Technology

and

Network Design Issues

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Introduction

In this lecture, I explore technology and network design issues for sustainable transportation networks. Research and development are progressing in such areas as electric drive vehicles powered by hydrogen-based fuels and vehicles with internal combustion engines operated on alcohol fuels derived from renewable biomass sources (see Transportation Research Board 1997 and Sperling 1995).

Although the design of such vehicles in a cost-effective and efficient manner has yet to be realized, such innovations lie on the horizon.

In addition, the introduction of alternative modes of transportation to an existing network may allow for emission reduction, since it is well-known that, on the average, older vehicles emit more than new vehicles.

Indeed, "super-emitters" – that is, those cars that emit much more now than when they were new, are 10% of those cars and light trucks that account for approximately half of the emissions. Interestingly, the cleanest 50 % produce less than 1% of the emissions. Super-emitters tend to be older vehicles, although some may be newer ones with inappropriately maintained engines, defective emission-control equipment, or with tampered emission controls (cf. Sperling 1995 and the references therein).

In this lecture, I model technology and network design instruments which can be used in conjunction with the market-based policy instruments of the preceding chapters to explore sustainability of transportation networks.

Here, as in other lectures in this course, I emphasize the network topology in the formulations and analyses. Note that technology innovations aimed at emission reduction, in practice, may be grouped in the following categories: new fuels, new vehicles, and/or new travel options.

Model summary

• Optimal budget allocation for emission reduction
• Optimal viability achievement
• New mode introduction
• Viable growth
• System optimality under mode allocation control

Advances in intelligent vehicle highway systems as well as in the development of emission sensors are making the realization of the implementation of the policy instruments described in this course possible.

Optimal Budget Allocation for Emission Reduction

I now construct a model which allows a transportation planner to determine how a given budget should be allocated in order to reduce the total emissions by the greatest possible amount.

Let Δh_a denote the change in emission factor on a link a, where Δh_a is assumed to be nonnegative since it represents the reduction in the emission factor h_a on the link a.

Let B denote the budget available to the authority for network improvements in the form of emission reductions (due, for example, to technology innovations).

Let k_a denote the cost associated with making a reduction on link a.

I assume here that a flow pattern on the transportation network is given and I denote the induced link load pattern by f^*.

Note that, for the sake of generality, I do not explicitly state whether the solution f^* is system- or user-optimized (or neither).

Then the optimization problem facing the transportation authority is:

$$\text{Maximize} \quad \sum_{a \in L} f_a^* \Delta h_a \tag{1}$$

subject to:

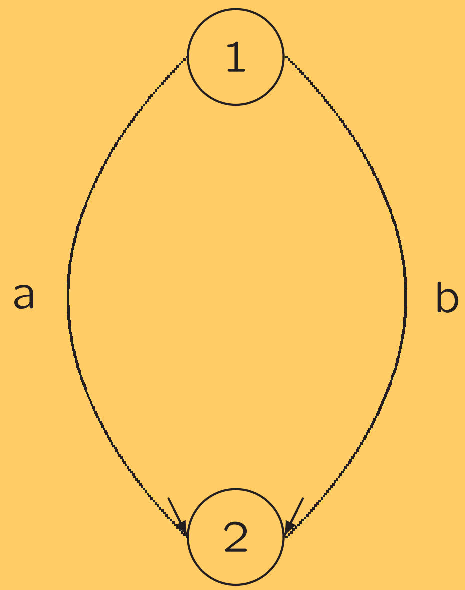
$$\sum_{a \in L} k_a \Delta h_a \leq B, \tag{2}$$

$$0 \leq \Delta h_a \leq u_a, \quad \forall a \in L, \tag{3}$$

where u_a denotes the upper bound possible on the reduction in the emission factor on link $a \in L$.

The objective function (1) represents the total emission reduction. The constraint (2) expresses the budget constraint, whereas the constraints (3) ensure that the emission reductions are within technologically feasible limits.

Note that in the above framework, one could also investigate a "second best" policy in which only certain links' emission factors can be reduced, in which case one would simply have to sum in constraint (2) over those links and bound them according to (3).



Network topology for Examples 1 and 2

Observe that the problem is a linear programming problem, which may be solved by the well-known simplex method. Moreover, there exist commercial software codes for this algorithm for practical applications.

I now present a simple example for illustrative purposes.

Example 1

The network is the two node, two link network depicted in the Figure.

Let the user link travel cost functions be:

$$c_a(f_a) = f_a + 5, \quad c_b(f_b) = f_b + 5,$$

with a travel demand $d_{w_1} = 10$ associated with O/D pair $w_1 = (1, 2)$.

Assume that the travel behavior is that of user-optimization and that the emission factors on the links are:

$$h_a = 0.5, \quad h_b = 0.5.$$

Hence, one has that $f_a^* = f_b^* = 5$, with incurred path costs on path $p_1 = a$, $C_{p_1} = 10$, and on path $p_2 = b$, $C_{p_2} = 10$.

Suppose that the authority has 1 dollar in his budget for emission reduction and that the unit costs for reduction are $k_a = 2$, $k_b = 2$.

Furthermore, assume that the upper bounds on emission factor reductions are $u_a = 0.25$ and $u_b = 0.25$.

The problem can then be formulated as:

$$\text{Maximize} \quad 5\Delta h_a + 5\Delta h_b$$

subject to:

$$2\Delta h_a + 2\Delta h_b \leq 1,$$

$$0 \leq \Delta h_a \leq 0.25,$$

$$0 \leq \Delta h_b \leq 0.25.$$

The solution of this problem yields: $\Delta h_a = 0.25$, $\Delta h_b = 0.25$, with a total reduction in emissions of 2.5 and an exhaustion of the budget.

Multimodal Version

I now construct a multimodal version of the above budget allocation problem. Let j denote a typical mode. Δh_a^j denotes the reduction in the emission factor of the mode, whereas f_a^{j*} denotes the link load of mode j on link a. Note that, again, I permit flexibility in terms of how this load pattern is achieved.

Also, let u_a^j denote the upper bound on link a associated with the reduction of the emission factor for mode j.

The multimodal budget allocation problem is then:

$$\text{Maximize} \quad \sum_j \sum_{a \in L} f_a^{j*} \Delta h_a^j \tag{4}$$

subject to:

$$\sum_j \sum_{a \in L} k_a^j \Delta h_a^j \leq B, \tag{5}$$

$$0 \leq \Delta h_a^j \leq u_a^j, \quad \forall j, \quad \forall a \in L. \tag{6}$$

Optimal Viability Achievement

I now address the cost associated with the achievement of the viability of a transportation network.

Specifically, I am interested in formulating the problem whose solution will provide the reduction of the emission factors at minimal total cost so that the environmental quality standard is achieved, given a specific flow pattern, which is denoted, again, by f^*.

Assume, as given, a cost function $\hat{k}_a$ associated with reducing the emission factor on link a. The function is a function of the change (reduction) in the emission factor.

Using the same notation as in the preceding model (single-modal version), one has the following optimization problem, whose solution yields the optimal emission factor reductions at minimal total cost, given the desired environmental quality standard $\bar{Q}$:

$$\text{Minimize} \quad \sum_{a \in L} \hat{k}_a(\Delta h_a) \tag{7}$$

subject to:

$$\sum_{a \in L} f_a^*(h_a - \Delta h_a) \leq \bar{Q}, \tag{8}$$

$$0 \leq \Delta h_a \leq u_a, \quad \forall a \in L. \tag{9}$$

The objective function (7) expresses the total cost associated with emission factor reduction, whereas constraint (8) expresses the environmental quality standard achievement.

Finally, constraints (9) ensure that the emission reductions are within technologically feasible limits.

Note that there may not exist a solution to this problem if the feasible set governed by constraints (8) and (9) is empty.

Indeed, this would mean that there is no technologically feasible manner (given the state of the art) to achieve the standard with the given flow pattern.

Of course, one could also formulate the problem which would consist of nonlinear constraints (and a nonlinear objective function) that permits the flows to vary as well.

Indeed, this problem would then take the form:

$$\text{Minimize} \quad \sum_{a \in L} \widehat{k}_a(\Delta h_a) \tag{10}$$

subject to:

$$\sum_{a \in L}(h_a - \Delta h_a) f_a \leq \bar{Q}, \tag{11}$$

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{12}$$

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w \in W, \tag{13}$$

$$x_p \geq 0, \quad \forall p \in P, \tag{14}$$

$$0 \leq \Delta h_a \leq u_a, \quad \forall a \in L. \tag{15}$$

Example 2

An example for the model (7)–(9) is now presented. The network is the one as depicted in the first Figure with user link cost functions, travel demand, and emission factors as in Example 1.

Assume that the cost functions $\hat{k}_a$ and $\hat{k}_b$ are linear and fixed, that is, $\hat{k}_a = 10$ and $\hat{k}_b = 10$.

The optimization problem is, hence:

$$\text{Minimize} \quad 10\Delta h_a + 10\Delta h_b$$

subject to:

$$(0.5 - \Delta h_a)5 + (0.5 - \Delta h_b)5 \leq 2.5,$$

$$0 \leq \Delta h_a \leq 0.25,$$

$$0 \leq \Delta h_b \leq 0.25.$$

The optimal solution to this problem is:

$$\Delta h_a = 0.25, \quad \Delta h_b = 0.25.$$

The total cost associated with such an emission reduction is 5 and the environmental quality standard is precisely met.

Introduction of a New Mode

I now present models to assist in the evaluation of whether or not a new mode should be introduced. Two distinct cases are considered:

1. in the first case, mode switching is allowed, that is, a portion of the users of the original mode of transportation can switch to the new mode so as to achieve the tightest possible environmental quality standard and

2. in the second case, no such mode switching is permitted.

I first present some needed notation. Let d_w^0 for all $w \in W$ denote the original travel demands associated with the O/D pairs in the transportation network.

Let superscript 1 refer to mode 1 and superscript 2 to mode 2.

The problem I am interested in formulating is, hence:

$$\text{Minimize} \quad Q \tag{16}$$

subject to:

$$d_w^1 + d_w^2 = d_w^0, \quad \forall w \in W, \tag{17}$$

$$\sum_{p \in P_w} x_p^1 = d_w^1, \quad \forall w, \tag{18}$$

$$\sum_{p \in P_w} x_p^2 = d_w^2, \quad \forall w \tag{19}$$

$$\sum_{a \in L} h_a^1 \sum_{p \in P} x_p^1 \delta_{ap} + \sum_{a \in L} h_a^2 \sum_{p \in P} x_p^2 \delta_{ap} \leq Q, \tag{20}$$

$$x_p^1 \geq 0, \quad x_p^2 \geq 0, \quad \forall w \in W. \tag{21}$$

The objective function (16) denotes the tightest environmental quality standard to be achieved. In constraints (17), on the other hand, the demands d_w^1 and d_w^2 for all w are variables and they must sum for each O/D pair to the (original) demand for that O/D pair, which is assumed to be given.

Also, constraints (18) and (19) ensure that the path flows associated with the original mode and the new mode satisfy the (to be determined) optimal travel demands for each mode.

Constraint (20) guarantees that the path flow pattern will not exceed the environmental quality standard, which is to be determined at the tightest value possible.

Note that here one can also consider a second best policy in which a new mode of transportation only connects one or more (but not necessarily all) O/D pairs of travel, in which case the constraints (17)–(20) would be modified accordingly.

Mode Switching Not Permitted

I now consider the case in which the introduction of a new mode generates its own travel demand and no switching takes place from the original mode to the new one. Again, I am interested in determining the tightest environmental quality standard that can be achieved in this situation.

The problem is, thus:

$$\text{Minimize} \quad Q \tag{22}$$

subject to:

$$\sum_{p \in P_w} x_p^1 = d_w^1, \quad \forall w \in W, \tag{23}$$

$$\sum_{p \in P_w} x_p^2 = d_w^2, \quad \forall w \in W, \tag{24}$$

$$\sum_{a \in L} h_a^1 \sum_{p \in P} x_p^1 \delta_{ap} + \sum_{a \in L} h_a^2 \sum_{p \in P} x_p^2 \delta_{ap} \leq Q, \tag{25}$$

$$x_p^1 \geq 0, \quad x_p^2 \geq 0 \quad \forall p \in P. \tag{26}$$

Of course, in this case, one can also construct a model in which one has the second-best policy in the sense that only a subset of the O/D pairs are introduced to the new mode.

Viable Growth

I now turn to the determination of the maximal growth of a transportation network that is possible, in terms of the travel demand, while adhering to the desired environmental quality standard $\bar{Q}$.

Specifically, let Δd_w denote the nonnegative change associated with the travel demand for O/D pair $w \in W$.

I seek to maximize the total change in the travel demand for the entire network.

The optimization problem is given by:

$$\text{Maximize} \quad \sum_{w \in W} \Delta d_w \tag{27}$$

subject to:

$$\sum_{p \in P_w} x_p = d_w + \Delta d_w, \quad \forall w \in W. \tag{28}$$

$$\sum_{a \in L} h_a \sum_{p \in P} x_p \delta_{ap} \leq \bar{Q}, \tag{29}$$

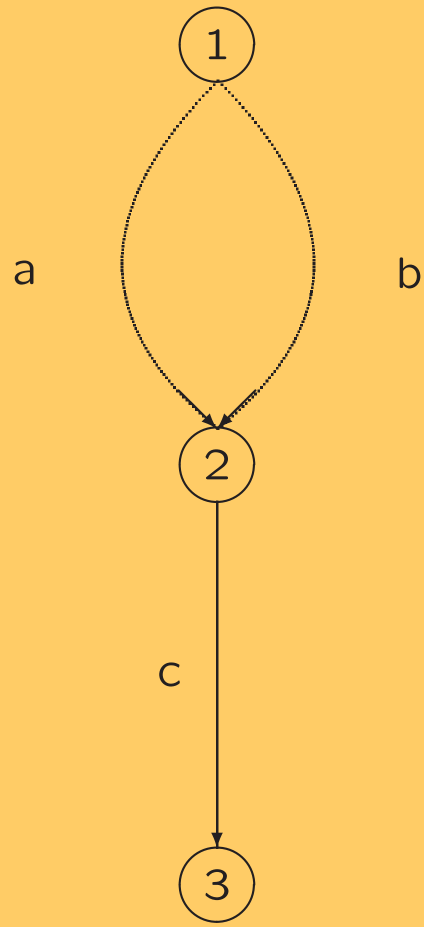
$$x_p \geq 0, \quad \forall p \in P, \tag{30}$$

$$\Delta d_w \geq 0, \quad \forall w \in W. \tag{31}$$

The objective function (27) represents the total change or growth in travel demand. The constraint (28) guarantees that the path flow pattern satisfies the new travel demands, whereas constraint (29) ensures that the environemental quality standard will not be exceeded by the path flow pattern.

Constraints (30) and (31) guarantee nonnegativity of the path flows and the travel demand changes.

This optimization problem is a linear programming problem and, hence, amenable to solution via the simplex method.



Network for Example 3

Example 3

The network is depicted in the Figure and is comprised of three nodes: 1, 2, 3; three links: a, b, c; and a single O/D pair $w_1 = (1, 3)$. Let path $p_1 = (a, c)$ and path $p_2 = (b, c)$.

The travel demand is $d_{w_1} = 100$ and the emission factors on the links are $h_a = 0.1$, $h_b = 0.1$, and $h_c = 0.1$. Assume that the environmental quality standard $\bar{Q} = 30$. The optimization problem for optimal viable growth is then:

$$\text{Maximize} \quad \Delta d_w$$

subject to:

$$x_{p_1} + x_{p_2} = 100 + \Delta d_{w_1},$$

$$0.1x_{p_1} + 0.1x_{p_2} + 0.1(x_{p_1} + x_{p_2}) \leq 30,$$

$$x_{p_1} \geq 0, \quad x_{p_2} \geq 0,$$

$$\Delta d_{w_1} \geq 0.$$

A solution to this problem yields $\Delta d_{w_1} = 50$, resulting in a new travel demand of 150. Note that there are alternative optimal path flow patterns, given for example by $x^*_{p_1} = 150$, $x^*_{p_2} = 0$, and $x^*_{p_1} = x^*_{p_2} = 75$.

System-Optimization with Mode Allocation Control

I now present a model which allows the controller to allocate the flows on the network across modes while still satisfying the total demand associated with each O/D pair. Hence, one now no longer can assume that the travel demand associated with each mode and each O/D pair are fixed.

For example, the controller may select modes of transportation which are lower emitters so as to not violate the environmental quality standard by allocating more of the flows to the lower emitters.

Let j denote mode j, which is used as a superscript for the link cost functions, the path flows and the travel demands.

Here, I present a path flow formulation, but now I allow for switching among the modes for an O/D pair.

Let D_w denote the total demand for O/D pair.

The multimodal system-optimization problem with mode allocation control is, hence:

$$\text{Minimize} \quad S(x) = \sum_{j} \sum_{p \in P} \widehat{C}_p^j \tag{32}$$

subject to:

$$\sum_{p \in P_w} x_p^j = d_w^j, \quad \forall w \in W, \quad \forall j, \tag{33}$$

$$\sum_{j} d_w^j = D_w, \quad \forall w \in W, \tag{34}$$

$$x_p^j \geq 0, \quad \forall p \in P, \quad \forall j, \tag{35}$$

$$d_w^j \geq 0, \quad \forall w \in W, \quad \forall j. \tag{36}$$

The vector x in the objective function in (32) is the vector of path flows for all the modes. Note that constraint (34) reflects that mode allocation is permitted across the modes for each O/D pair.

The references for this lecture (including the text) are given below.

References

Sperling, D. (1995), *Future Drive*, Island Press, Washington, DC.

Transportation Research Board (1997), *Toward A Sustainable Future: Addressing the Long-Term Effects of Motor Vehicle Transportation on Climate and Ecology*, Special Report 251, National Academy Press, Washington, DC.