Viable and Sustainable Transportation Networks

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Viability and Sustainability

In this lecture, the fundamental concepts of *viability* and *sustainability* of transportation networks are made explicit.

These concepts are rigorously defined through formal definitions and rest against the backdrop of environmental goals.

Generally speaking, in this framework, viability is concerned with whether or not the environmental goals are achievable, given the transportation network topology and travel demands as well as the environmental parameters.

Sustainability, on the other hand, is concerned with the attainment of the goals through a variety of policy instruments, given not only the transportation network and the environmental parameters, but also the cost structure and the travel behavior.

Clearly, in this framework, viability of a transportation network is a precursor to sustainability.

First, I consider viability of a transportation network, given that the origin/destination pairs are known as well as the associated travel demands. Subsequently, I take what may be viewed as a longer perspective and provide formal definitions of viability in the following situations:

Situation 1

Given the origin nodes and the total number of trips produced in each origin node, the travelers select their destinations as well as their travel paths.

Situation 2

Given the destination nodes and the total number of trips attracted to each destination, the travelers select both their origins and their travel paths.

Situation 3

Given the origin nodes, the destination nodes, and the total number of trips generated in all the origin nodes, which is equal to the number of trips attracted to each destination, the travelers are free to select their origins, their destinations, as well as their travel paths.

Situation 1 models the scenario in which travelers have predetermined origins, typically places of residence, and seek to determine their work destinations as well as the routes of travel to the destinations.

Situation 2, on the other hand, models the scenario in which travelers have predetermined destinations, such as places of work, and seek to determine their residential locations, as well as their paths of travel.

Finally, Situation 3 is the most flexible one in a sense in that travelers determine their places of employment, their places of residence, as well as the routes of travel between the origin/destinations. In the Table, I summarize the classification of fixed demand networks in distinct situations.

Classification of fixed demand networks in different situations

Given		To be Determined
Baseline:	• O/D pairs and demands	• routes of travel
Situation 1:	• Origin nodes and trip productions	• destinations and routes of travel
Situation 2:	• Destination nodes and trip attractions	• origins and routes of travel
Situation 3:	• Origins nodes, destination nodes, and total number of trips	• origins, destinations, and routes of travel

Viability of Networks with Known O/D Pairs and Travel Demands

Here transportation networks with fixed travel demands are considered. I assume, as in the classical traffic network models, that the origin/destination pairs are known and given a priori as are the associated travel demands. Note that, in regards to viability of a transportation network, I do not consider the travel cost structure on the network nor the behavior of the travelers.

The cost parameters and behavior of the travelers enter into the equation when we investigate sustainability of a network.

I motivate the definition of viability by first presenting two problems.

Problem 1 (Determination of the Tightest Achievable Environmental Quality Standard)

Given a network topology $G = [N, L]$, the set of origin/destination pairs W, the vector of travel demands d, and the vector of link emissions h, what is the tightest environmental quality standard, denoted by Q^*, that can be achieved by the transportation network?

The solution to this question can be formulated as a mathematical programming problem, in particular, a *linear programming* problem, given by:

$$\text{Minimize} \quad Q \tag{1}$$

subject to:

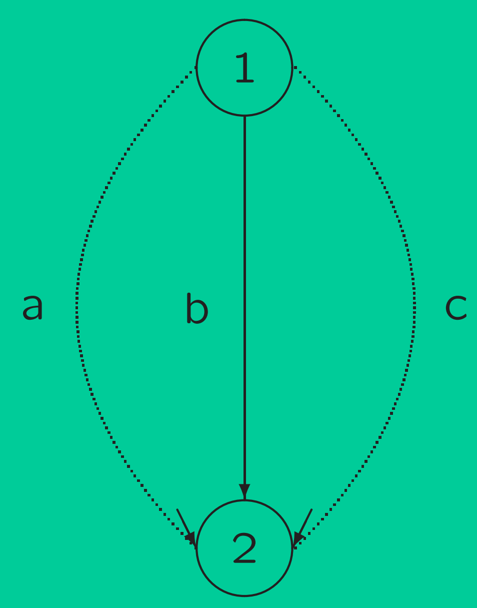
$$\sum_{p \in P_w} x_p = d_w, \quad \forall w \in W, \tag{2}$$

$$\sum_{a \in L} h_a \sum_{p \in P} x_p \delta_{ap} \leq Q, \tag{3}$$

$$x_p \geq 0, \quad \forall p \in P. \tag{4}$$

The first set of constraints (2) guarantees that the path flow pattern satisfies the travel demands, whereas the second one (3) guarantees that the path flow pattern does not exceed the tightest environmental quality standard (which is endogenous to this problem).

Finally, the last set of constraints ensures that the path flows are nonnegative.



Network topology for Example 1

I now solve this problem for a specific network example depicted in the Figure.

Example 1

I present a small example for illustrative purposes. The network is depicted in the first Figure and consists of two nodes: 1, 2; three links: a, b, c, and one origin/destination pair: $w_1 = (1, 2)$. There are three paths connecting the O/D pair, each of which consists of a single link, that is, $p_1 = a$, $p_2 = b$, and $p_3 = c$. The travel demand is $d_{w_1} = 10$.

The emission factors are: $h_a = 0.1$, $h_b = 0.2$, and $h_c = 0.3$.

A straightforward application of the simplex method (see Bazarra, Jarvis, and Sherali (1990)) to Problem 4.1 for this example, where one seeks to solve:

$$\text{Minimize} \quad Q$$

subject to:

$$x_{p_1} + x_{p_2} + x_{p_3} = 10,$$

$$0.1x_{p_1} + 0.2x_{p_2} + 0.3x_{p_3} \leq Q,$$

$$x_{p_1} \geq 0, \quad x_{p_2} \geq 0, \quad x_{p_3} \geq 0,$$

yields: $Q^* = 1$, that is, the tightest emission quality standard that is achievable by this network is given by 1.

I next ask the question as to what is the maximum demand that is achievable, given a desired environmental quality standard.

Problem 2 (Determination of the Maximum Achievable Travel Demand, Given the Environmental Quality Standard)

Given a network topology $G = [N, L]$, the set of origin/destination pairs W, the vector of link emissions h, and the desired environmental quality standard $\bar{Q}$, what is the maximum total demand that is achievable?

The solution to this question can also be formulated as a linear programming problem given by:

$$\text{Maximize} \quad \sum_{w \in W} d_w \tag{5}$$

subject to:

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w \in W, \tag{6}$$

$$\sum_{a \in L} h_a \sum_{p \in P} x_p \delta_{ap} \leq \bar{Q}, \tag{7}$$

$$x_p \geq 0, \quad \forall p \in P. \tag{8}$$

Note that, in this problem, the constraints (6) (as (2) in Problem 1) guarantee that the path flow pattern satisfies the travel demands, but now the travel demands are no longer known but are variables whose optimal values are to be determined. Constraint (7) guarantees that the environmental quality standard is met by the path flow pattern, but now, in contrast to the analogous constraint in Problem 1, there is a known environmental quality standard, denoted by $\bar{Q}$, which one wishes not to exceed.

Hence, in this problem, the travel demands, rather than the environmental quality standard, are endogenous and to be determined by the solution of the optimization problem.

Example 2

I now consider, again, the network in the first Figure, with emission factors: $h_a = 0.1$, $h_b = 0.2$, and $h_c = 0.3$.

The desired environmental standard, denoted by $\bar{Q}$, is equal to 1. Problem 2 applied to the example, is given by:

$$\text{Maximize} \quad d_{w_1}$$

subject to:

$$x_{p_1} + x_{p_2} + x_{p_3} = d_{w_1},$$

$$0.1x_{p_1} + 0.2x_{p_2} + 0.3x_{p_3} \leq 1,$$

$$x_{p_1} \geq 0, \quad x_{p_2} \geq 0, \quad x_{p_3} \geq 0,$$

and its solution is obtained via an application of the simplex method yielding: $d^*_{w_1} = 10$.

Hence, the highest achievable travel demand for the network, given the desired environmental standard of 1, is equal to 10.

Henceforth, I will term a transportation network with given origin/destination pairs and travel demands *viable* if there exists a solution to the following linear system:

Linear System 1

Determine a vector x satisfying:

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w \in W, \tag{9}$$

$$\sum_{a \in L} h_a \sum_{p \in P} x_p \delta_{ap} \leq \bar{Q}, \tag{10}$$

$$x_p \geq 0, \quad \forall p \in P. \tag{11}$$

Note that the existence of a solution to this linear system of equations and inequalities guarantees that the demand associated with the O/D pairs can be satisfied by a path flow pattern, which also simultaneously satisfies the environmental quality standard. Clearly, there may be more than one such path flow pattern.

If, for a particular transportation network, the imposed environmental quality standard is not smaller than the tightest achievable standard for the network, that is: $\bar{Q} \geq Q^*$ (where Q^* is the solution to Problem 1), then the network is viable.

In other words, there exists a feasible path flow pattern, that is, one that is nonnegative and satisfies the travel demands, while, at the same time, not exceeding the imposed environmental quality (emissions) standard for the network equal to $\bar{Q}$.

The focus in this course is on environmental quality standards as regards air quality and vehicular emissions. Of course, the definition of viability may be modified to capture other environmental quality standards (and goals) as well, provided that the system of equations and inequalities are modified/expanded accordingly.

Hence, one has the following definition:

Definition 1 (Viable Transportation Network with Known O/D Pairs and Travel Demands)

A transportation network with known origin/destination pairs and fixed travel demands associated with the origin/destination pairs is viable if there exists a solution to the Linear System 1. Alternatively, a transportation network is viable if $Q^* \leq \bar{Q}$*.*

Note that viability is an achievability/feasibility concept and it is relative to the desired environmental target or quality standard $\bar{Q}$ under the given demand structure and network topology.

Returning to the network depicted in Figure 1, and assuming now that the emission parameters remain as in Example 1, but the travel demand doubles to 20, one can see that the network is no longer viable, given an environmental quality standard of 1.

Subsequently, I present a transportation network example which is not viable under the fixed O/D pair and travel demand scenario but is viable in Situation 1. This suggests that environmental goals may be achievable if one allows for an additional degree of flexibility such as, for example, by also allowing for the selection of destination nodes.

Viability of Traffic Networks in Other Situations

I now consider the viability of transportation networks in the three distinct situations outlined above, each of which is discussed separately. As mentioned earlier, these situations reflect a longer time perspective for the network since travelers now select not only their travel paths but also their destinations, or their origins, or both their origins and their destinations.

Note that, previously, I do not consider the cost structure on the network or the behavior of the travelers in making their decisions. Such issues enter in when one assesses whether or not a network is sustainable.

Hence, the determination of the precise travel demands associated with the ultimate O/D pairs (as well as the "ultimate" path flows from the portfolio of feasible flows) can only be accomplished after the cost structure and travelers' behavior are superimposed on the network.

Known Origins and Trip Productions

Given known origins corresponding to particular nodes in a fixed demand traffic network, and which correspond to locations in which trips are produced, and the total number of trips produced at the origin nodes, we are interested in determining whether such a network is viable or not.

Specifically, I assume as given the origin nodes, with a typical origin node denoted by y, and the number of trips at each origin node, with the number of trips at origin node y denoted by O_y.

Let P_{O_y} denote the set of paths originating at origin node y and ending at any of the destination nodes.

Let Y denote the set of such paths in the network.

Assuming, as before, that one is given the environmental quality standard $\bar{Q}$ for the network, one can define the following linear system:

Linear System 2

Determine a vector x satisfying the following linear system:

$$\sum_{p \in P_{O_y}} x_p = O_y, \quad \forall y \in Y, \tag{12}$$

$$\sum_{a \in L} h_a \sum_{y \in Y} \sum_{p \in P_{O_y}} x_p \delta_{ap} \leq \bar{Q}, \tag{13}$$

$$x_p \geq 0, \quad \forall p \in P_{O_y}, \quad \forall y \in Y. \tag{14}$$

Also, consider the solution to Problem 3.

Problem 3 (Determination of the Tightest Achievable Quality Standard in the Case of Known Origins and Trip Productions)

$$\text{Minimize} \quad Q \tag{15}$$

subject to:

$$\sum_{y \in P_{O_y}} x_p = O_y, \quad \forall y \in Y, \tag{16}$$

$$\sum_{a \in L} h_a \sum_{y \in Y} \sum_{p \in P_{O_y}} x_p \delta_{ap} \leq Q, \tag{17}$$

$$x_p \geq 0, \quad \forall y \in P_{O_y}, \quad \forall y \in Y, \tag{18}$$

and denote the solution to this problem by Q^{1^*}.

Then one can state the following definition of viability of a transportation network in the case of Situation 1:

Definition 2 (Viability of a Transportation Network with Known Origins and Trip Productions)

A network is said to be viable in the case of known origins and trip productions if there exists a solution to Linear System 2; equivalently, if $Q^{1^*} \leq \bar{Q}$.

I now present an example which illustrates that a network may not be viable in the case of fixed origin/destination pairs and travel demands but, nevertheless, may be viable in the case of known origins and trip productions.

Example 3

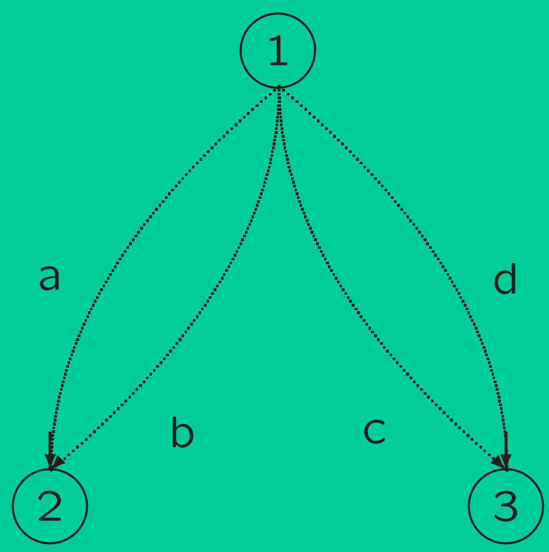
This example illustrates the fact that a network may not be viable in the case of known O/D pairs and travel demands but, nevertheless, may be viable in Situation 1.

Consider the network depicted in the Figure, in which there are three nodes: $1, 2, 3$, and 4 links: a, b, c, d. There are two O/D pairs: $w_1 = (1, 2)$ and $w_2 = (1, 3)$.

The paths connecting O/D pair w_1 are: $p_1 = a$ and $p_2 = b$, whereas the paths connecting O/D pair w_2 are: $p_3 = c$ and $p_4 = d$.

The travel demands are: $d_{w_1} = 6$ and $d_{w_2} = 4$.

The emission factors are: $h_a = 0.5$, $h_b = 0.5$, $h_c = 0.3$, and $h_d = 0.3$.



Network topology for Example 3

It is straightforward to verify that the minimum total emissions, that is, the solution to Problem 1, are equal to $Q^* = 4.2$ for this network.

Assume, however, that the desired environmental quality standard is $\bar{Q} = 4$.

Then, clearly, since $\bar{Q} < Q^*$, according to the definition of viability given in Definition 1, the network is not viable.

Indeed, the environmental quality standard of 4 cannot be achieved by this network.

Suppose now that, rather than having the O/D pairs being fixed with known travel demands, one now has Situation 1 applying to the network example. There is only a single origin node and that is node 1 (since it was the origin for both O/D pairs in the original problem) and the travelers are free to choose their destinations, which can be either node 2 and/or node 3. The total number of trips at node 1 is given by the sums of the original travel demands and, hence, $O_1 = 6 + 4 = 10$.

If one can provide one solution to Linear System 2, then the network is viable under Situation 1 since one only needs to establish the existence of a solution. One can enumerate the paths as before.

Let $x_{p_1} = x_{p_2} = 2.5$ and $x_{p_3} = x_{p_4} = 2.5$. This flow pattern is nonnegative, and satisfies both the first equation in Linear System 4.2 since $2.5 + 2.5 + 2.5 + 2.5 = 10$ as well as the second constraint since $0.5 \times 2.5 + 0.5 \times 2.5 + 0.3 \times 2.5 + .3 \times 2.5 = 4 = \bar{Q}$.

Hence, according to Definition 2, the network (which was not originally viable in the case of known O/D pairs and associated travel demands) is now viable in Situation 1.

Known Destinations and Trip Attractions

Now consider the problem in which the destinations are predetermined, but the travelers select their origins (and their travel paths).

Hence, I assume now that one has, as given, the total number of trips D_z attracted to each destination z and let P_{D_z} denote the paths from the origin nodes to the destination node z.

Let Z denote the set of such paths in the network.

In order to determine whether a network is viable in Situation 2, I consider the following linear system, which is analogous to Linear Systems 1 and 2, but for the case when only the destinations and the trip attractions are given.

Linear System 3

Determine a vector x satisfying the following linear system:

$$\sum_{p\in P_{D_z}} x_p = D_z, \quad \forall z \in Z, \tag{19}$$

$$\sum_{a\in L} h_a \sum_{z\in Z} \sum_{p\in P_{D_z}} x_p \delta_{ap} \leq \bar{Q}, \tag{20}$$

$$x_p \geq 0, \quad \forall p \in P_{D_z}, \quad \forall z \in Z. \tag{21}$$

Also, consider the solution to the following problem:

Problem 4 (Determination of the Tightest Achievable Environmental Quality Standard in the Case of Known Destinations and Trip Attractions)

$$\text{Minimize} \quad Q \tag{22}$$

subject to:

$$\sum_{p\in P_{D_z}} x_p = D_z, \quad \forall z \in Z, \tag{23}$$

$$\sum_{a\in L} h_a \sum_{z\in Z} \sum_{p\in P_{D_z}} x_p \delta_{ap} \leq Q, \tag{24}$$

$$x_p \geq 0, \quad \forall p \in P_{D_z}, \quad \forall z \in Z, \tag{25}$$

and denote the optimal value of the objective function by Q^{2^*}.

One can now state the definition of viability of a transportation under Situation 2.

Definition 3 (Viability of a Transportation Network with Known Destinations and Trip Attractions)

A network is said to be viable in the case of known destinations and trip attractions, if there exists a solution to Linear System 3; equivalently, if $Q^{2^*} \leq \bar{Q}$, *where recall that* $\bar{Q}$ *denotes the environmental quality standard.*

Known Total Number of Trips

I assume now that the total number of trips T generated in all origin nodes of the network (and equal to the total number of trips attracted to the destination nodes in the network) is given.

I am interested in determining whether or not the transportation network is viable in Situation 3.

The linear system that one needs to investigate the existence of a solution to, hence, is:

Linear System 4

Determine a vector x satisfying:

$$\sum_{p \in P} x_p = T, \tag{26}$$

$$\sum_{a \in L} h_a \sum_{p \in P} x_p \delta_{ap} \leq \bar{Q}, \tag{27}$$

$$x_p \geq 0, \quad \forall p \in P. \tag{28}$$

Equivalently, one can investigate the solution to the following problem and see whether the solution exists and whether the optimal value of the objective function, denoted by Q^{3^*}, satisfies: $Q^{3^*} \leq \bar{Q}$.

Problem 5 (Determination of Tightest Achievable Environmental Quality Standard in the Case of Known Total Number of Trips)

$$\text{Minimize} \quad Q \tag{29}$$

subject to:

$$\sum_{p \in P} x_p = T, \tag{30}$$

$$\sum_{a \in L} h_a \sum_{p \in P} x_p \delta_{ap} \leq Q, \tag{31}$$

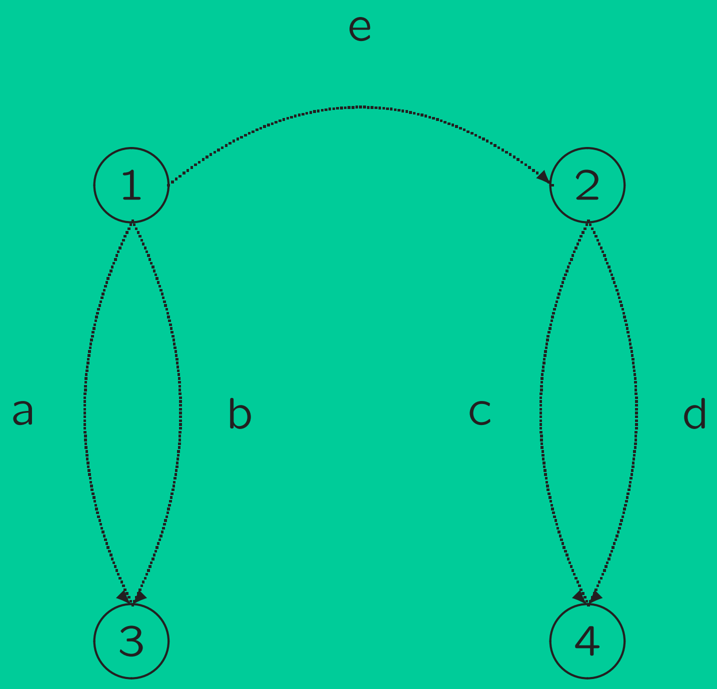
$$x_p \geq 0, \quad \forall p \in P. \tag{32}$$

The optimal value of the objective function is denoted by Q^{3^*}.

Analogously to the definitions of viability of a network in Situations 1 and 2 above, I now state:

Definition 4 (Viability of a Transportation Network in the Case of Known Total Number of Trips)

A network is said to be viable in the case of known total number of trips if there exists a solution to Linear System 4; equivalently, if $Q^{3^*} \leq \bar{Q}$.



Network topology for Example 4

An example is now presented which illustrates that a transportation network may not be viable in the case of given travel O/D pairs and travel demands; may not be viable in the case of known origins and trip productions: but, nevertheless, may be viable in the case of known total number of trips.

Example 4

Consider the transportation network depicted in the Figure consisting of four nodes: 1, 2, 3, 4, and five links: a, b, c, d, and e. Consider, first, the case of known O/D pairs and travel demands and assume that there are 3 O/D pairs: $w_1 = (1, 3)$, $w_2 = (1, 2)$, and $w_3 = (2, 4)$.

Assume that the travel demands are: $d_{w_1} = 2$, $d_{w_2} = 2$, and $d_{w_3} = 2$.

Let the paths be as follows: For O/D pair w_1: $p_1 = a$, $p_2 = b$; for O/D pair w_2: $p_3 = e$, and for O/D pair w_3: $p_4 = c$, and $p_5 = d$.

The emission factors are: $h_a = 0.1$, $h_b = 0.1$, $h_c = 0.2$, $h_d = 0.2$, and $h_e = 0.7$. Assume that the desired environmental quality standard $\bar{Q} = 0.6$.

It is easy to see that this environmental quality standard is not achievable by this network under fixed O/D pairs by noting that the emissions generated just by travelers between O/D pair w_2 are equal to 1.4.

Suppose now that one wishes to investigate whether the network is viable in Situation 1.

Note that one now has two origin nodes: 1 and 2 (which corresponded to the origin nodes in the O/D pairs originally), and three possible destination nodes: 2, 3, and 4.

The trip production at node 1, O_1, is equal to $d_{w_1} + d_{w_2}$ since O/D pairs w_1 and w_2 both had origin nodes 1 or $2 + 2 = 4$, whereas the trip production at node 2, O_2, is equal to 2.

One notes that the solution to Problem 3 yields $Q^{1^*} = 0.8$, which still exceeds the desired environmental quality standard of 0.6.

Finally, consider Situation 3, where the total number of trips $T = 2+2+2 = 6$ (and corresponds to $d_{w_1} + d_{w_2} + d_{w_3}$ in the original network).

It is easy to verify that the environmental quality standard of 0.6 is now achievable.

For example, a path flow pattern that satisfies Linear System 4 is given by: $x_{p_1} = x_{p_2} = 3$, with all other path flows equal to 0.

Hence, the first equation in Linear System 4 is satisfied as well as the second constraint. Furthermore, the path flows are nonnegative.

Alternative path flow patterns which would satisfy the system are: $x_{p_1} = 6$, with all other path flows being zero, and $x_{p_2} = 6$, with all other path flows being zero.

Note that viability of transportation networks was defined in the context of the network topology, the given emission factors, as well as the environmental quality standard.

Examples 3 and 4 illustrate that a network may be viable in one situation but not in another situation. Of course, one may also make a network viable through technological improvements such as through the reduction of the emission factors.

We leave the discussion of technological and network design issues for a letare lecture in this course.

Viability of Elastic Demand Traffic Networks

I now consider transportation networks in which the travl demands no longer take on fixed values but are now variables. Hence, in the case of known origin/destination pairs, the associated travel demands would now be variables.

In the case of Situation 1, the trip productions would no longer be fixed but would be allowed to vary.

Similarly, in the case of Situation 2, the trip attractions would no longer be fixed but could vary, and in Situation 3, the total number of trips would also be allowed to vary.

Indeed, one may immediately write down the elastic counterparts to the Linear Systems 1–4, given, respectively, by Linear Systems 5–8.

Linear System 5

Determine a vector of travel demands d and a vector of path flows x satisfying:

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w \in W, \tag{33}$$

$$\sum_{a \in L} h_a \sum_{p \in P} x_p \delta_{ap} \leq \bar{Q}, \tag{34}$$

$$x_p \geq 0, \quad \forall p \in P. \tag{35}$$

Note that, in contrast to Linear System 1, one now needs to determine not only the vector of path flows x but also the vector of travel demands d.

Linear System 6

Determine a vector of trip productions O and a vector of path flows x satisfying the following linear system:

$$\sum_{p\in P_{O_y}} x_p = O_y, \quad \forall y \in Y, \tag{36}$$

$$\sum_{a\in L} h_a \sum_{y\in Y} \sum_{p\in P_{O_y}} x_p \delta_{ap} \leq \bar{Q}, \tag{37}$$

$$x_p \geq 0, \quad \forall p \in P_{O_y}, \quad \forall y \in Y. \tag{38}$$

Linear System 7

Determine a vector of trip attractions D and a vector of path flows x satisfying the following linear system:

$$\sum_{p\in P_{D_z}} x_p = D_z, \quad \forall z \in Z, \tag{39}$$

$$\sum_{a\in L} h_a \sum_{z\in Z} \sum_{p\in P_{D_z}} x_p \delta_{ap} \leq \bar{Q}, \tag{40}$$

$$x_p \geq 0, \quad \forall p \in P_{D_z}, \quad \forall z \in Z. \tag{41}$$

Linear System 8

Determine the total number of trips T and a vector of path flows x satisfying:

$$\sum_{p \in P} x_p = T, \tag{42}$$

$$\sum_{a \in L} h_a \sum_{p \in P} x_p \delta_{ap} \leq \bar{Q}, \tag{43}$$

$$x_p \geq 0, \quad \forall p \in P. \tag{44}$$

It is clear that there *always* exists a solution to each of the Linear Systems 4–8, since the trivial solution of setting the path flows equal to zero (with the demands, or trip productions or attractions, also equal to zero) will always guarantee that even the tighest environmental quality standard is met. However, this implies that there will be no travel on the transportation network!

One can also establish that there always exists a nontrivial solution to each of the Linear Systems 5–8, although the total number of demands, or trip productions, or trip attractions, or total number of trips may be very small.

Hence, we have the following observation:

Observation 1

A transportation network in the case of elastic demands, or elastic trip productions, or elastic trip attractions, or elastic total number of trips is always viable in that the corresponding linear system always has a solution.

In the case of such "elastic" traffic networks, Problem 2, and its counterparts to the elastic versions of Situations 1, 2, and 3, are relevant.

Indeed, a transportation authority may wish, for example, to solve Problem 2 in order to determine whether or not the total demand would be sufficient in order to maintain the transportation network.

In the case of the elastic version of Situation 1 (see also Linear System 5) the relevant problem would be the following:

Problem 6 (Determination of Maximum Achievable Trip Productions, Given the Environmental Quality Standard)

Determine a vector of trip productions O and a vector of path flows x satisfying:

$$\text{Maximize} \quad \sum_{y \in Y} O_y \tag{45}$$

subject to:

$$\sum_{p \in P_{O_y}} x_p = O_y, \quad \forall y \in Y, \tag{46}$$

$$\sum_{a \in L} h_a \sum_{y \in Y} \sum_{p \in P_{O_y}} x_p \delta_{ap} \leq \bar{Q}, \tag{47}$$

$$x_p \geq 0, \quad \forall p \in P_{O_y}, \quad \forall y \in Y. \tag{48}$$

The solution to this problem would determine the total number of trip productions that can be handled by the network, given the emission factors and the desired environmental quality standard.

Analogously, one can construct the mathematical programming problems in order to determine the maximal trip attractions, or the maximal total number of trips that can be handled by the network.

Sustainable Transportation Networks

I now turn to the presentation of one of the most fundamental concepts in this course, that of *sustainability*.

The term "sustainability" necessarily implies that a system will *last*.

Moreover, it should last in the context of the environmental setting in which it is situated and within the framework of the users of the system, which, unless an authority imposes some policies in order to modify their behavior, may be in conflict with the environmental goals which serve to determine the ultimate existence and usage of the system itself.

In the context of a transportation network, sustainability of the network is crucially linked to the behavior of the users of the network system, which are the travelers, and their interaction on the network through the travel demand structure, the cost structure, as well as any environmental policies.

I now provide a motivating example.

Example 5

I return to the network depicted in the first Figure, consisting of two nodes, three links, and a single O/D pair $w_1 = (1, 2)$. Assume, as in Examples 1 and 2, that the emission factors are: $h_a = 0.1$, $h_b = 0.2$, and $h_c = 0.3$, and the desired environmental quality standard, as in Problem 2, is $\bar{Q} = 1$.

I now impose a cost structure on the network consisting of user travel link cost functions as follows:

$$c_a(f_a) = 2f_a + 5, \quad c_b(f_b) = f_b + 8, \quad c_c(f_c) = 1.5f_c + 5.$$

Assuming the behavioral principle (also commonly referred to as Wardrop's first principle) of user-optimization, the travelers, in order to minimize their path travel costs (and in the absence of any policy interventions in order to guarantee that the behavior complies with the desired environmental quality standard), select their paths, where $p_1 = a$, $p_2 = b$, and $p_3 = c$, as follows:

$$x^*_{p_1} = 3, \quad x^*_{p_2} = 3, \quad x^*_{p_3} = 4,$$

which corresponds to the link load pattern:

$$f^*_a = 3, \quad f^*_b = 3, \quad f^*_c = 4,$$

and incurred user travel costs on the paths:

$$C_{p_1} = C_{p_2} = C_{p_3} = 11.$$

Indeed, this path flow pattern satisfies the traffic network equilibrium conditions given in (15).

Note, however, that the total emissions under this user-optimized path flow pattern is: $h_a f^*_a + h_b f^*_b + h_c f^*_c = 0.3 + 0.6 + 1.2 = 2.1$, which clearly exceeds the environmental quality standard of $\bar{Q} = 1$.

Nevertheless, we know that the network is viable in that there exists a solution to the Linear System 1, with the path flow pattern: $x_{p_1} = 10$, $x_{p_2} = x_{p_3} = 0$.

Suppose now that one considers a policy of adding tolls to the roads so that the travelers still behave in a user-optimized manner, but now one also wishes to guarantee that the environmental quality standard will be met.

If one assigns tolls t_a, t_b, and t_c, on links a, b, and c, respectively, as: $t_a = 0$, $t_b = 17$, and $t_c = 20$, then the new user-optimized pattern will be: $x^*_{p_1} = 10$, $x^*_{p_2} = x^*_{p_3} = 0$, with associated *generalized* user path travel costs: $\bar{C}_{p_1} = C_{p_1} + t_a = 25$, $\bar{C}_{p_2} = C_{p_2} + t_b = 25$, and $\bar{C}_{p_3} = C_{p_3} + t_c = 25$.

Hence, in the presence of tolls as assigned above, this user-optimized path flow pattern satisfies the environmental target since the total emissions generated are equal to 1.

However, note that travelers on link a, although they emit all the emissions, are not tolled since the toll on link a, t_a, is equal to zero.

In subsequent lectures, I provide policy instruments, including tolls and pollution permits, which make users of the transportation networks pay according to their emissions.

I am now ready to state the definition of a sustainable transportation network.

Definition 5 (Sustainable Transportation Network)

Henceforth, a transportation network is said to be sustainable if the flow pattern satisfies the conservation of flow equations and does not exceed the imposed environmental quality standard, subject to the operating behavioral principle(s) underlying the network.

Note that the definition of a sustainable transportation network does not exclude the imposition of policies; in fact, policies will be viewed as an essential tool in directing transportation networks to sustainability. Indeed, the Table emphasizes this important point.

Relationship between viability and sustainability of a network

Viability + Behavioral Principle	Appropriate Policy $\Rightarrow$	Sustainability

The principal reference for this lecture is the text for this course *Sustainable Transportation Networks* (2000), Anna Nagurney, Edward Elgar Publishers, Cheltenham, England.