Network Equilibrium Models: Varied and Ambitious

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The applications of network equilibrium models are varied:
They range from simple to the very complex

- single mode, single class equilibrium assignment
- multi-class equilibrium assignment
- generalized cost on road and transit networks
- equilibrium assignment on congested transit networks
- path analyses
- complex multi-modal equilibration
- combined mode trips
- dynamic network equilibrium models
The single mode single class network equilibrium model

\[ \min \sum_{a \in A} \int_{0}^{v_a} s_a(x) \, dx \]

subject to \( \sum_{k \in K_i^c} h_k = g_i, i \in I \),

\( h_k \geq 0, k \in K_i, i \in I \)

\( (v_a = \sum_{k \in K_i} \delta_{ak} h_k, a \in A) \)

Numerous algorithms have been developed for its solution; As is well known the arc flows \( v_a \) are unique, but the path flows \( h_k \) are not unique.
Some selected applications

- The presentation includes examples of the application of various network equilibrium models carried out around the world.

- The first set of applications was carried out with static models of increasing complexity both for road and transit networks.

- The second set of applications presents new results with a dynamic network equilibrium model.
Some Straightforward Applications

• Madrid, Spain
• Pretoria, South Africa
• Auckland, New Zealand
Madrid- AM Peak Flows and Speeds
Regional Study of the Province of Gauteng, South Africa

Study carried out by
Vela VKE – Pretoria, South Africa,
Scenario Without New facility

K58 Scenario 4: 2010 am Peak Hour Volumes
Scenario with New facility

K58 Scenario 1: 2010 am Peak Hour Volumes
Scenario comparison
Planning Transport in Auckland

Study carried out by Auckland Regional Council, Auckland, NZ
AM Vehicle Flows, 2001
Complex Variable Demand Network Equilibrium Models

- The “Step Size Mechanism” may take different forms depending on the knowledge that one has of the underlying model.
- Does the model have an equivalent convex cost optimization formulation?
- Can the model be formulated as a variational inequality?
- The model is very complicated and one carries out “ad-hoc” feedback by some averaging scheme.
Some well known variants of such models are the Combined Distribution-Assignment Model, Combined Distribution-Assignment-Mode Choice Model, Equilibrium Assignment Model with Variable Demand,...

One can use adaptations of nonlinear programming algorithms to obtain the solution of such models.

The “Step Size Mechanism” may be trivially stated to be the result of a line search on the objective function

\[
\min_{0 \leq \lambda \leq 1} F(x^k + \lambda(d^k - x^k)), x^{k+1} = x^k + \lambda(d^k - x^k)
\]

where \(x^k\) is a current solution and \(d^k\) is a direction of descent.
Network Equilibrium Models: Variational Inequality Formulations

It is well known that models are with asymmetric cost functions, such as intersection delays, transit travel time depending on auto travel times in multi-mode models can be formulated as:

\[ s_a (v^*_a)(v^*_a - v^*_a) \geq 0 \]

subject to

\[ \sum_{k \in K_i} h_k = g_i, i \in I, \]

\[ h_k \geq 0, k \in K_i, i \in I \]

\[ (v_a = \sum_{k \in K_i} \delta_{ak} h_k, a \in A) \]
Network Equilibrium Models: Variational Inequality Formulations

• Such models may be solved by a variety of algorithms. Often the sufficient conditions for convergence are impossible to verify.

• A common heuristic method used in practice is the Method of Successive Averages.

• The “Step Size Mechanism” may be related to the averaging of the link costs or the averaging of link flows:

\[ x^{k+1} = x^k + \alpha_k (x^{k+1} - x^k) ; \quad x^{k+1} = T(x^k) \]

\[ 0 < \alpha < 1 ; \quad \sum_{k=1}^{\infty} \alpha_k (1 - \alpha_k) = +\infty \]

where \( T(x^k) \) is the computed procedure that is used to obtain the next iterate.
A Complex Model:
Rigorous Formulation-Heuristic Solution Algorithm

• Santiago, Chile
• Complex demand model
• Road Network Equilibrium
• Transit Network Equilibrium
• Combined modes: road-transit; transit-transit
Santiago, Chile Strategic Planning Model

- Base network
  - 409 centroids including 49 parking locations
  - 1808 nodes, 11,331 directional links
  - 1116 transit lines and 52468 line segments
  - 11 modes, including 4 combined modes
    (bus-metro, txc-metro, auto-metro and auto passenger-metro)

- The demand
  - subdivided into 13 socio-economic classes
  - 3 trip purposes (work, study, other)
  - driving license holders can access to 11 modes
  - no license holders can access to 9 modes
Base Network of Santiago, Chile

Base Network of Santiago, Chile
(Year 2001)

Plot generated by Enif 2002-01-23 19:30:20 at INRO
Variational Inequality Formulation

Find \((h^*, T^*) \in \Omega\) such that

\[
\sum_{pn} \sum_{(ij)} \sum_{g \subseteq G^p} \sum_{m \in g} \sum_r \phi_g^p C_r^{pnm}(h^*, T^*)(h_r - h_r^*) + \frac{1}{\beta^{pn}} \ln T_{ij}^{png*}(T_{ij}^{png} - T_{ij}^{png*}) + \\
\sum_{m \in g} \phi_g^p \ln(T_{ij}^{pnm*} / T_{ij}^{png*})(T_{ij}^{pnm} - T_{ij}^{pnm*}) \geq 0, \quad \forall (h, T) \in \Omega.
\]
Trip Ends and Conservation of Flow Constraints

\[ \sum_{g} \sum_{j} T_{ij}^{png} = O_{i}^{pn}, \forall i, p, n \quad (\alpha_{i}^{pn}) \]

\[ \sum_{g} \sum_{n} \sum_{i} T_{ij}^{png} = D_{j}^{p}, \forall j, p \quad (\xi_{i}^{p}) \]

\[ \sum_{m \in g} T_{ij}^{pnm} - T_{ij}^{png} = 0, \forall j, p, n, g \quad (L_{ij}^{png}) \]

\[ \phi_{g}^{p} \left( \sum_{r \in R^{m}} h_{r}^{pnm} - T_{ij}^{pnm} \right) = 0, \forall j, p, n, g \quad (\mu_{ij}^{pnm}) \]

\[ h_{r}^{pnm} \geq 0, \forall r, p, n, m \quad (\gamma_{r}^{pnm}) \]

\[ T_{ij}^{pnm} > 0, \forall ij, p, n, m \]

\[ T_{ij}^{png} > 0, \forall ij, p, n, g \]
Network Equilibrium Models: car and transit

- Multi-class network equilibrium model
- Multi-class transit network equilibrium model
- Heuristic equilibration that resorts to averaging of flows and travel impedances
Solution Procedure

Start and Initialize

Trip Distribution and Mode Choice

Equil. Auto Assignment

MSA Auto Volume

Auto Skims

Convergence?

end

Multiple Transit Assignments

Standard Transit Assignment for buses

Equilibrium Transit Assignment for metro

Aut Skims

Transit Auto equivalent flow

Congested time

Auto Impedance

Bus Impedance

Metro MSA Impedance

Park-and-Ride Model for auto-metro and bus-metro

All Impedances
Santiago, Chile Strategic Planning Model

• The next slide shows the convergence of the MSA algorithm that uses link flow averaging for the car network and travel time averaging for the transit network.

• The convergence of both the car demand and link flows are given for two variants: uncongested transit assignment and equilibrium transit assignment.
Convergence of equilibration

Normalized Gap vs. Iterations
congested vs. non-congested metro assignment with metro capacity

- Auto demand
- Auto link volume

Iteration
Metro Volume (non-congested vs. congested version)

<table>
<thead>
<tr>
<th>Line</th>
<th>Metro Volume (in thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>14342.67</td>
</tr>
<tr>
<td>2A</td>
<td>12394.67</td>
</tr>
<tr>
<td>5A</td>
<td>15676.67</td>
</tr>
<tr>
<td>8A</td>
<td>14557.33</td>
</tr>
</tbody>
</table>

Legend:
- Green: within capacity
- Red: slightly overcapacitated
- Orange: overcapacitated
Metro Volume Changes

Metro Volume Difference on Links by using congested metro assignments

Plot generated by Enl 2002-02-12 12:02:48 at INRIO
Auto-metro volume (non-congested vs. congested version)

STGO 2 Auto-metro Volume (Parking and ride)

Congested

Non-congested
Metro Volume - metro 5A, non-congested version

Metro Line 5A
(non-congested version)

Plot generated by Enl 2002-02-12 16:50:30 at INRO
Metro Volum2 - metro 5A, congested version

Metro Line 5A
(congested version)

Plot generated by Enif 2002-02-12 15:09:32 at INRO
Some Complex Applications

- Los Angeles, California
- Toll Highways Poznan, Poland
- Toll Bridge, Montreal,
The SCAG Regional Transportation Planning Model

- A complex and very large scale model
- Lack of rigorous formulation; network equilibrium sub-models
- A multi-class multi-mode network equilibrium model with asymmetric cost function is part of the model
- Heuristic solution algorithm based on an outer averaging scheme
SCAG MODEL
FLOW CHART:

START

auto skims for PK

transit skims for PK

transit skims for OP

auto skims for OP

Trip generation

HBW Logsums for PK (mc)

HBW Logsums for OP (mc)

trip distribution for PK (gravity)

trip distribution for OP (gravity)

mode choice model for PK

mode choice model for OP

demand computations (time-of-day model)

auto-truck assignments for AM

auto-truck assignments for MD

successive average link volume for outer loop of AM

successive average link volume for outer loop of MD

is the number of outer loops satisfied?

auto-truck assignments for PM

auto-truck assignments for NT

transit assignments for AM

transit assignments for MD

END
Network Overview - Highway Network
by facility type

2000 Highway Base Network
Network Overview - Highway Network with parking lots

2000 Highway Base Network with Park-and-Ride Lots

Legend:
- parking lot
- ramps
- freeway
- principal arterial
- minor arterial
- major collector
- HOV
- centroid connectors
Equilibration Algorithm
Convergence Results

SCAG Model Convergence (AM peak)
Assignment Results
AM peak volume

AM Peak Link Volume Vs. Average Daily Link Volume

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Assignment Results
AM link speed
Assignment Results
AM truck volume by class
Assignment Results
AM HOV VMT Grid Map

HOV VMT Grid Map
low
high

SOUTHERN CALIFORNIA ASSOCIATION OF GOVERNMENTS

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Assignment Results
HCM Level of Service

Southern California Association of Governments
(base year 2000, AM, freeway)
Scenario Comparison
VMT changes

Scenario Comparison between Base Year and a 2030 Plan, AM
(VMT changes)
Toll highway analysis – Poznan, Poland

It involves the following models:

- Multi-class equilibrium assignment with generalized costs

- Demand models for toll-no toll choice and future year demands

- Equilibration of the demand for toll highway and network performance
The multi-class equilibrium model with tolls

\[
\min \sum_{a \in A} \int_{0}^{v_a} s_a(x) \, dx + \sum_{c \in C} \sum_{a \in A} v_a^c \theta^c t_a^c
\]

subject to 
\[
\sum_{k \in K_i^c} h_k = g_i^c, i \in I, c \in C
\]
\[
h_k \geq 0, k \in K_i^c, i \in I
\]
\[
(v_a^c = \sum_{k \in K_i^c} \delta_{ak} h_k, a \in A, c \in C)
\]

The numerical solution of this model is well known; it is worthwhile to point out that the flows by class, \( v_a^c \), are not unique, nor are the path flows \( h_k \), but the arc flows \( t_a^c \) are unique.
Base year – No Toll
Base year – Medium Cost Toll
Base year – High Toll
Year 2010 – High Cost Toll
Year 2020 – Medium Cost Toll

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Year 2010 – Low Cost Toll
Toll Bridge Study – Montreal, Canada

• Study carried out by the Ministry of Transportation of Quebec
• The analysis relied heavily on the analysis of paths generated by the assignment algorithm
The Montreal Region
The new proposed bridge
Zones around Bridge
Demand for Current and Future Year
The Current Bridge Flows
Bridge Flows with New facility
Toll Income with Various Toll Levels
National Models

- These are very large multi-modal multi-class models
- The underlying demand models are rather complex and the running times are very high
- An example of a national model is the PLANET model developed for British Rail
The PLANET Model – British Rail

Zones

Rail

Road

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Dynamic Network Equilibrium Model

• Solved by a hybrid optimization-simulation model a discretized version of a variational inequality formulation of a dynamic network equilibrium model

• The theoretical properties of the model are difficult to establish

• Wardrop’s user equilibrium in a temporal framework is a basis for the model
Dynamic equilibrium

variables

\[(0, T) = \text{demand period}\]
\[I = \text{set of OD pairs}\]
\[K_i = \text{set of paths for } i\]
\[h_k(t) = \text{flow } (t) \text{ on path } k\]

constraints

\[\sum_{k \in K_i(t)} h_k(t) = g_i(t)\]
\[h_k(t) \geq 0\]

equilibrium conditions

\[u_i(t) = \min_{k \in K_i} \{s_k(t)\}\]
\[s_k(t)\begin{cases} = u_i(t) & \text{if } h_k(t) > 0 \\ \geq u_i(t) & \text{otherwise} \end{cases}\]
Dynamic assignment model

- Network definition
- Time-dependent OD matrices
- Traffic Control Data

1. Choose initial paths
2. Calculate path flows (t)
3. Run traffic simulation
4. Determine path travel times (t)
5. Convergence achieved?
   - Yes: STOP
   - No: Modify path sets: add a new path or remove an existing path (each O-D pair & each interval)
Traffic simulation model

simplified model of vehicle interactions allows for an efficient event-based simulation

- car following
- lane changing
- gap acceptance

sophisticated lane selection heuristics

- local lane selection rules
- stochastic look-ahead strategy
fundamental diagram

\[ V = \text{free-flow speed} \]

\[ Q = \frac{1}{L/V + R} \]

\[ K = 1/L \]

\[ W = L/R \]
An application in Stockholm
An application in Stockholm
An application in Montreal
An application in Auckland, NZ
Ending Remarks

• The equilibrium model of route choice is here to stay for both static and dynamic models

• We have a lot to thank to the landmark contribution of 1956