Network Equilibrium Models: Varied and Ambitious

Michael Florian

Center for Research on Transportation

University of Montreal

Centre
de recherche
sur les transports

The applications of network equilibrium models are varied:

They range from simple to the very complex

- single mode, single class equilibrium assignment
- multi-class equilibrium assignment
- generalized cost on road and transit networks
- equilibrium assignment on congested transit networks
- path analyses
- complex multi-modal equilibration
- combined mode trips
- dynamic network equilibrium models

The single mode single class network equilibrium model

$$\min \sum_{a \in A} \int_0^{v_a} s_a(x)\,dx$$

subject to $\sum_{k \in K_i^c} h_k = g_i \,, i \in I,$

$$h_k \geq 0, k \in K_i \,, i \in I$$

$$(v_a = \sum_{k \in K_i} \delta_{ak} h_k \,, a \in A)$$

Numerous algorithms have been developped for its solution;
As is well known the arc flows , v_a are unique, but the path flows h_k , are not unique.

Some selected applications

- The presentation includes examples of the application of various network equilibrium models carried out around the world
- The first set of applications was carried out with static models of increasing complexity both for road and transit networks
- The second set of applications presents new results with a dynamic network equilibrium model

Some Straightforward Applications

- Madrid, Spain
- Pretoria, South Africa
- Auckland, New Zealand

Madrid- AM Peak Flows and Speeds

Regional Study of the Province of Gauteng, South Africa

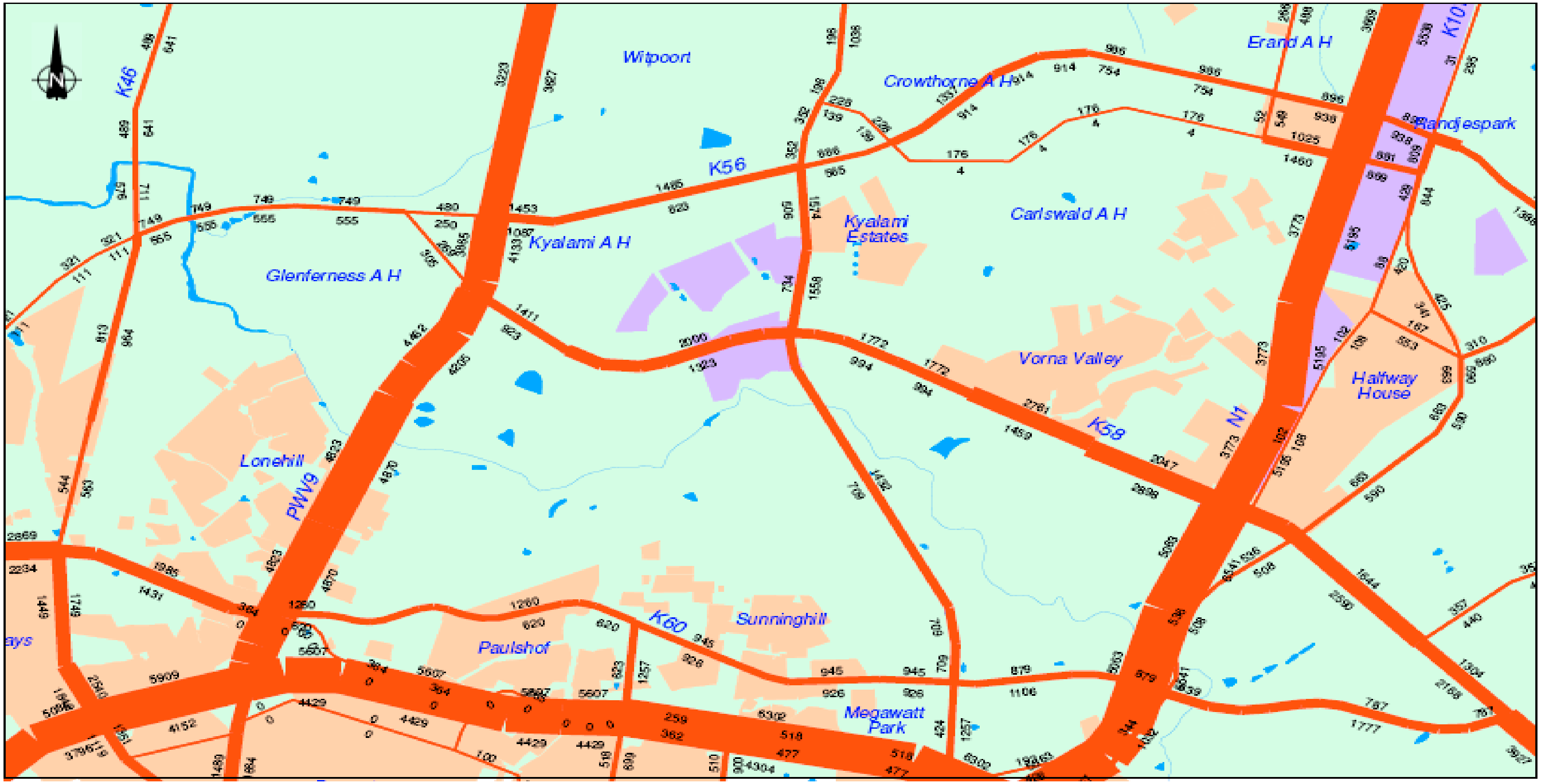
Study carried out by

Vela VKE – Pretoria, South Africa,

Scenario Without New facility

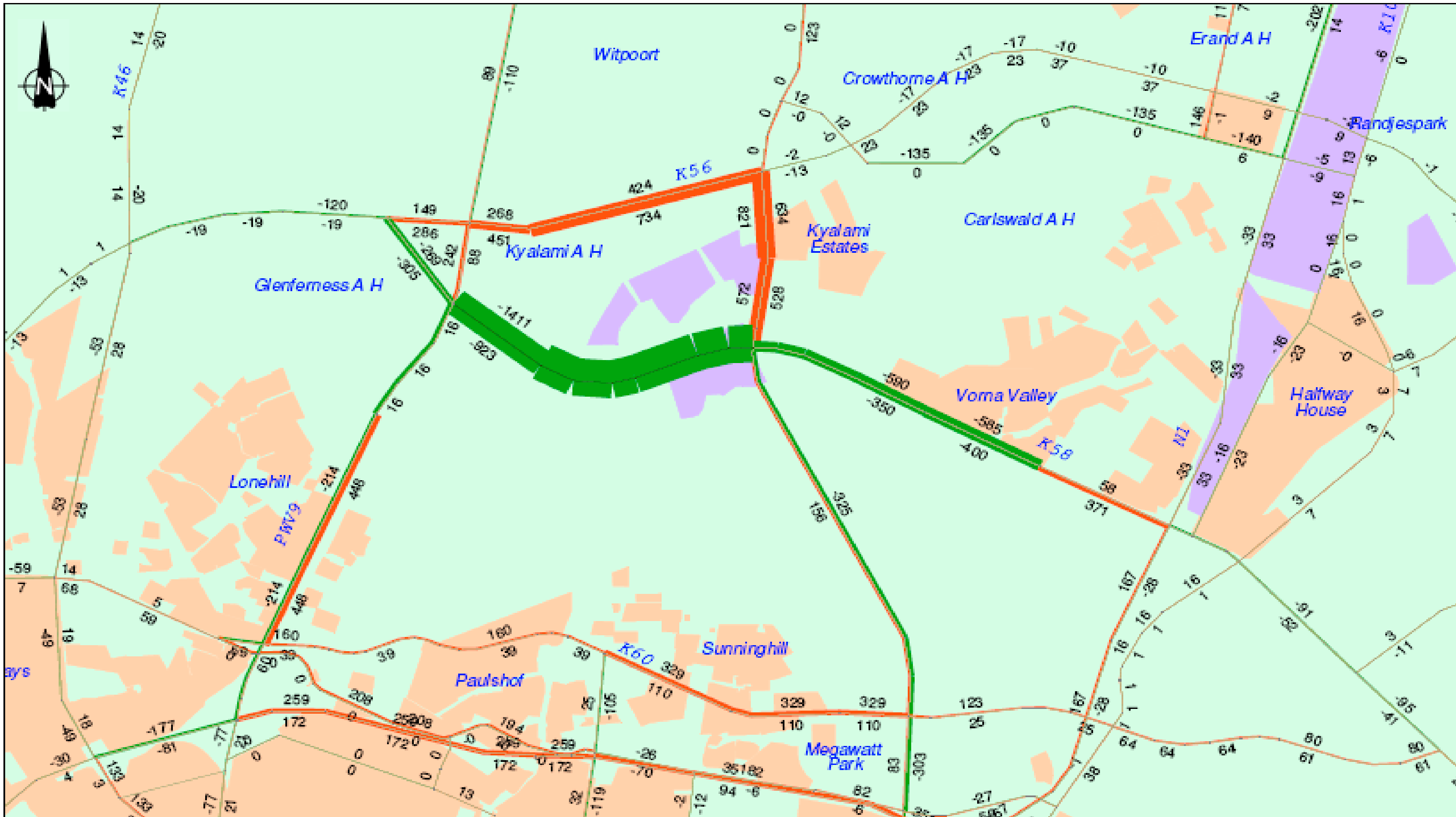
K58 Scenario 4: 2010 am Peak Hour Volumes

Scenario with New facility



K58 Scenario 1: 2010 am Peak Hour Volumes

Scenario comparison



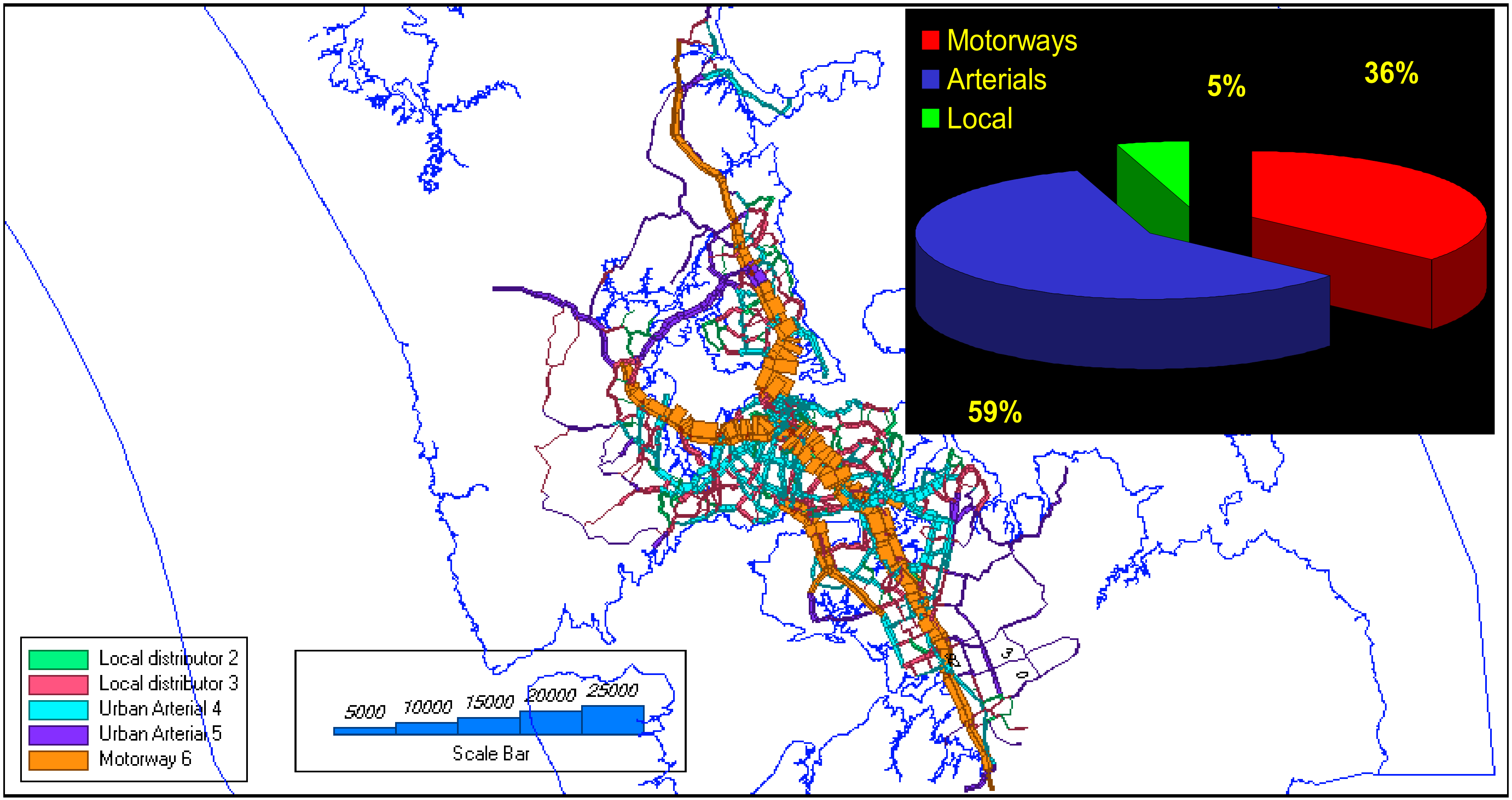
Difference in Volumes (Scenario 4 - Scenario 1): 2010 am Peak Hour Volumes

Planning Transport in Auckland

Study carried out by Auckland Regional Council, Auckland, NZ

AM Vehicle Flows, 2001

Bare network



2001 ART Model
Scenario 110: Base 2001 AM Scenario
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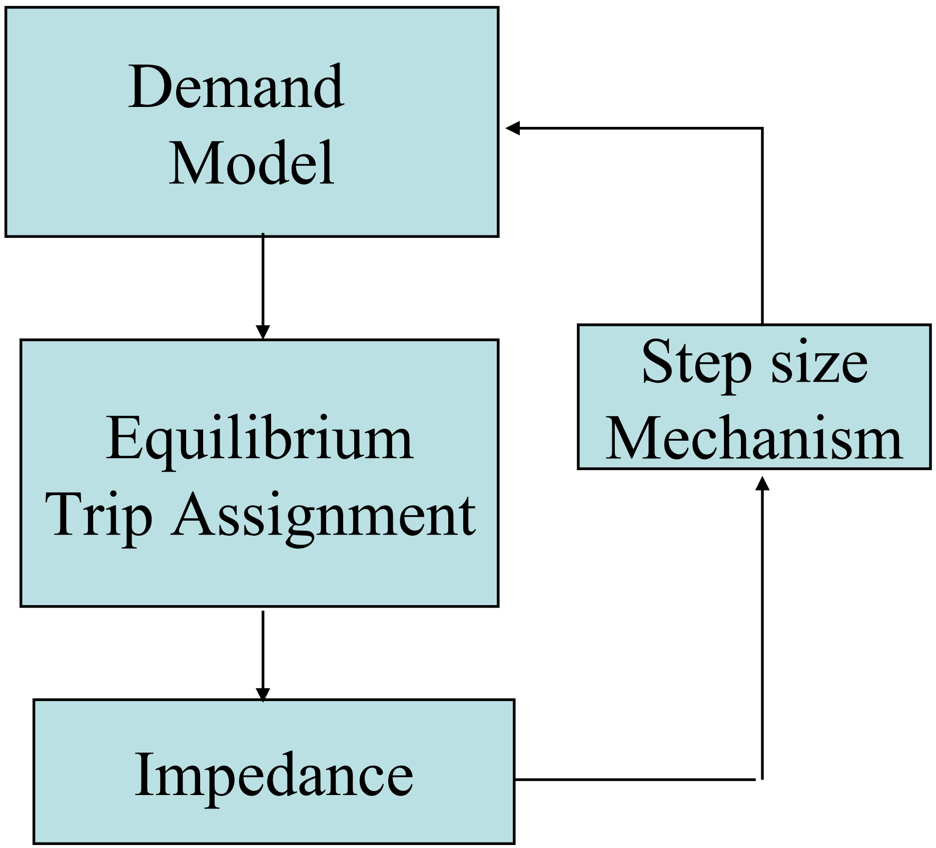
Complex Variable Demand Network Equlibrium Models



- The "Step Size Mechanism" may take different forms depending on the knowledge that one has of the underlying model.
- Does the model have an equivalent convex cost optimization formulation?
- Can the model be formulated as a variational inequality?
- The model is very complicated and one carries out "ad-hoc" feedback by some averaging scheme.

Complex Variable Demand Network Equilibrium Models:
Equivalent Convex Cost Optimization Formulations

- Some well known variants of such models are the Combined Distribution-Assignment Model , Combined Distribution-Assignment-Mode Choice Model, Equilibrium Assignment Model with Variable Demand,...
- One can use adaptations of nonlinear programming algorithms to obtain the solution of such models
- The "Step Size Mechanism" may be trivially stated to be the result of a line search on the objective function

$$\min_{0 \le \lambda \le 1} F(x^k + \lambda(d^k - x^k)),\ x^{k+1} = x^k + \lambda(d^k - x^k)$$

where x^k is a current solution and d^k is a direction of descent

Network EquilibriumModels:
Variational Inequality Formulations

It is well known that models are with asymmetric cost functions, such as intersection delays, transit travel time depending on auto travel times in multi-mode models can be formulated as :

$$s_a(v^*)(v_a - v_a^*) \geq 0$$

subject to $\sum_{k \in K_i^c} h_k = g_i \,, i \in I,$

$$h_k \geq 0, k \in K_i \,, i \in I$$

$$(v_a = \sum_{k \in K_i} \delta_{ak} h_k, \, a \in A)$$

Network EquilibriumModels:
Variational Inequality Formulations

•Such models may be solved by a variety of algorithms. Often the sufficient conditions for convergence are impossible to verify

• A common heuristic method used in practice is the Method of Successive Averages

•The "Step Size Mechanism" may be related to the averaging of the link costs or the averaging of link flows

$$x^{k+1} = x^k + \alpha_k (x^{k+1} - x^k) \ ; \quad x^{k+1} = T(x^k)$$

$$0 < \alpha < 1 \ ; \quad \sum_{k=1}^{\infty} \alpha_k (1 - \alpha_k) = +\infty$$

where $T(x^k)$ is the computed procedure that is used to obtain the next iterate

A Complex Model:
Rigorous Formulation-Heuristic Solution Algorithm

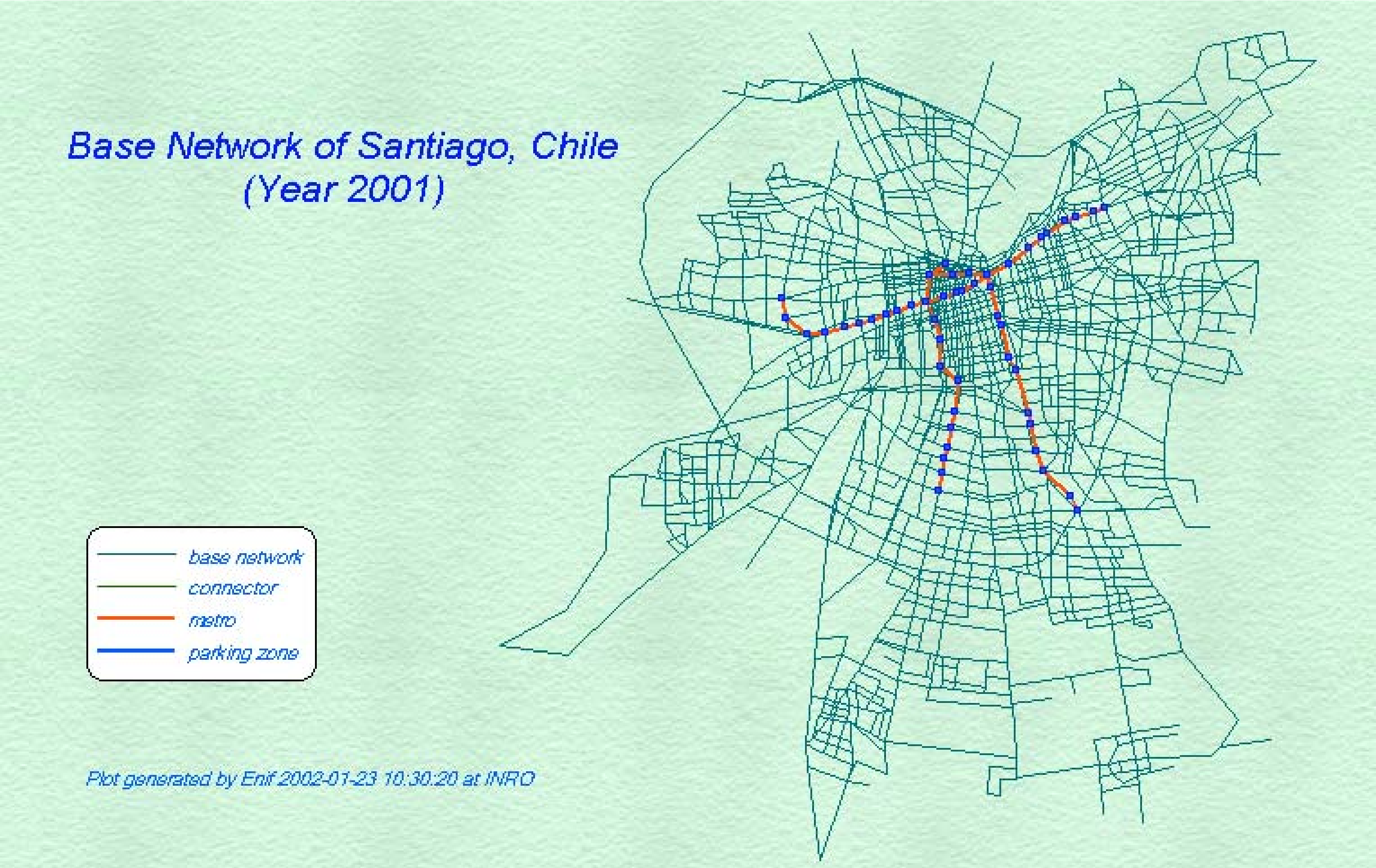
- Santiago, Chile
- Complex demand model
- Road Network Equilibrium
- Transit Network Equilibrium
- Combined modes: road-transit; transit-transit

Santiago, Chile Strategic Planning Model

- Base network
  - 409 centroids including 49 parking locations
  - 1808 nodes, 11,331 directional links
  - 1116 transit lines and 52468 line segments
  - 11 modes, including 4 combined modes

    (bus-metro, txc-metro, auto-metro and auto passenger-metro)
- The demand
  - subdivided into 13 socio-economic classes
  - 3 trip purposes (work, study, other)
  - driving license holders can access to 11 modes
  - no license holders can access to 9 modes

Base Network of Santiago, Chile



Variational Inequality Formulation

Find $(h^*, T^*) \in \Omega$ such that

$$\sum_{pn} \sum_{(ij)} \sum_{g \subseteq G^p} [\sum_{m \in g} \sum_{r} \phi_g^p C_r^{pnm}(h^*, T^*)(h_r - h_r^*) + \frac{1}{\beta^{pn}} \ln T_{ij}^{png*} (T_{ij}^{png} - T_{ij}^{png*})) +$$

$$\sum_{m \in g} \phi_g^p \ln(T_{ij}^{pnm*} / T_{ij}^{png*})(T_{ij}^{pnm} - T_{ij}^{pnm^*})] \geq 0, \qquad \forall (h, T) \in \Omega.$$

Trip Ends and Conservation of Flow Constraints

$$\sum_{g}\sum_{j} T_{ij}^{png} = O_i^{pn}, \forall i, p, n \qquad (\alpha_i^{pn})$$

$$\sum_{g}\sum_{n}\sum_{i} T_{ij}^{png} = D_j^{p}, \forall j, p \qquad (\xi_i^{p})$$

$$\sum_{m \in g} T_{ij}^{pnm} - T_{ij}^{png} = 0, \forall j, p, n, g \qquad (L_{ij}^{png})$$

$$\phi_g^p \left(\sum_{r \in R^m} h_r^{pnm} - T_{ij}^{pnm}\right) = 0, \forall j, p, n, g \qquad (\mu_{ij}^{pnm})$$

$$h_r^{pnm} \geq 0, \quad \forall r, p, n, m \qquad (\gamma_r^{pnm})$$

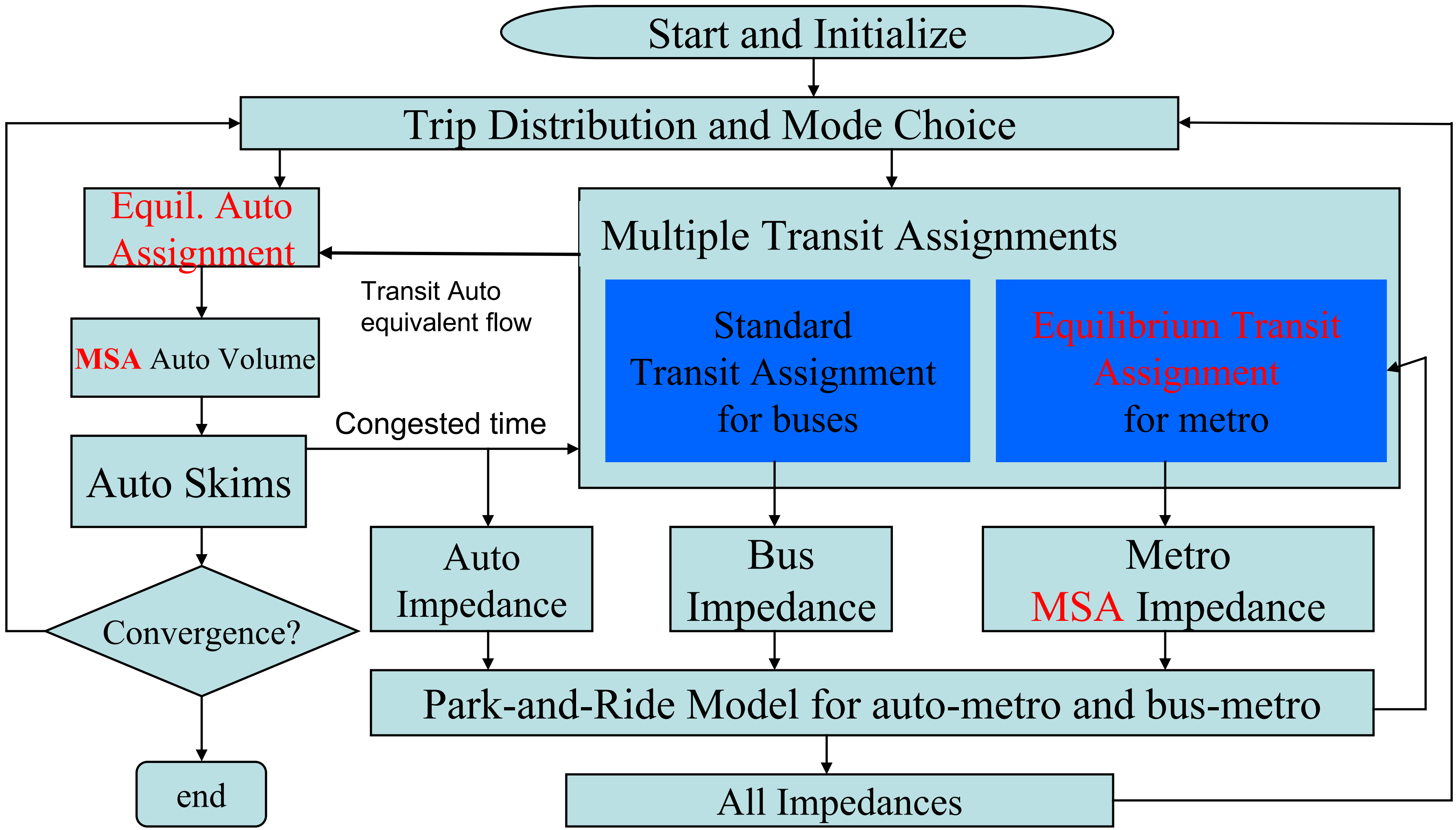
$$T_{ij}^{pnm} > 0, \quad \forall ij, p, n, m$$

$$T_{ij}^{png} > 0, \quad \forall ij, p, n, g$$

Network Equilibrium Models: car and transit

- Multi-class network equlibrium model
- Multi-class transit network equlibrium model
- Heuristic equilibration that resorts to averaging of flows and travel impedances

Solution Procedure

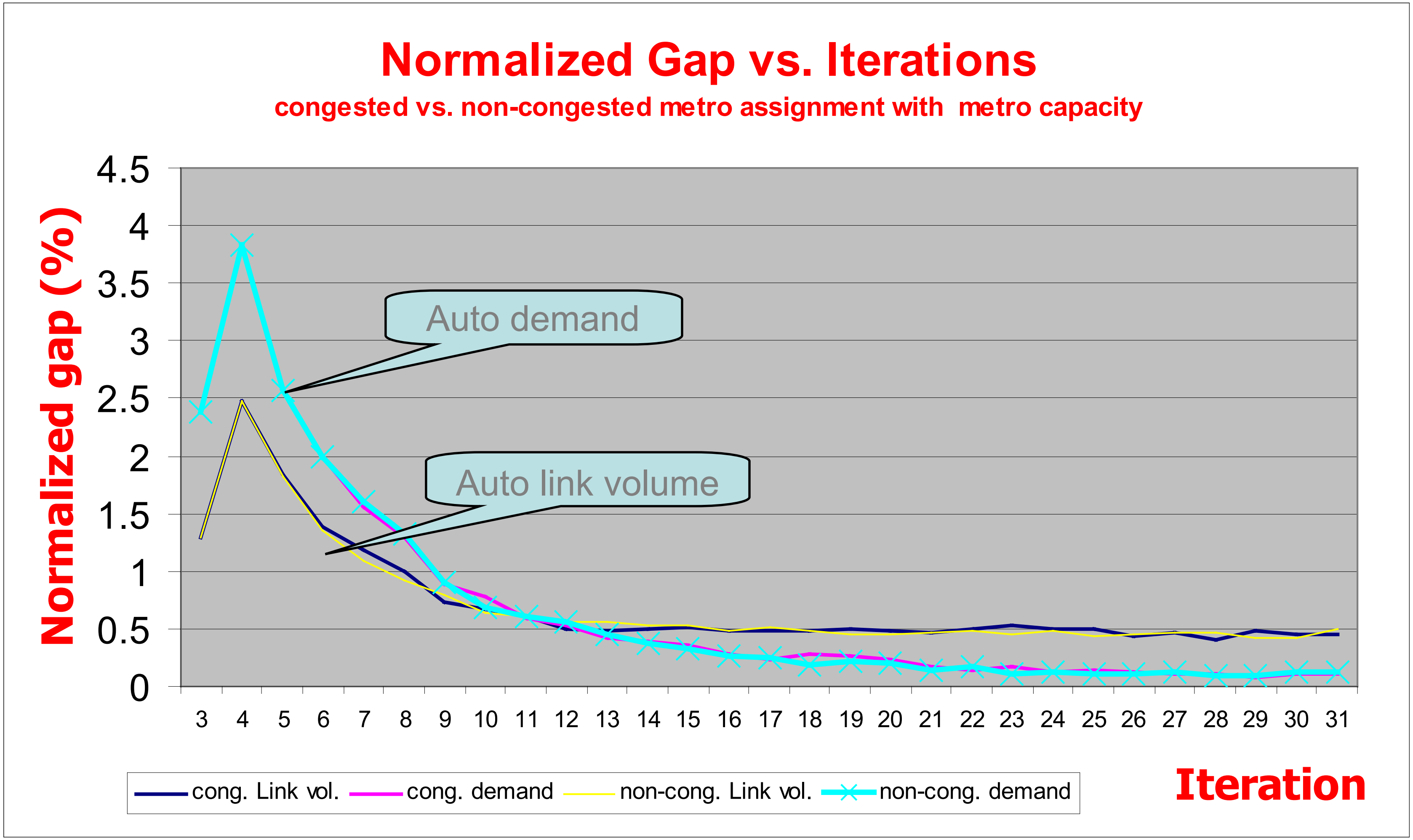


Santiago, Chile Strategic Planning Model

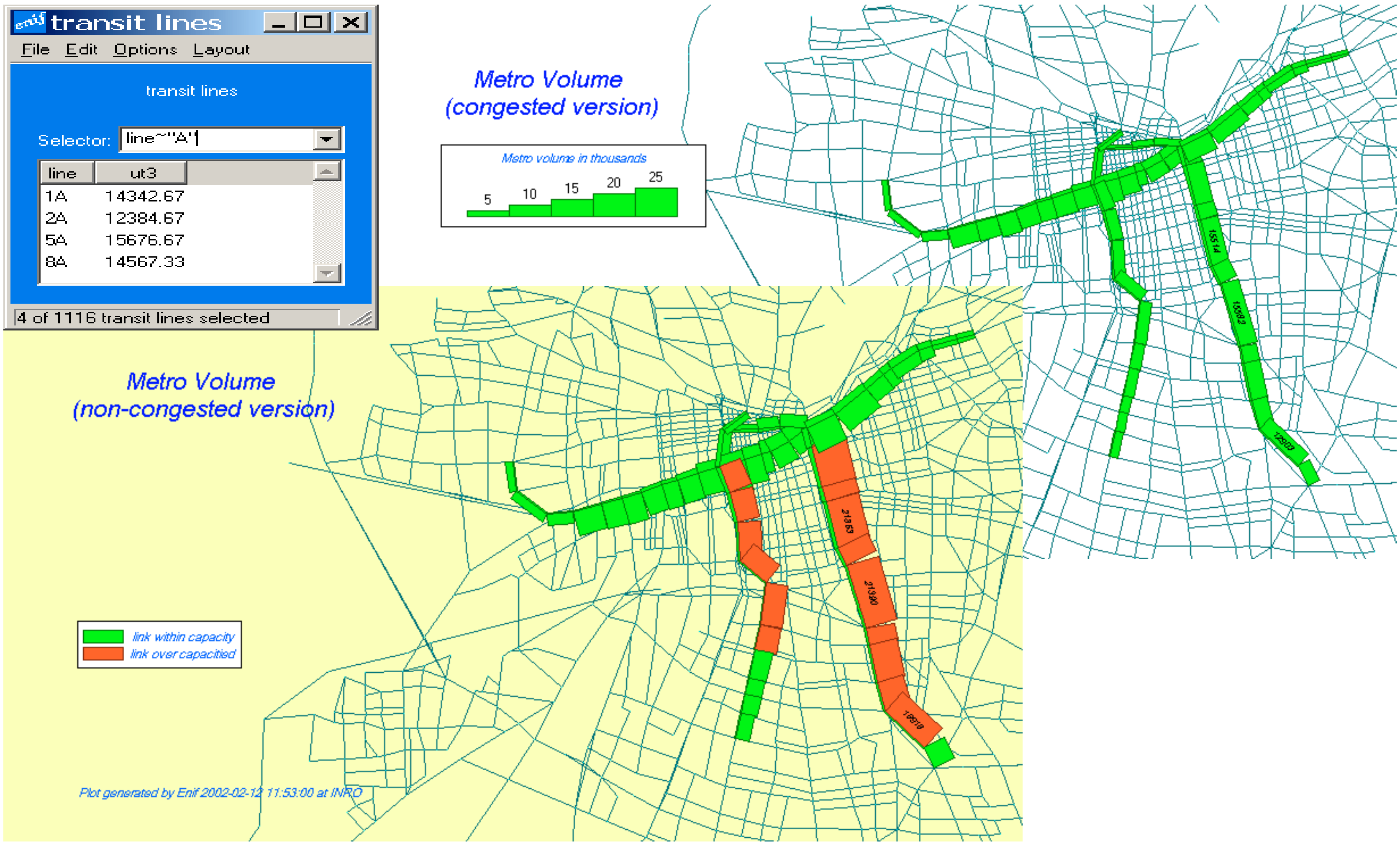
- The next slide shows the convergence of the MSA algorithm that uses link flow averaging for the car network and travel time averaging for the transit network.

- The convergence of both the car demand and link flows are given for two variants: uncongested transit assignment and equilibrium transit assignment.

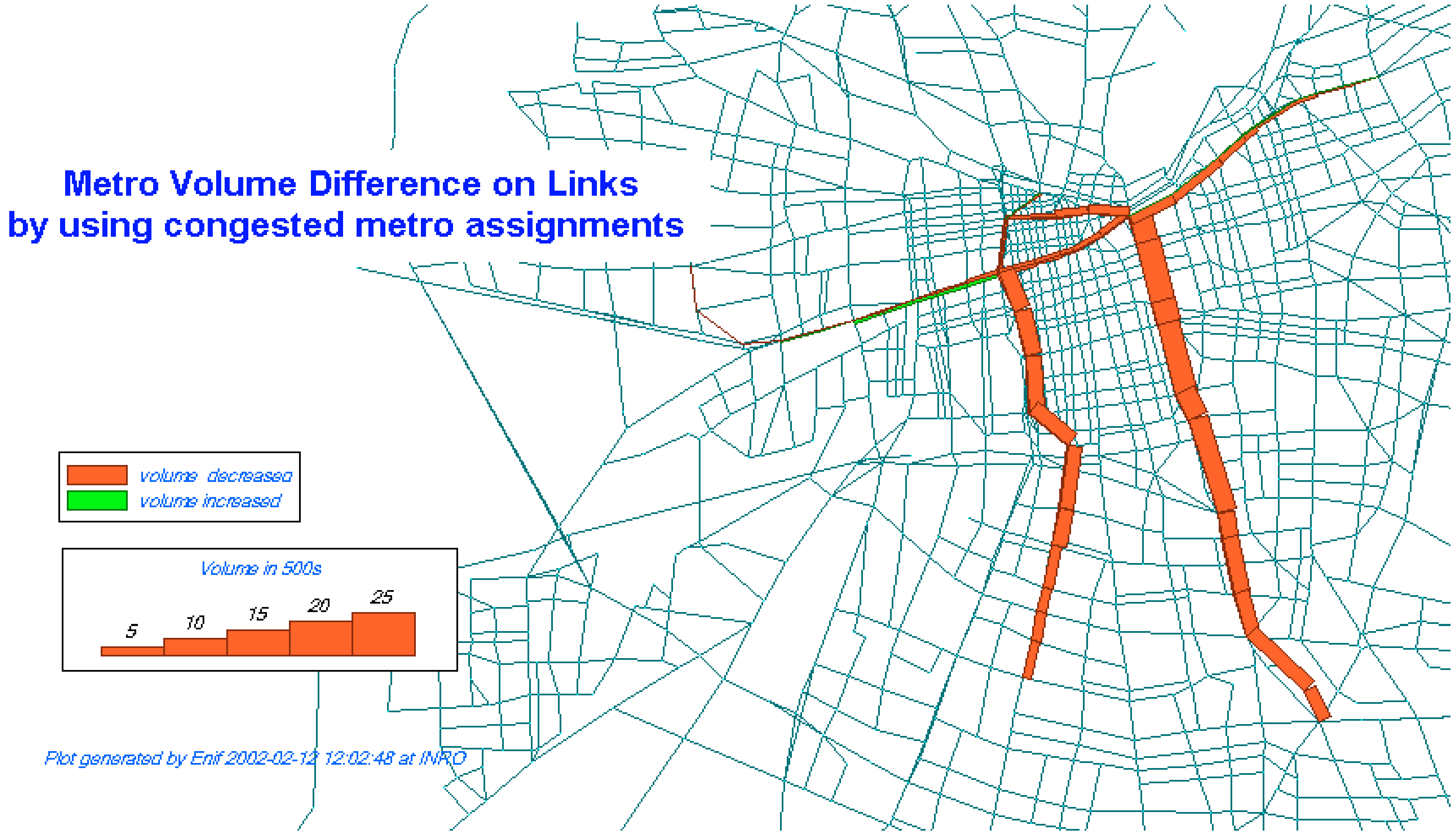
Convergence of equilibration



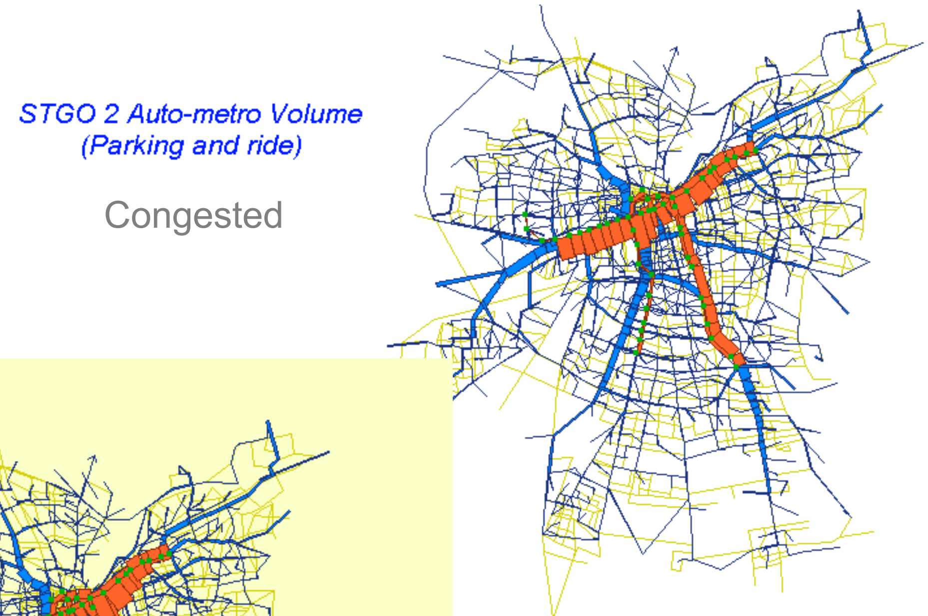
Metro Volume (non-congested vs. congested version)

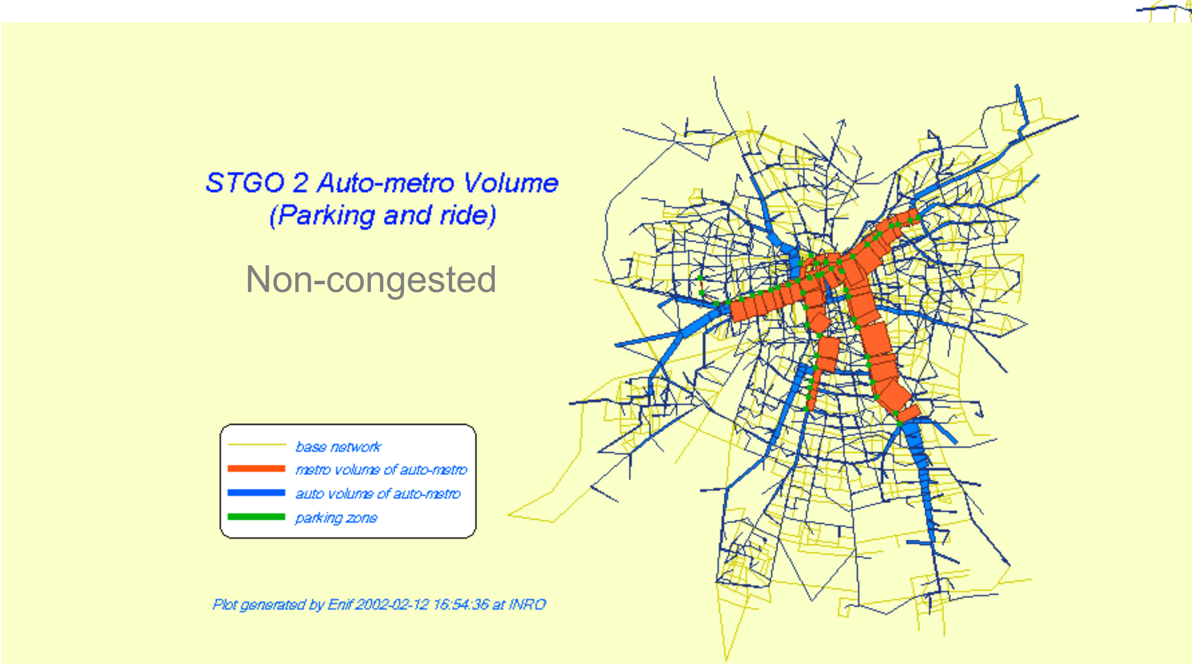


Metro Volume Changes

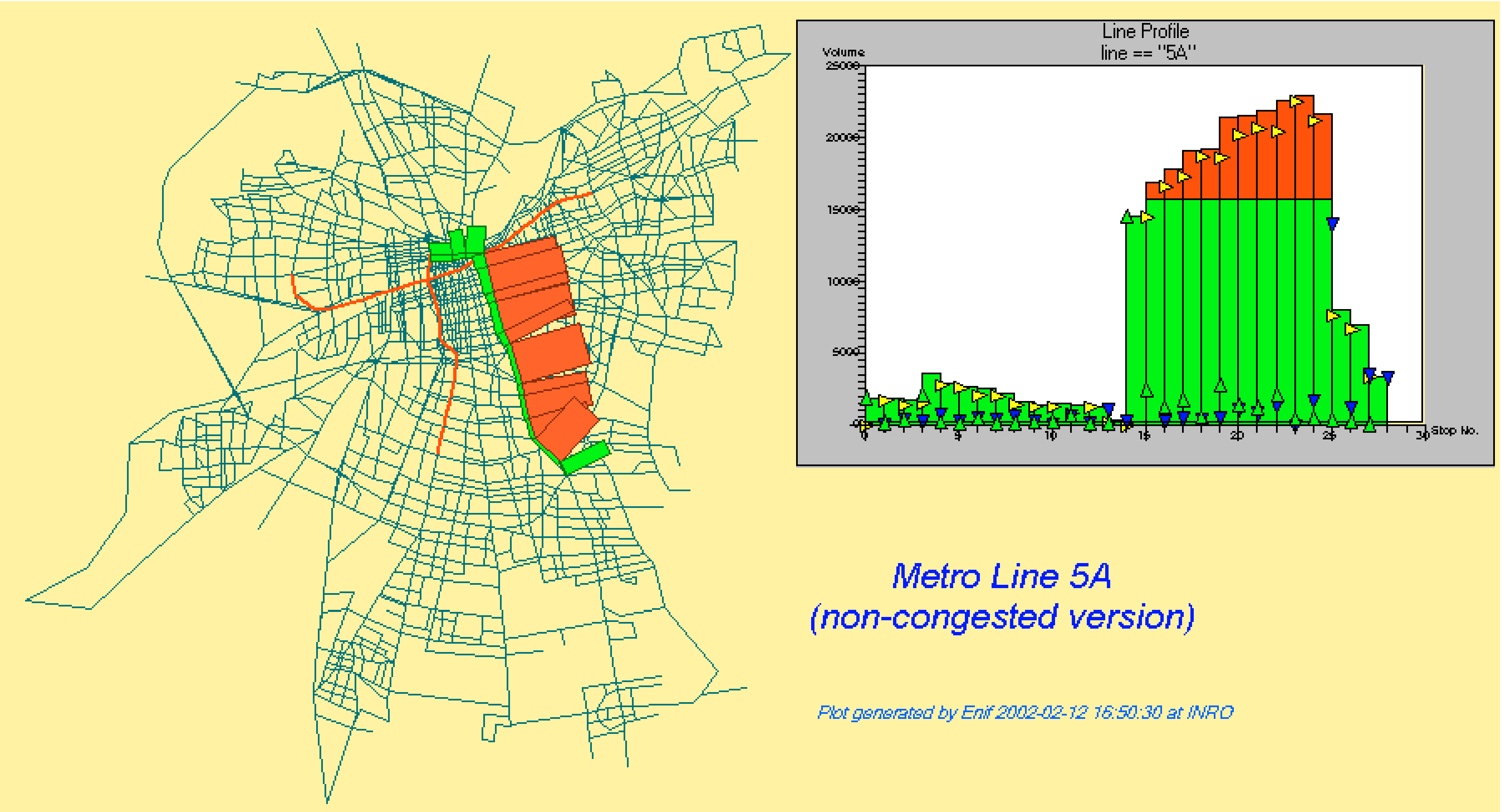


Auto-metro volume (non-congested vs. congested version)

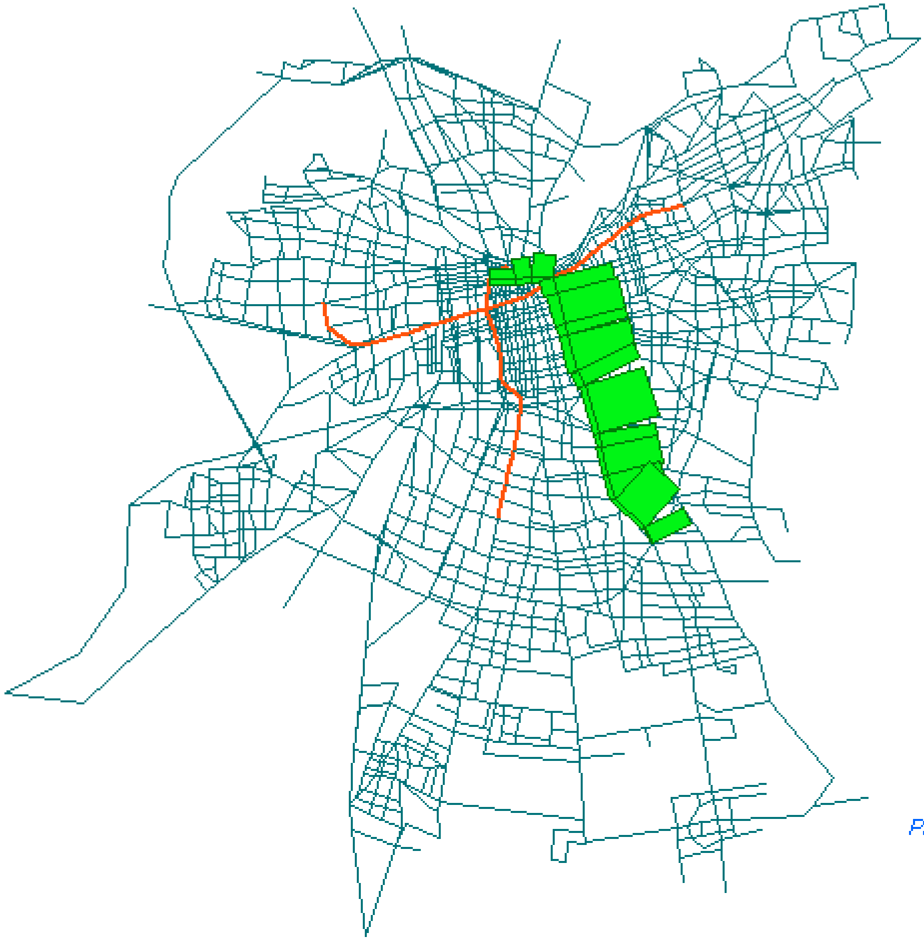


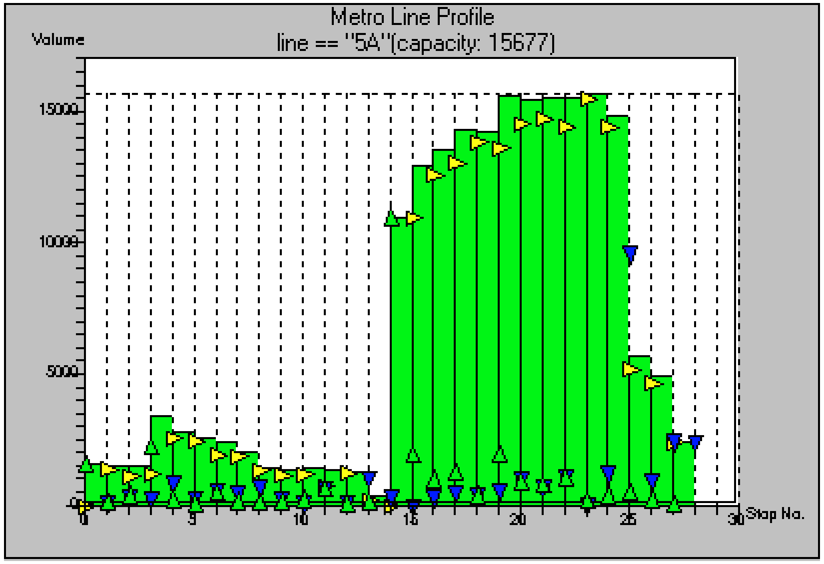


Metro Volume - metro 5A, non-congested version



Metro Volum2 - metro 5A, congested version





Metro Line 5A
(congested version)

Plot generated by Enif 2002-02-12 15:09:32 at INRO

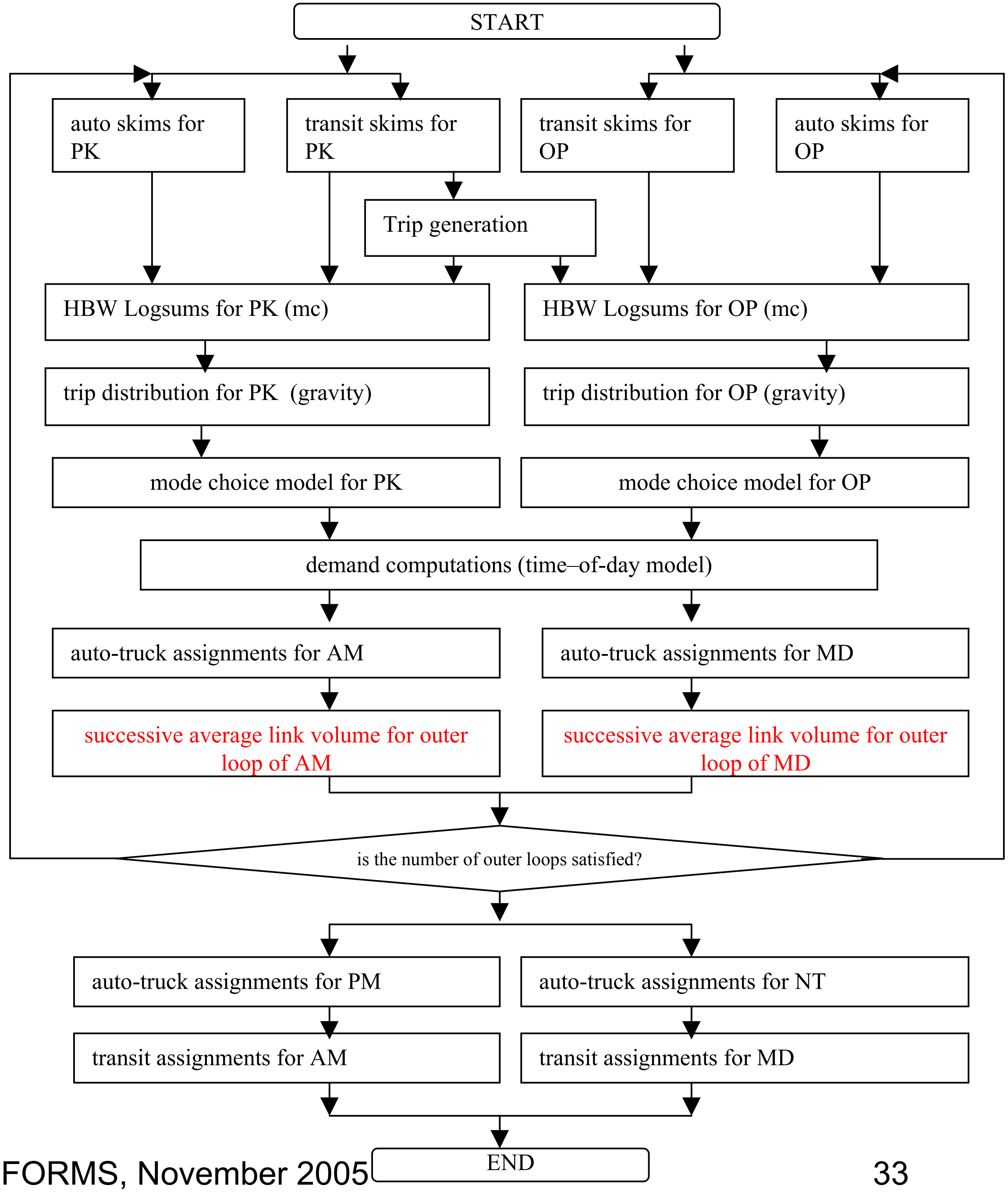
Some Complex Applications

- Los Angeles, California
- Toll Highways Poznan, Poland
- Toll Bridge, Montreal,

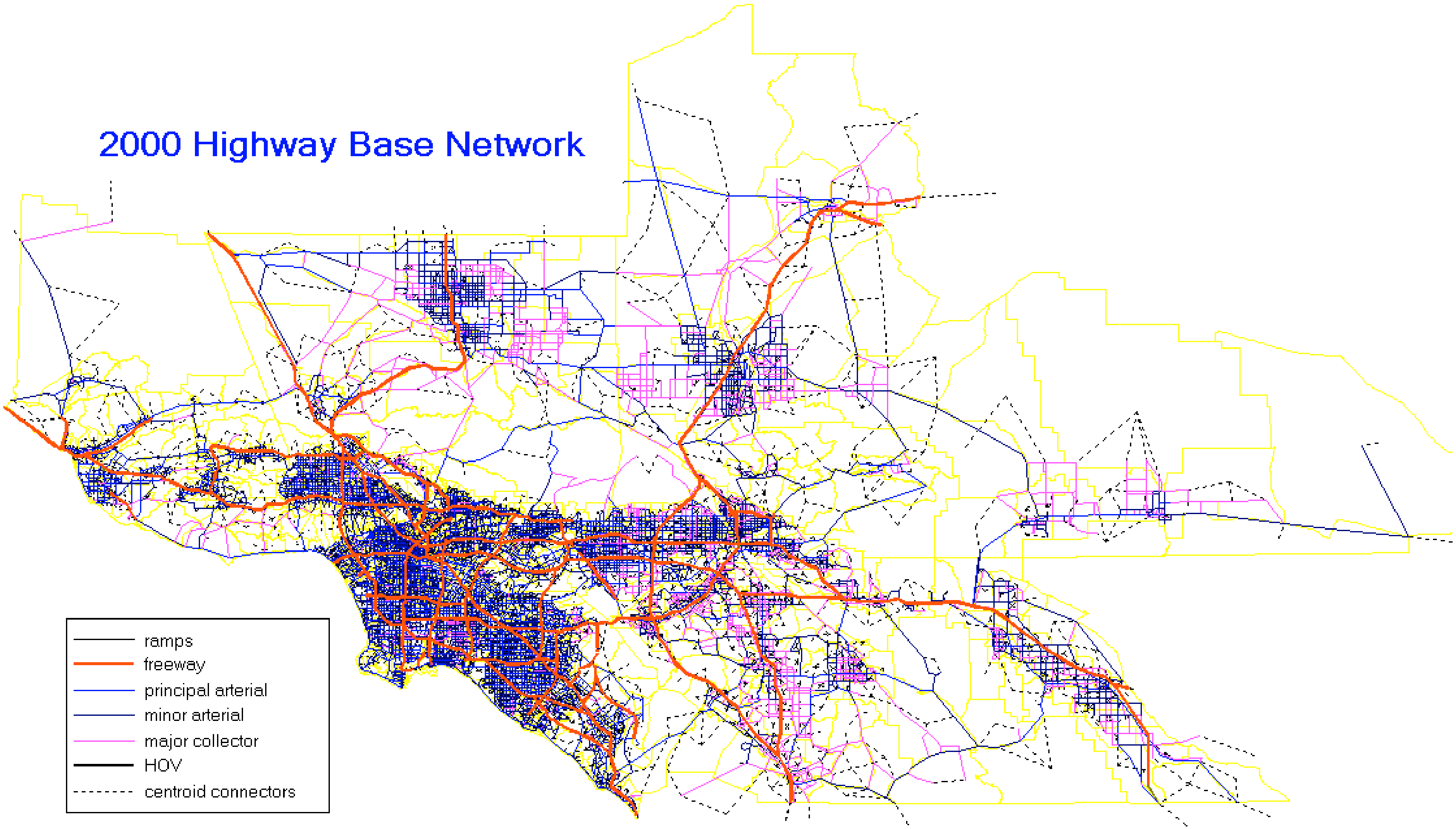
The SCAG Regional Transportation Planning Model

- A complex and very large scale model
- Lack of rigorous formulation; network equilibrium sub-models
- A multi-class multi-mode network equilibrium model with asymmetric cost function is part of the model
- Heuristic solution algorithm based on an outer averaging scheme

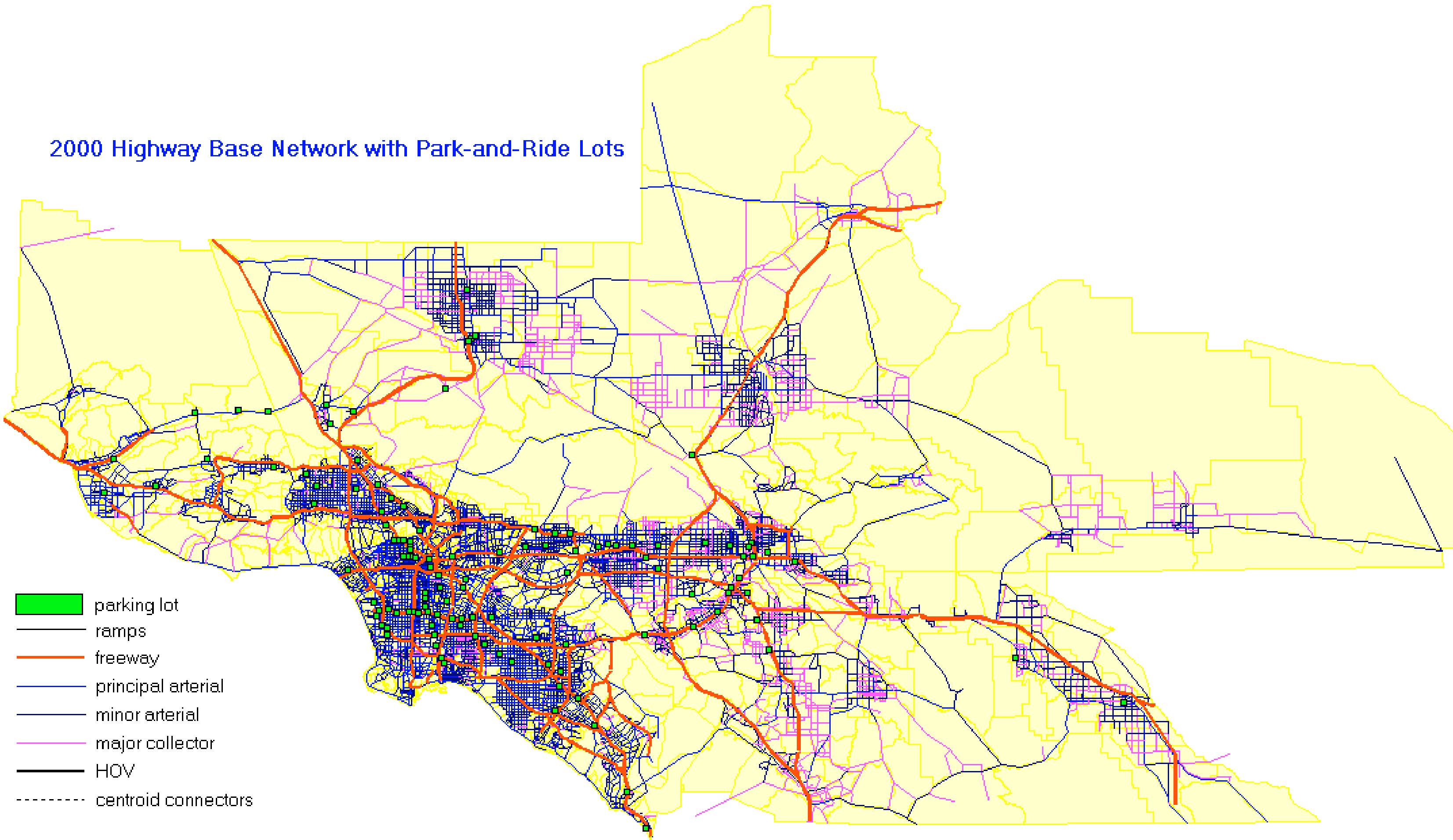
SCAG MODEL FLOW CHART:



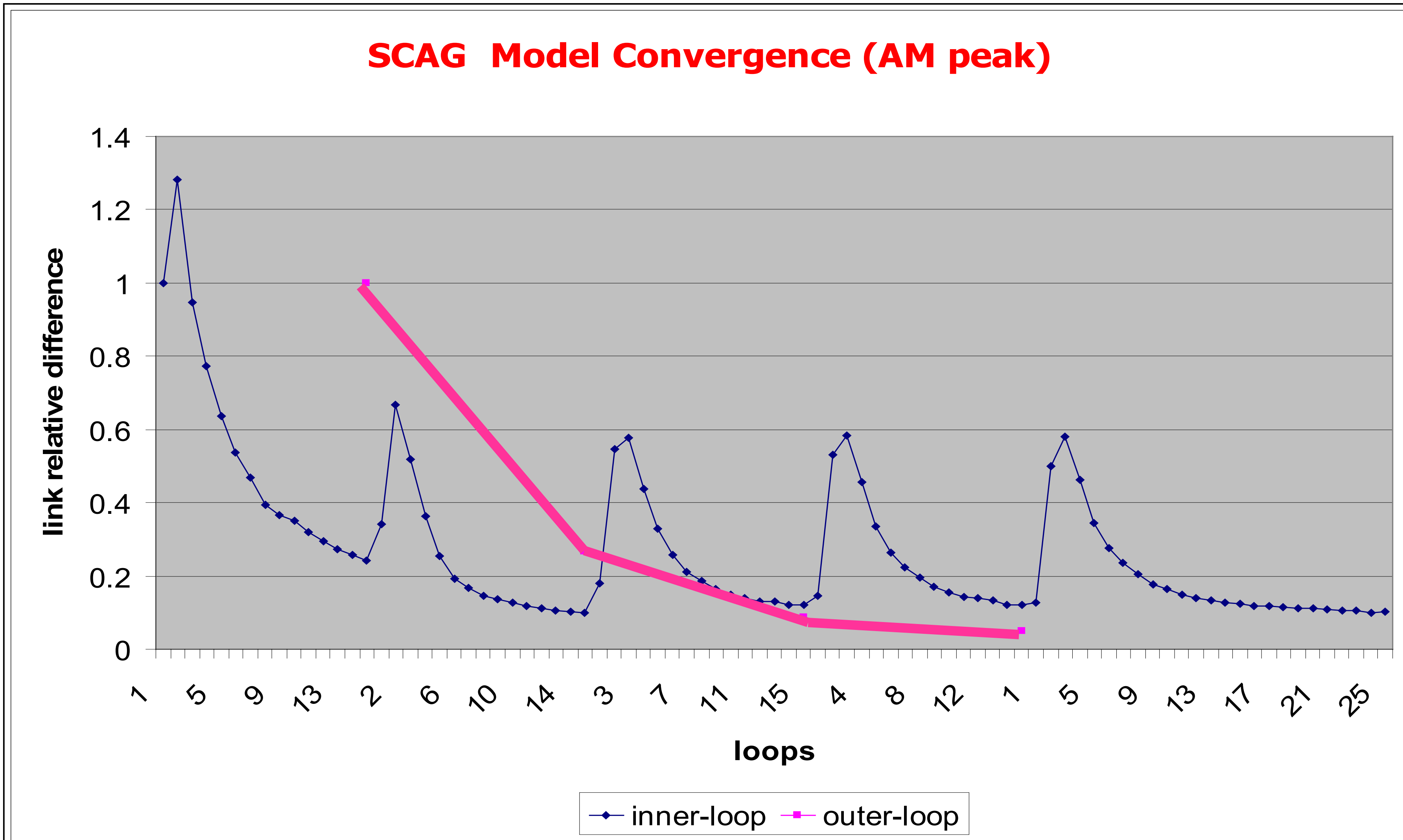
Network Overview - Highway Network
by facility type



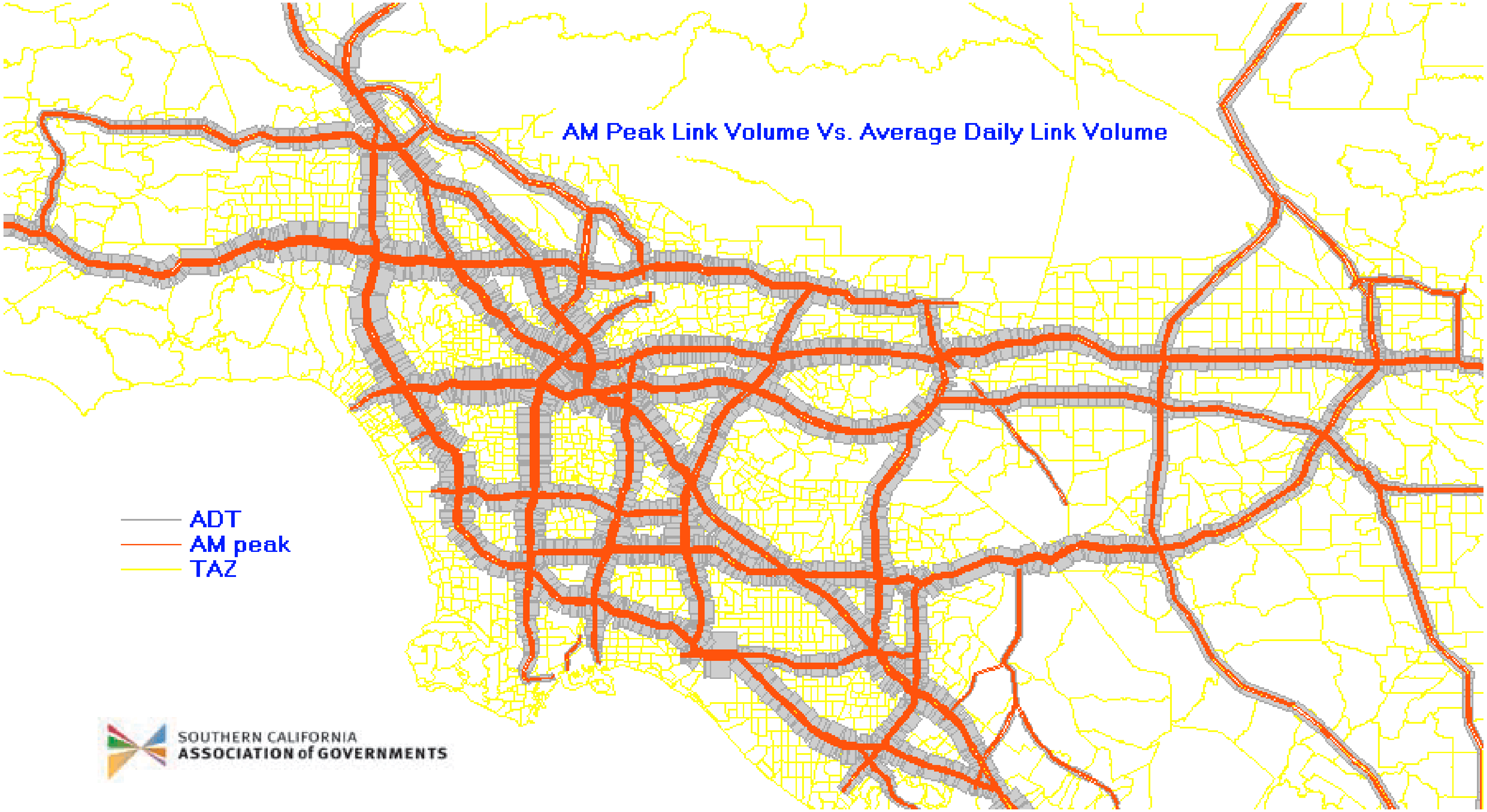
Network Overview - Highway Network
with parking lots



Equilibration Algorithm Convergence Results

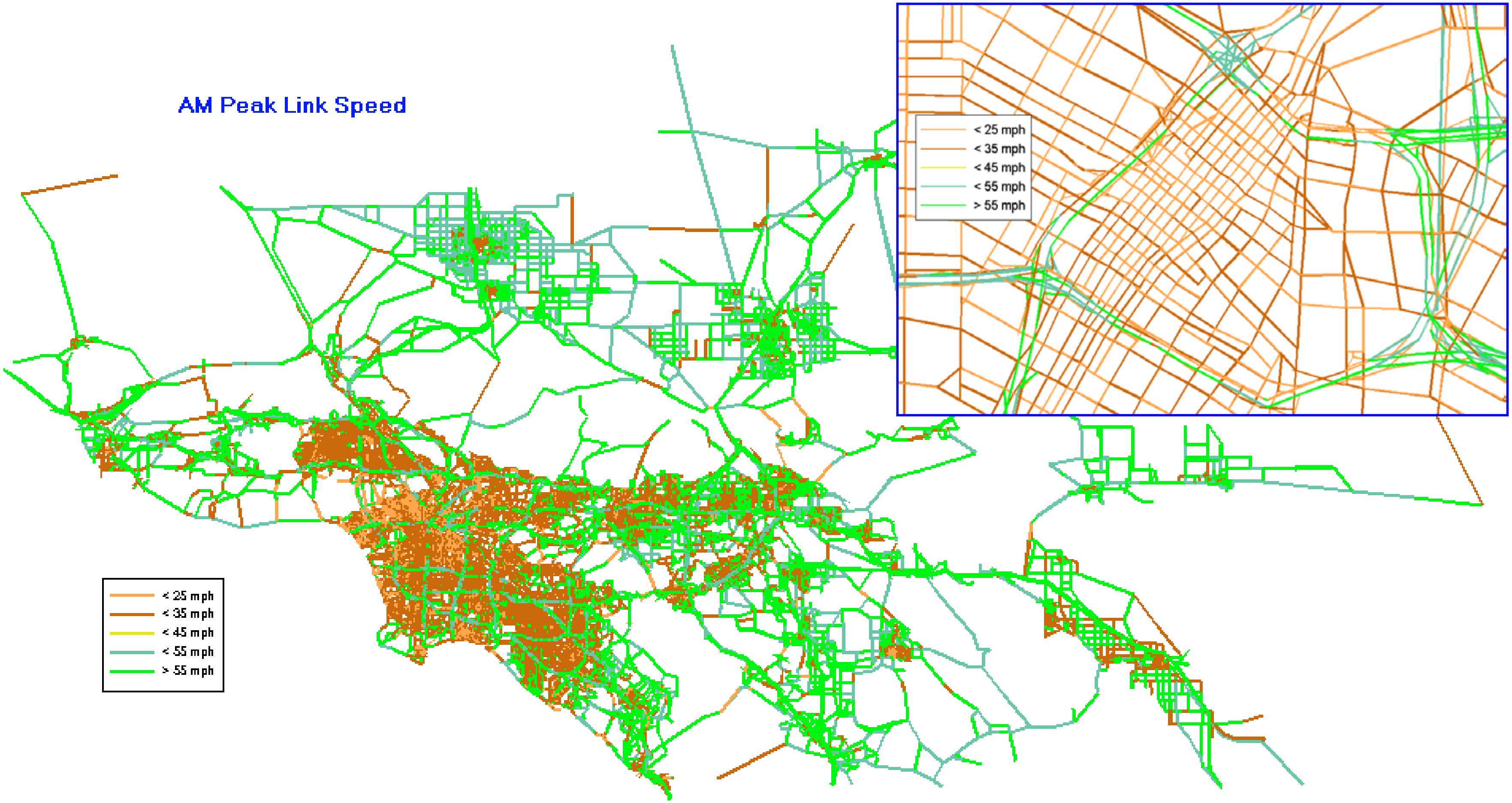


Assignment Results
AM peak volume



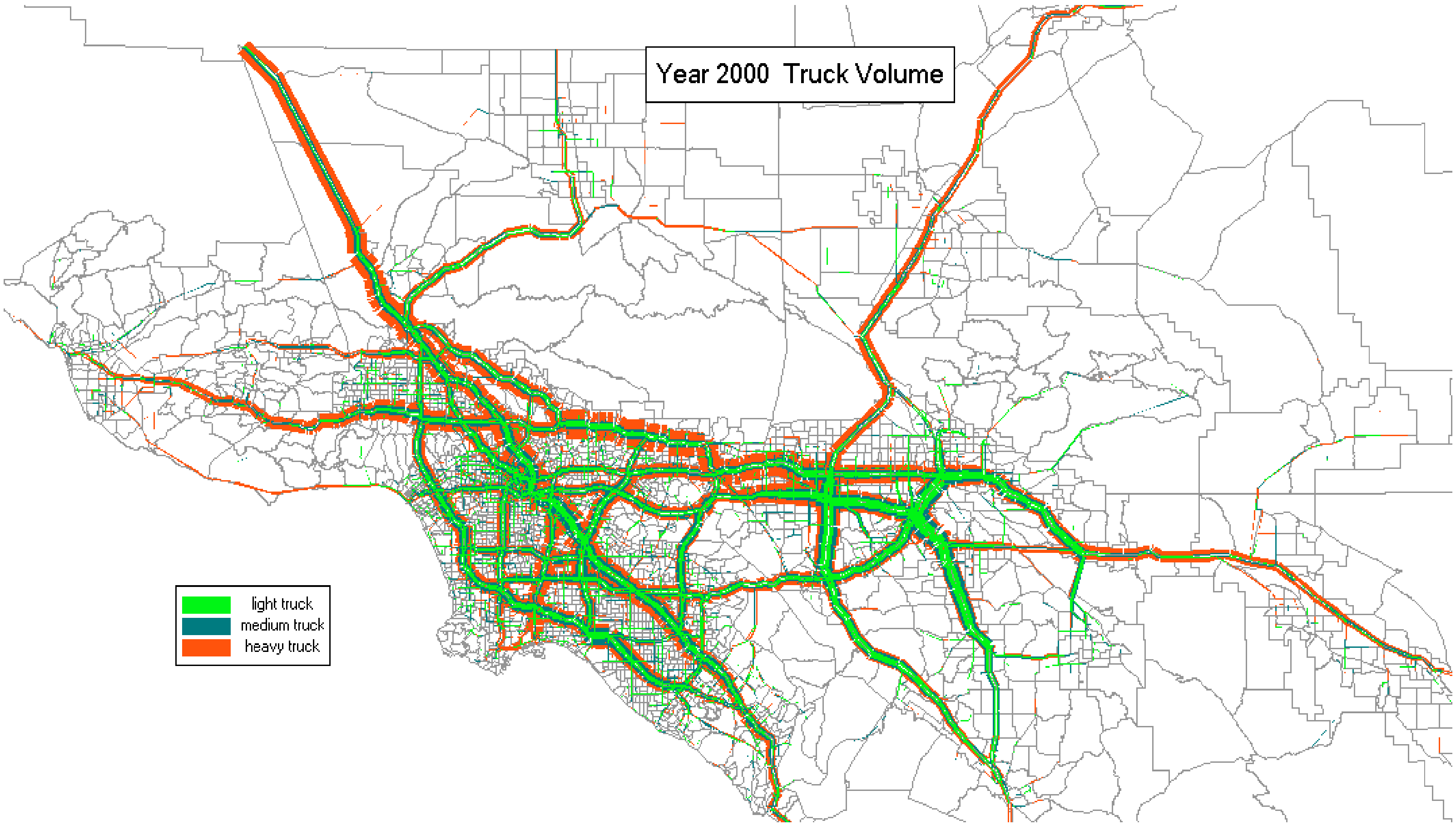
Assignment Results

AM link speed

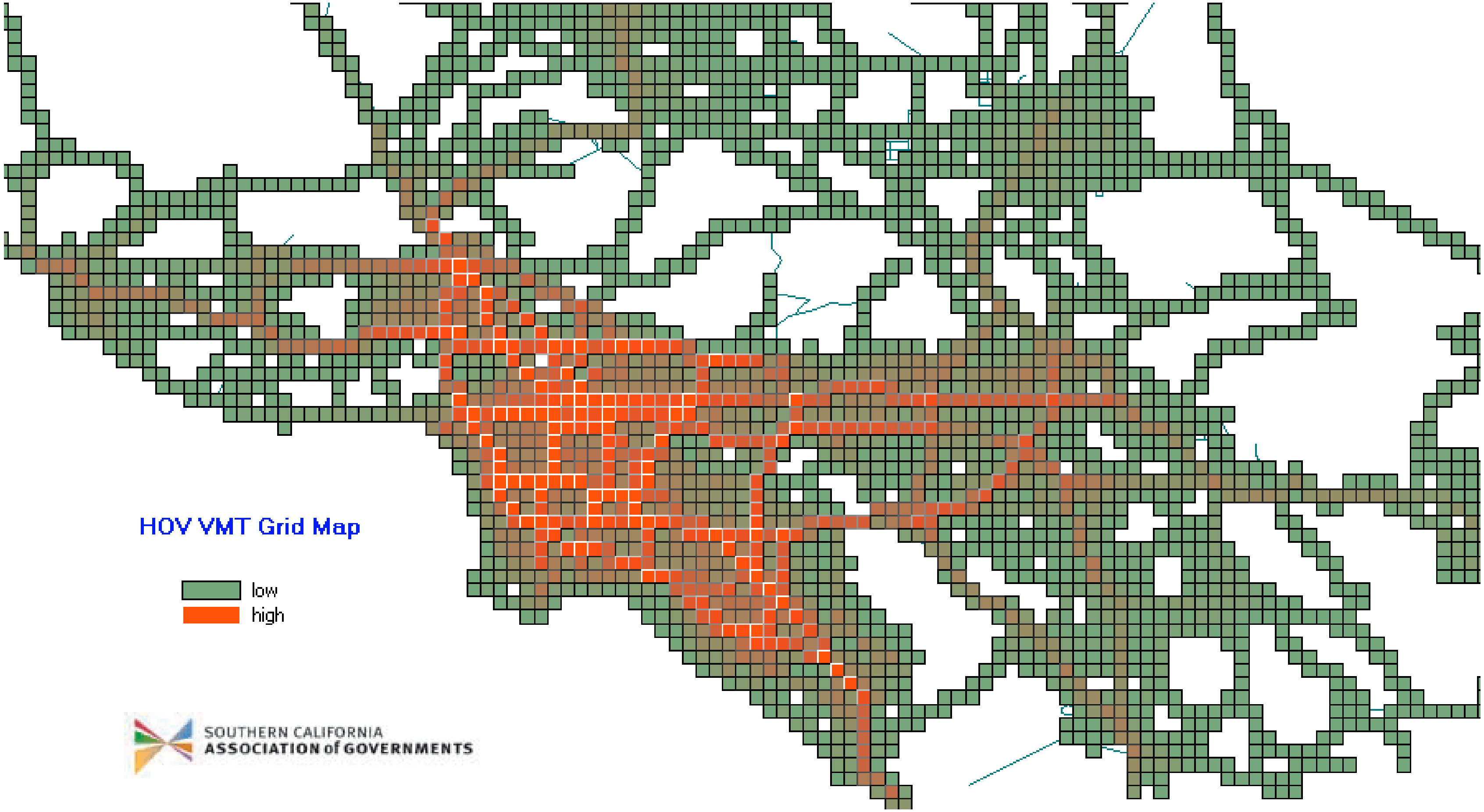


Assignment Results

AM truck volume by class

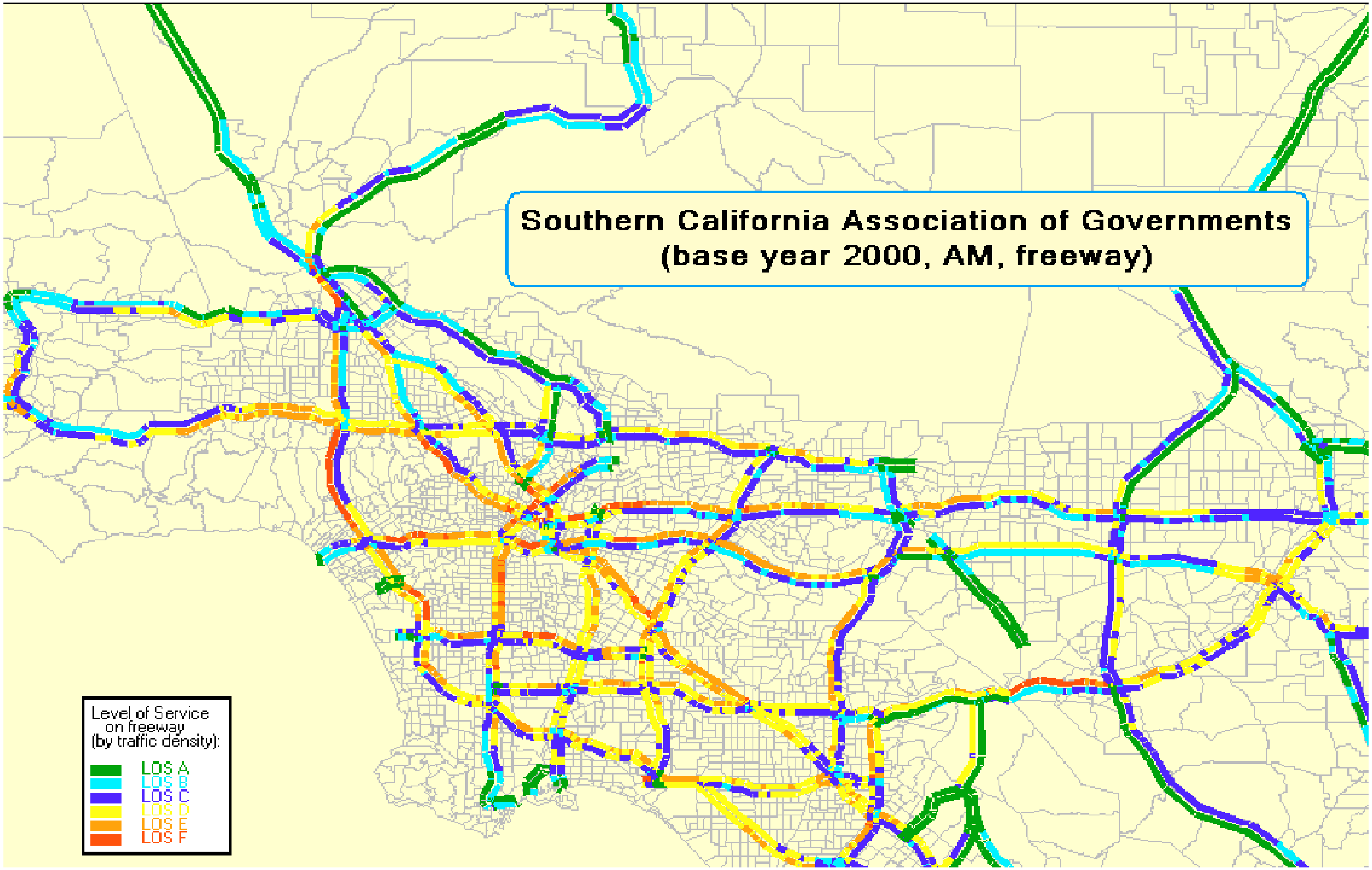


Assignment Results
AM HOV VMT Grid Map

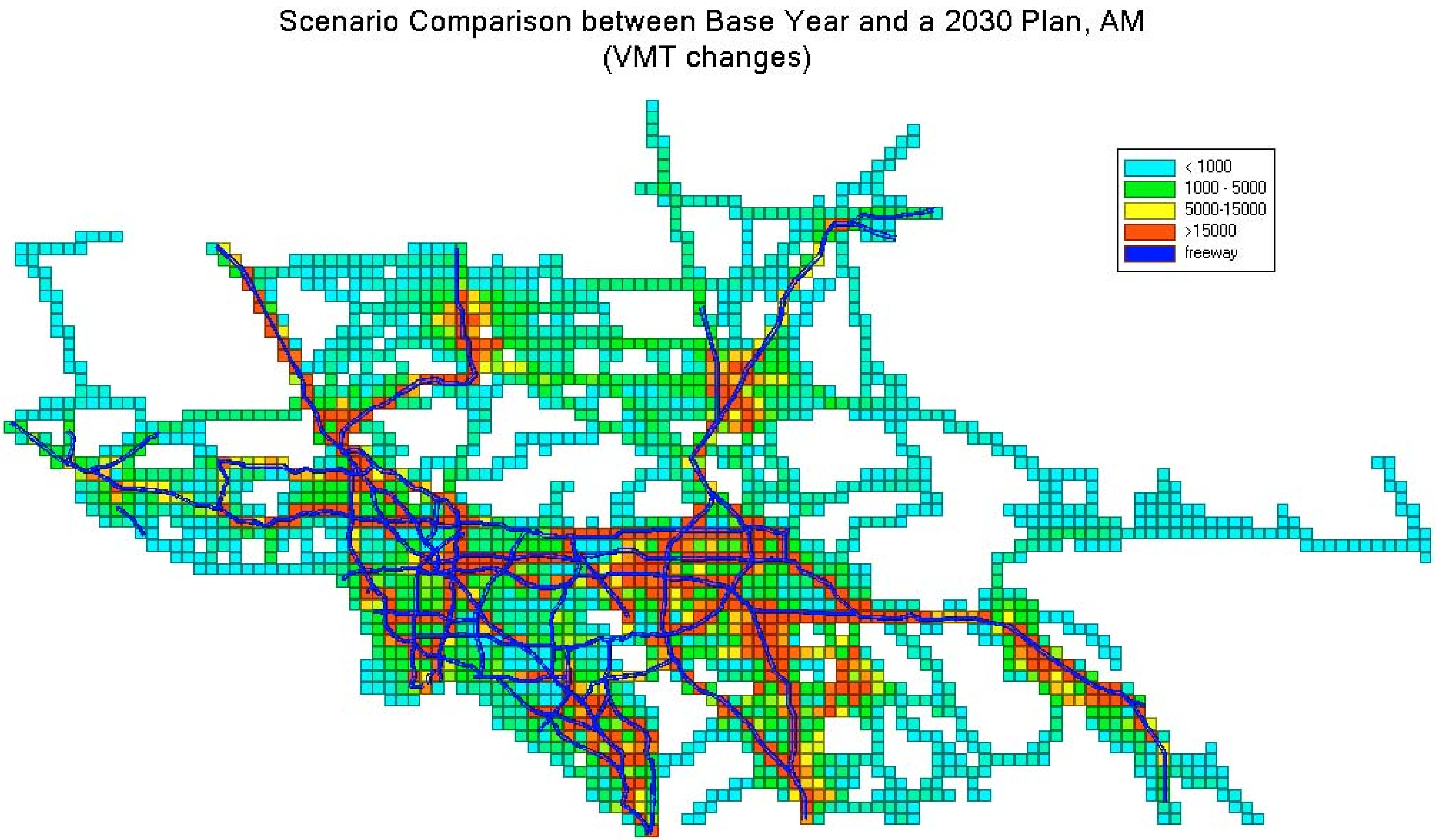


SOUTHERN CALIFORNIA
ASSOCIATION of GOVERNMENTS

Assignment Results
HCM Level of Service



Scenario Comparison
VMT changes



Toll highway analysis – Poznan, Poland

It involves the following models:

-Multi-class equilibrium assignment with generalized costs

-Demand models for toll-no toll choice and future year demands

-Equilibration of the demand for toll highway and network performance

The multi-class equilibrium model with tolls

$$\min \sum_{a \in A} \int_0^{v_a} s_a(x)\,dx + \sum_{c \in C} \sum_{a \in A} v_a^c \theta^c t_a^c$$

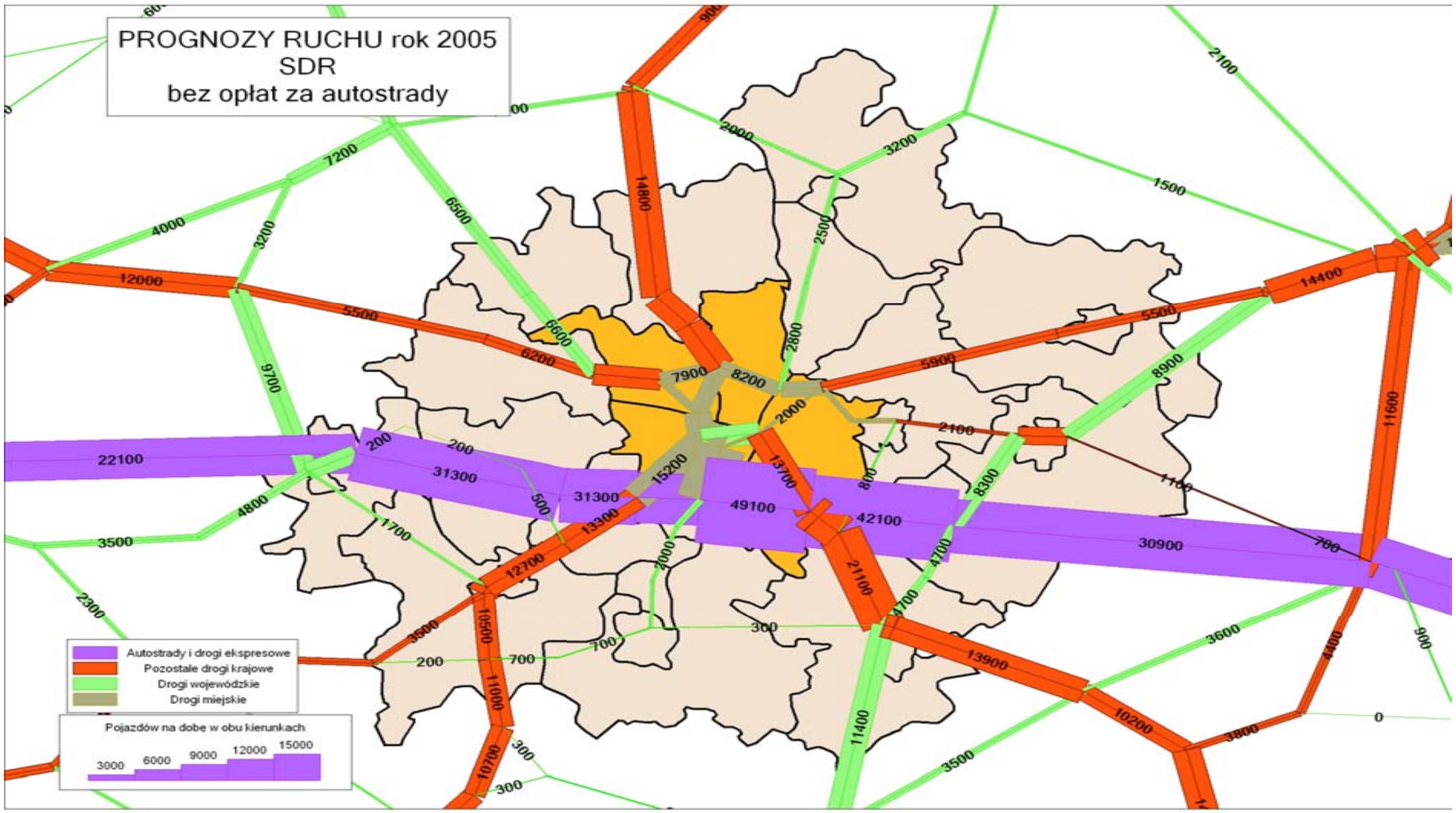
subject to $\displaystyle\sum_{k \in K_i^c} h_k = g_i^c, i \in I, c \in C$

$$h_k \geq 0, k \in K_i^c, i \in I$$

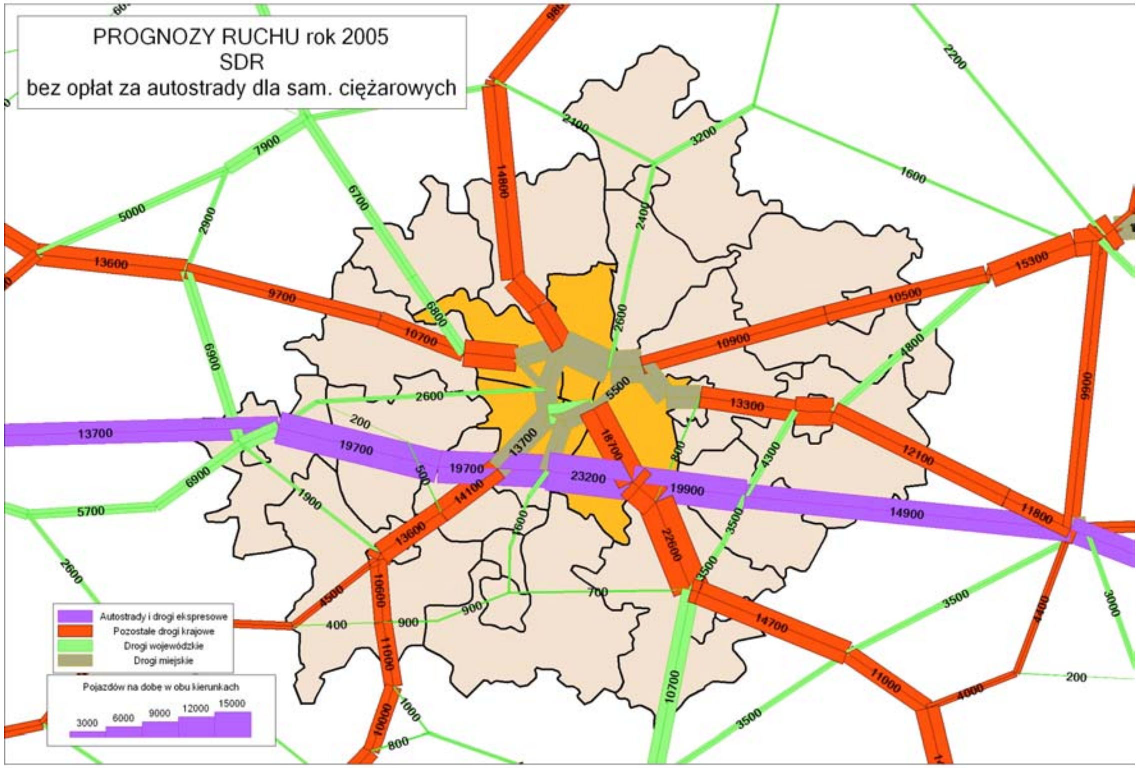
$$(v_a^c = \sum_{k \in K_i^c} \delta_{ak} h_k, a \in A, c \in C)$$

The numerical solution of this model is well known;
It is worthwhile to point out that the flows by class, v_a^c are not unique, nor are the path flows h_k, but the arc flows v_a are unique.

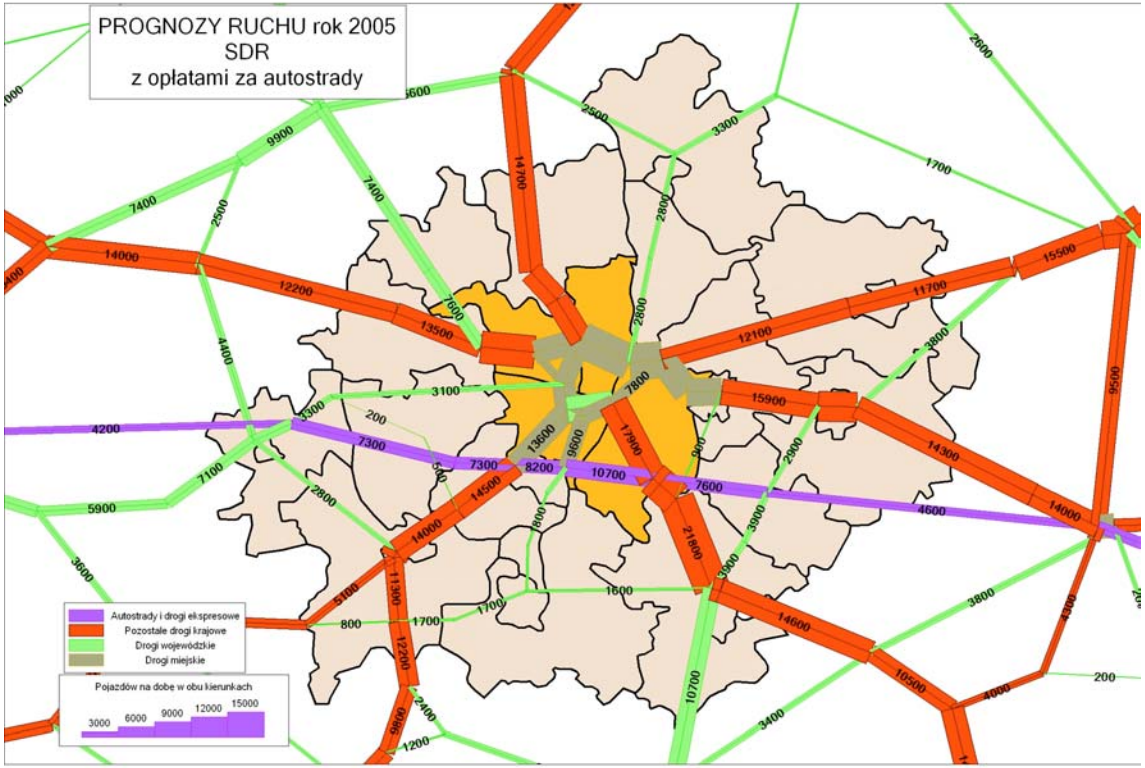
Base year – No Toll



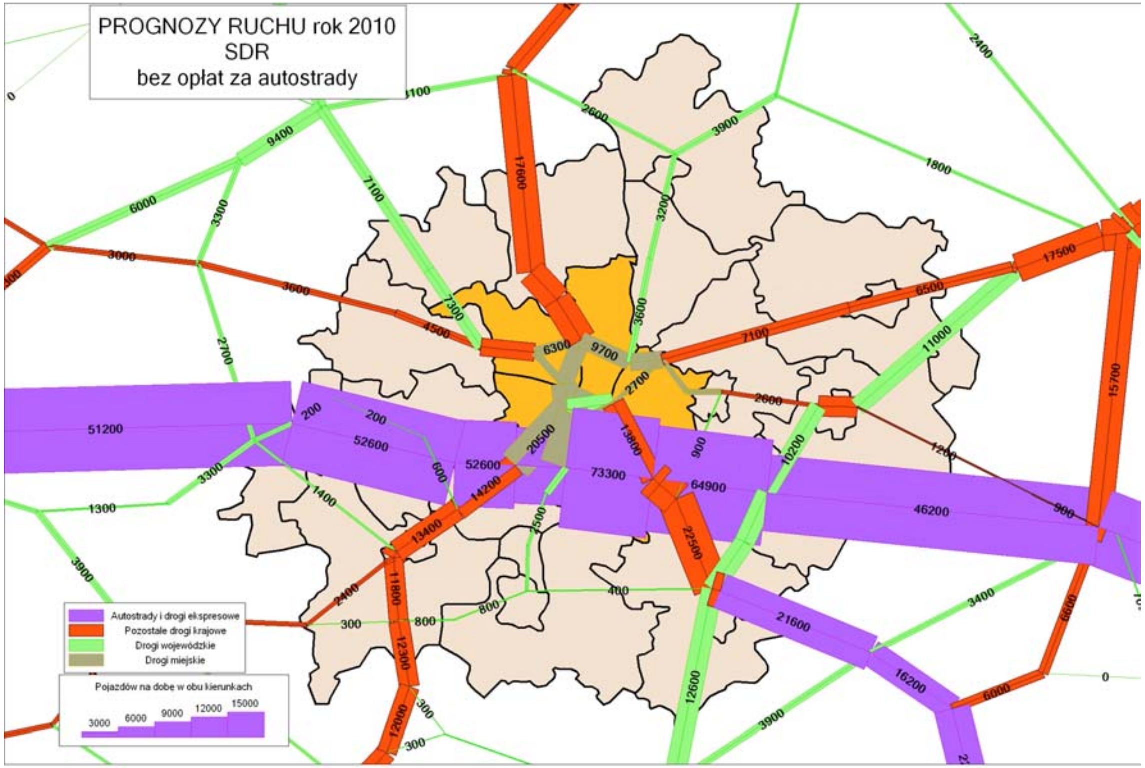
Base year – Medium Cost Toll



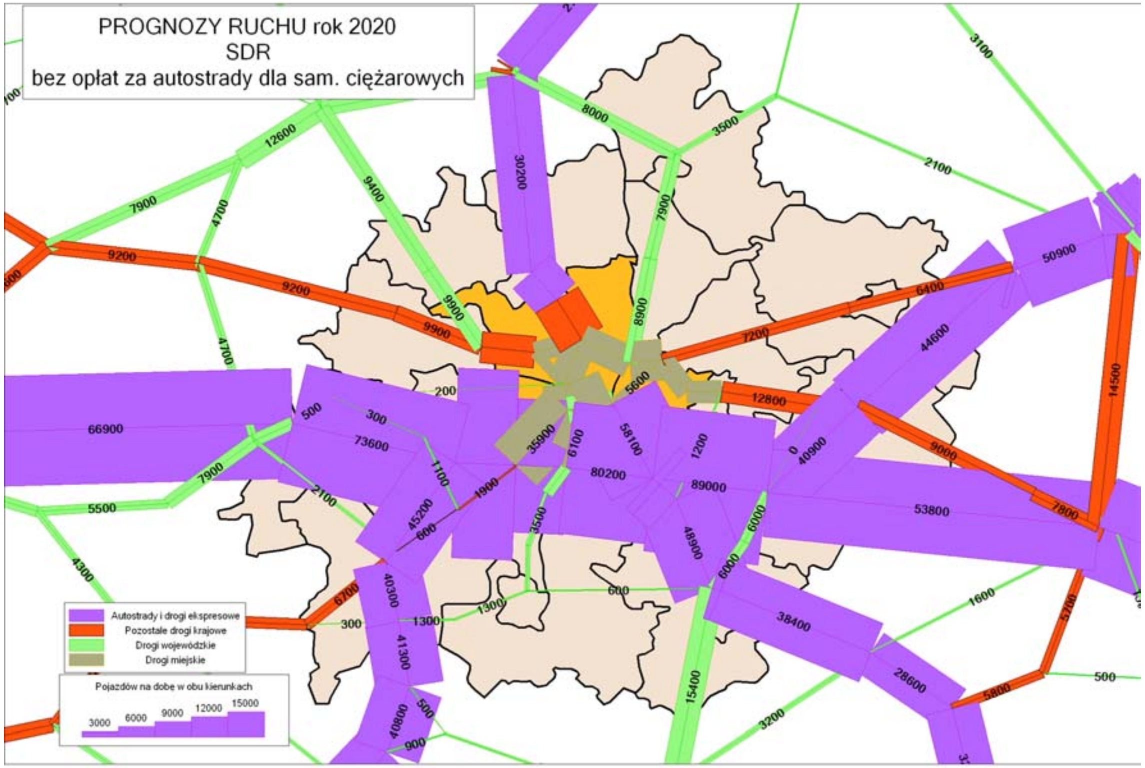
Base year – High Toll



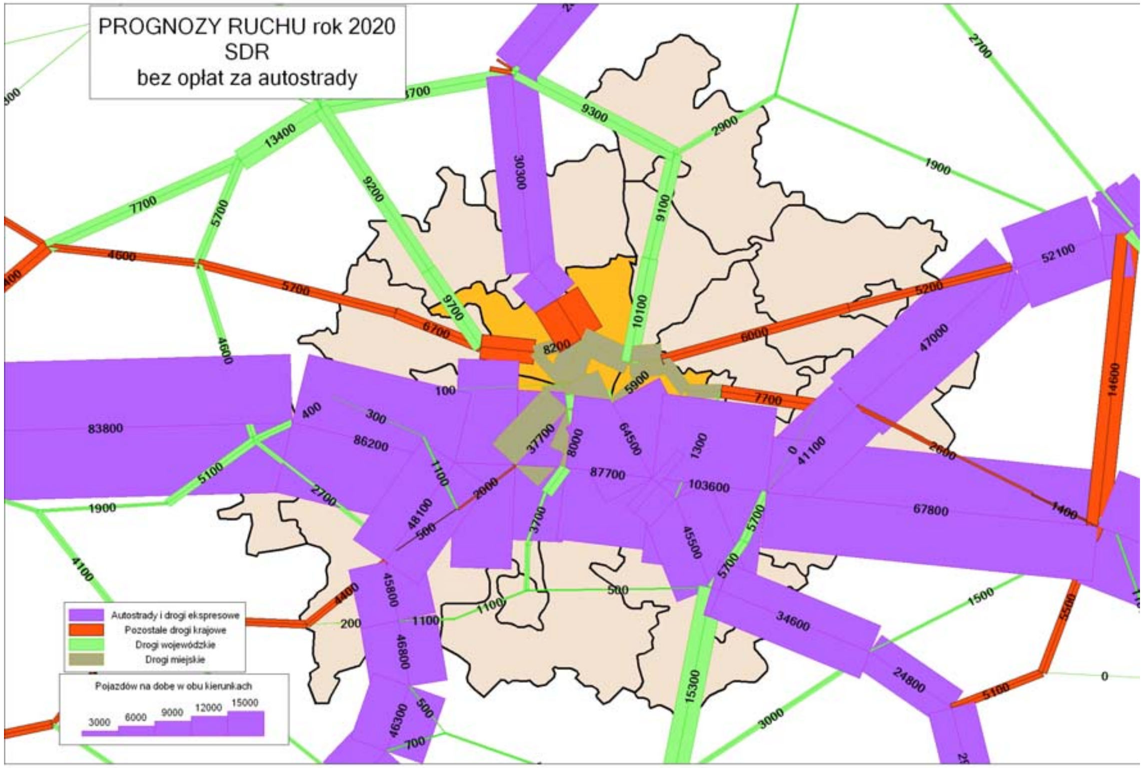
Year 2010 – High Cost Toll



Year 2020 – Medium Cost Toll



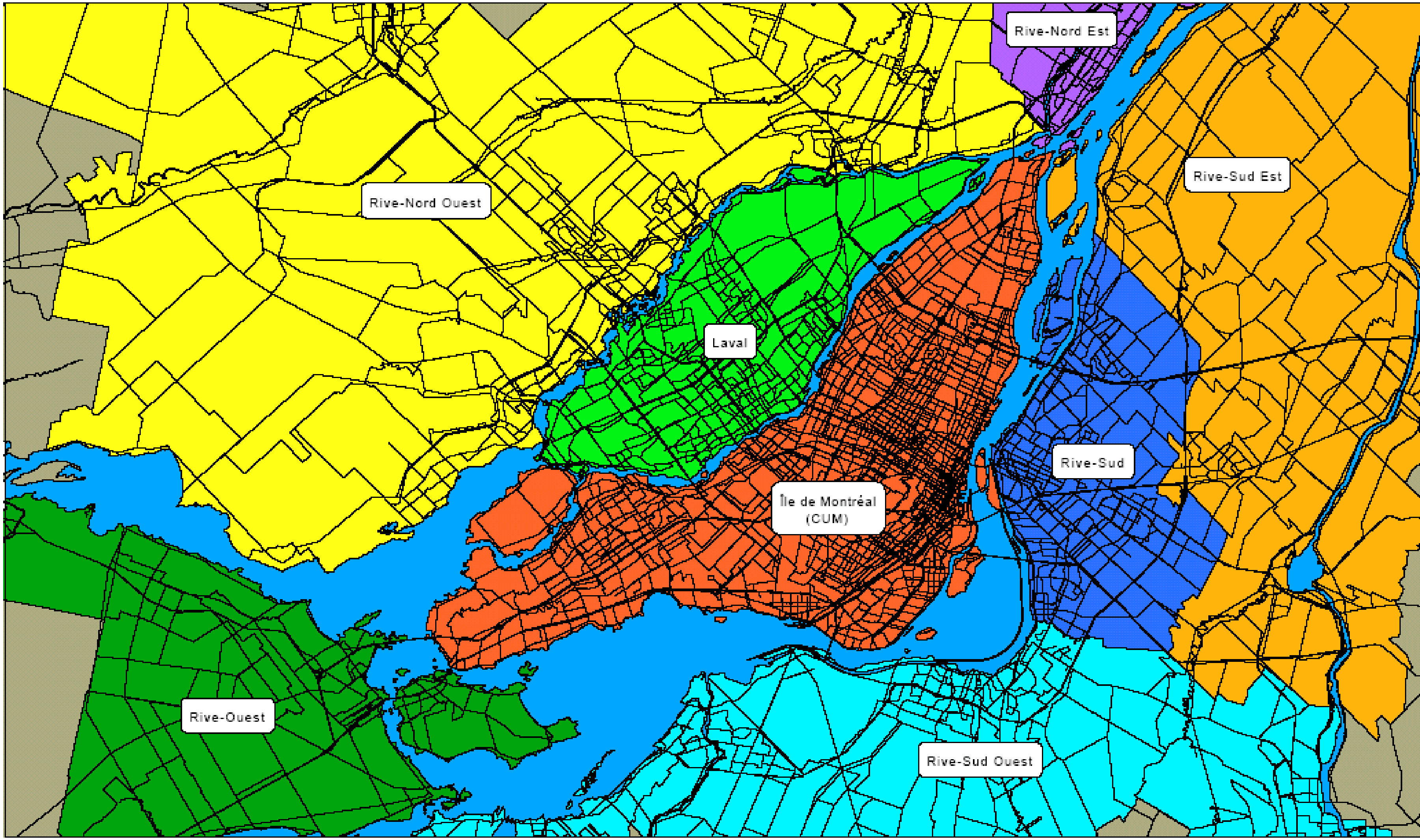
Year 2010 – Low Cost Toll



Toll Bridge Study – Montreal, Canada

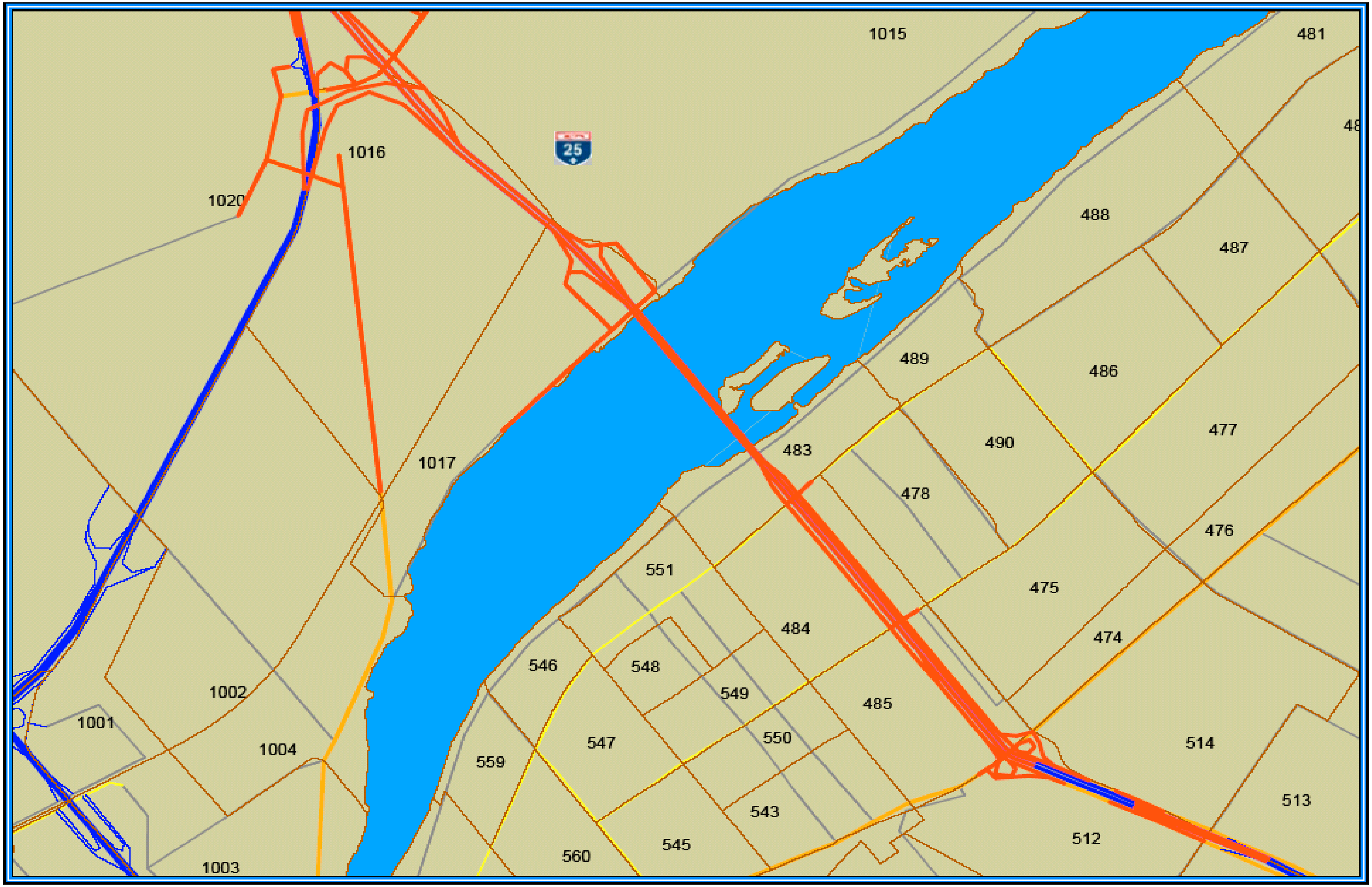
- Study carried out by the Ministry of Transportation of Quebec
- The analysis relied heavily on the analysis of paths generated by the assignment algorithm

The Montreal Region

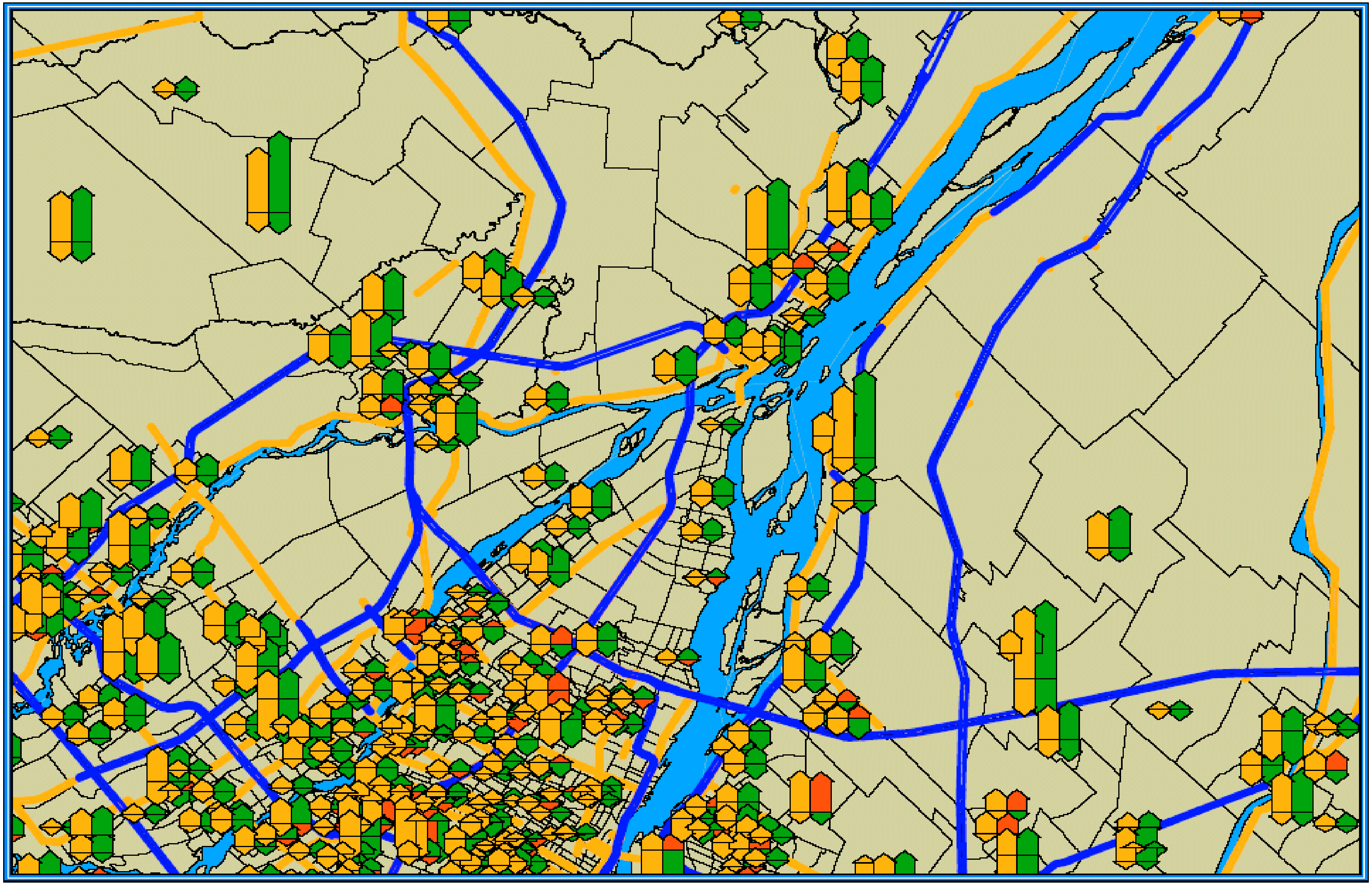


The new proposed bridge

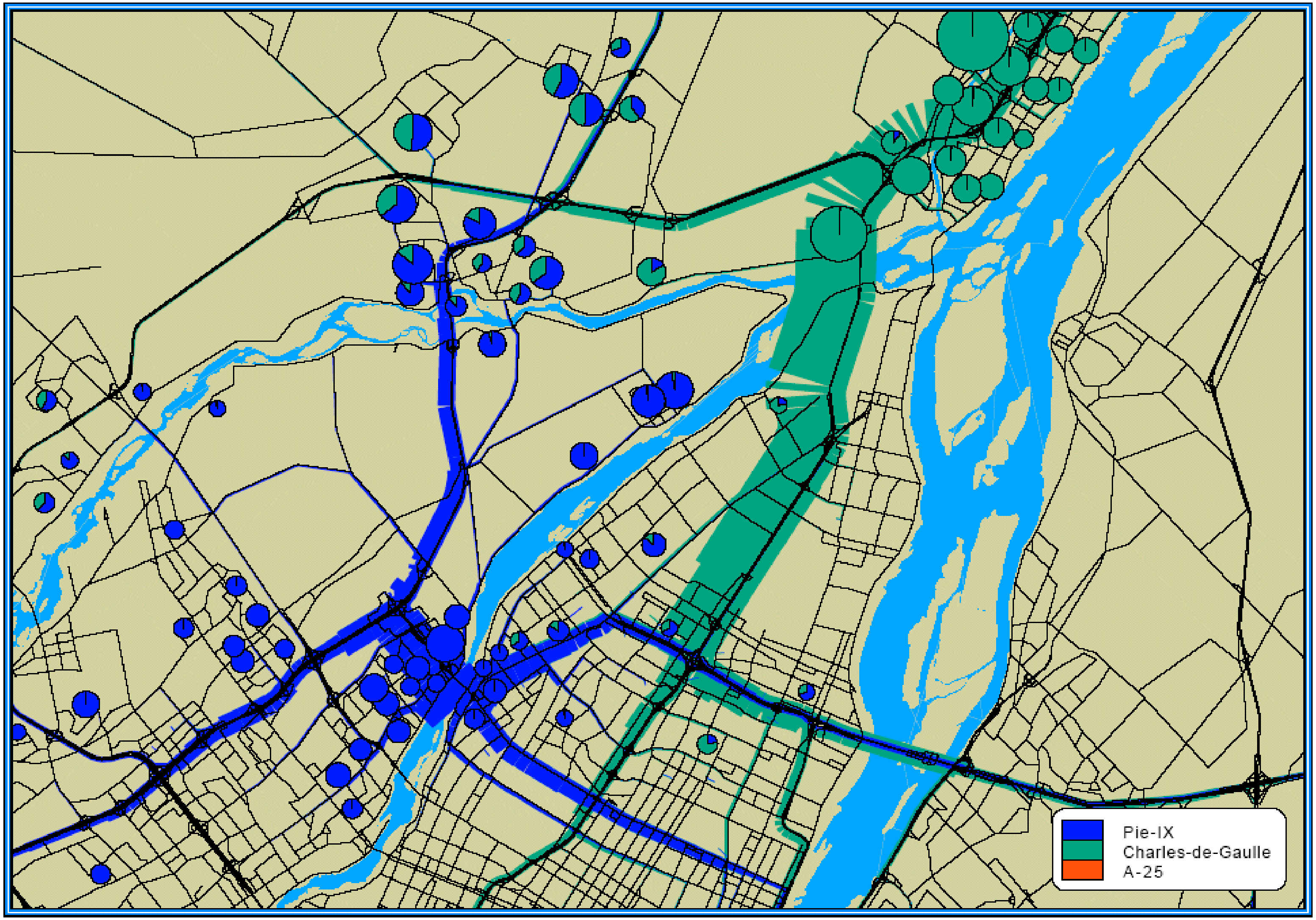
Zones around Bridge



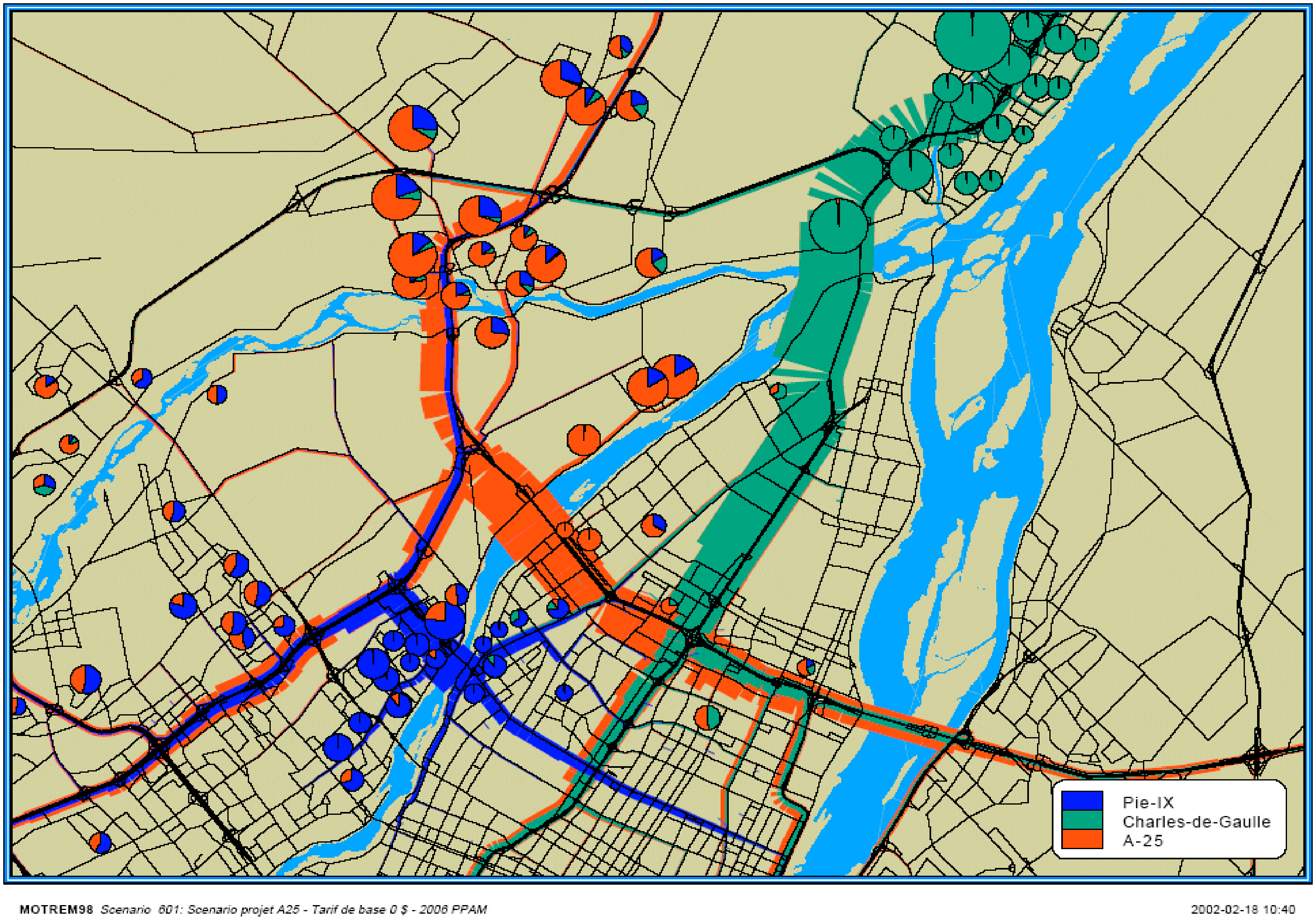
Demand for Current and Future Year



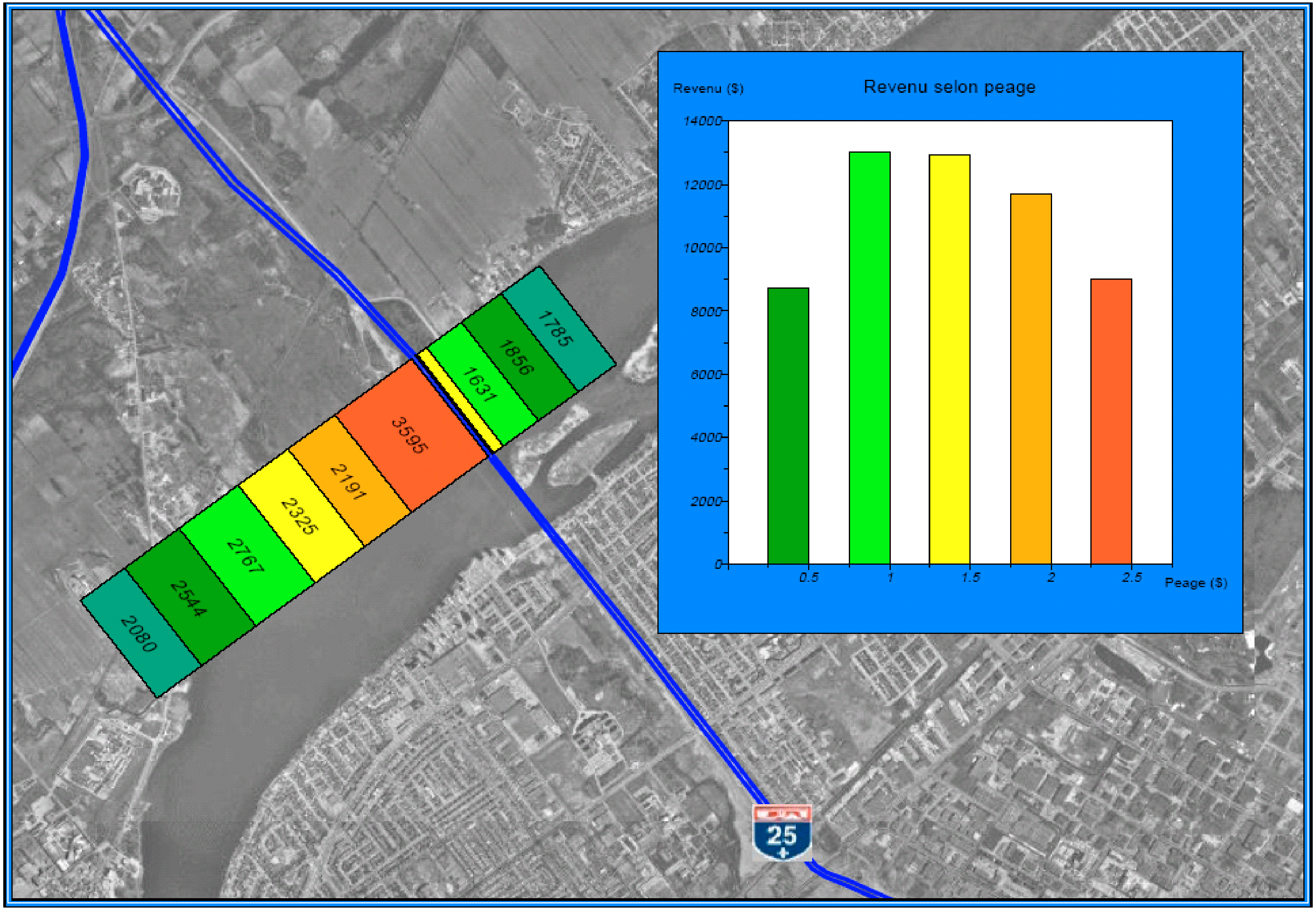
The Current Bridge Flows



Bridge Flows with New facility



Toll Income with Various Toll Levels

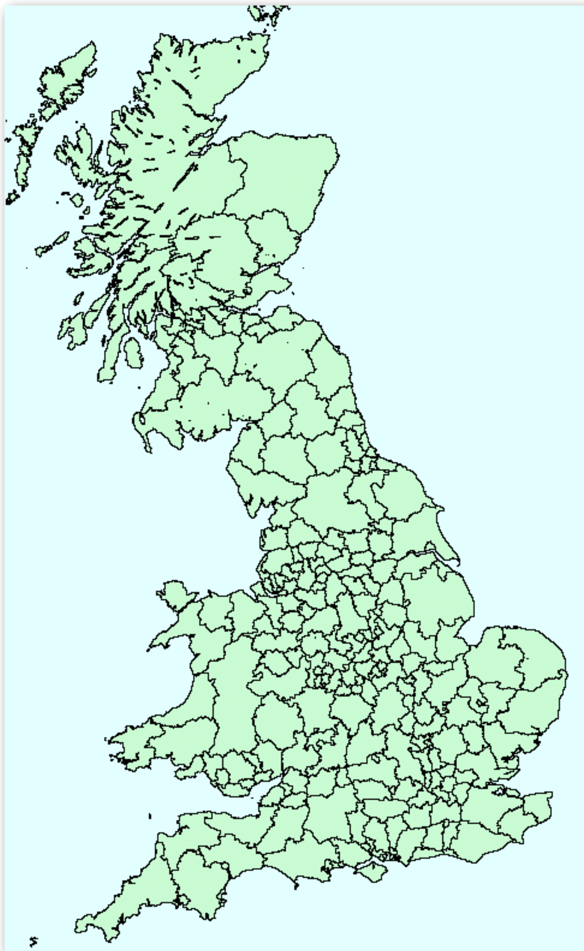


National Models

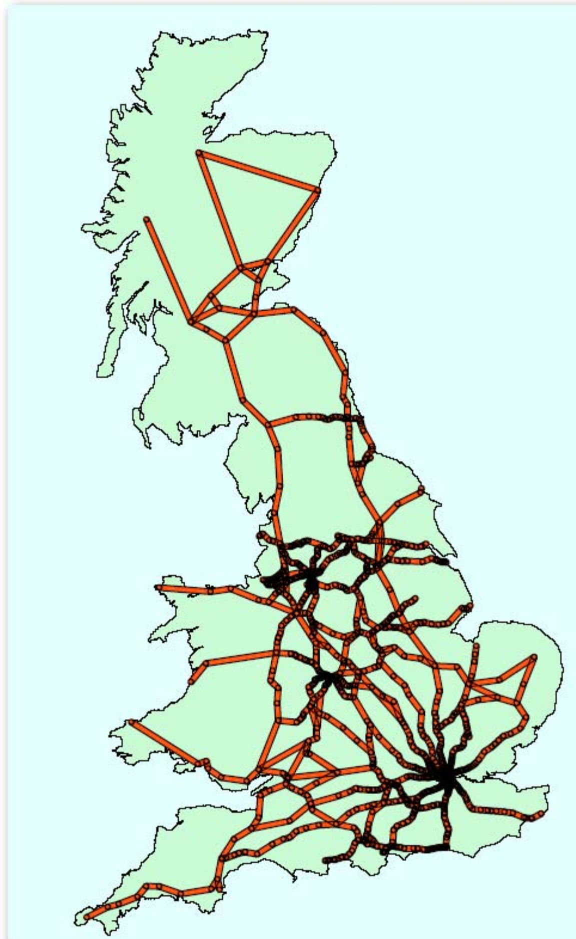
- These are very large multi-modal multi-class models
- The underlying demand models are rather complex and the running times are very high
- An example of a national model is the PLANET model developed for British Rail

The PLANET Model – British Rail

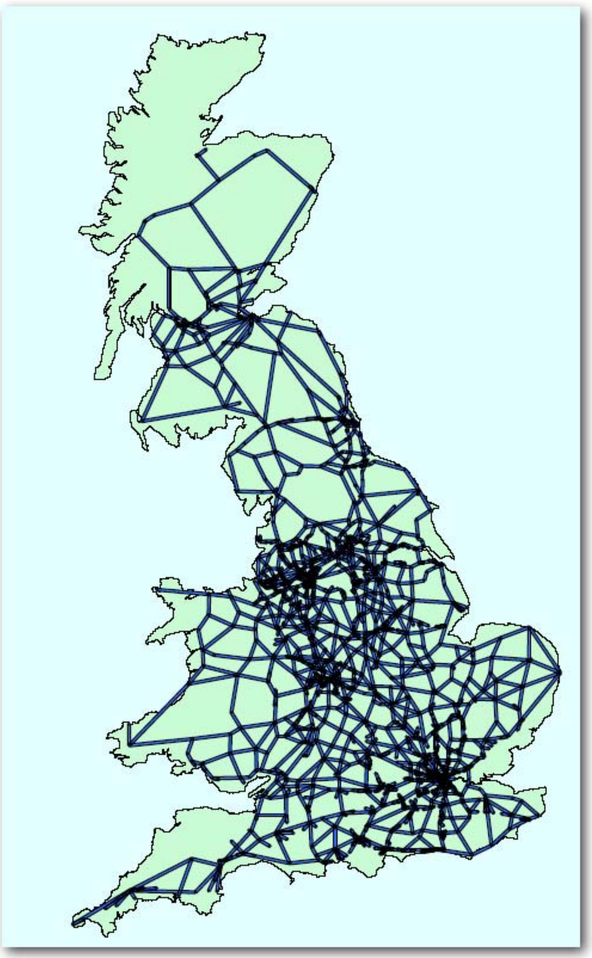
Zones



Rail



Road



Dynamic Network Equilibrium Model

- Solved by a hybrid optimization-simulation model a discretized version of a variational inequality formulation of a dynamic network equilibrium model
- The theoretical properties of the model are difficult to establish
- Wardrop's user equilibrium in a temporal framework is a basis for the model

Dynamic equilibrium

variables

$(0,T) =$ demand period

$t =$ assignment interval $t \in (1,T)$

$I =$ set of OD pairs

$i =$ OD pair, $i \in I$

$K_i =$ set of paths for i

$g_i(t) =$ demand for OD pair i

$h_k(t) =$ flow (t) on path k

$s_k(t) =$ travel time (t) on path k

constraints

$$\sum_{\forall k \in K_i(t)} h_k(t) = g_i(t)$$

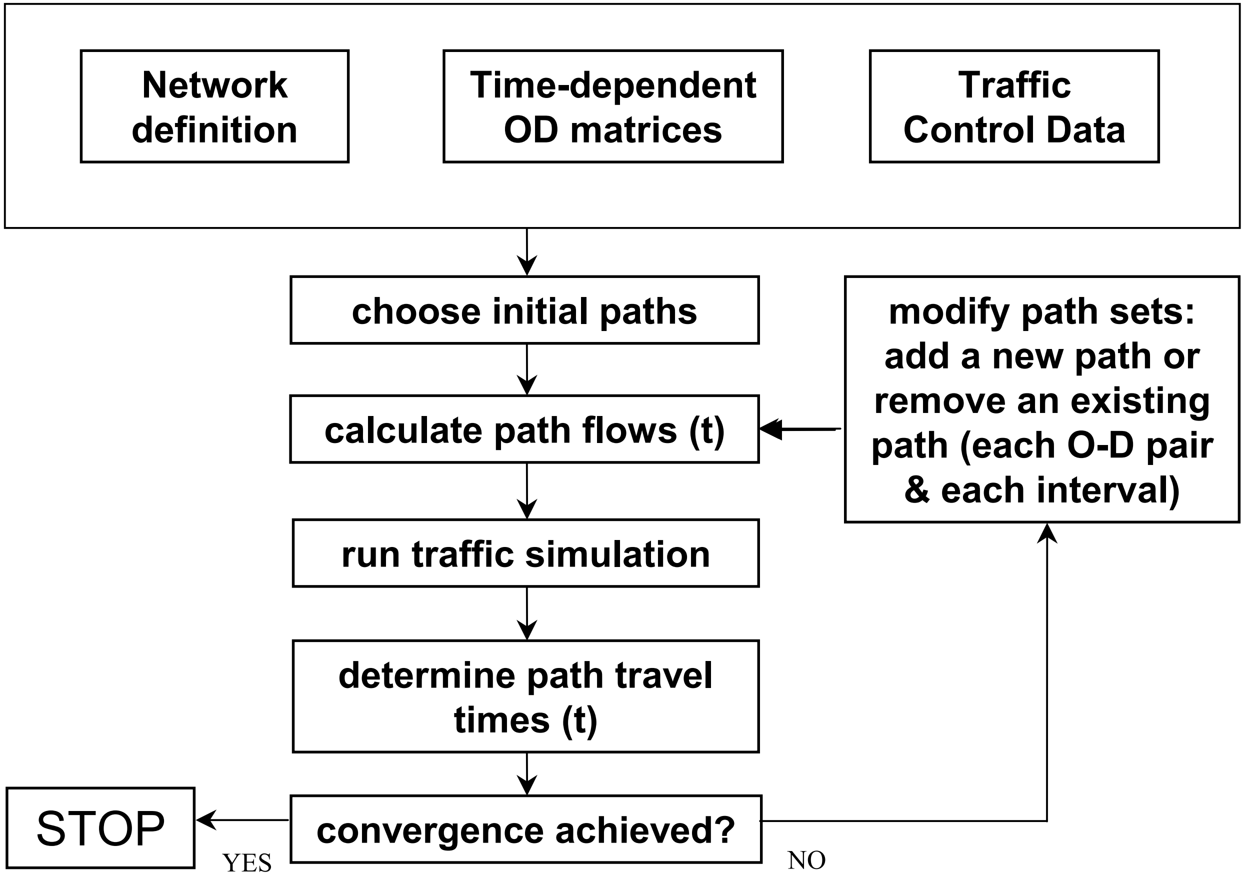
$$h_k(t) \geq 0$$

equilibrium conditions

$$u_i(t) = \min_{k \in K_i} \{s_k(t)\}$$

$$s_k(t) \begin{cases} = u_i(t) & \text{if } h_k(t) > 0 \\ \geq u_i(t) & \text{otherwise} \end{cases}$$

Dynamic assignment model



Traffic simulation model

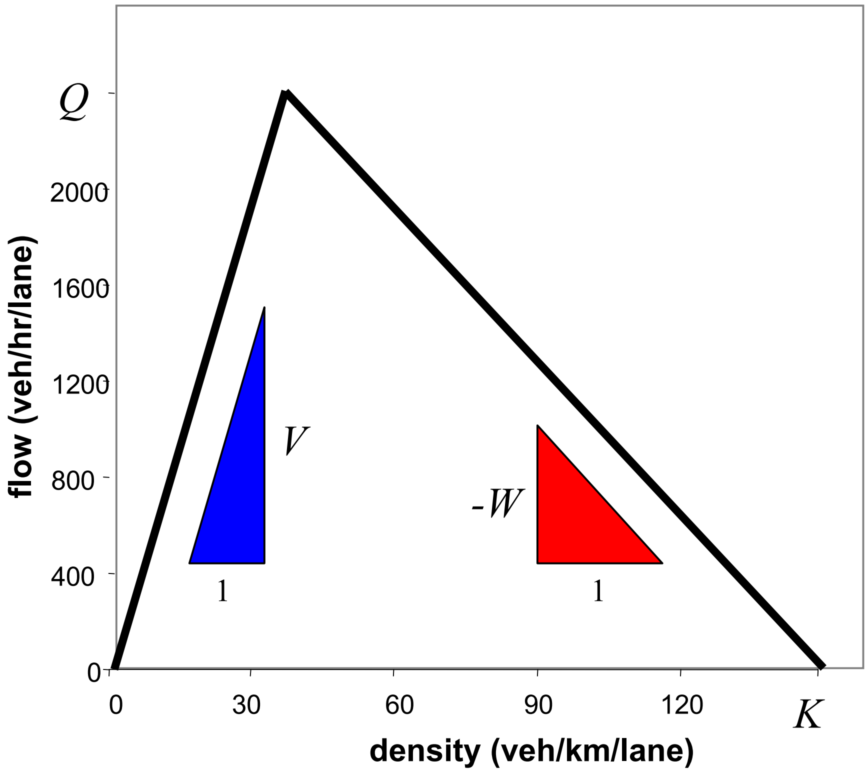
simplified model of vehicle interactions allows for an efficient event-based simulation

- car following
- lane changing
- gap acceptance

sophisticated lane selection heuristics

- local lane selection rules
- stochastic look-ahead strategy

fundamental diagram



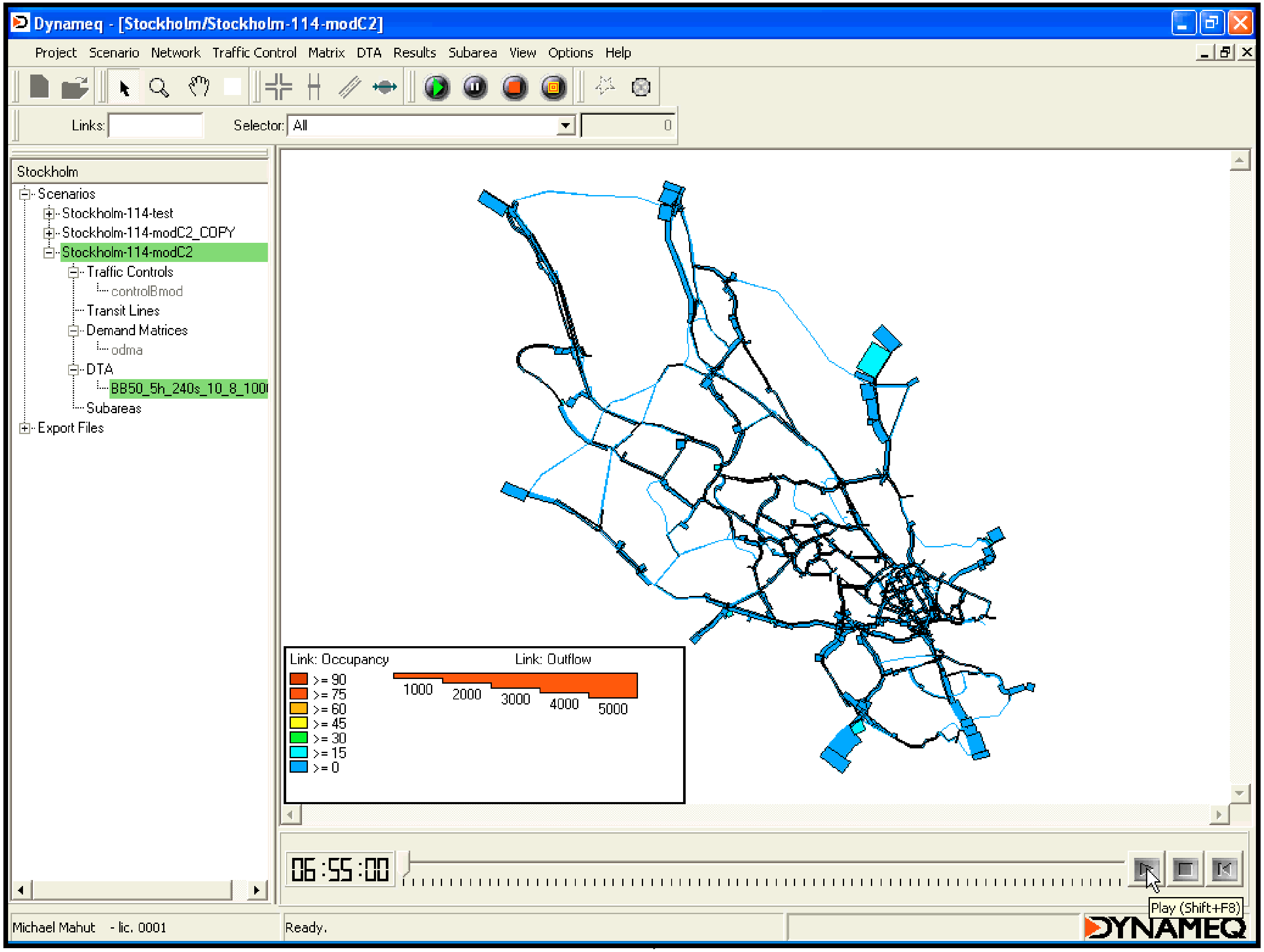
$$V = \text{free-flow speed}$$

$$Q = \frac{1}{L/V + R}$$

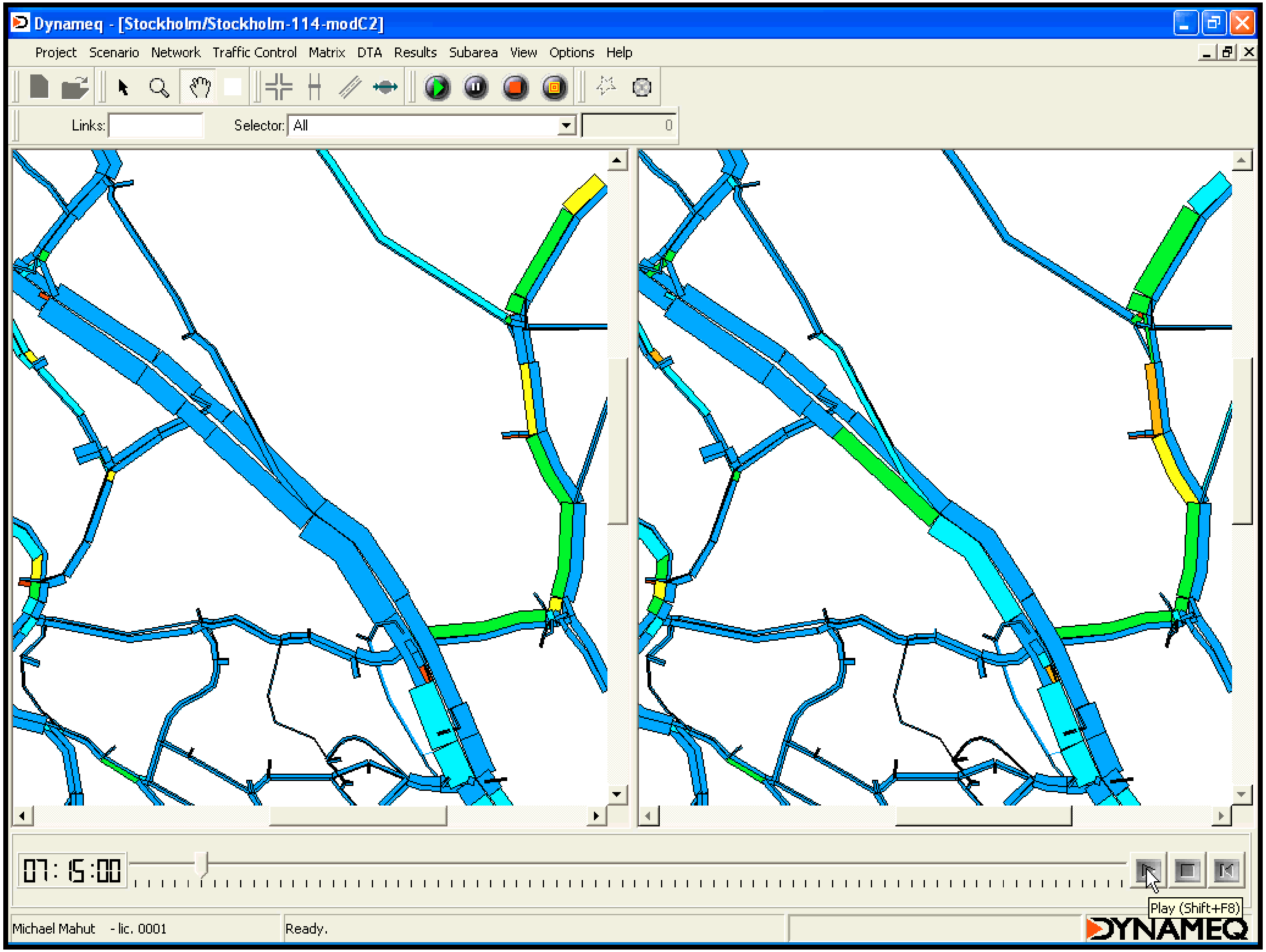
$$K = 1/L$$

$$W = L/R$$

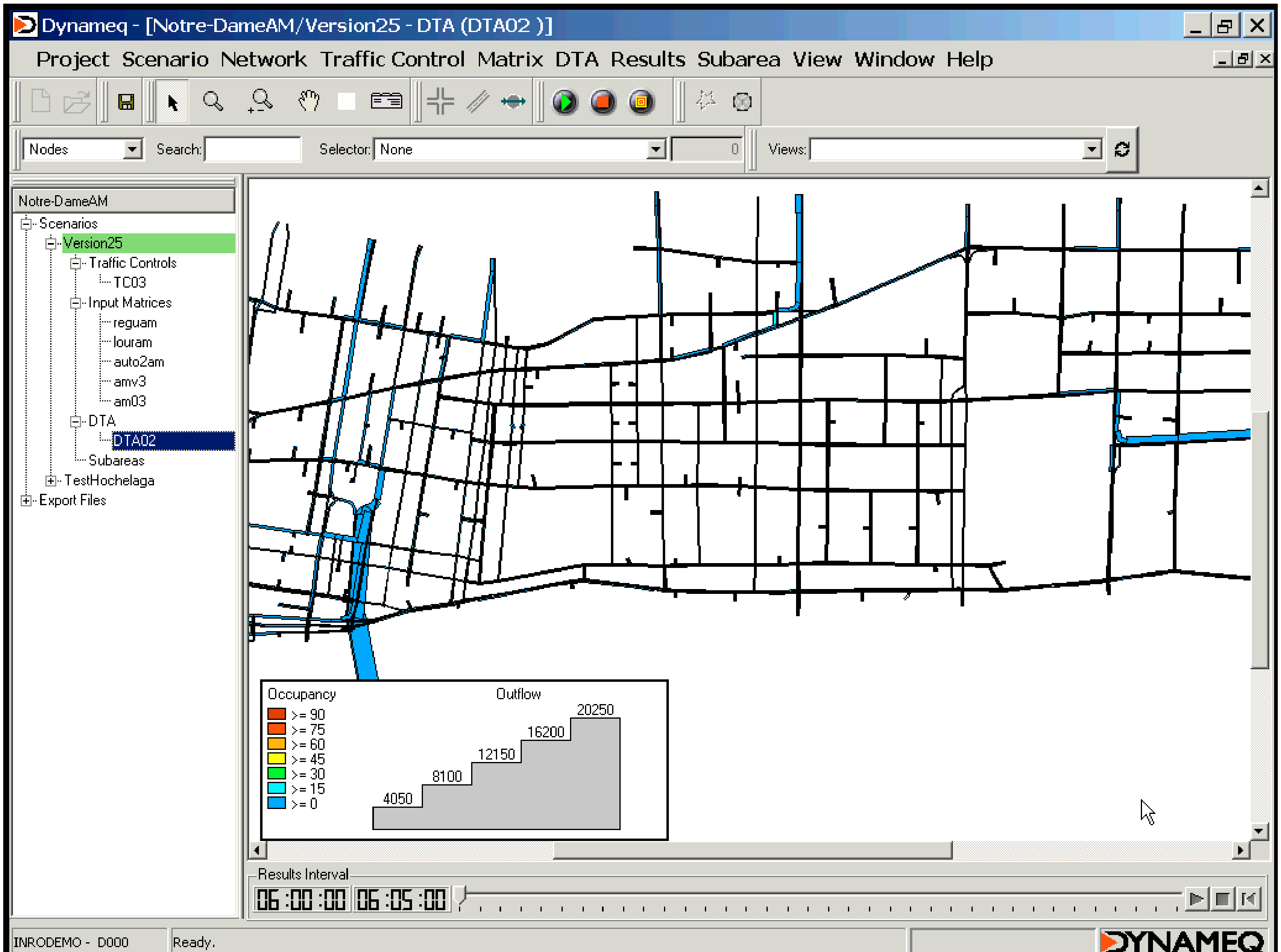
An application in Stockholm



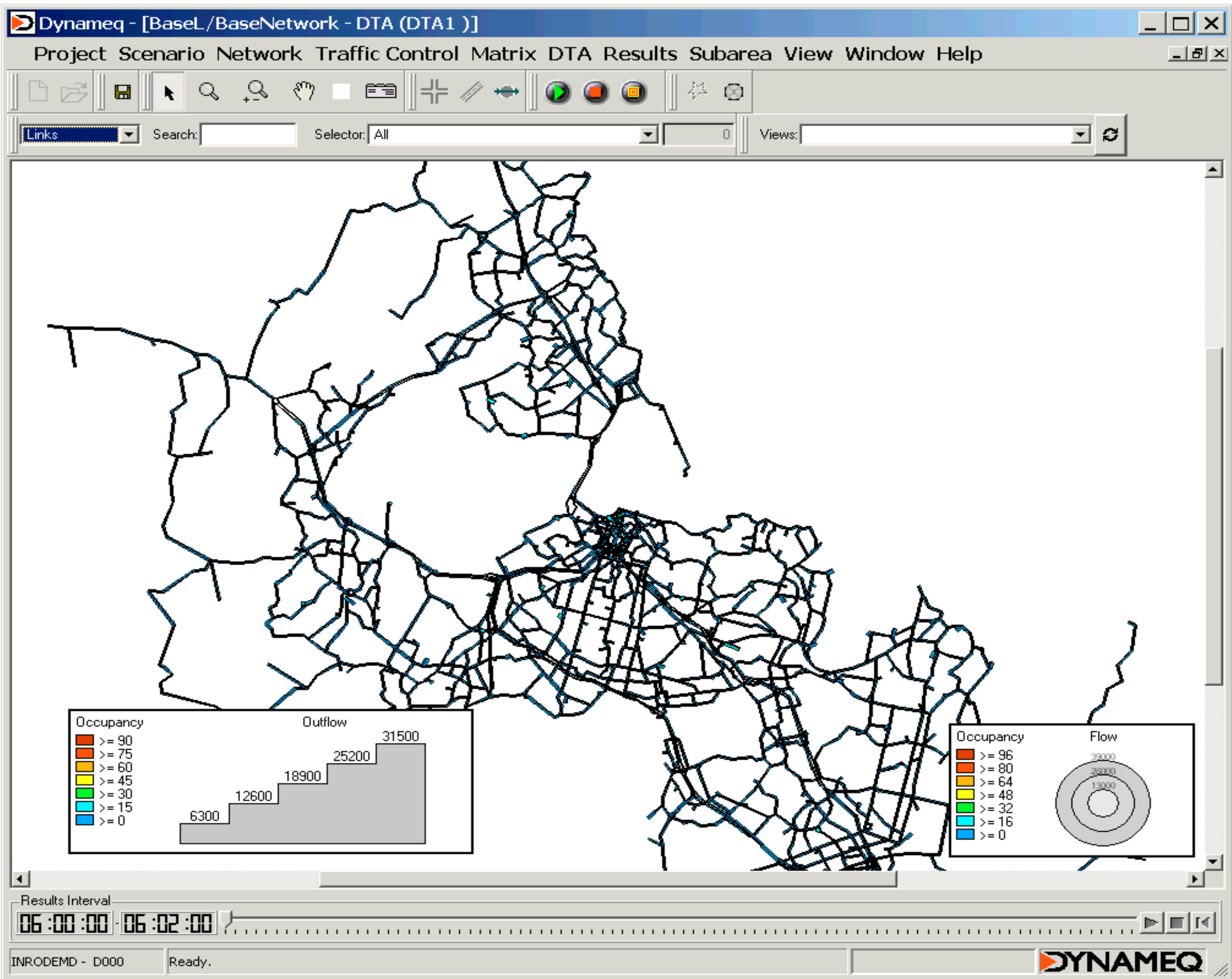
An application in Stockholm



An application in Montreal



An application in Auckland,NZ



Ending Remarks

- The equilibrium model of route choice is here to stay for both static and dynamic models
- We have a lot to thank to the landmark contribution of 1956