Insights into the Why, How and When of the Traffic Network Equilibrium Model

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STUDIES IN THE ECONOMICS OF TRANSPORTATION

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Introduction by TJALLING C. KOOPMANS

Published for the COWLES FOUNDATION
for Research in Economics at
Yale University

Dramatis Personae of *Studies*

- Tjalling C. Koopmans, 1910 - 1985
  - Doctorate, University of Leiden, The Netherlands, 1936
  - Research Director, Cowles Commission for Research in Economics, and Professor of Economics, University of Chicago, 1948-1954
  - Recipient of 1975 Nobel Memorial Prize in Economic Science, with Leonid Kantorovich, USSR, for their contributions to the theory of the optimum allocation of resources
- Martin J. Beckmann, 1924 -
  - Doctorate, Economics, University of Freiburg, Germany, 1950
  - Research Associate, Cowles Commission, 1951-1954
- C. Bartlett McGuire, 1925 -
  - A. M., Economics, University of Chicago, 1952
  - Research Associate, Cowles Commission, 1952-1954
- Christopher B. Winsten, 1923 - 2005
  - B. A., Cambridge University
  - Research Associate, Cowles Commission, 1952-1954

Cowles Commission for Research in Economics

- *Cowles Commission for Research in Economics* was founded by Alfred Cowles, head of an investment counseling firm in Colorado Springs, was its principal benefactor and first president.
- The Commission undertook research on mathematical economics, including activity analysis, a term in use at the time for mathematical programming, or optimization subject to constraints.
- In 1939, the Commission moved to the University of Chicago. In 1954, it was reorganized as the *Cowles Foundation for Research in Economics* at Yale University, where it continues to this day.
- In 1952-1954, the Cowles Commission was the leading academic research center in mathematical economics and applications of math programming to a broad range of economic problems. Cowles-affiliated economists received 12 Nobel Prizes in Economic Science, of which 8 were for research during 1940-54.

Research Leading up to *Studies*

- During WW II, Koopmans had conducted research on maritime shipping, which led to the formulation and solution of the Transportation Problem of Linear Programming, also formulated by Hitchcock, and known as the Koopmans-Hitchcock problem.
- Koopmans, a leading innovator of the new field of activity analysis, edited the influential monograph, *Activity Analysis of Production and Allocation*, published in 1951 from the field's first conference.
- Beckmann came to the University of Chicago from Germany in 1950 as a post-doctoral student, and conducted theoretical research on transportation and location problems.
- McGuire, a graduate student in economics at Chicago, had participated in Cowles research on highway capacity and use.
- Winsten joined the staff in 1952, and worked on problems related to capacity of roads and queuing behavior of traffic.

Study of the Allocation of Resources

- Research was initiated in 1951 with support of RAND Corporation on the "Theory of Resources Allocation," with applications to transportation, location and dispersal problems. RAND's principal interest was railway capacity analysis, perhaps motivated by a desire to estimate the capacity of the USSR railway system.
- The research team worked on the application of activity analysis to transportation and location problems. A study of efficiency in road networks led to the discovery of "network equilibrium" of travel demand in the context of road and intersection capacities. An OR analysis of railway freight yard operations led to the concept of "accumulation delay," and efficient allocation of switching work among freight yards in route. (Hildreth, 1986)
- In a 1954 letter to Morgenstern at Princeton University, McGuire related: "Our original hope was that this work would give us some insight into the economics of city layout, so that if a long-run policy of city dispersal were initiated, primarily for defense purposes, we could say something about where things should be dispersed to, and the costs or benefits thereof. While not very successful, I do think the work has led to a better understanding of highway economics."

Cowles Commission Discussion Papers

Beckmann, McGuire and Winsten completed 8 Discussion Papers, which I examined to identify the origins and direction of their research. In order of date of preparation, they were:

- Beckmann, *Optimum Transportation on Networks*, Aug 1951
- McGuire, *Highway Capacity and Traffic Congestion*, Jul 1952
- Beckmann with McGuire, *Efficient Transportation in Networks*, Aug 1952
- Beckmann, *Efficient Transportation in Networks*, Nov 1952
- Beckmann, *Efficient Transportation in Networks Cont'd*, Apr 1953
- Beckmann and McGuire, *Determination of Traffic in a Road Network – An Economic Approach*, RAND Corporation, Oct 1953
- Winsten, *Some Models of Traffic Congestion*, 1953
- Winsten, *Mean Delay at a Traffic Intersection*, 1953
- Also, Beckmann and McGuire, *Road Utilization Under Conditions of Individual Choice*, was referenced by McGuire (1952), but evidently never issued.

Optimum Transportation on Networks, Aug 1951

- In this exploratory and theoretical paper, Beckmann introduced a transportation network into a one-dimensional space economy. The overall objective was to determine optimal location of production, subject to limits on local availability of resources.
- In this work, Beckmann is moving from his training and past experience in the German school of location theory (Weber and Lösch) to the new developments of activity analysis introduced by Koopmans and his collaborators.

Highway Capacity and Traffic Congestion, Jul 1952

- McGuire reviewed the status of highway capacity analysis, especially the pioneering studies of Normann of the U.S. Bureau of Public Roads.
- Capacity functions for traffic flows with uniform speeds and desired speeds of drivers were examined in detail. Concepts of speed, density and flow were introduced and interrelated, and measures of congestion were explored.
- In a description of joint work with Beckmann on "road utilization under conditions of individual choice," McGuire seemed unsure how to represent a capacity function for a two-lane road, while recognizing that the two lanes could be treated as separate roads. This discussion reveals they were exploring the user-equilibrium problem at that time.

1. *Efficient Transportation in Networks*, Aug 1952

- As the principal, surviving precursor discussion paper to *Studies*, this paper shows the development of the authors conceptual and analytical thinking. Clearly, their motivation was congested transportation networks.
- Given the network of transportation lines, whose capacities are functions of speeds, the capacities of line intersections, independent of speeds, and a transportation program specifying net amounts of commodities to be shipped from or received at given points,

  "What is the allocation of traffic flows to roads and the system of traffic speeds that minimizes the total money costs of transportation? We have here the problem with which traffic authorities are concerned in their daily activities."
- Notes: the initial representation of a transportation network included intersection delay as well as link delay; traffic was described in terms familiar to those engaged in activity analysis of transportation networks: requirements for commodities at originating and terminating nodes.

2. *Efficient Transportation in Networks*, Aug 1952

Extension of Koopmans's formulation to a network with congestion

- Definition of efficiency: a combination of transportation activities is efficient if its input vector (costs) is minimal, subject to the given program of commodities
- Formulation of a nonlinear objective function of link flows and multi-dimensional link costs as functions of unknown link speeds
- Non-negativity conditions on flows and speeds
- Intersection-based conservation of flow constraints
- Link capacity and intersection capacity functions
- Representation of nodal inflows and outflows as a commodity transportation program, which included empty vehicles

Tools of analysis

- Main theorem of linear activity analysis of Koopmans (1949)
- Conditions for optimality with non-negativity requirements, based on Kuhn-Tucker (1951) and Slater (1950)

3. *Efficient Transportation in Networks*, Aug 1952

Conclusions

- Relation of average costs, marginal social costs, and tolls
- Marginal social costs on links the same for all commodities
- Equality of marginal social costs among all used routes and among commodities flowing on a given route
- Computational problems were the familiar problem of finding saddle points of a function convex in one set of variables and concave in the complementary set
- Uniqueness and non-uniqueness properties understood well

Understanding of this Model

- Problem formulation in terms of program requirements; no mention of origin-destination demand, fixed or variable
- Network formulation with variable speeds and capacities
- Marginal social cost principle implemented as tolls
- Familiarity with Kuhn-Tucker analysis, through M. Slater
- Non-uniqueness of route flows and link commodity flows
- Several fixed input sources recognized, such as money, but transit time not mentioned explicitly

Efficient Transportation in Networks, 1952-53

- The first part of the revised paper (Nov. 1952) primarily added examples to the original discussion paper (Aug. 1952).
- The second part (April 1953) was more ambitious and general. It considered the construction cost of increasing road capacity, and restated the transportation cost function in terms of the flow to capacity ratio. The analysis is deeper, and some specific attention was given to computational solution of the problem.
- As in the original paper, there is no mention of a demand function, or of origin-destination demand.

Determination of Traffic in a Road Network, 1953

- Submitted to *Traffic Quarterly* in late 1953, and promptly rejected, this paper shows the authors had progressed substantially by late 1953 beyond the extension of the Koopmans-Hitchcock problem to include network capacities.
- The paper demonstrates substantial insights into the effect of traffic conditions on travel choices, including best routes from an individual viewpoint, discusses demand as a function of travel cost, and states user and system route choice conditions. In particular, the authors stated: "all traffic will distribute itself in such a way that if a particular flow between a given origin and destination uses more than one route, the travel times for these different routes will be equalized." The paper closes with a call for an equilibrium model, and with an outline of a solution procedure.
- Some sections of the paper were included in the text of *Studies*, such as the introduction to Ch. 2.

Some Models of Traffic Congestion, and *Mean Delay at a Traffic Intersection*, 1953

- Winsten authored two discussion papers related to network supply, extending McGuire's earlier analysis of 1952; only abstracts are available, and summarized here.
- In the first paper, the proposed models sought to represent congestion at unsignalized intersections, and congestion of cars traveling at various speeds on a road. A model of a minor road with queuing intersecting a major road is also described.
- In the second paper, details of the mean waiting time defined in the first paper are described.
- Ch. 1 of *Studies* draws on these two papers.

Next Steps toward a Draft of *Studies*

- In May 1953, the authors and their team leader exchanged short memos regarding the next period of work pertaining to both the highway and railway research. Here, McGuire refers for the first time in the surviving documents to a monograph.
- Then, in October 1953 several outlines for a monograph were prepared; one by Beckmann bears a close correspondence to *Studies*. Evidently, there was much discussion pro and con, since at one point Beckmann stated: "Even if the plan of a book is shelved, I should argue for an integrated, though not so broadly written, article or series of articles on highway transportation." The attached outline has sections on demand as a function of delay and risk, and equilibrium conditions.
- In an extended abstract dated Dec. 1953, Beckmann suggests he understands the problem of equilibrium, but has not yet found the user-equilibrium formulation. An undated draft entitled "Road Utilization under Conditions of Individual Choice," also suggests he is still struggling for a viable formulation. Undoubtedly, these fragments of outlines and drafts are only a small part of the story.

Suddenly, a Draft of *Studies* Was Completed

- On May 19, 1954, just seven months later, Koopmans wrote to William Vickrey requesting his help in appraising "a manuscript of what we call a report." Koopmans explained the project had been terminated by RAND, and that "the main research staff is leaving on July 1." He offered "the amount of $150 as a fee in partial compensation for this work." The outline attached to the letter is effectively the same as the Contents of *Studies*.
- The first half of the manuscript was mailed to Vickrey on July 15. Beckmann left for Germany in late June or early July; his first letter conveyed some corrections, as well as his impressions upon returning to Germany after an absence of four years.
- Review and corrections continued through the summer and early fall, involving Koopmans, from his summer home in Vermont, Beckmann from various holiday spas in Europe, Vickrey from his summer home near New York City, and McGuire back in Chicago, until McGuire departed for RAND about Sept. 18. At that time a complete, corrected manuscript was ready for typing.

Publication by RAND and Yale University Press

- RAND Corporation issued RM-1488, *Studies in the Economics of Transportation*, on May 12, 1955. Corrections and final editing of the manuscript accepted for publication by Yale University Press continued through the remainder of the year, including preparation of an index. http://www.rand.org/publications/RM/RM1488.pdf
- On Dec. 27, 1955, letters were sent to McGuire and Beckmann advising them that *Studies* was now in print, with a price of $4; the book was also issued by Oxford University Press with a price of 32 shillings. The book enjoyed three printings by 1959; a Spanish edition was also published in that year.
- A search in Sept. 2005 of *WorldCat List of Records* showed 373 libraries throughout the world hold copies of *Studies*. In addition, 13 libraries hold the RAND Corp. version, and 7 hold the Spanish edition, *Economía del transporte*.
- A search of *Web of Science* in October 2005 listed 321 journal articles that cited *Studies*; as a basis for comparison, Kuhn and Tucker (1951) *Nonlinear Programming* had 956 citations.

Commentary on *Studies*: Introduction

The Introduction by Koopmans usefully reveals some of his thinking about the work.

- A "report" addressed to various professions, including OR and MS
- Characterizations:
  – "while the aim of these studies is modest and provisional, they are offered in hopes of stimulating further factual, conceptual, mathematical, and computational research into efficient utilization of transportation systems"
  – "purpose of these studies is to develop and illustrate concepts, methods and models that may be useful as points of departure"
  – "method is to construct and study simple (mathematical) models"
- Koopmans's attribution of authorship of Part I:
  – "Beckmann … contributed most of the chapters of the highway traffic analysis, except for Chapter 1 on capacity" by Winsten.
  – "McGuire was assigned primary responsibility for the degree of 'realism' of the models developed by the group; in earlier phases he gave most of his time to study of literature on traffic analysis"

Commentary on Chapters 1 and 2

Chapter 1 – Road and Intersection Capacity

- This 43 page chapter is relatively separate from the rest of Part I, but it has a similar style and approach.
- Much emphasis is placed on two-lane roads and the effect of passing maneuvers on delay and capacity.
- The intersection analysis is original, difficult and somewhat speculative, but rather deep.
- A conclusion, seemingly ignored subsequently, is that the relationship between total flow and desired speed is monotone decreasing, in contrast with the usual non-monotone assumption.

Chapter 2 – Demand

- This 13 page chapter is clear, perceptive and conceptually helpful.
- Notation and properties of the demand function are introduced, including its inverse.

Commentary on Chapter 3, Equilibrium (21 pages)

- Clear verbal characterization of the traffic equilibrium problem: "there is one level of flow at which traffic conditions give rise to a demand just equal to the prevailing flow, … what we shall call the equilibrium flow."
- Formulation of user-equilibrium conditions, with variable demand
- Proof of the existence of an equilibrium by the formulation of an "extremum problem," thereby introducing the sum of the integrals of link cost, but with no interpretation or comment.
- Analysis and proof of uniqueness properties of the formulation:
  - Link flows are unique for strictly decreasing demand functions, and strictly increasing link cost functions;
  - Route flows are not unique.
- Sensitivity analyses, including the effect of changes in demand functions and link cost functions on the solution.
- Stability analyses of the solution with respect to small deviations in model inputs, and the ability of "the system to reach an equilibrium from any initial position."
- Computational studies for a toy network, but no solution algorithm.

Commentary on Chapter 4, Efficiency (22 pages)

- "We attempt to clarify the economic meaning of 'best utilization,' (of a road network) and to evaluate traffic equilibrium in light of it"
  - "Achieve allocation of available road capacity to those competing for its use."
  - Considered a single class model only, but recognized the need for multiclass models
  - Introduced and explained the concept of consumers' surplus using the now-classic two-link diagram
  - Graphical derivation of marginal cost curve; distinction between private costs and social costs in relation to marginal cost
- Derived the necessary and sufficient conditions for the maximization of a concave function subject to linear constraints, based on Kuhn and Tucker (1951), and presumably Slater (1950).

(continued)

- Formulated and analyzed the problem of minimizing total transportation cost with fixed demand, followed by consideration of variable demand function and computation of consumers' surplus and other properties of the problem.
- Explained efficiency tolls and explored their properties in detail, with many insights:

  > "efficient utilization can, at least in theory, be achieved through a state of equilibrium in which suitable taxes or tolls are levied on the use of all congested roads."

- Reconsidered toll road analysis for the case of class-specific values of time, showing that total benefits of road users can be improved by providing a choice between faster travel at higher money cost, and slower travel at a smaller money cost, thereby anticipating the implementation of California State Route 91.

Commentary on Chapter 5, Unsolved Problems

In this short chapter of 9 pages, the authors:

- Recognized that further study of theoretical capacity functions would be useful, tying the model back to Winsten's analysis, and posing questions related to delays at intersections, queuing, multi-class formulations, two-lane roads and free speed distributions.
- Described extensions to commodity flow models, relating back to the 1952 discussion papers, but now with 'transportation programs' depending on equilibrium prices at the nodes.
- Stated that the key issue in making these models dynamic is understanding of demand substitution over time, now known as departure time choice and choice of time period of travel.
- Identified the relation of long-run models to locational decisions of travelers, anticipating need for integrated location (land use) and travel choice models.

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- Noted the relation of the traffic equilibrium model to problems related to electrical power networks, later studied by McGuire.
- Described "the most fundamental problem in traffic economics: determining the proper extent and layout of a road network. The difficulty of this problem springs from the fact that choices between a great number of all-or-nothing alternatives are involved," thereby recognizing that the road network design problem is NP-hard. This problem remains today as formidable as in 1954, despite many attempts at its solution by researchers and practitioners.
- Stated that the "optimal apportionment of general vehicle and fuel taxes with respect to equity and efficiency poses an interesting problem which will be the subject of discussion for a long time to come. Still true today!
- After 50 years, advanced students searching for a research problem would still benefit from reading Chapter 5 of *Studies*.

Chronology of Contributions

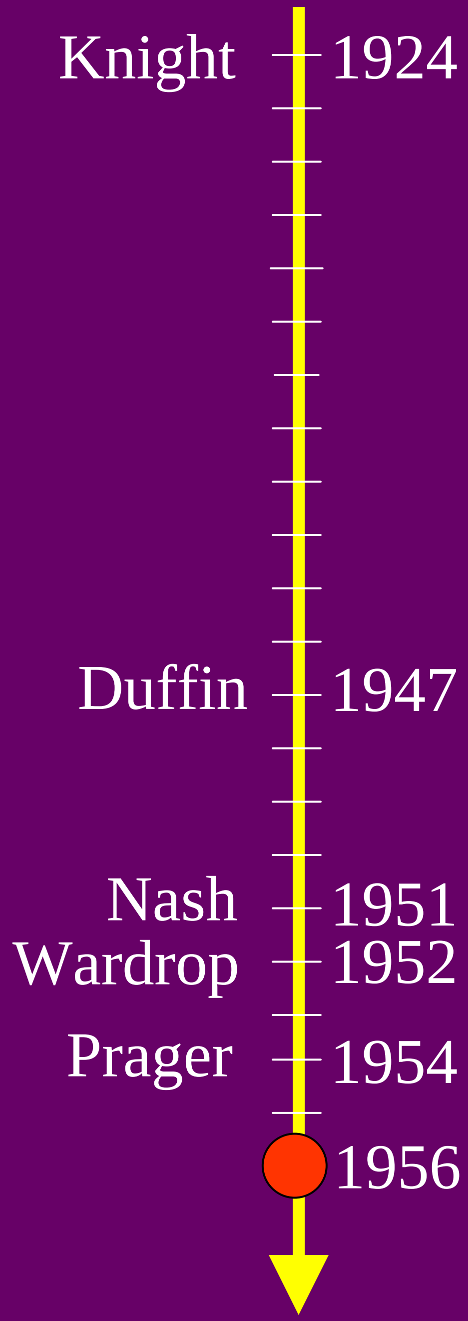
- Project initiation — 1951
- Discussion Papers — 1952
- 1953
- Completed equilibrium and efficiency formulations — 1954
- Publication of *Studies* by RAND — 1955
- Publication of *Studies* by Yale University Press — 1956
- Publication of *Studies* in Spanish — 1959
- Beckmann paper in *Traffic Quarterly* — 1967
- Recognition of Beckmann at the Montréal Symposium — 1974

Book Reviews of *Studies*

- *Operational Research Quarterly*, 7 (1956), by D. J. R.
- *American Economic Review*, 46 (1956), book notice only.
- *Wall Street Journal*, January 2, 1957, p. 8, column 6: ".. this is a 'heavy' theoretical work by a group of economists searching for the optimum efficiency of highway systems."
- *The Economic Journal*, 67 (1957) by R. J. Smeed
- *Quarterly J. of Applied Math.*, 14 (1957) by W. Prager
- *Econometrica*, 26 (1958) by R. M. Thrall
- *KYKLOS*, 11 (1958) by C. Ponsard
- *Operations Research*, 7 (1959) by G. D. Camp
- *Journal of Political Economy*, 67 (1959) by E. Mansfield

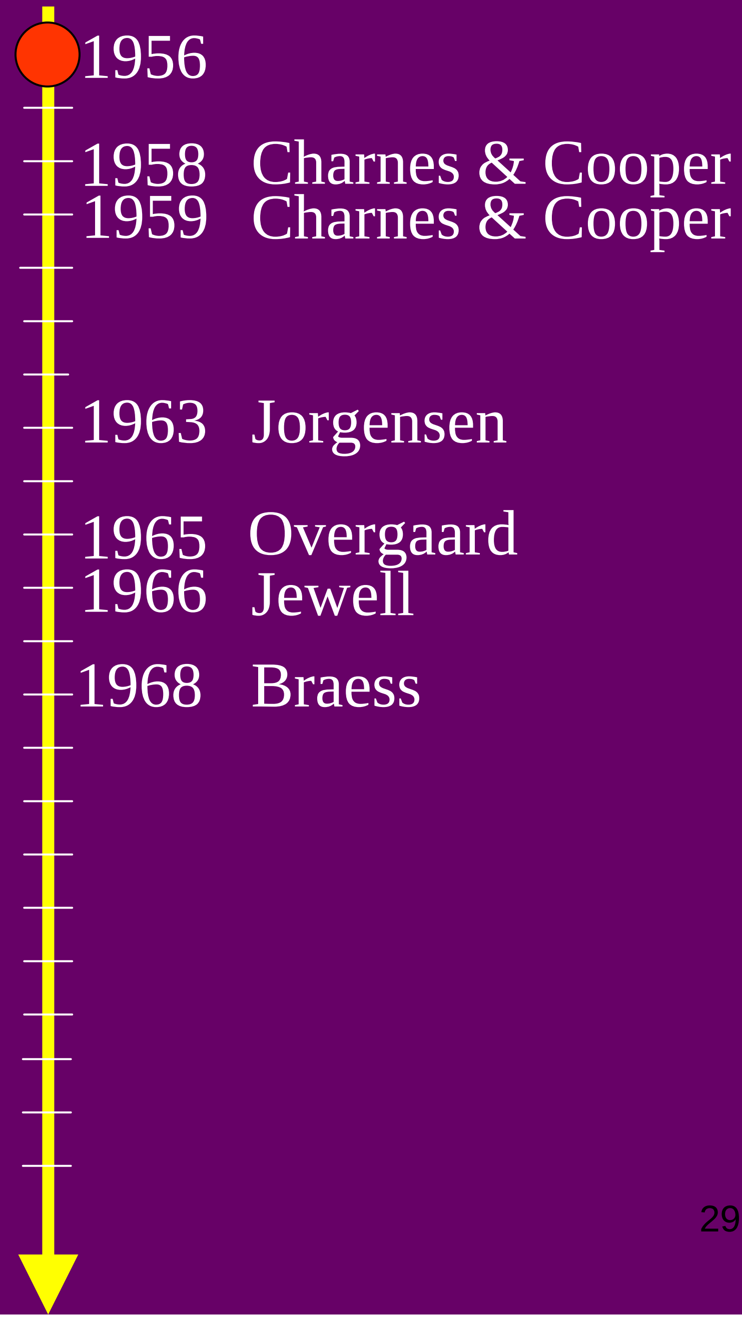
Among 9 reviews published, no reviewer identified the significance of the formulation achieved in Part I, and none linked the authors' formulation to travel forecasting for urban transportation planning.

Network Equilibrium Research Preceding and Concurrent with Research Leading to *Studies*



Network Equilibrium Research

Independent of *Studies*



Network Equilibrium Research

Based on *Studies*

Fixed OD Flows	Year	Variable OD Flows
	1956	
	1961	Walters
	1964	Johnson
Almond	1965	
	1967	Tomlin
Dafermos	1968	
Bruynooghe et al.	1969	Murchland
Netter	1971	
Leventhal /LeBlanc	1973	Evans
Nguyen	1974	Potts & Oliver
	1975	Florian et al.
	1977	Erlander

Concurrent Urban Transportation Studies

- The first urban transportation study in Detroit was directed by J. Douglas Carroll, Jr., 1953-1955; initially, an origin-destination desire line analysis with a secondary network analysis.
- First urban transportation study using models of origin-destination and route flows (trip distribution and traffic assignment) was in Chicago, 1955-1962, also led by Carroll. There, Schneider formulated and applied an intervening opportunities model and a tree-based algorithm to assign OD flows to shortest routes.
- Carroll was active in searching for research innovations to strengthen the computer-based analysis he envisaged, which led to the identification and application of shortest route methods. Evidently, he was not aware of *Studies*, and did not grasp the significance of Charnes's formulation of a "traffic network" model in 1958-59.

Travel Forecasting by Early Practitioners

- Early analyses of the impact of major road improvements applied an empirical diversion curve approach, which sought to reallocate traffic flows on the basis of time savings.
- As larger computers and shortest route methods became available in the U.S. in the late 1950s, solving the Traffic Assignment Problem was viewed as a procedure for "loading" origin-destination flows onto shortest routes of the road network
- Wardrop's principles of route choice were unknown to practitioners in the U.S. Likewise, the network equilibrium model formulations of Beckmann, Prager, and Charnes-Cooper were generally unknown to practitioners.

(continued)

- Carroll and his associates also initiated the concept of the sequential travel forecasting procedure; as reported at the January 1957 Highway Research Board Meeting: "a continuous, integrated analysis and planning process consists of three major parts, each a considerable advance:
  - Estimating traffic generation from land use;
  - Predicting future lines of travel desire;
  - Predicting flows on a transportation network."

  Although mode choice was not included, testing of a mass transit facility was described.
- Irwin and von Cube, Capacity Restraint in Multi-Travel Mode Assignment Programs (*Bulletin 347*, Highway Research Board, 1962) described a sequential procedure including assignment to links whose costs increase with flow, and a "feedback procedure repeated until equilibrium is reached." They are perhaps the first practitioners to cite *Studies*.

Summary and Lessons Learned

- A brilliant theoretical formulation of urban travel was achieved in the early 1950s; its significance was not appreciated, perhaps even by its authors, and not understood by others for nearly 20 years.
- Related and partial formulations were proposed by others during the same period, but poor communication likely restricted their assimilation; until 1967, there was no scientific journal in this field.
- Transportation planners, unaware of these developments, sought to make travel forecasts by empirical or heuristic methods, some with striking similarities to convergent solution methods proposed later.
- Solution algorithms, model implementation, parameter estimation, validation and useful software systems have slowly emerged, providing a more rigorous basis for the solution of practical planning problems.

(continued)

- This story describes the meandering path that research sometimes follows in the evolution of a field. Interdisciplinary by nature, and faced with pressures of real problems requiring decisions, this field may well have experienced more chaos than others during its first 50 years.
- Hopefully, as the field matures during the next 50 years, through the publication of reviews, textbooks, improved software, and better training for practitioners, the promising findings of these young, insightful researchers, as well as pioneering practitioners, will finally be realized.