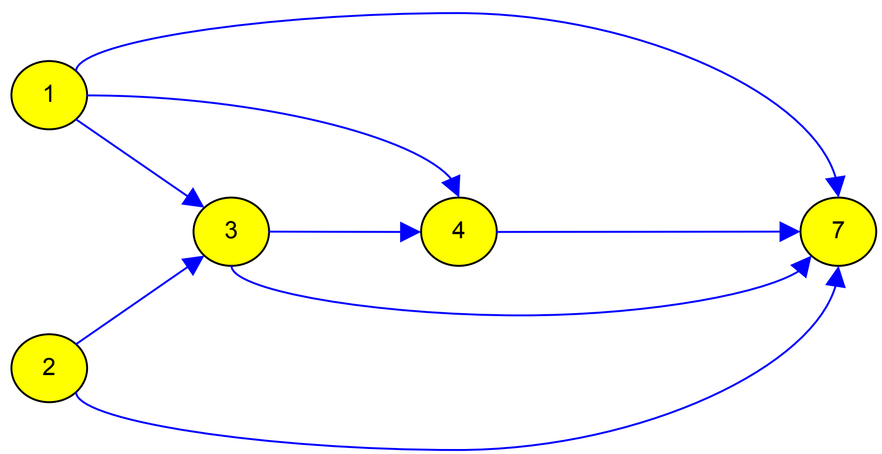
Walk-Metro: Travelers walk to metro stations, use metro lines to reach the final metro stations, and walk from there to their destinations.

Auto-Metro: Travelers drive to metro stations, use metro lines to reach the final stations, and walk from there to their destinations.

Y. Hamdouch, M. Florian, D. W. Hearn, and S. Lawphongpanich, "Congestion Pricing for Multi-Modal Transportation Systems," accepted and under revision for *Transportation Research B*.

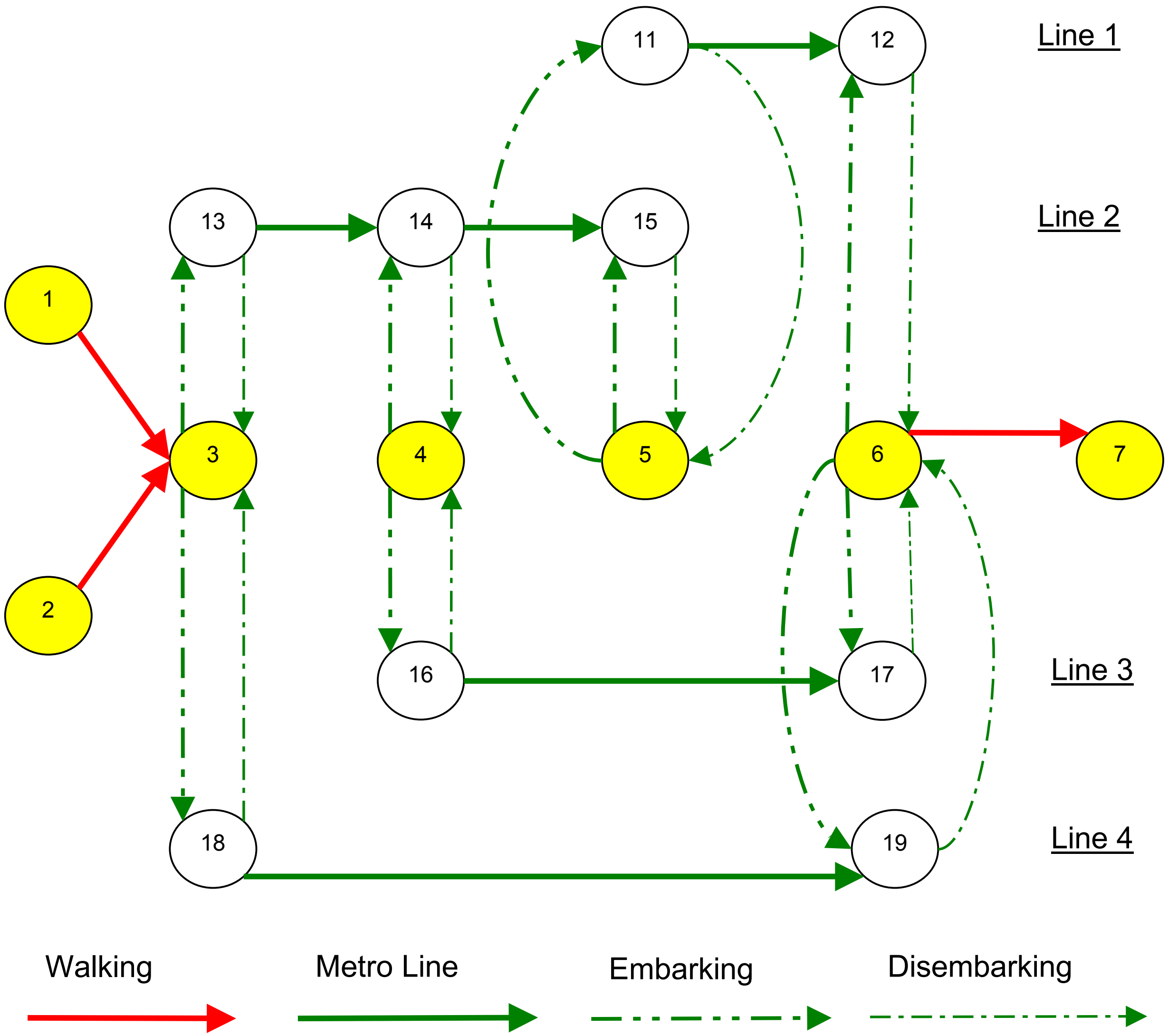
Underlying Networks

- There is an underlying network for each travel option.
  - Auto only option



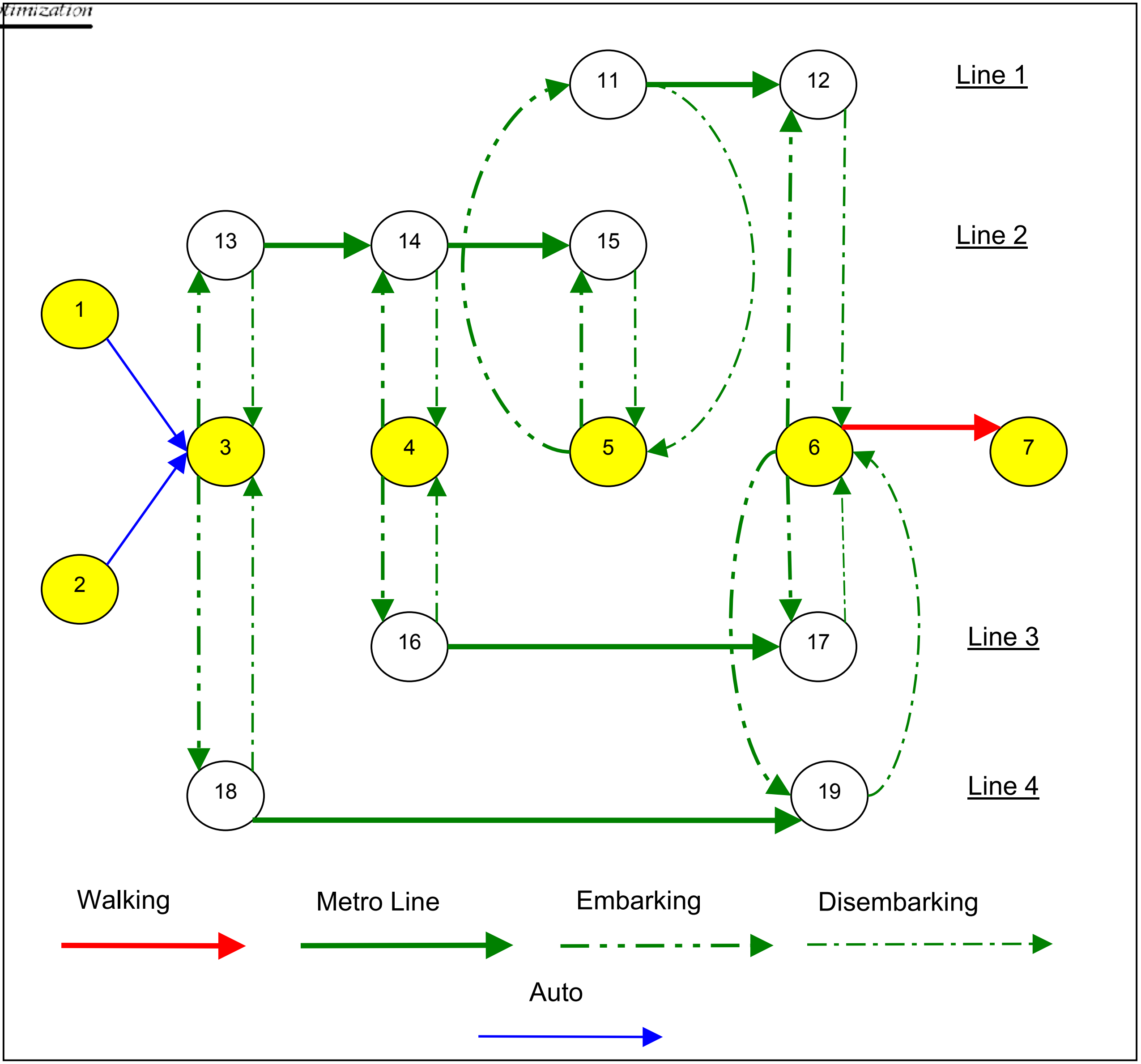
Nodes = road intersections
Arcs = roads and highways

Underlying Networks: Walk-Metro Option



![CAO Center for Applied Optimization logo]

Underlying Networks: Auto-Metro Option



Center for Applied Optimization

Multi-mode Pricing - Summary

- Developed a system problem that leads to the toll pricing framework
  - SOPT maximizes partial (metro benefit) NUB
  - UOPT problem is a VI
- Show that 2nd best pricing is not needed for zero tolls on walk, embark, disembark links
- Propose secondary toll selection problems unique to the multi-mode case.
  - For example, auto tolls are used to reduce transit fares

End