Supply Chain Networks and Electronic Commerce

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Background

The study of supply chain networks that we are conducting is in the context of the Information Age with the innovations brought about by electronic commerce (e-commerce), which has had an enormous effect on the manner in which businesses as well as consumers order goods and have them transported.

Electronic commerce is defined as a "trade" that takes place over the Internet usually through a buyer visiting a seller's website and making a transaction there.

B2B Transactions

The major portion of e-commerce transactions is in the form of business-to-business (B2B) with estimates ranging from approximately \$1 billion to \$1 trillion in 1998 and with forecasts reaching as high as \$4.8 trillion in 2003 in the United States.

B2C Transactions

The business-to-consumer (B2C) component, on the other hand, has seen tremendous growth in recent years but its impact on the US retail activity is still relatively small. Nevertheless, this segment should grow to \$80 billion per year.

Supply Chains

As noted by the National Research Council (2000), the principal effect of B2B commerce, estimated to be 90 percent of all e-commerce by value and volume, is in the creation of new and more profitable **supply chain networks**.

A supply chain is a chain of relationships which synthesizes and integrates the movement of goods between suppliers, manufacturers, distributors, retailers, and consumers.

The **topic of supply chain analysis is multidisciplinary** by nature since it involves aspects of manufacturing, transportation and logistics, retailing/marketing, as well as economics.

It has been the subject of a growing body of literature with researchers focusing both on the conceptualization of the underlying problems, due to the complexity of the problem and the numerous agents, such as manufacturers, retailers, or consumers involved in the transactions, as well as on the analytics.

New Opportunities

The introduction of e-commerce has unveiled new opportunities in terms of research and practice in supply chain analysis and management. Indeed, the primary benefit of the Internet for business is its open access to potential suppliers and customers both within a particular country and beyond national boundaries. Consumers, on the other hand, may obtain goods which they physically could not locate otherwise.

In our research, we have developed a supernetwork framework for the study of supply chains with electronic commerce in the form of B2C and B2B transactions.

- The framework is sufficiently general to allow for the modeling, analysis, and computation of solutions to such problems.

- The focus is on the network interactions of the underlying agents and on the underlying competitive processes.

- Moreover, the emphasis is placed on the **equilibrium aspects** of the problems rather than, simply, the optimization ones. Of course, it is assumed that the agents in the supply chain behave in some optimal fashion.

- An equilibrium approach is necessary and valuable since it provides a benchmark against which one can evaluate both prices and product flows. Moreover, it captures the independent behavior of the various decision-makers as well as the effect of their interactions. Finally, it provides for the development of dynamic models, with possible disequilibrium behavior, to enable the study of the evolution of supply chains.

Publications:

Supernetworks: Decision-Making in the Information Age, with J. Dong (2002), Edward Elgar Publisher.

"A Supply Chain Network Equilibrium Model," with J. Dong and D. Zhang, *Transportation Research E* (2002), in press.

"Dynamics of Supply Chains: A Multilevel (Logistical/ Informational/Financial) Network Perspective," with K. Ke, J. Cruz, K. Hancock, and F. Southworth, revised and resubmitted to *Environment & Planning B* (2001).

"Supply Chain Networks and Electronic Commerce: A Theoretical Perspective," (2001), with J. Loo, J. Dong, and D. Zhang, submitted to *Netnomics* (2001).

The Manufacturers

In our framework, manufacturers are considered who are involved in the production of a homogeneous commodity, referred to also as the product, which can then be shipped to the retailers or to the consumers directly or to both.

The manufacturers obtain a price for the product (which is endogenous) and seek to determine their optimal production and shipment quantities, given the production costs as well as the *transaction* costs associated with conducting business with the different retailers and demand markets.

Here a transaction cost is considered to be sufficiently general, for example, to include the transportation/ shipping cost.

On the other hand, in the case of an e-commerce link, the transaction costs can include the cost associated with the use of such a link, the lack of productivity due to congestion, an associated risk, etc.

The Retailers

The retailers, in turn, must agree with the manufacturers as to the volume of shipments, either ordered physically or through the Internet, since they are faced with the handling cost associated with having the product in their retail outlet.

In addition, they seek to maximize their profits with the price that the consumers are willing to pay for the product being endogenous.

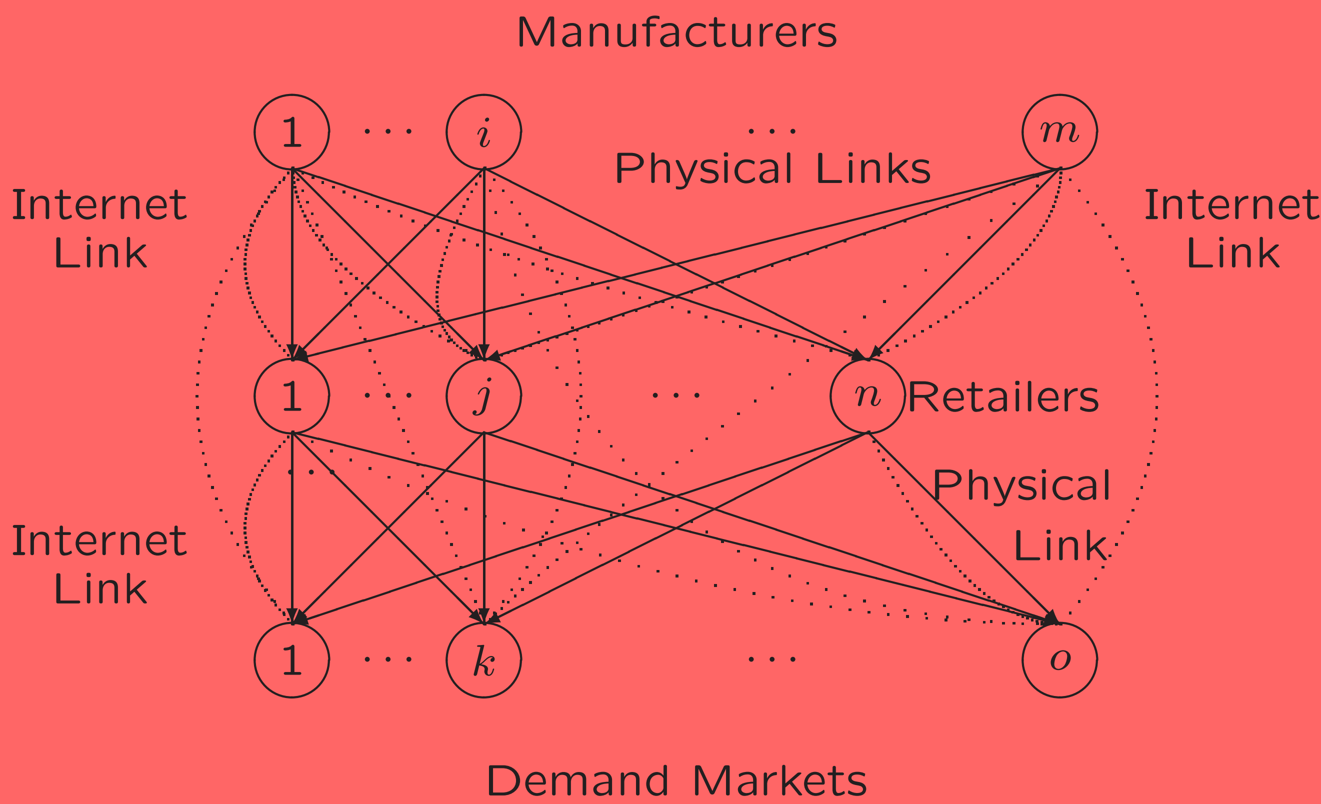
The Consumers

Finally, in this supply chain, the consumers provide the "pull" in that, given the demand functions at the various demand markets, they determine their optimal consumption levels from the various retailers (transacted either physically or through the Internet) and from the manufacturers (transacted through the Internet), subject both to the prices charged for the product as well as the cost of conducting the transaction (which, of course, may include the cost of transportation associated with obtaining the product from the manufacturer or the retailer).

The demand for the product is a central part of the supply chain framework.

It is shown that, in equilibrium, the structure of the supply chain network is that of a three-tiered network, with links connecting the top tier (the manufacturers) with the bottom tier (the demand markets) to represent e-commerce links and additional links from the top tier to the middle tier (the retailers) and from the middle tier to the bottom tier nodes to also represent the e-commerce links.

The variational inequality formulation of the governing equilibrium conditions is then utilized in order to obtain both qualitative properties as well as an algorithm for the computation of the equilibrium flows and prices.



The Multitiered Supernetwork Structure of the Supply Chain Network with E-Commerce at Equilibrium

Details Concerning the Supernetwork Construction

The links in the supply chain supernetwork in the Figure include classical physical links as well as Internet links to allow for e-commerce.

The introduction of e-commerce allows for “connections” that were, heretofore, not possible, such as enabling consumers, for example, to purchase a product directly from the manufacturers.

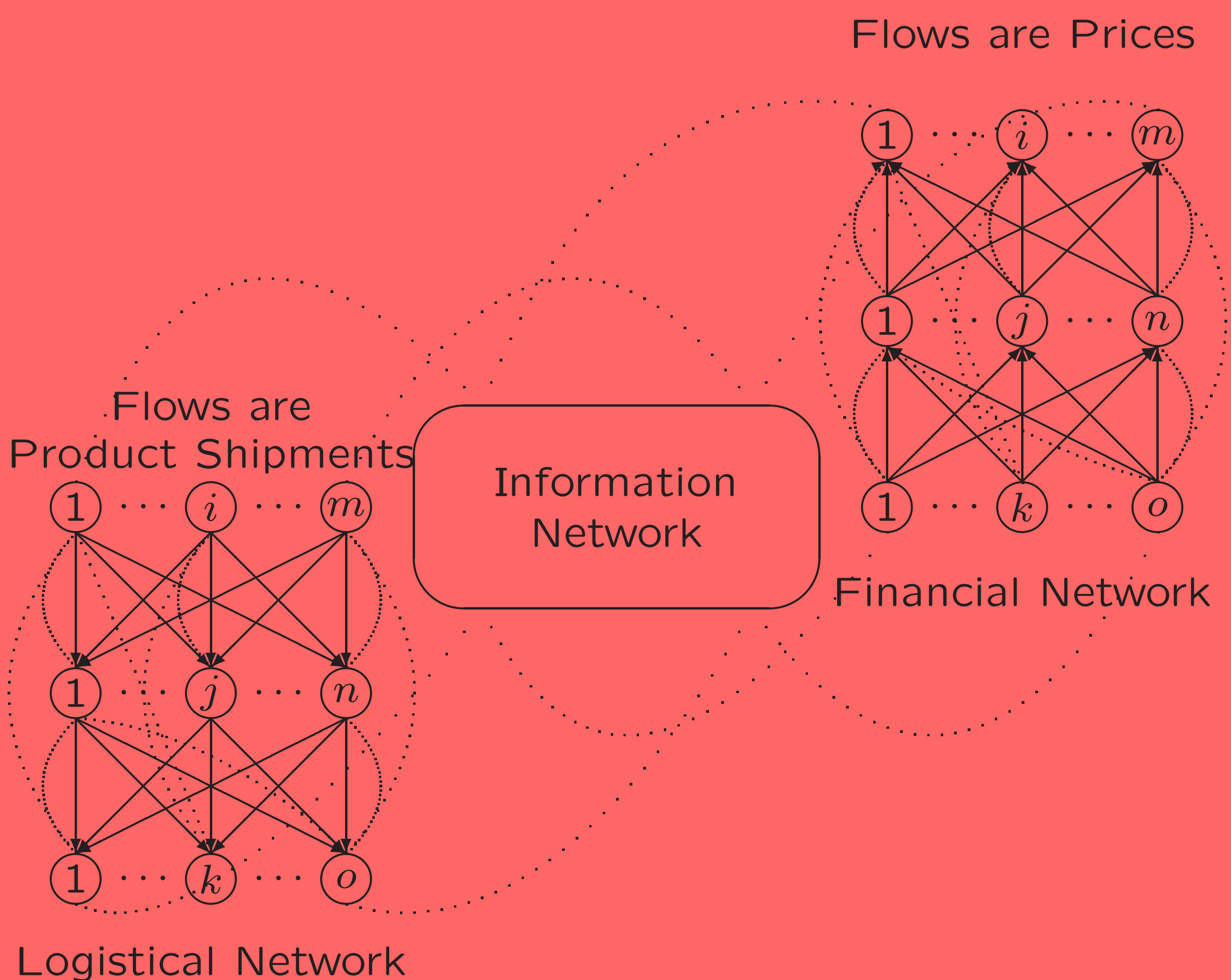
In order to conceptualize this B2C type of transaction, a direct link has been constructed from each top tier node to each bottom tier node.

In addition, since manufacturers can transact not only with the consumers directly but also with the retailers through the Internet, an additional link is added (to represent such a possible B2B transaction) between each top tier node and each middle tier node.

Hence, a manufacturer may transact with a retailer through either a physical link or through an Internet link, or through both. Finally, consumers can transact with retailers either via a physical link, or through an Internet link, or through both.

Conceptualization of the Dynamics

The supernetwork is a multilevel network consisting of: the logistical network, the information network, and the financial network. Such a perspective allows one to visualize and to identify the interrelationships between the individual networks. For example, in the case of e-commerce, orders over the Internet trigger shipments over logistical and transportation networks, and financial payments, in turn, over a financial network.



Multilevel Network Structure of the Supply Chain System with Electronic Commerce

The novelty of the proposed multilevel network framework allows one to capture distinct flows, in particular, *logistical*, *information*, and *financial* within the same network system.

Moreover, since both the logistical and financial networks are multitiered in structure, one is able to observe, through a discrete-time process, how the prices as well as the product shipments are adjusted from iteration to iteration (time period to time period), until the equilibrium state is reached.

Although the focus here is on a supply chain consisting of competing manufacturers, retailers, and consumers, the framework is sufficiently general to include other levels of decision-makers in the network such as suppliers and/or owners of distribution centers, for example.

The Dynamics

We now describe the dynamics by which the manufacturers adjust their product shipments over time, the consumers adjust their consumption amounts based on the prices of the product at the demand markets, and the retailers operate between the two. We also describe the dynamics by which the prices adjust over time.

The product flows evolve over time on the logistical network, whereas the prices do so over the financial network.

The information network stores and provides the product shipment and price information so that the new product shipments and prices between tiers of network agents can be computed.

The dynamics are derived from the bottom tier of nodes on up since, as mentioned previously, it is the demand for the product (and the corresponding prices) that actually drives the supply chain dynamics.

The Demand Market Price Dynamics

We begin by describing the dynamics underlying the prices of the product associated with the demand markets (see the bottom-tiered nodes in the financial network). Assume, as given, a demand function d_k, which can depend, in general, upon the entire vector of prices ρ_3.

Assume that the rate of change of the price ρ_{3k}, denoted by $\dot{\rho}_{3k}$, is equal to the difference between the demand at the demand market k, as a function of the demand market prices, and the amount available from the retailers and the manufacturers at the demand market.

Hence, if the demand for the product at the demand market (at an instant in time) exceeds the amount available, the price at that demand market will increase; if the amount available exceeds the demand at the price, then the price at the demand market will decrease. The dynamics of the price ρ_{3k} associated with the product at demand market k can be expressed as:

$$\dot{\rho}_{3k} = \begin{cases} d_k(\rho_3) - \sum_{j=1}^n \sum_{l=1}^2 q_{jkl} - \sum_{i=1}^m q_{ik}, & \text{if} \quad \rho_{3k} > 0 \\ \max\{0, d_k(\rho_3) - \sum_{j=1}^n \sum_{l=1}^2 q_{jkl} - \sum_{i=1}^m q_{ik}\}, \\ \text{if} \;\; \rho_{3k} = 0. \end{cases}$$

The Dynamics of the Product Shipments between the Retailers and the Demand Markets

The dynamics of the product shipments in the logistical network taking place over the links joining the retailers to the demand markets are now described. There is a unit transaction cost $\hat{c}_{jkl}$ associated with transacting between retailer j and the consumers at demand market k, via mode l, where $\hat{c}_{jkl}$ and can depend upon, in general, all the product shipments to all the demand markets.

The rate of change of the product shipment q_{jkl} is assumed to be equal to the difference between the price the consumers are willing to pay for the product at demand market k minus the unit transaction cost and the price charged for the product at the retail outlet. One may write:

$$\dot{q}_{jkl} = \begin{cases} \rho_{3k} - \hat{c}_{jkl}(Q^2, Q^3) - \rho_{2j}, & \text{if } q_{jkl} > 0 \\ \max\{0, \rho_{3k} - \hat{c}_{jkl}(Q^2, Q^3) - \rho_{2j}\}, & \text{if } q_{jkl} = 0, \end{cases}$$

where $\dot{q}_{jkl}$ denotes the rate of change of the product shipment q_{jkl}.

The Dynamics of the Product Shipments between the Manufacturers and the Demand Markets

It is assumed that each manufacturer i is faced with a production cost f_i, which can depend, in general, upon all the product shipments from all the manufacturers to the retailers and demand markets, that is,

$$f_i = f_i(Q^1, Q^2), \quad \forall i.$$

In addition, c_{ik} is the transaction cost associated with manufacturer i transacting with demand market k. The consumers at the demand markets, in turn, are also faced with a transaction cost associated with transacting with a manufacturer directly. For manufacturer/demand market pair (i, k), this function is denoted by $\hat{c}_{ik}$ and can depend, in general, upon all the product shipments to all the demand markets from all the manufacturers or retailers.

The consumers at demand market k also incur a unit transaction cost associated with transacting with manufacturer i. Thus, the following rate of change for the product shipments between the top tier of nodes and the bottom tier of nodes in the logistical network is proposed:

$$\dot{q}_{ik} = \begin{cases} \rho_{3k} - \frac{\partial f_i(Q^1,Q^2)}{\partial q_{ik}} - \frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}} - \hat{c}_{ik}(Q^2, Q^3), & \text{if} \;\; q_{ik} > 0 \\ \max\{0, \rho_{3k} - \frac{\partial f_i(Q^1,Q^2)}{\partial q_{ik}} - \frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}} - \hat{c}_{ik}(Q^2, Q^3)\}, \\ \text{if} \;\; q_{ik} = 0, \end{cases}$$

where $\dot{q}_{ik}$ denotes the rate of change of the product shipment q_{ik}.

The Dynamics of the Prices at the Retail Outlets

The prices for the product at the retail outlets, in turn, must reflect supply and demand conditions as well (and as shall be shown shortly also reflect profit-maximizing behavior on the part of the retailers who seek to determine how much of the product they obtain from the different manufacturers for their outlet). In particular, assume that the price for the product associated with retail outlet j, ρ_{2j}, and computed at node j lying in the second tier of nodes of the financial network, evolves over time according to:

$$\dot{\rho}_{2j} = \begin{cases} \sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl} - \sum_{i=1}^{m} \sum_{l=1}^{2} q_{ijl}, \text{if } \rho_{2j} > 0 \\ \max\{0, \sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl} - \sum_{i=1}^{m} \sum_{l=1}^{2} q_{ijl}\}, \\ \text{if } \rho_{2j} = 0, \end{cases}$$

where $\dot{\rho}_{2j}$ denotes the rate of change of the retail price ρ_{2j}.

The Dynamics of Product Shipments between Manufacturers and Retailers

The dynamics underlying the product shipments between the manufacturers and the retailers are now described. As already noted, each manufacturer is faced with a production cost and transaction costs. Recall that the transaction cost associated with manufacturer i and retailer j transacting via mode l is denoted by c_{ijl}. A retailer j, in turn, is faced with a handling cost.

Since the product shipments sent from the manufacturers must be accepted by the retailers in order for the transactions to take place in the supply chain, we propose the following rate of change for the product shipments between the top tier of nodes and the middle tier in the logistical network:

$$\dot{q}_{ijl} = \begin{cases} \rho_{2j} - \frac{\partial f_i(Q^1,Q^2)}{\partial q_{ijl}} - \frac{\partial c_{ijl}(q_{ijl})}{\partial q_{ijl}} - \frac{\partial c_j(Q^1)}{\partial q_{ijl}} - \frac{\partial \hat{c}_{ijl}(q_{ijl})}{\partial q_{ijl}}, \\ \text{if } q_{ijl} > 0 \\ \max\{0, \\ \rho_{2j} - \frac{\partial f_i(Q^1,Q^2)}{\partial q_{ijl}} - \frac{\partial c_{ijl}(q_{ijl})}{\partial q_{ijl}} - \frac{\partial c_j(Q^1)}{\partial q_{ijl}} - \frac{\partial \hat{c}_{ijl}(q_{ijl})}{\partial q_{ijl}}\}, \\ \text{if } q_{ijl} = 0, \end{cases}$$

where $\dot{q}_{ijl}$ denote the rate of change of the product shipment between manufacturer i and retailer j transacted via mode l.

The Projected Dynamical System

Let X and $-F(X)$ be defined as: $X \equiv (Q^1, Q^2, Q^3, \rho_2, \rho_3)$, $F(X) \equiv (F_{ijl}, F_{ik}, F_{jkl}, F_j, F_k)$ for $\{i = 1, ..., m;\ j = 1, ..., n;\ l = 1, 2;\ k = 1, ..., o\}$, where the specific components of $-F$ are given by the functional terms preceding the first "if" term in the above dynamic expressions, respectively.

Here
$\mathcal{K} \equiv \{(Q^1, Q^2, Q^3, \rho_2, \rho_3) | (Q^1, Q^2, Q^3, \rho_2, \rho_3) \in R_+^{2mn+mo+2no+n+o}\}$.

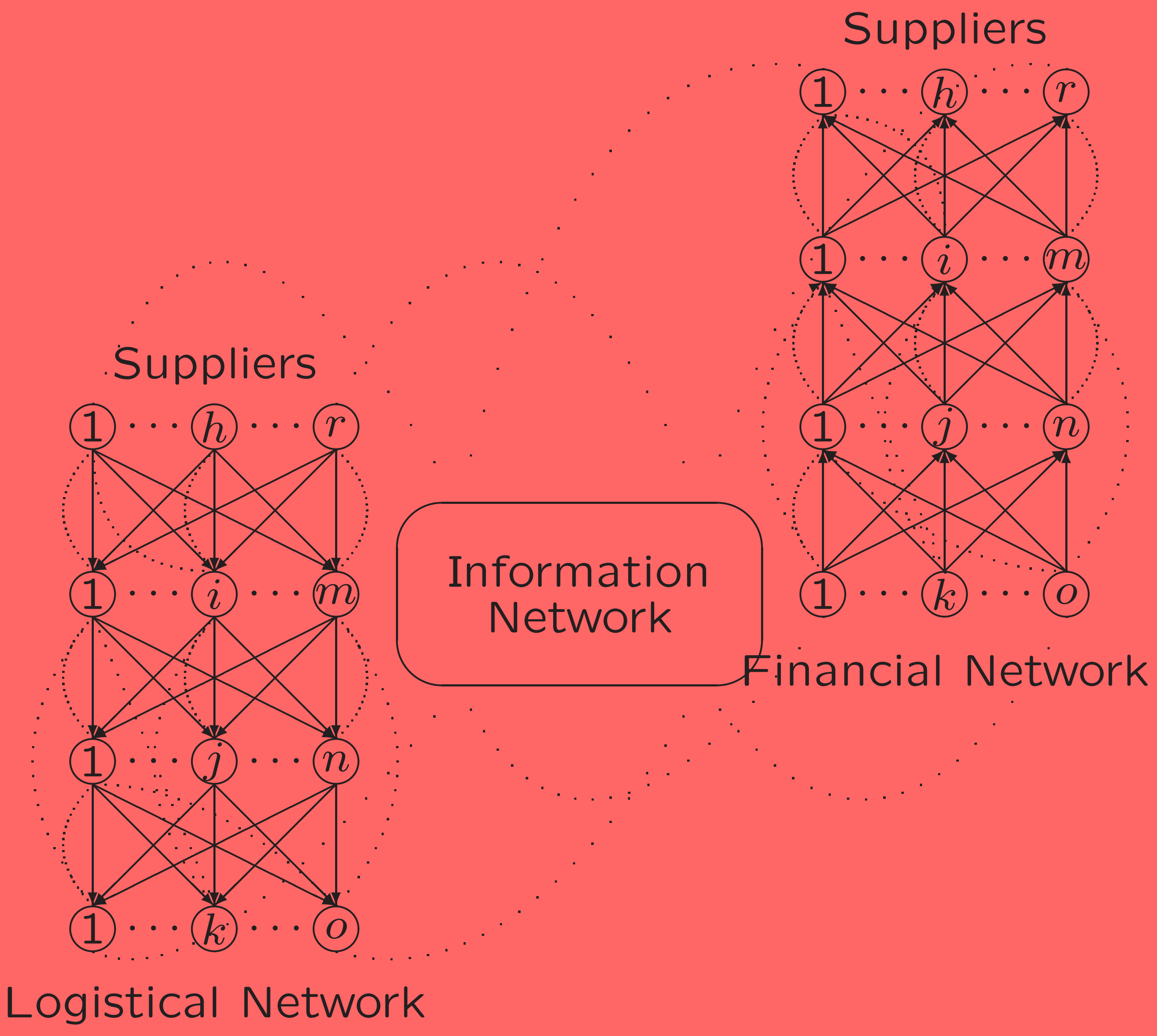
Then the dynamic model can be rewritten as the projected dynamical system (PDS) defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0,$$

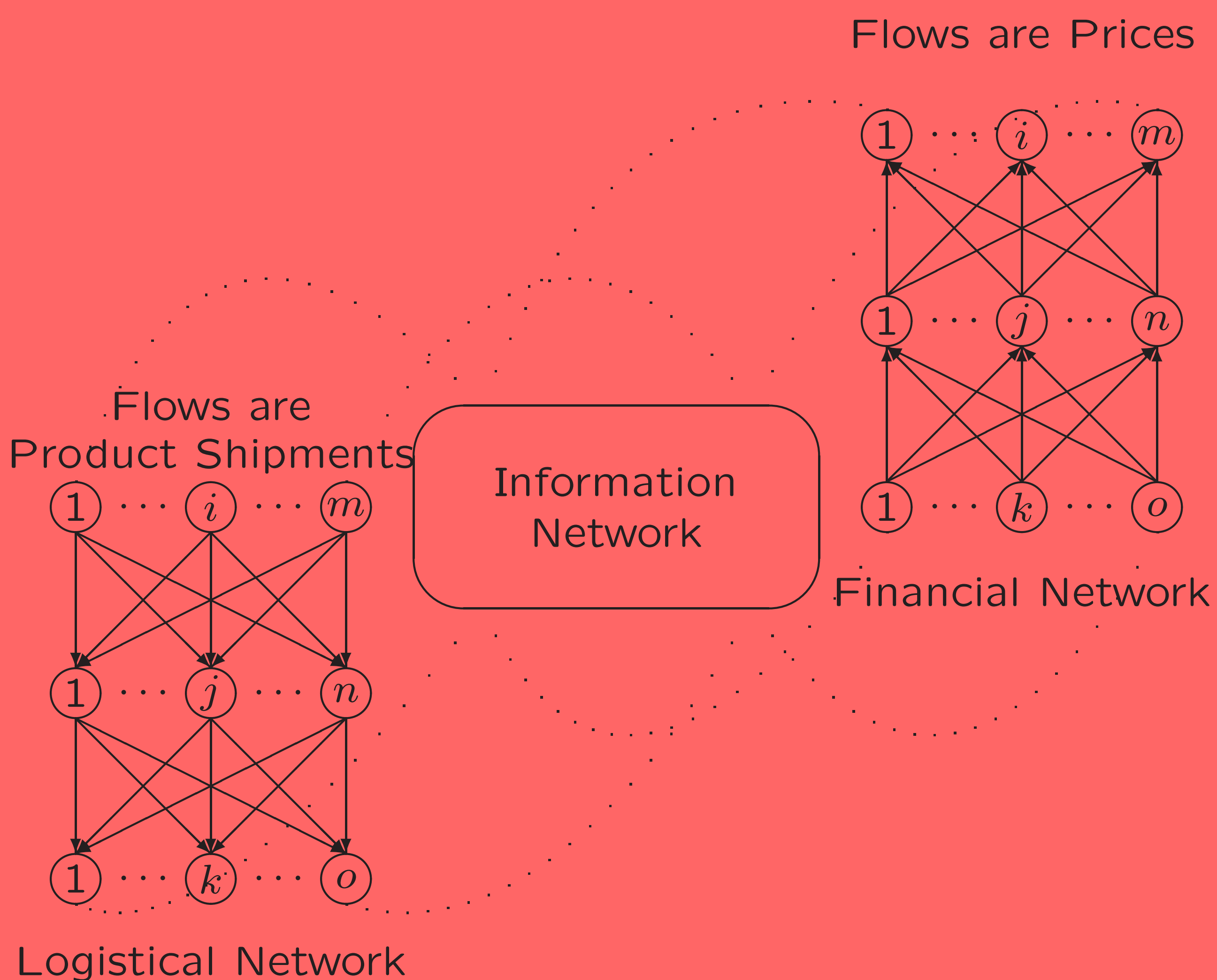
where $\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto $\mathcal{K}$ at X and $X_0 = (Q^{1^0}, Q^{2^0}, Q^{3^0}, \rho_2^0, \rho_3^0)$ is the initial point corresponding to the initial product shipments between the manufacturers and the retailers and the demand markets; the initial product shipments between the retailers and the demand markets; and the initial retailers' prices and the demand prices.

Stationary/Equilibrium Points

The following theorem states that the projected dynamical system evolves until it reaches a stationary point, that is, $\dot{X} = 0$, in which there is no change in the product shipments and prices, and that the stationary point coincides with the equilibrium point of the supply chain network model.



Multilevel Network Structure of the Supply Chain System with a Tier of Suppliers



Multilevel Network Structure of the Supply Chain System without Electronic Commerce

The Discrete-Time Adjustment Process

Step 0: Initialization Step

Set $(Q^{10}, Q^{20}, Q^{30}, \rho_2^0, \rho_3^0) \in \mathcal{K}$. Let $\tau = 1$ and set the sequence $\{\alpha_\tau\}$ so that $\sum_{\tau=1}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$.

Step 1: Computation

Compute $(Q^{1^\tau}, Q^{2^\tau}, Q^{3^\tau}, \rho_2^\tau, \rho_3^\tau) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[q_{ijl}^{\tau} + \alpha_\tau\left(\frac{\partial f_i(Q^{1^{\tau-1}}, Q^{2^{\tau-1}})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl}^{\tau-1})}{\partial q_{ijl}}\right.\right.$$

$$\left.\left.+\frac{\partial c_j(Q^{1^{\tau-1}})}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl}^{\tau-1})}{\partial q_{ijl}} - \rho_{2j}^{\tau-1}\right) - q_{ijl}^{\tau-1}\right] \times \left[q_{ijl} - q_{ijl}^{\tau}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[q_{ik}^{\tau} + \alpha_\tau\left(\frac{\partial f_i(Q^{1^{\tau-1}}, Q^{2^{\tau-1}})}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^{\tau-1})}{\partial q_{ik}}\right.\right.$$

$$\left.\left.+\hat{c}_{ik}(Q^{2^{\tau-1}}, Q^{3^{\tau-1}}) - \rho_{3k}^{\tau-1}\right) - q_{ik}^{\tau-1}\right] \times \left[q_{ik} - q_{ik}^{\tau}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\Big[q_{jkl}^{\tau}+\alpha_\tau(\rho_{2j}^{\tau-1}+\hat{c}_{jkl}(Q^{2^{\tau-1}},Q^{3^{\tau-1}})-\rho_{3k}^{\tau-1})$$

$$-q_{jkl}^{\tau-1}\Big]\times\left[q_{jkl}-q_{jkl}^{\tau}\right]$$

$$+\sum_{j=1}^{n}\left[\rho_{2j}^{\tau}+\alpha_\tau(\sum_{i=1}^{m}\sum_{l=1}^{2}q_{ijl}^{\tau-1}-\sum_{k=1}^{o}\sum_{l=1}^{2}q_{jkl}^{\tau-1})-\rho_{2j}^{\tau-1}\right]$$

$$\times\left[\rho_{2j}-\rho_{2j}^{\tau}\right]$$

$$+\sum_{k=1}^{o}\left[\rho_{3k}^{\tau}+\alpha_\tau(\sum_{j=1}^{n}\sum_{l=1}^{2}q_{jkl}^{\tau-1}+\sum_{i=1}^{m}q_{ik}^{\tau-1}-d_k(\rho_3^{\tau-1}))-\rho_{3k}^{\tau-1}\right]$$

$$\times[\rho_{3k}-\rho_{3k}^{\tau}]\geq 0,\quad \forall(Q^1,Q^2,Q^3,\rho_2,\rho_3)\in\mathcal{K}.$$

Step 2: Convergence Verification

If $|q_{ijl}^{\tau}-q_{ijl}^{\tau-1}|\leq\epsilon$, $|q_{ik}^{\tau}-q_{ik}^{\tau-1}|\leq\epsilon$, $|q_{jkl}^{\tau}-q_{jkl}^{\tau-1}|\leq\epsilon$, $|\rho_{2j}^{\tau}-\rho_{2j}^{\tau-1}|\leq\epsilon$, $|\rho_{3k}^{\tau}-\rho_{3k}^{\tau-1}|\leq\epsilon$, for all $i=1,\cdots,m$; $j=1,\cdots,n$; $l=1,2$; $k=1,\cdots,o$, with $\epsilon>0$, a pre-specified tolerance, then stop; otherwise, set $\tau:=\tau+1$, and go to Step 1.

Since $\mathcal{K}$ is the nonnegative orthant the solution of the above is accomplished exactly and in closed form.

For completeness and easy reference, we show how:

Computation of the Product Shipments

At iteration τ compute the q_{ijl}^{τ}s according to:

$$q_{ijl}^{\tau} = \max\left\{0, q_{ijl}^{\tau-1} - \alpha_{\tau}(\frac{\partial f_i(Q^{1^{\tau-1}}, Q^{2^{\tau-1}})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q_{ijl}^{\tau-1})}{\partial q_{ijl}}\right.$$

$$\left. + \frac{\partial c_j(Q^{1^{\tau-1}})}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl}^{\tau-1})}{\partial q_{ijl}} - \rho_{2j}^{\tau-1})\right\}, \forall i, j, l.$$

In addition, at iteration τ, compute the q_{ik}^{τ}s according to:

$$q_{ik}^{\tau} = \max\left\{0, q_{ik}^{\tau-1} - \alpha_{\tau}(\frac{\partial f_i(Q^{1^{\tau-1}}, Q^{2^{\tau-1}})}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^{\tau-1})}{\partial q_{ik}}\right.$$

$$\left. + \hat{c}_{ik}(Q^{2^{\tau-1}}, Q^{3^{\tau-1}}) - \rho_{3k}^{\tau-1})\right\}, \forall i, k.$$

Also, at iteration τ compute the q^{τ}_{jkl}s according to:

$$q^{\tau}_{jkl} = \max\{0, q^{\tau-1}_{jkl} - \alpha_{\tau}(\rho^{\tau-1}_{2j} + \hat{c}_{jkl}(Q^{2^{\tau-1}}, Q^{3^{\tau-1}}) - \rho^{\tau-1}_{3k})\}, \forall j, k, l.$$

Computation of the Prices

The prices, ρ^{τ}_{2j}, in turn, are computed at iteration τ explicitly according to:

$$\rho^{\tau}_{2j} = \max\{0, \rho^{\tau-1}_{2j} - \alpha_{\tau}(\sum_{i=1}^{m}\sum_{l=1}^{2} q^{\tau-1}_{ijl} - \sum_{k=1}^{o}\sum_{l=1}^{2} q^{\tau-1}_{jkl})\}, \quad \forall j,$$

whereas the prices, ρ_{3k}, are computed according to:

$$\rho^{\tau}_{3k} = \max\{0, \rho^{\tau-1}_{3k} - \alpha_{\tau}(\sum_{j=1}^{n}\sum_{l=1}^{2} q^{\tau-1}_{jkl} + \sum_{i=1}^{m} q^{\tau-1}_{ik} - d_k(\rho^{\tau-1}_{3}))\},$$

$$\forall k.$$

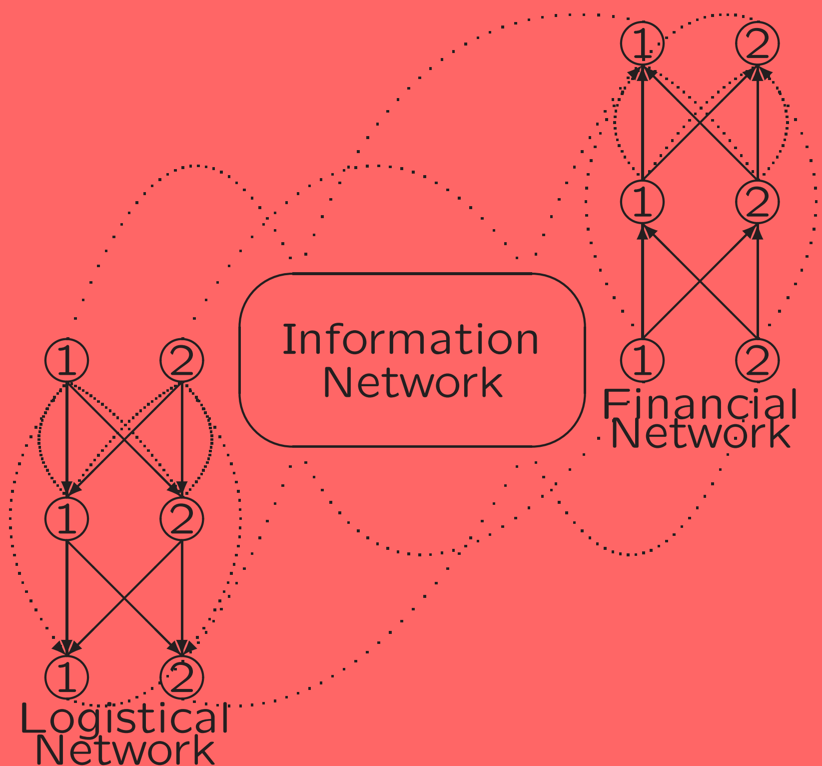
Numerical Examples

The discrete-time adjustment process (the Euler method) is now applied to several dynamic numerical supply chain examples. Two sets of examples were solved, consisting of three examples each. The first set of numerical examples consisted of supply chain network problems with e-commerce and these were solved via a FORTRAN implementation of the algorithm.

The second set of numerical examples consisted of supply chain network problems without e-commerce and these were solved via a FORTRAN implementation. The computer system used was a DEC Alpha system located at the University of Massachusetts at Amherst. The convergence criterion was that the absolute value of the flows and prices between two successive iterations differed by no more than 10^{-4}. The sequence $\{a_\tau\}$ was set to $\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$ for all the examples. The initial product shipments and prices were all set to zero for each example.

Examples 1, 2, and 3

The first three numerical examples had the multilevel network structure depicted in the Figure and consisted of two manufacturers, two retailers, and two demand markets with electronic commerce between manufacturers and retailers and manufacturers and the demand markets only. The data for the three examples were as follows.



Multilevel Network for Examples 1, 2, and 3

Example 1

The data for the first example were constructed for easy interpretation purposes. The production cost functions for the manufacturers were given by:

$$f_1(q) = 2.5q_1^2 + q_1 q_2 + 2q_1, \quad f_2(q) = 2.5q_2^2 + q_1 q_2 + 2q_2.$$

The transaction cost functions faced by the manufacturers and associated with transacting with the retailers using the physical link, that is, mode 1, were given by:

$$c_{111}(q_{111}) = .5q_{111}^2 + 3.5q_{111}, \quad c_{121}(q_{121}) = .5q_{121}^2 + 3.5q_{121},$$

$$c_{211}(q_{211}) = .5q_{211}^2 + 3.5q_{211}, \quad c_{221}(q_{221}) = .5q_{221}^2 + 3.5q_{221},$$

whereas the analogous transaction costs, but for mode 2, were given by:

$$c_{112}(q_{112}) = 1.5q_{112}^2 + 3q_{112}, \quad c_{122}(q_{122}) = 1.5q_{122}^2 + 3q_{122},$$

$$c_{212}(q_{212}) = 1.5q_{212}^2 + 3q_{212}, \quad c_{222}(q_{222}) = 1.5q_{222}^2 + 3q_{222},$$

The transaction costs of the manufacturers associated with dealing with the consumers at the demand markets via the Internet were given by:

$$c_{11}(q_{11}) = q_{11}^2 + 2q_{11}, \quad c_{12}(q_{12}) = q_{12}^2 + 2q_{12},$$

$$c_{21}(q_{21}) = q_{21}^2 + 2q_{21}, \quad c_{22}(q_{22}) = q_{22}^2 + 2q_{22}.$$

The handling costs of the retailers, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i1l})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i2l})^2.$$

The transaction costs of the retailers associated with transacting with the manufacturers via mode 1 and mode 2 were, respectively, given by:

$$\hat{c}_{111}(q_{111}) = 1.5q_{111}^2 + 3q_{111}, \quad \hat{c}_{121}(q_{121}) = 1.5q_{121}^2 + 3q_{121},$$

$$\hat{c}_{211}(q_{211}) = 1.5q_{211}^2 + 3q_{211}, \quad \hat{c}_{221}(q_{221}) = 1.5q_{221}^2 + 3q_{221},$$

$$\hat{c}_{112}(q_{112}) = 1.5q_{112}^2 + 3q_{112}, \quad \hat{c}_{122}(q_{122}) = 1.5q_{122}^2 + 3q_{122},$$

$$\hat{c}_{212}(q_{212}) = 1.5q_{212}^2 + 3q_{212}, \quad \hat{c}_{222}(q_{222}) = 1.5q_{222}^2 + 3q_{222}.$$

The demand functions at the demand markets were:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000$$

$$d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000,$$

and the transaction costs between the retailers and the consumers at the demand markets (denoted for a typical pair by $\hat{c}_{jkl}$ with the associated shipment by q_{jkl} with $l = 1$) were given by:

$$\hat{c}_{111}(Q^2, Q^3) = q_{111} + 5, \quad \hat{c}_{121}(Q^2, Q^3) = q_{121} + 5,$$

$$\hat{c}_{211}(Q^2, Q^3) = q_{211} + 5, \quad \hat{c}_{221}(Q^2, Q^3) = q_{221} + 5,$$

whereas the transaction costs associated with transacting with the manufacturers via the Internet for the consumers at the demand markets (denoted for a typical such pair by $\hat{c}_{ik}$ with the associated shipment of q_{ik}) were given by:

$$\hat{c}_{11}(Q^2, Q^3) = q_{11} + 1, \quad \hat{c}_{12}(Q^2, Q^3) = q_{12} + 1,$$

$$\hat{c}_{21}(Q^2, Q^3) = q_{21} + 1, \quad \hat{c}_{22}(Q^2, Q^3) = q_{22} + 1.$$

The Euler method yielded the following equilibrium pattern: the product shipments between the two manufacturers and the two retailers associated with the physical links, and with the Internet links, respectively, that is, with transacting via mode 1 and mode 2 were:

$$Q^{1^*}: q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 3.4611,$$

$$q^*_{112} = q^*_{122} = q^*_{212} = q^*_{222} = 2.3907.$$

The product shipments between the two manufacturers and the two demand markets with transactions conducted through the Internet were:

$$Q^{2^*}: q^*_{11} = q^*_{12} = q^*_{21} = q^*_{22} = 13.3033.$$

The product shipments (consumption volumes) between the two retailers and the two demand markets were:

$$Q^{3^*}: q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 5.8513.$$

The vector ρ_2^*, which was equal to the prices charged by the retailers γ^*, had components:

$$\rho^*_{21} = \rho^*_{22} = 263.9088,$$

and the demand prices at the demand markets were:

$$\rho^*_{31} = \rho^*_{32} = 274.7701.$$

It is easy to verify that the optimality/equilibrium conditions were satisfied with good accuracy.

Note that the price charged by the manufacturers to the consumers at the demand markets, approximately 260, was higher than the price charged to the retailers, regardless of the mode of transacting. The price charged to the retailers for the product transacted via the Internet, in turn, exceeded that charged using the classical physical manner.

Example 2

Example 1 was then modified as follows: The production cost function for manufacturer 1 was now given by:

$$f_1(q) = 2.5q_1^2 + q_1 q_2 + 12q_1,$$

whereas the transaction costs for manufacturer 1 were now given by:

$$c_{11}(Q^1) = q_{11}^2 + 3.5q_{11}, \quad c_{12}(Q^1) = q_{12}^2 + 3.5q_{12}.$$

The remainder of the data was as in Example 4.1. Hence, both the production costs and the transaction costs increased for manufacturer 1.

The Euler method yielded the following equilibrium pattern: the product shipments between the two manufacturers and the two retailers associated with the physical links, and with the Internet links, respectively, that is, with transacting via mode 1 and mode 2 were:

$$Q^{1^*} : q_{111}^* = q_{121}^* = 3.3265, \quad q_{211}^* = q_{221}^* = 3.5408,$$

$$q_{112}^* = q_{122}^* = 2.3010, \quad q_{212}^* = q_{222}^* = 2.4438.$$

The product shipments between the two manufacturers and the two demand markets with transactions conducted through the Internet were:

$$Q^{2^*} : q_{11}^* = q_{12}^* = 12.5781, \quad q_{21}^* = q_{22}^* = 13.3638.$$

The product shipments (consumption volumes) between the two retailers and the two demand markets were:

$$Q^{3^*}: q_{11}^* = q_{12}^* = q_{21}^* = q_{22}^* = 5.8056.$$

The vector ρ_2^* had components:

$$\rho_{21}^* = \rho_{22}^* = 264.1706,$$

and the demand prices at the demand markets were:

$$\rho_{31}^* = \rho_{32}^* = 274.9861.$$

The optimality/equilibrium conditions were, again, satisfied at the desired accuracy.

Note that, again, the prices charged by the manufacturers to the consumers at the demand markets were higher than the prices charged to the retailers. Of course, the demand price was, nevertheless, equal for all consumers at a given demand market. In fact, both in this and in the preceding example the equilibrium demand prices were the same for each demand market.

Hence, manufacturer 1 now produced less than it did in Example 1, whereas manufacturer 2 increased its production output. The prices charged by the retailers to the consumers increased, as did the prices at the demand markets, with a decrease in the incurred demand.

Example 3

Example 3 was constructed by changing Example 2 as follows. The data were identical to that in Example 2 except that the demand function for demand market 1 was now:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 2000.$$

The Euler method yielded the following equilibrium pattern: the product shipments between the two manufacturers and the two retailers associated with the physical links, and with the Internet links, respectively, that is, with transacting via mode 1 and mode 2 were:

$$Q^{1^*}: q^*_{111} = q^*_{121} = 16.1444, \quad q^*_{211} = q^*_{221} = 16.4974,$$

$$q^*_{112} = q^*_{122} = 10.8463, \quad q^*_{212} = q^*_{222} = 11.0816.$$

The product shipments between the two manufacturers and the two demand markets with transactions conducted through the Internet were:

$$Q^{2^*}: q^*_{11} = 60.2397, \quad q^*_{12} = 0.0000,$$

$$q^*_{21} = 61.2103, \quad q^*_{22} = 0.0000.$$

The product shipments (consumption volumes) between the two retailers and the two demand markets were:

$$Q^{3*}: q^*_{111} = 54.5788, \quad q^*_{121} = 0.0000,$$

$$q^*_{211} = 54.5788, \quad q^*_{221} = 0.0000,$$

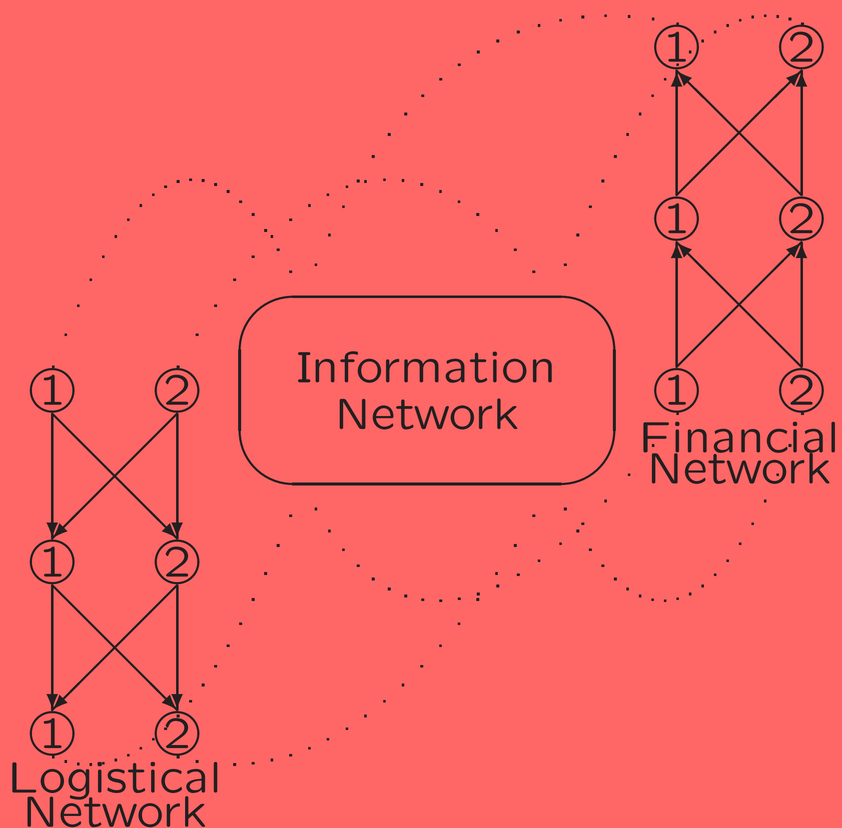
the vector ρ^*_2, which was equal to the prices charged by the retailers γ^*, had components:

$$\rho^*_{21} = \rho^*_{22} = 825.1216,$$

and the demand prices at the demand markets were:

$$\rho^*_{31} = 884.694, \quad \rho^*_{32} = 0.0000.$$

The Euler method converged for each of these three examples. For the first two examples, the Euler method required 256 iterations for convergence, whereas for the third example, it required 304 iterations.



Multilevel Network for Example 4

In the next three examples, supply chain network problems with no e-commerce were solved via the Euler method.

Example 4

The first example in the second set consisted of two manufacturers, two retailers, and two demand markets, and its multilevel network structure was, hence, as depicted in the Figure. There was no e-commerce in this and the next two examples.

The production cost functions for the manufacturers were given by:

$$f_1(q) = 2.5q_1^2 + q_1q_2 + 10q_1, \quad f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2.$$

The transaction cost functions faced by the manufacturers and associated with transacting with the retailers were given by:

$$c_{11}(q_{11}) = q_{11}^2 + 3.5q_{11}, \quad c_{12}(q_{12}) = .5q_{12}^2 + 3.5q_{12},$$

$$c_{21}(q_{21}) = .5q_{21}^2 + 3.5q_{21}, \quad c_{22}(q_{22}) = .5q_{22}^2 + 3q_{22}.$$

The handling costs of the retailers, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .75(\sum_{i=1}^{2} q_{i2})^2.$$

The demand functions at the demand markets were:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1200,$$

$$d_2(\rho_3) = -2.5\rho_{32} - 1\rho_{31} + 1000,$$

and the transaction costs between the retailers and the consumers at the demand markets were given by:

$$\hat{c}_{11}(Q^3) = q_{11} + 5, \quad \hat{c}_{12}(Q^3) = q_{12} + 5,$$

$$\hat{c}_{21}(Q^3) = 3q_{21} + 5, \quad \hat{c}_{22}(Q^3) = q_{22} + 5.$$

All other functions were set to zero.

The Euler method converged in 196 iterations and yielded the following equilibrium pattern.

The product shipments between the two manufacturers and the two retailers were:

$$Q^{1^*}: q^*_{11} = 19.002,\ q^*_{12} = 16.920,$$

$$q^*_{21} = 30.225,\ q^*_{22} = 9.6402,$$

the product shipments (consumption volumes) between the two retailers and the two demand markets were:

$$Q^{3^*}: q^*_{11} = 49.228,\ q^*_{12} = 0.000,$$

$$q^*_{21} = 26.564,\ q^*_{22} = 0.000,$$

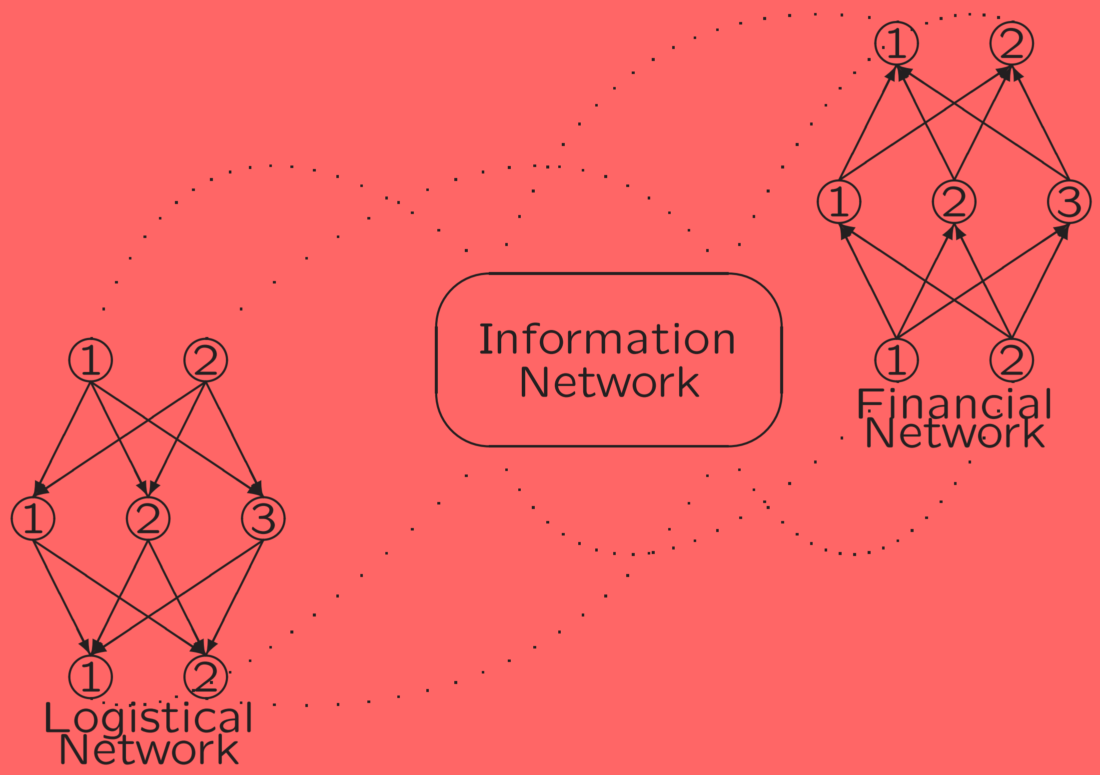
the vector ρ^*_2 had components:

$$\rho^*_{21} = 320.2058, \quad \rho^*_{22} = 289.7407,$$

and the demand prices at the demand markets were:

$$\rho^*_{31} = 374.433, \quad \rho^*_{32} = 250.227.$$

Note that there were zero shipments of the product from both retailers to demand market 2, where the demand for the product was zero.



Multilevel Network for Example 5

Example 5

The second supply chain problem in this set of numerical examples consisted of two manufacturers, three retailers, and two demand markets. Its multilevel network structure is given in the Figure.

The production cost functions for the manufacturers were given by:

$$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1, \quad f_2(q) = 2.5q_2^2 + q_1q_2 + 12q_2.$$

The transaction cost functions faced by the two manufacturers and associated with transacting with the three retailers were:

$$c_{11}(q_{11}) = q_{11}^2 + 3.5q_{11}, \; c_{12}(q_{12}) = q_{12}^2 + 3.5q_{12},$$

$$c_{13}(q_{13}) = .5q_{13}^2 + 5q_{13},$$

$$c_{21}(q_{21}) = .5q_{21}^2 + 3.5q_{21}, \; c_{22}(q_{22}) = .5q_{22}^2 + 3.5q_{22},$$

$$c_{23}(q_{23}) = .5q_{23}^2 + 5q_{23}.$$

The handling costs of the retailers, in turn, were:

$$c_1(Q^1) = .5(\sum_{i=1}^{2} q_{i1})^2, \; c_2(Q^1) = .5(\sum_{i=1}^{2} q_{i2})^2,$$

$$c_3(Q^1) = .5(\sum_{i=1}^{2} q_{i3})^2.$$

The demand functions at the demand markets were:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000,$$

$$d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000,$$

and the transaction costs between the retailers and the consumers at the demand markets were given by:

$$\hat{c}_{11}(Q^3) = q_{11} + 5, \quad \hat{c}_{12}(Q^3) = q_{12} + 5,$$

$$\hat{c}_{21}(Q^3) = q_{21} + 5, \quad \hat{c}_{22}(Q^3) = q_{22} + 5,$$

$$\hat{c}_{31}(Q^3) = q_{31} + 5, \quad \hat{c}_{32}(Q^3) = q_{32} + 5.$$

The Euler method converged in 215 iterations and yielded the following equilibrium pattern.

The product shipments between the two manufacturers and the three retailers were:

$$Q^{1^*}: q^*_{11} = q^*_{12} = 9.243, \quad q^*_{13} = 14.645,$$

$$q^*_{21} = q^*_{22} = 13.567, \quad q^*_{23} = 9.726,$$

the product shipments between the three retailers and the two demand markets were:

$$Q^{3^*}: q^*_{11} = q^*_{12} = q^*_{21} = q^*_{22} = 11.404,$$

$$q^*_{31} = q^*_{32} = 12.184.$$

The vector of retail prices ρ^*_2 had components:

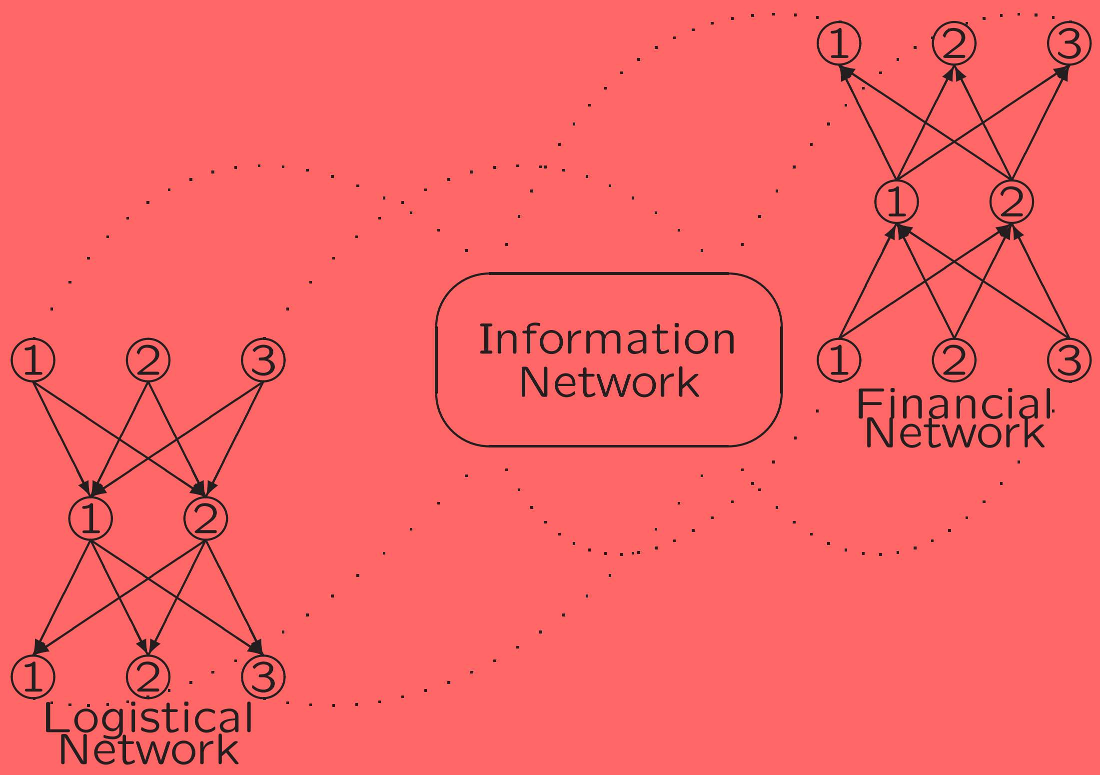
$$\rho^*_{21} = \rho^*_{22} = 259.310, \quad \rho^*_{23} = 258.530,$$

and the prices at the demand markets were:

$$\rho^*_{31} = \rho^*_{32} = 275.717.$$

Example 6

The third numerical example in this set of examples without e-commerce consisted of three manufacturers, two retailers, and three demand markets. The multilevel network structure for this supply chain problem is given in the Figure.



Multilevel Network for Example 6

The production cost functions for the manufacturers were given by:

$$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1, \; f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2,$$

$$f_3(q) = .5q_3^2 + .5q_1q_3 + 2q_3.$$

The transaction cost functions faced by the manufacturers and associated with transacting with the retailers were given by:

$$c_{11}(q_{11}) = .5q_{11}^2 + 3.5q_{11}, \quad c_{12}(q_{12}) = .5q_{12}^2 + 3.5q_{12},$$

$$c_{21}(q_{21}) = .5q_{21}^2 + 3.5q_{21}, \quad c_{22}(q_{22}) = .5q_{22}^2 + 3.5q_{22},$$

$$c_{31}(q_{31}) = .5q_{31}^2 + 2q_{31}, \quad c_{32}(q_{32}) = .5q_{32}^2 + 2q_{32}.$$

The handling costs of the retailers, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2} q_{i2})^2.$$

The demand functions at the demand markets were:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000,$$

$$d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000,$$

$$d_3(\rho_3) = -2\rho_{33} - 1.5\rho_{31} + 1000,$$

and the transaction costs between the retailers and the consumers at the demand markets were given by:

$$\hat{c}_{11}(Q^3) = q_{11} + 5, \quad \hat{c}_{12}(Q^3) = q_{12} + 5,$$

$$\hat{c}_{13}(Q^3) = q_{13} + 5, \quad \hat{c}_{21}(Q^3) = q_{21} + 5,$$

$$\hat{c}_{22}(Q^3) = q_{22} + 5, \quad \hat{c}_{23}(Q^3) = q_{23} + 5.$$

All other functions were set to zero.

The Euler method converged in 175 iterations and yielded the following equilibrium pattern.

The product shipments between the three manufacturers and the two retailers were:

$$Q^{1^*}: q_{11}^* = q_{12}^* = q_{21}^* = q_{22}^* = 12.395,$$

$$q_{31}^* = q_{32}^* = 50.078.$$

The product shipments (consumption levels) between the two retailers and the three demand markets were computed as:

$$Q^{3^*}: q_{11}^* = q_{12}^* = q_{13}^* = q_{21}^* = q_{22}^* = q_{23}^* = 24.956,$$

whereas the retail prices were now equal to:

$$\rho_{21}^* = \rho_{22}^* = 241.496,$$

and the demand prices at the three demand markets were:

$$\rho_{31}^* = \rho_{32}^* = \rho_{33}^* = 271.454.$$

Summary and Conclusions

• We have presented a supernetwork framework for the study of supply chain networks which allows up to capture shipment, financial, and information flows.

• The approach is theoretically rigorous and allows for both qualitative analysis and the computation of product prices and shipments as they adjust over time towards their equilibrium values.

• Ongoing research includes: the incorporation of multicriteria decision-making into this framework, the inclusion of uncertainty, as well as actual implementation using physical transportation networks, and the development of visualization techniques for displaying the dynamics and the results.

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